# Computing Topological Field Theories

## Lecture 1: Introduction to Lattice Field Theories

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School “Recent Advances in Fundamental Physics”, Tbilisi

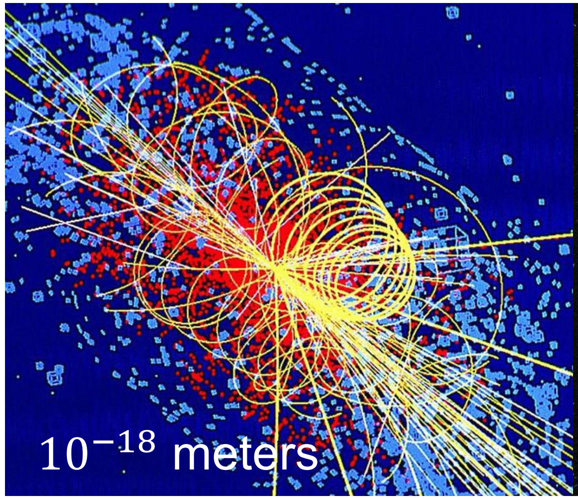
# Motivation: open questions of particle physics

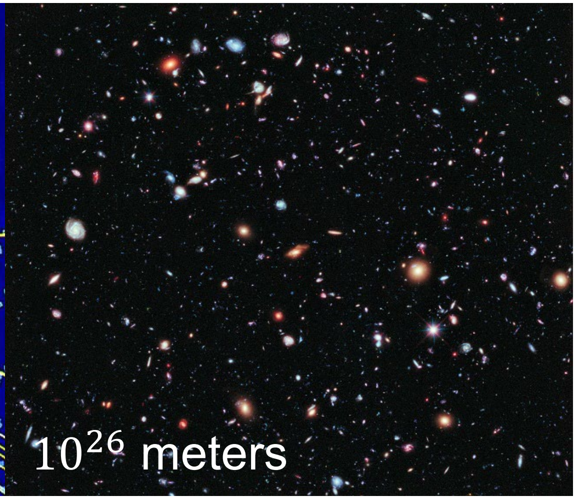
## What we know…

**Standard Model of particle physics**

**Content**: all particles & forces (except for gravity)

**Range**:





**Precision**: 0.00000000001 (electron $g$-factor)

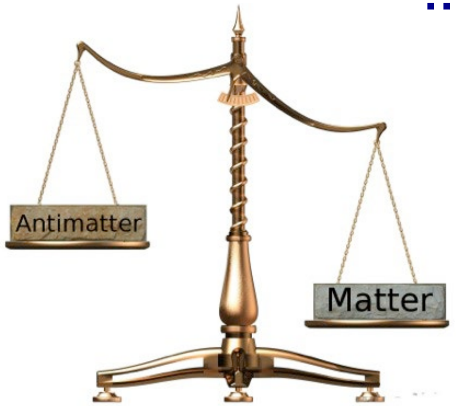
## … and what we don't know

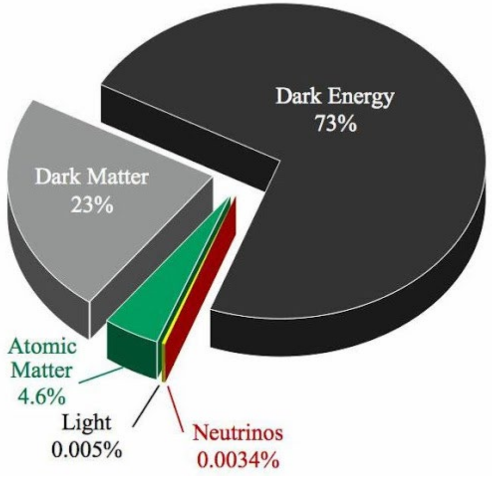
Why is there more matter
than antimatter in the universe?

Why doesn't the strong force distinguish
between matter and antimatter?

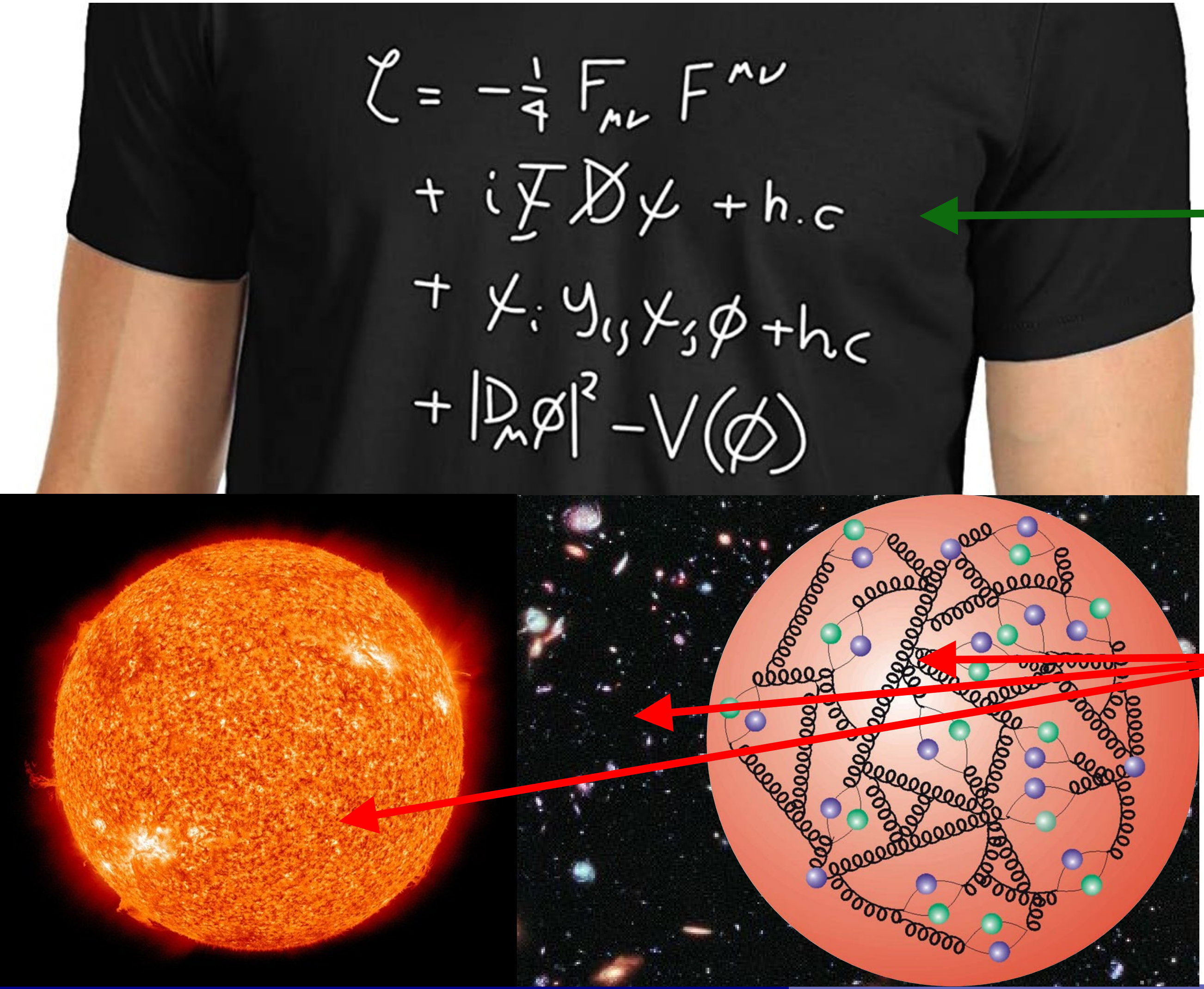
What are dark matter and dark energy?

…





# How can we answer these open questions?



## Simple equation

Beautiful but incomplete!
Need *new models* → **lectures later today**

## Complex phenomena

Emergent structures!
Need *numerical computations* → **this lecture**

# Overview: computing topological field theories

## Numerical computations

**Lecture 1: Monte Carlo method**
High-precision lattice computations
Computational issues

**Lecture 2: Machine learning**
Efficient sampling,
thermodynamic observables…

**Lecture 3: Tensor networks & quantum computing**
Topological θ-terms,
chemical potentials…

## Theoretical models

**Standard Model**
“Real world”
Quarks, gluons, Higgs…

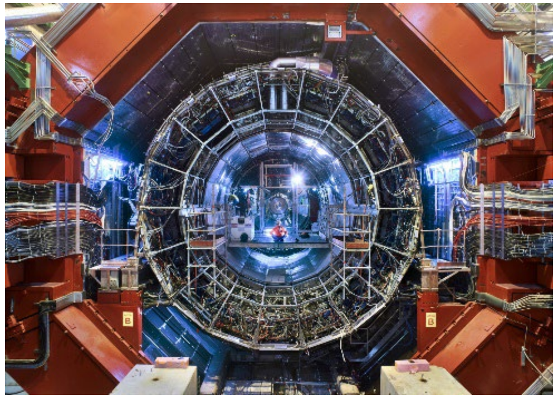
**1+1D $\phi^4$ theory**
Higgs toy model
Symmetry breaking…

**1+1D Schwinger model**
QCD toy model
θ-term, confinement…

## Experiments

**Observables**
Spectrum, free energy,
entropy, pressure…

**LHC, cosmology, …**
Heavy-ion collisions,
Early-universe physics…

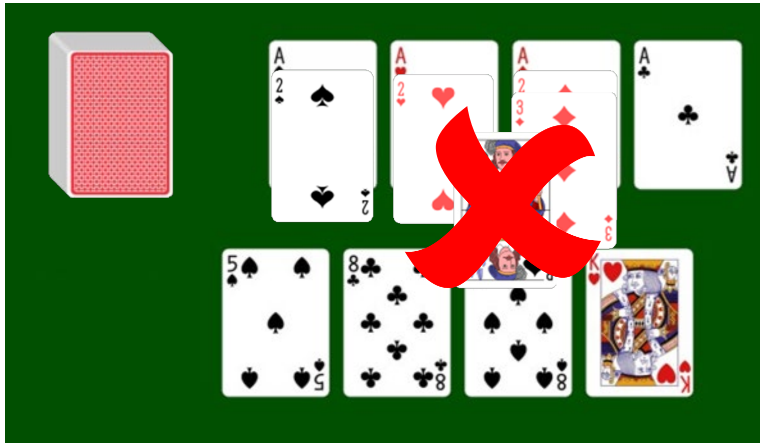


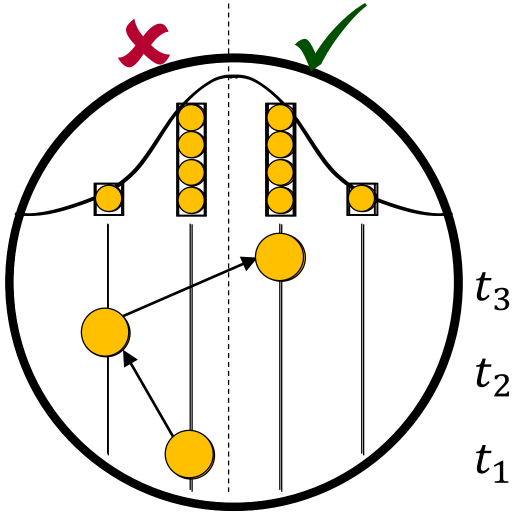
(Image credit: ALICE Collaboration / CERN)

# The (solitaire) origin of the Monte Carlo method

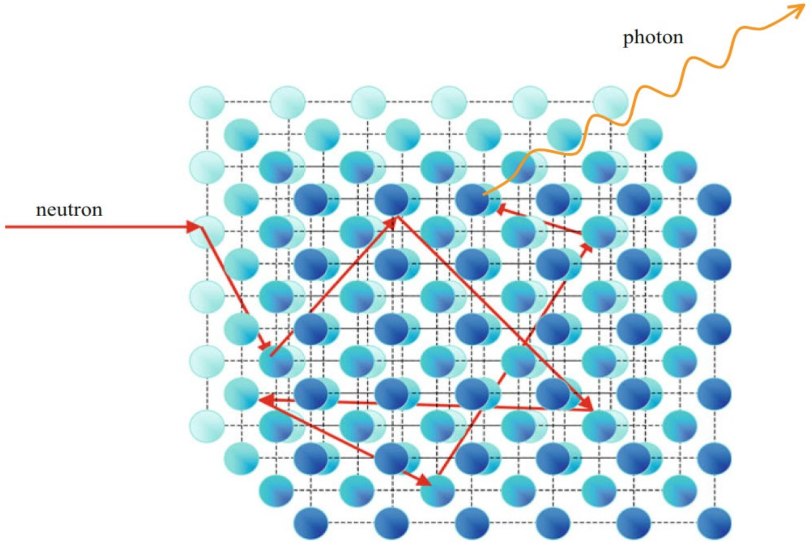
**Solitaire**

**Neutron Diffusion**





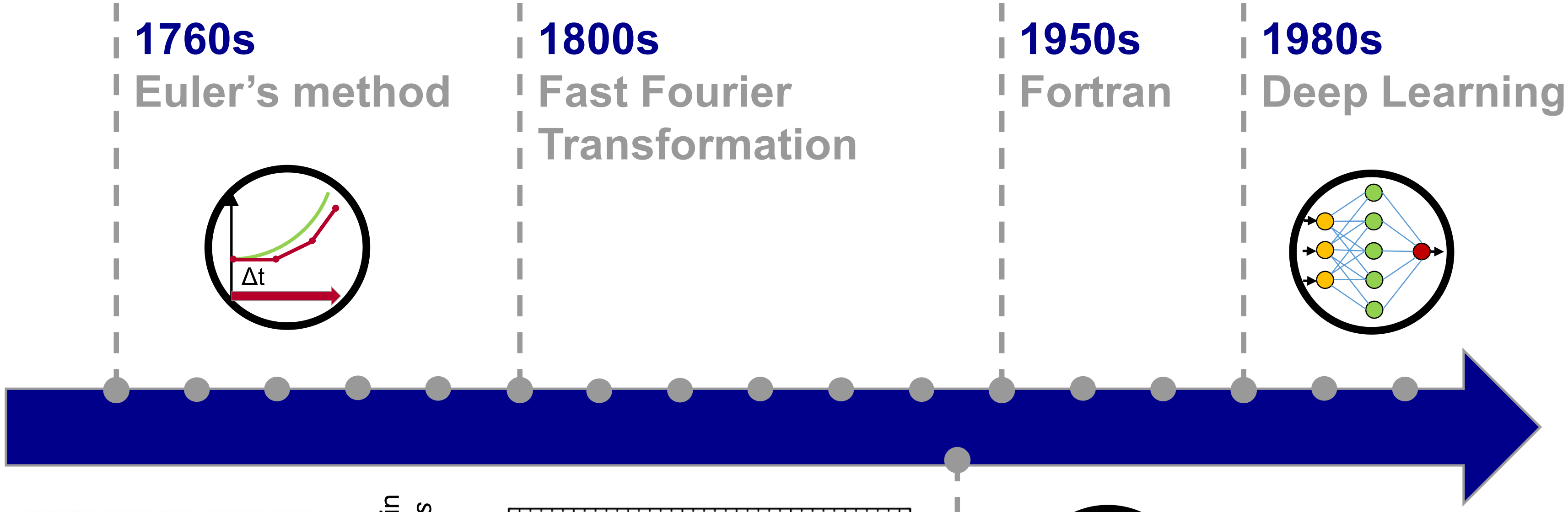
**Monte Carlo method**
(Markov chain → MCMC)



Chance that a solitaire will come out successfully?

“I immediately thought of problems of neutron diffusion and other questions of mathematical physics”
Stanislaw Ulam, 1946

# Monte Carlo and other methods drive science

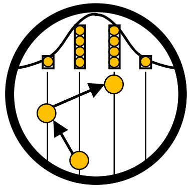


**1760s**
Euler's method

**1800s**
Fast Fourier Transformation

**1950s**
Fortran

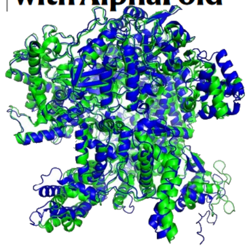
**1980s**
Deep Learning

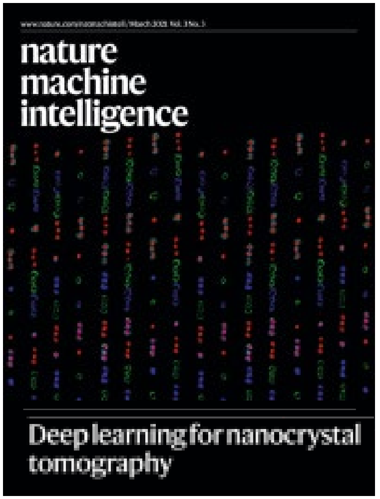


**1940s**
Monte Carlo simulations

Article

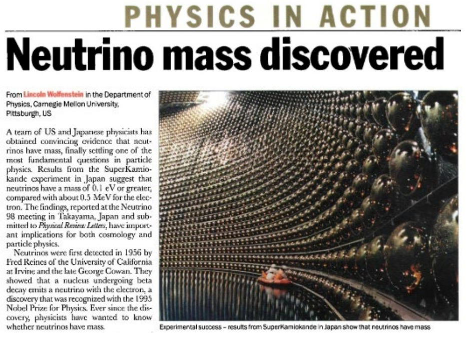
**Highly accurate protein structure prediction with AlphaFold**





nature machine intelligence

Deep learning for nanocrystal tomography



PHYSICS IN ACTION

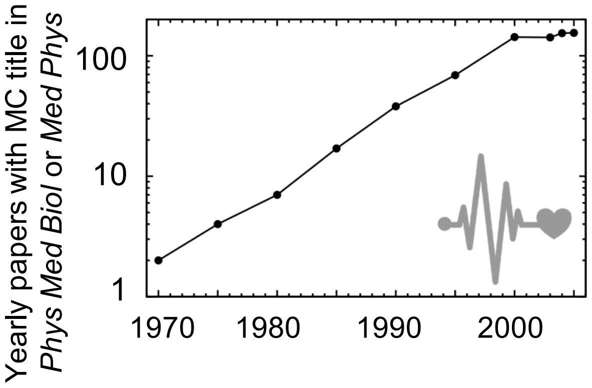
**Neutrino mass discovered**

From Lincoln Wolfenstein in the Department of Physics, Carnegie Mellon University, Pittsburgh, US

A team of US and Japanese physicists has obtained convincing evidence that neutrinos have mass, finally settling one of the most fundamental questions in particle physics. Results from the SuperKamiokande experiment in Japan suggest that neutrinos have a mass of 0.1 eV or greater, compared with about 0.5 MeV for the electron. The findings, reported at the Neutrino 98 meeting in Takayama, Japan and submitted to *Physical Review Letters*, have important implications for both cosmology and particle physics.

Neutrinos were first detected in 1956 by Fred Reines of the University of California at Irvine and the late George Cowan. They showed that a nucleus undergoing beta decay emits a neutrino with the electron, a discovery that was recognized with the 1995 Nobel Prize for Physics. Ever since the discovery, physicists have wanted to know whether neutrinos have mass.

Experimental success – results from SuperKamiokande in Japan show that neutrinos have mass



Ann. Rev. Fluid Mech. 1978. 10: 11–31
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MONTE CARLO SIMULATION
OF GAS FLOWS

PHYSICAL REVIEW D 66, 103511 (2002)

**Cosmological parameters from CMB and other data: A Monte Carlo approach**

**Ensemble methods for meteorological predictions**

National Oceanic and Atmospheric Administration (NOAA)
National Weather Service

Science

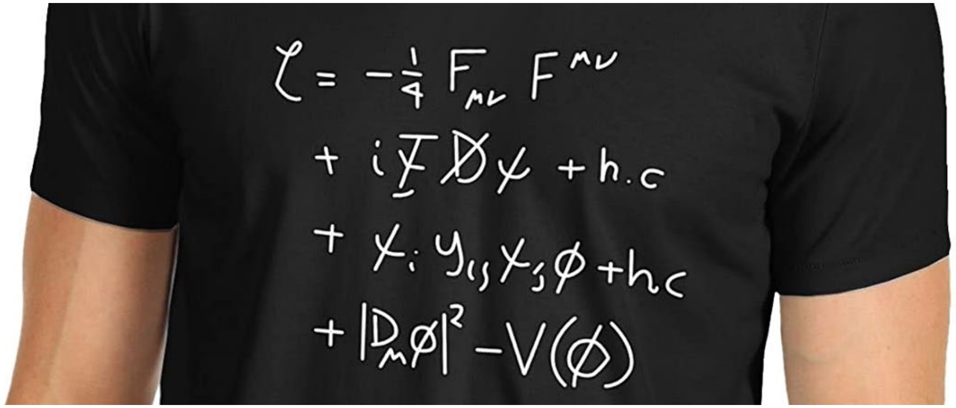
BREAKTHROUGH of the YEAR

The HIGGS BOSON

# Why do we need the Monte Carlo method in field theory?

$$\int$$

Integrate



$$dF\, d\psi\, d\phi$$

over forces ($F$), matter ($\psi$), Higgs field ($\phi$)

Too complex:
no exact computation!

→

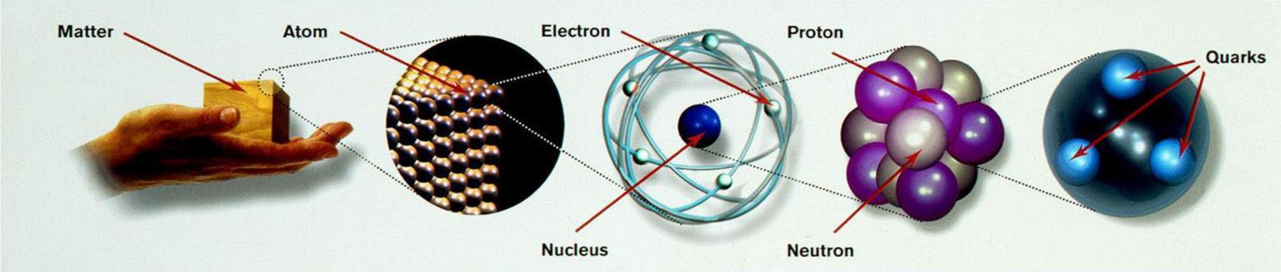
Way out:
approximation!

# The most prominent example: “Lattice QCD”

## What is Lattice QCD?

### Quantum Chromodynamics (QCD)

Theory that describes how strong force (gluons) glues quarks into protons and neutrons



### High energies

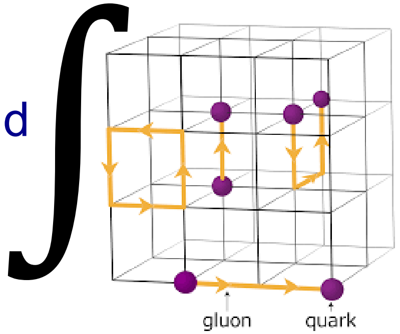
Perturbation theory: *small-coupling expansion*

### Low energies

Non-perturbative regularization through *discretization*

**→ “Lattice” QCD**

## Why do we need Monte Carlo?



### Computational trick

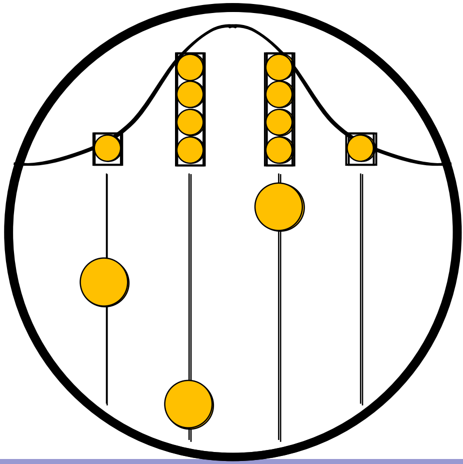
Put quarks and gluons on spacetime grid and integrate over field configurations

### Size of spacetime lattice

Very large: up to $192 \times 96^3 \sim 10^8$ lattice points!

### How to compute such integrals?

Monte Carlo: *sample* configurations



# More details: let's start with the basics…

## Quantum mechanics of point particle in 1+1D

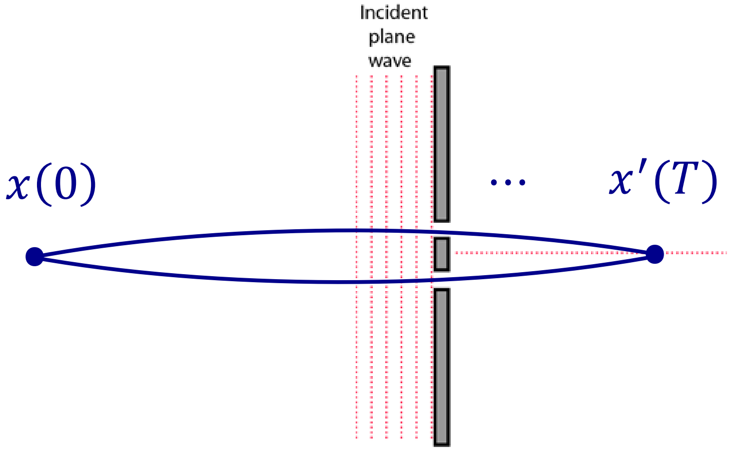
**Transition amplitude**

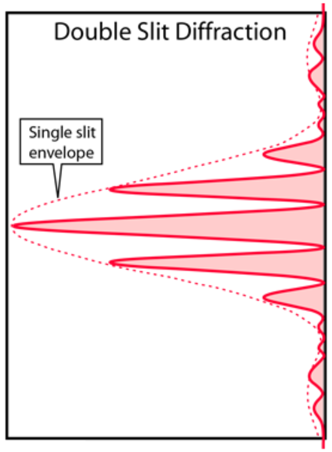
$$\langle x'|e^{-iHT}|x\rangle = \int_x^{x'} Dx\, e^{iS} = \int_x^{x'} Dx\, e^{i\int_0^T dt\, L(x,\dot{x})}$$

→ integral over all possible paths $x(t)$ from $x$ to $x'$

→ weighted by classical action $S$ evaluated along path

→ in 1+1D: $Dx = \prod_t dx(t)$, in 3+1D: $Dx = \prod_{t,i} dx_i(t)$





## Scalar quantum field theory in 1+1D

**Time evolution**

$\phi(\vec{x},t) = e^{iHt}\phi(\vec{x},t=0)e^{-iHt}$, where $x \to x = (\vec{x},t)$

**Greens functions**

$$\langle 0|\phi(x_1)\cdots\phi(x_n)|0\rangle = \frac{1}{Z}\int \mathcal{D}\phi\, \phi(x_1)\cdots\phi(x_n)e^{iS}$$

→ VEVs of products of field operators, e.g. propagators: $\langle 0|\phi(x)\phi(y)|0\rangle$ or $2 \to 2$ scattering: $\langle 0|\phi(x_1)\cdots\phi(x_4)|0\rangle$

→ partition function: $Z = \int \mathcal{D}\phi e^{iS}$, where $t_1 > t_2 > \cdots > t_n$

$$\boxed{\begin{array}{c} x_i(t) \leftrightarrow \phi(\vec{x},t) \\ i \leftrightarrow \vec{x} \\ \prod_{t,i} dx_i(t) \leftrightarrow \prod_{t,\vec{x}} d\phi(\vec{x},t) \equiv \mathcal{D}\phi \\ S = \int dt\, L \leftrightarrow S = \int dt\, d^3x\mathcal{L} \end{array}}$$

# Going from Minkowski to Euclidean spacetime

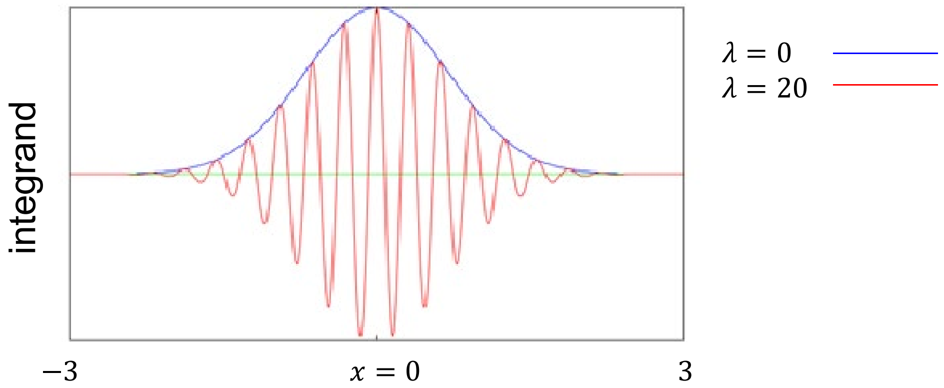
## Minkowski spacetime

**“Sign problem”**

Complex integrand $\propto \exp(iS[x(t)])$ is highly oscillatory

→ Near-cancellation of positive & negative contributions

→ MCMC requires *exponentially* large sample number



$$\int dx \exp(-x^2 + i\lambda x) \to \int dx \exp(-x^2)\cos(\lambda x)$$

## Euclidean spacetime

**No sign problem***

Wick rotation: $t \to i\tau$, path integral gets *real* weight factor

Propagator: $\langle x'|U(\tau',\tau)|x\rangle = \int_x^{x'}[dx(\tau)]\exp\{-S_E[x(\tau)]\}$

**Euclidean spacetime = unphysical?**

Can be advantageous, e.g. compute low-lying spectrum:

$$\begin{aligned}\langle 0|Ae^{-H\tau}A|0\rangle &= \frac{1}{Z}\int \mathcal{D}\phi e^{-S_E}A(\tau)A(0) \\ &= \sum_n \langle 0|A|n\rangle e^{-E_n\tau}\langle n|A|0\rangle \\ &\xrightarrow{\tau\gg 0} |\langle 0|A|1\rangle|^2 e^{-m_1\tau} + \cdots\end{aligned}$$

assuming $A(\tau) = \int d^3x\phi(\vec{x},\tau)$, such that $\langle 0|A|1\rangle \neq 0$ for one-particle state $|1\rangle$ with momentum $\vec{p} = 0$ and mass $m_1$

* actually, there can still be a sign problem (see later)

# Going from continuous to discretized spacetime

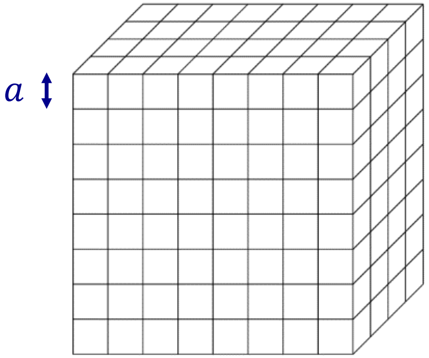
## Infinite- vs. finite-dim. integration

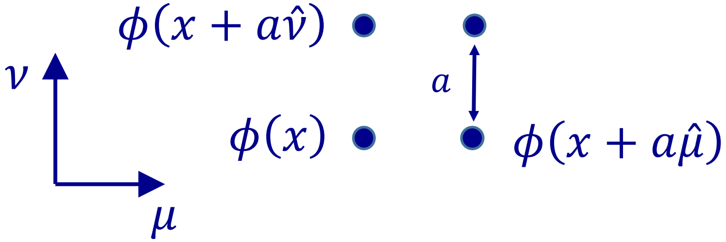
**Problem: infinite-dimensional integration**

… over all field configurations: $\mathcal{D}\phi = \prod_x d\phi(x)$

**Solution: space-time discretization**

… on hypercubic lattice: $x_\mu = an_\mu,\, n_\mu \in \mathbb{Z}$





## Example: lattice scalar field theory

**Lattice scalar field**

… is defined on lattice points only: $\phi(x),\ x \in \text{lattice}$

**Partial derivatives**

… become finite differences: $\partial_\mu \phi \equiv \frac{1}{a}[\phi(x + a\hat{\mu}) - \phi(x)]$

**Space-time integrals**

… are replaced by sums: $\int d^4x \to \sum_x a^4$

**Action of lattice $\phi^4$-theory**

… reads: $S = \sum_x a^4 \left\{ \frac{1}{2} \sum_\mu \left[\partial_\mu \phi(x)\right]^2 + \frac{m_0^2}{2} \phi(x)^2 + \frac{g_0}{4!} \phi(x)^4 \right\}$

**Previously infinite-dimensional integrals**

… now become finite → discrete set of variables!

# This looks familiar…

## Fourier transforms

**Fourier-transformed lattice field**

$\tilde{\phi}(p) = \sum_x a^4 e^{-ipx} \phi(x)$ is periodic in momentum-space

**Lattice momentum**

$p_\mu \cong p_\mu + \frac{2\pi}{a}$ restricted to first Brillouin zone: $-\frac{\pi}{a} < p_\mu \leq \frac{\pi}{a}$

**Inverse-Fourier-transformed lattice field**

$\phi(x) = \int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4} e^{ipx} \tilde{\phi}(p)$, ultraviolet cutoff: $|p_\mu| \leq \frac{\pi}{a}$

**Finite lattice volume**

Hypercubic lattice with length $L_1 = L_2 = L_3 = L$ in spatial direction and length $L_4 = T$ in Euclidean time:

$V = L^3 T,\ x_\mu = a n_\mu,\ n_\mu = 0,1,2, \dots, L_\mu - 1$

## …

**Periodic boundary conditions**

$\phi(x) = \phi(x + aL_\mu \hat{\mu})$

**Discretized lattice momentum**

$p_\mu = \frac{2\pi}{a} \frac{l_\mu}{L_\mu}$ with $l_\mu = 0,1,2, \dots, L_\mu - 1$

**Momentum-space integration**

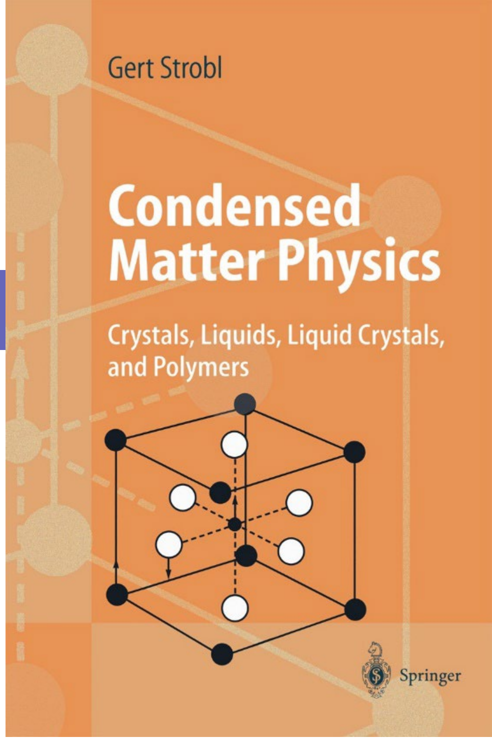
$\int \frac{d^4p}{(2\pi)^4} \rightarrow \frac{1}{a^4 L^3 T} \sum_{l_\mu}$

**Previously infinite integrals…**

… are now regularized (finite $a$) and finite (finite $V$)

**Recover "true" physics…**

… by taking limits $L, T \rightarrow \infty$ and $a \rightarrow 0$

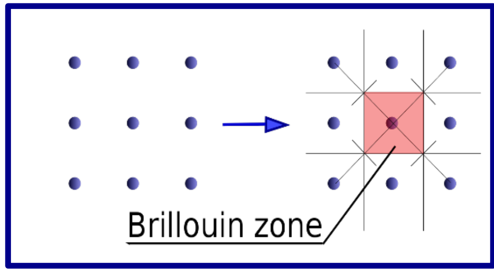


Gert Strobl

Condensed Matter Physics

Crystals, Liquids, Liquid Crystals, and Polymers

Springer



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$V = L^3 T$, $x_\mu = an_\mu$, $n_\mu = 0,1,2,\ldots,L_\mu - 1$

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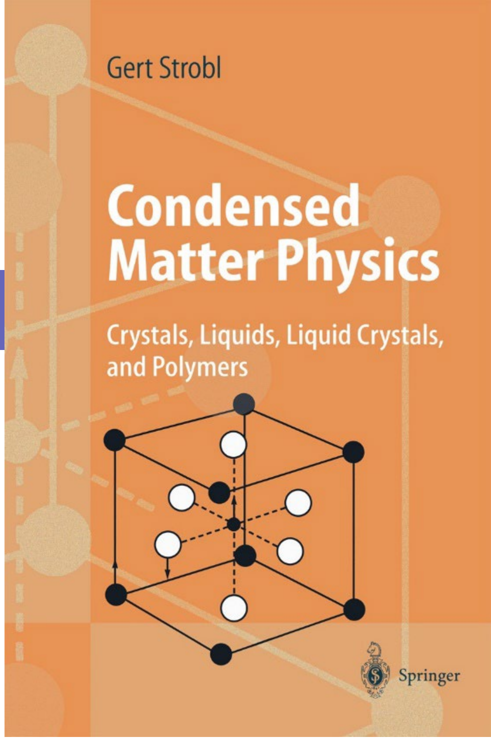
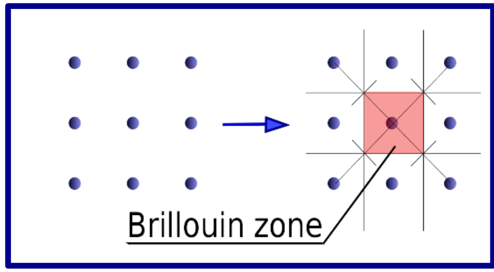
### Previously infinite integrals…

… are now re

### Recover “tru

… by taking l





We can use similar computational methods (MCMC sampling, machine learning, tensor networks, quantum computing, …) in particle *and* condensed matter physics!

# Going beyond $\phi(x)$… How to deal with fermions?

## Grassmann variables

**Fermionic field operators**

$\{\psi_\alpha(x), \psi_\beta(x)\} = 0$ → anti-commuting

**Grassmann variables**

Fulfil $\{\eta_i, \eta_j\} = \{\eta_i, \bar\eta_j\} = \{\bar\eta_i, \bar\eta_j\} = 0 \rightarrow \eta_i^2 = \bar\eta_i^2 = 0$

**Integral and derivative**

$$\int d\eta_i (a + b\eta_i) = \frac{\partial}{\partial\eta_i}(a + b\eta_i) = b$$

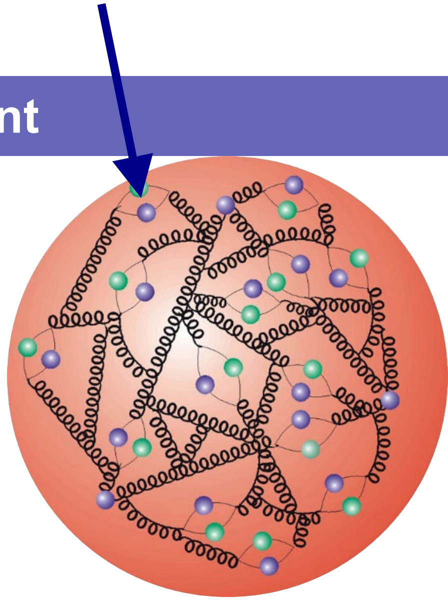
**Multiple integrals and derivatives**

$$\int d\eta_j \int d\eta_i \eta_i \eta_j = \frac{\partial}{\partial\eta_j}\frac{\partial}{\partial\eta_i}\eta_i\eta_j = 1$$

Derivation: $e^{\sum_{i,j}\bar\eta_i Q_{ij}\eta_j} = \prod_i e^{\bar\eta_i \sum_j Q_{ij}\eta_j} = \prod_i\left(1 + \bar\eta_i \sum_j Q_{ij}\eta_j\right)$

then use integration rules and: $\det Q = \sum_\sigma\left(\mathrm{sgn}(\sigma)\prod_i Q_{i,\sigma_i}\right)$

## Fermion determinant



**Fermionic Greens function**

$$\langle 0|A|0\rangle = \frac{1}{Z}\int \mathcal{D}\psi\mathcal{D}\bar\psi\, A\, e^{-S_F}$$

**Fermionic integration measure**

$$\mathcal{D}\psi\mathcal{D}\bar\psi = \prod_x \prod_\alpha d\psi_\alpha(x) d\bar\psi_\alpha(x)$$

**Fermionic action**

$$S_F = \int d^4x\, \bar\psi(x)\left(\gamma_\mu\partial_\mu + m\right)\psi(x)$$

**Fermionic path integral**

$\int \mathcal{D}\psi\mathcal{D}\bar\psi\, e^{-\int d^4x\, \bar\psi(x)\, Q\, \psi(x)} = \det Q$ → huge matrix!

**Quenched approximation**

Up to recently: neglected fermion dynamics ($\det Q = 1$)

# Problems with (too many) fermions on the lattice

## Naïve lattice fermions

**Fermionic lattice action**

$$S_F = \frac{1}{2}\sum_x \sum_\mu \bar{\psi}(x)\left(\gamma_\mu \partial_\mu + m\right)\psi(x) + \mathrm{h.c.}$$

**Resulting fermionic propagator…**

$$\tilde{\Delta}(p) = \left[i \sum_\mu \gamma_\mu \frac{1}{a} \sin\left(p_\mu a\right) + m\right]^{-1}$$

**… has too many poles**

Expected pole at $p_\mu = (m, 0{,}0{,}0)$ but 15 additional poles at $p_\mu = (m, 0{,}0{,}0) + \pi^\mu / a$ (corners of Brillouin zone)!

**Fermion "doubling" problem**

$S_F$ describes $2^d = 16$ instead of $1$ particle flavors!

**Nielsen-Ninomiya theorem**

No *local*, *chiral* fermionic lattice actions without doublers

## Wilson vs. staggered fermions

**Wilson fermions (non-chiral)**

Add Wilson term: $S_F \rightarrow S_F^W = S_F - \frac{r}{2} a^2 \sum_x \bar{\psi}(x)\, \partial_\mu^2\, \psi(x)$

**Modified propagator**

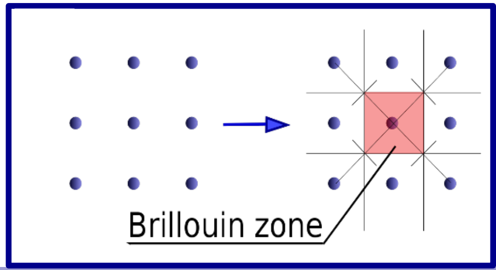
$$\tilde{\Delta}(p) = \left[i \sum_\mu \gamma_\mu \frac{1}{a} \sin\left(p_\mu a\right) + m + \sum_\mu \frac{2r}{a} \sin^2\left(\frac{p_\mu a}{2}\right)\right]^{-1}$$

Doublers acquire masses $\propto r/a$ and decouple for $a \rightarrow 0$

**Staggered fermions (non-local)**

Distribute 4 components of $\psi_\alpha$ on different lattice points

→ reduction to 4 flavors → take 4th root of determinant



# Going beyond fermions… How to deal with gauge fields?

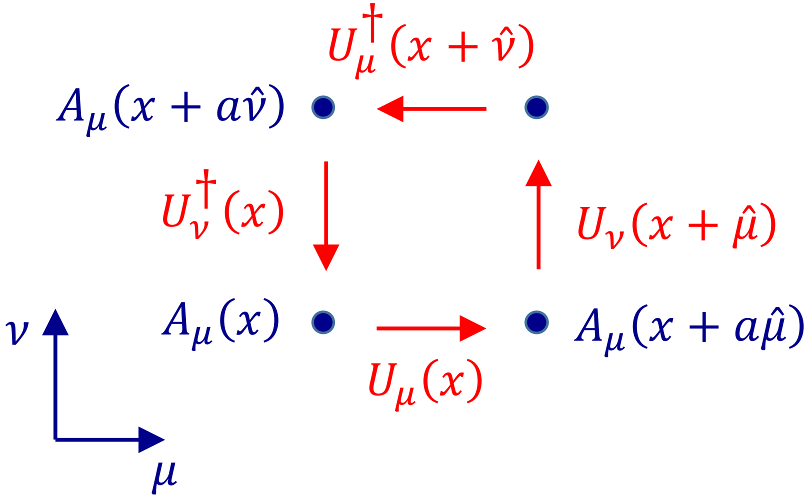
## Naïve lattice discretization

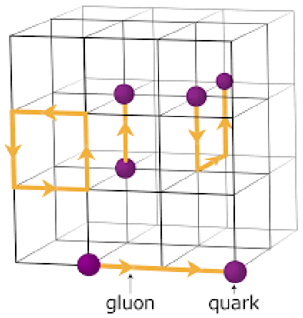
### Naïve approach

Define vector gauge field $A_\mu(x)$ at each lattice point

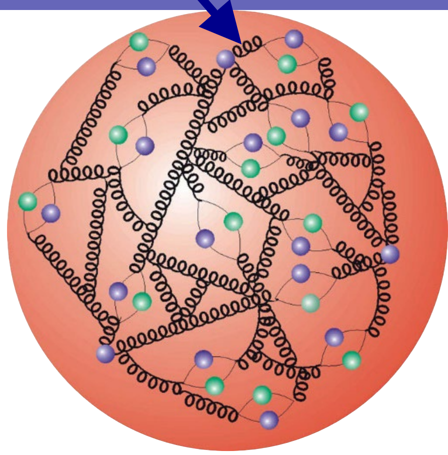
### Problem

Finite differences and sums of vector gauge fields don't preserve gauge symmetry: $F_{\mu\nu}(x) \to \underbrace{\Omega(x)}_{\text{position-dependent } SU(3) \text{ matrix}} F_{\mu\nu}(x)\Omega^\dagger(x)$





## Gauge-invariant discretization



### Link variable

$$U_\mu(x) \equiv e^{iaA_\mu(x)} \in SU(3)$$

### Gauge transformation

$$U_\mu(x) \to \Omega(x)U_\mu(x)\Omega^\dagger(x+a\hat{u})$$

### Simplest gauge invariant quantity

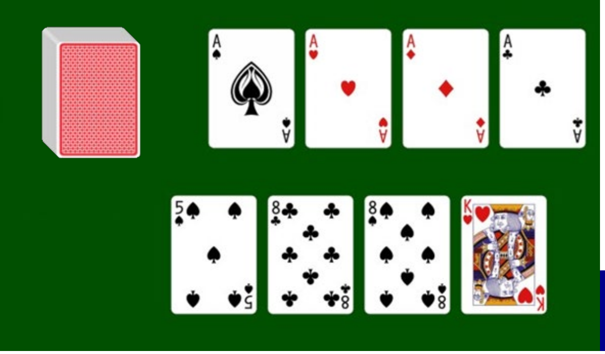
$$U_{\mu\nu}(x) = U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x)$$

$$= e^{ia^2F_{\mu\nu}(x)+\mathcal{O}(a^3)}$$ ⇒ called "**plaquette**"

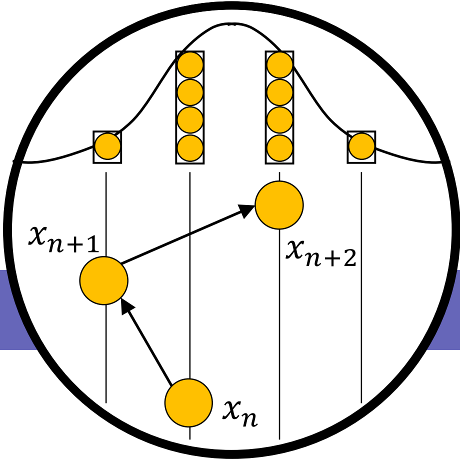
### "Wilson" lattice gauge action

$$S_W = \frac{1}{g^2}\sum_{x,\mu>\nu} \mathrm{Re}\,\mathrm{Tr}[1-U_{\mu\nu}(x)] + \mathcal{O}(a^2)$$

$$\to \frac{1}{2g^2}\int d^4x\, \mathrm{Tr}\big[F_{\mu\nu}(x)F_{\mu\nu}(x)\big]$$ ⇒ gauge−invariant



# How to compute the integrals?



## Monte Carlo method (MC)

**Naïve approach**

*Randomly* generate ensemble of “configurations” $\{x\}$

$$\rightarrow \langle x_f|U(\tau',\tau)|x_i\rangle = \underbrace{\frac{1}{N}}_{\substack{\text{number of configurations}\\ \text{in ensemble}}}\sum_{\{x\}} e^{-S(x)} = \underbrace{\langle e^{-S(x)}\rangle}_{\substack{\text{average value}\\ \text{within ensemble}}}$$

**Problem**

Generate lots of irrelevant configurations → inefficient!

**Solution**

Generate configurations such that probability $P(x_n)$ of obtaining configuration $x_n$ is $P(x) \propto \exp[-S(x)]$

→ configurations have high probability of being relevant!

## Markov Chain … method (MCMC)

**Starting point**

Initialization: Choose arbitrary starting point $x_n$

Proposal density: $g(x'|x_n)$ [e.g. Gaussian centered at $x_n$] suggests candidate $x'$ for $x_{n+1}$, given previous value $x_n$

**For each iteration $n$:**

Generate $x'$ and calculate $\alpha = \exp[-S(x')]\,/\,\exp[-S(x_n)]$

Accept or reject:

Generate uniform random number $u \in [0,1]$

If $u \leq \alpha$, accept candidate by setting $x_{n+1} = x'$

If $u \geq \alpha$, reject candidate and set $x_{n+1} = x_n$ instead

→ **Metropolis-Hastings algorithm**

# Phenomenological results of lattice QCD

## Computing the light QCD spectrum

### QCD Lagrangian

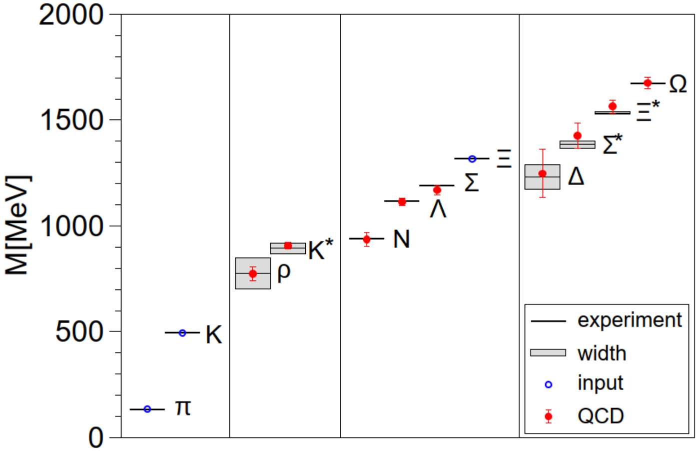
$$\mathcal{L} = \frac{1}{2g^2}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_i\left[\gamma_\mu\left(\partial_\mu + A_\mu\right) + m_i\right]\psi_i$$

### Input parameters of lattice calculation

Masses (e.g. assume $m_u = m_d \equiv m_{ud} \ll m_s$), coupling $g$

### Input quantities from experiments

Precisely measured and computable: e.g. $\pi$, $K$, $\Xi$ masses



BMW Collaboration, Dürr et al. (2009)

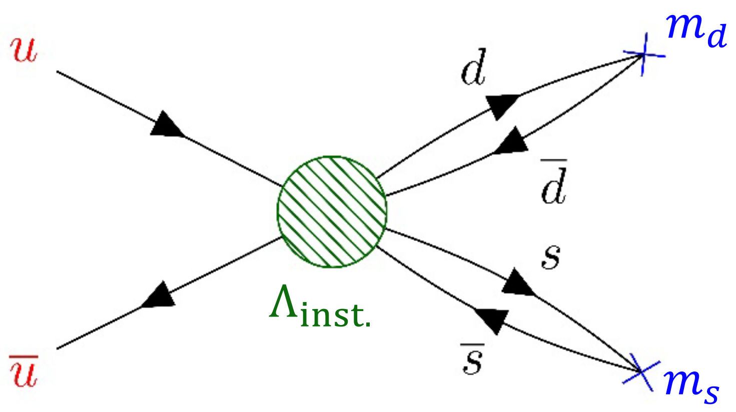
## Computing topological phenomena

### Axion mass at high temperatures

Temperature dependence of topological susceptibility

### Topological mass contribution to up-quark

Compute $\frac{m_d m_s}{\Lambda_{\text{inst.}}}$, compare to $m_{u,\text{exp.}} = m_u + \frac{m_d m_s}{\Lambda_{\text{inst.}}} > 0$



# How to probe BSM physics with lattice computations?

## Example: strong CP problem

**Theory**

QCD vacuum: non-trivial topology → $\theta$-term

$S_{\text{QCD}} \supset \frac{\theta}{16\pi^2} \int d^4x\, G\tilde{G}$ → violates CP symmetry

**Experiment**

$G\tilde{G}$ yields mass: $m_{\eta'} \gg m_\eta$ ✓

$\theta G\tilde{G}$ yields neutron electric dipole moment ✗

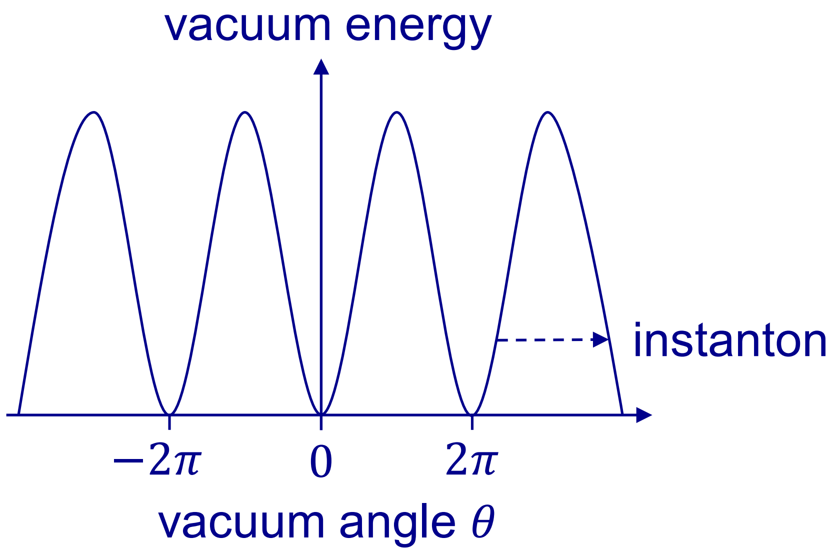
→ unobserved! $\theta < 10^{-10}$

## New physics beyond the Standard Model?

**BSM model building**

Small number → new symmetry → new physics

(see lectures later today)



# Possible solutions: (1) $m_u = 0$ or (2) axion

**Theory**

### "Massless up-quark" solution

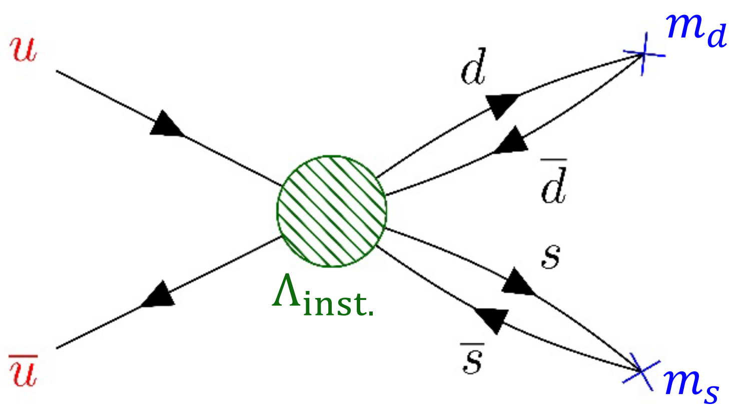
$m_u = 0$ eliminates $\theta \to \theta + \alpha$ with $U(1)_u$ rotation

Observed up-quark mass: $m_{u,\text{exp.}} = m_u + \frac{m_d m_s}{\Lambda_{\text{inst.}}} > 0$

### "Axion" solution

Axion eliminates $\theta \to \theta + \frac{a}{f_a}$ with $U(1)_{\text{PQ}}$ rotation

Non-perturbative axion mass: $m_a = \frac{\Lambda_{\text{inst.}}}{f_a^2}$



**Experimental tests**

### Massless up-quark solution

Lattice QCD: can compute both $m_u$ and $\frac{m_d m_s}{\Lambda_{\text{inst.}}}$

### Axion solution

Lattice QCD: can compute axion properties, e.g. $m_a(T)$

Dark matter experiments: search for axion(-like) particles

(Image credit: XENON Collaboration)

# Testing $m_u = 0$ with lattice QCD

## Computing both mass contributions

### Two approaches

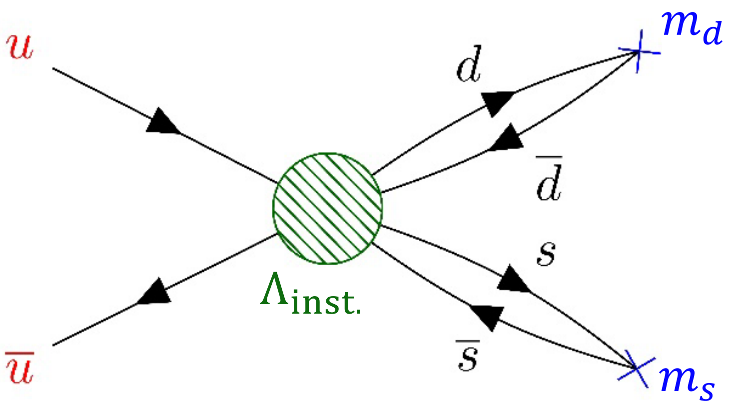
Computation of $m_u$ yields $m_u(2\text{ GeV}) \sim 2.130(41)\text{ MeV}$

Computation of topological mass contribution

### Details of second approach

Compute $\frac{m_d m_s}{\Lambda_{\text{inst.}}}$, compare to $m_{u,\text{exp.}} = m_u + \frac{m_d m_s}{\Lambda_{\text{inst.}}} > 0$

→ if $m_{u,\text{exp.}} \gg \frac{m_d m_s}{\Lambda_{\text{inst.}}}$, we can rule out $m_u = 0$



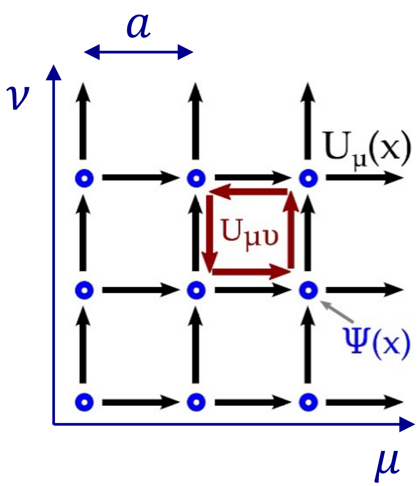
## Details of method

### Lattice QCD + refinements

Reduce lattice errors: Iwasaki improved gauge action

→ uses standard + *rectangular* plaquettes of size $2a \times a$

Twisted mass: $\psi(x) \to e^{-i\omega\gamma_5\tau^3/2}\,\psi(x)$, $m_\psi \to e^{i\omega\gamma_5\tau^3} m_\psi$



Extended Twisted Mass Collaboration

Alexandrou et al. (2020)

# Lattice QCD results for topological mass contribution

## Computing $\beta_2/\beta_1$

### Procedure

Test how $m_{u,\mathrm{exp.}} = m_u + \frac{m_d m_s}{\Lambda_{\mathrm{inst.}}} > 0$ contributes to $M_\pi^2$

Compute $M_{\pi,i}^2 = \beta_1(m_u + m_d) + \beta_2 m_{s,i}(m_u + m_d) + \ldots$

### Results

$$\frac{\beta_2}{\beta_1} = \left.\frac{M_{\pi,1}^2 - M_{\pi,2}^2}{m_{s,1}^2 M_{\pi,2}^2 - m_{s,2}^2 M_{\pi,1}^2}\right|_{M_\pi^2 \to 0} = 0.63(39)\mathrm{GeV}^{-1}$$

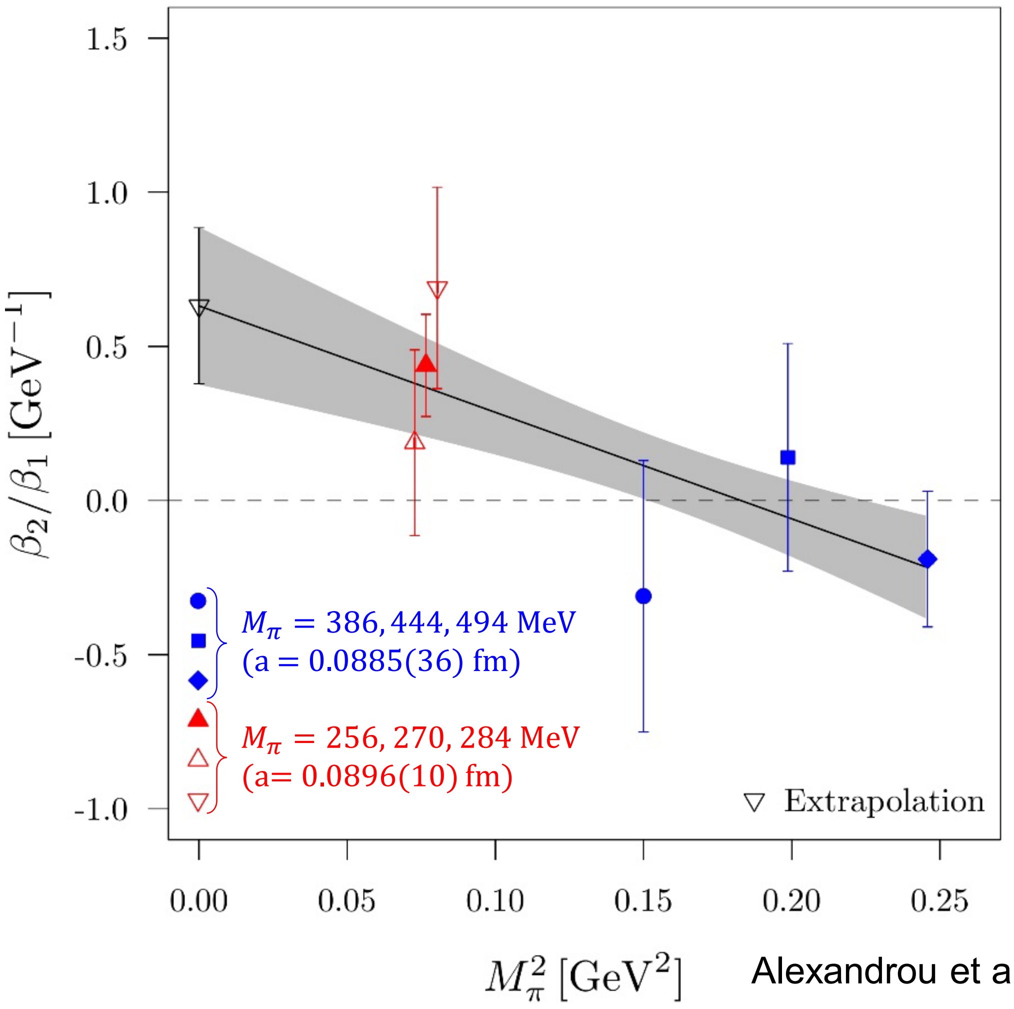
$\left.\frac{\beta_2}{\beta_1}\right|_{M_\pi^2 \to 0} \approx 5\ \mathrm{GeV}^{-1}$ required for $m_u = 0$ solution

### Implication for strong CP problem

Implies $\frac{m_d m_s}{\Lambda_{\mathrm{inst.}}} \ll m_{u,\mathrm{exp.}}$ and rules out $m_u = 0$

Agrees with computation of $m_u(2\ \mathrm{GeV}) \sim 2\ \mathrm{MeV}$

## Results



Alexandrou et al. (2020)

# Outlook: MCMC is hungry & challenging

## Computational costs of lattice field theory

Supercomputer usage for different fields (INCITE 2019)

→ **Lattice QCD:** $\sim \mathbf{40\%}$

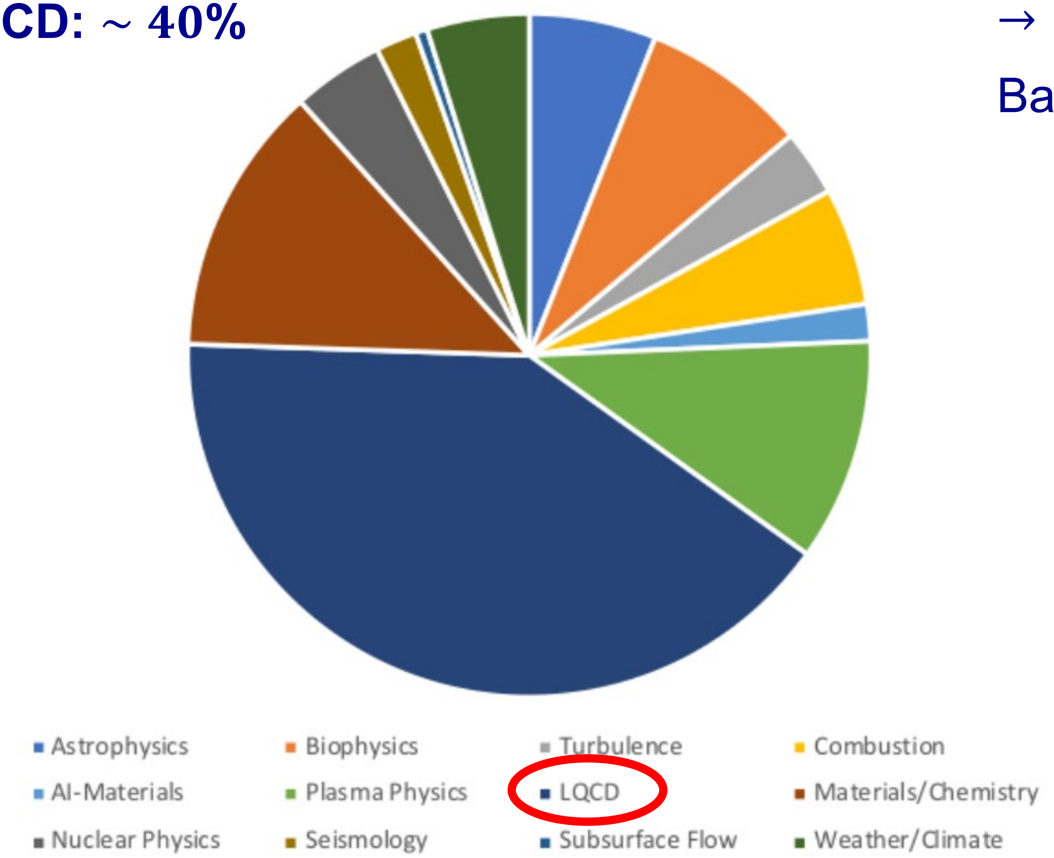


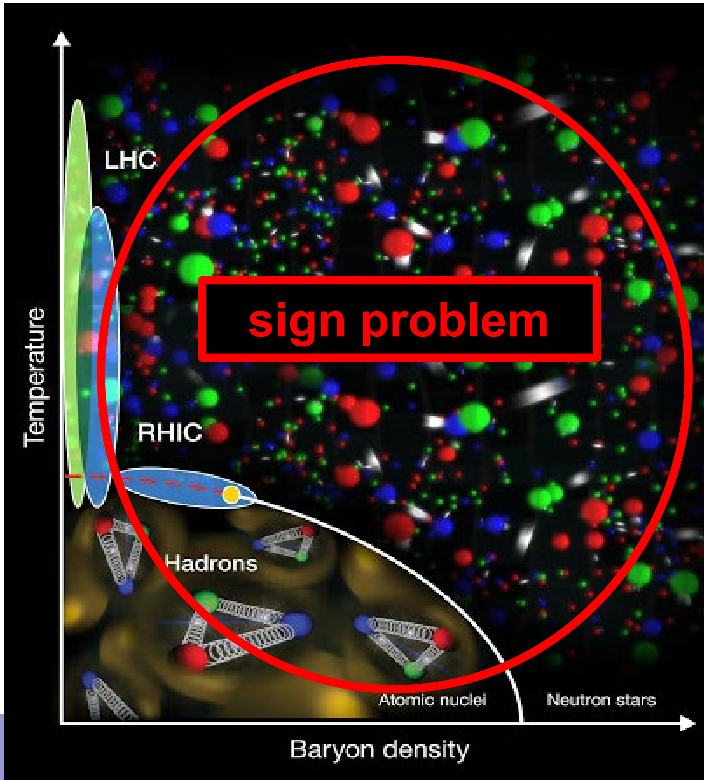
Figure credit:
Jack Wells,
Kate Clark

## Computational challenges of lattice field theory

No direct computation of thermodynamic observables, …

→ Machine learning (lecture tomorrow)

Baryon chemical potential, $\theta$-term, real-time evolution, …



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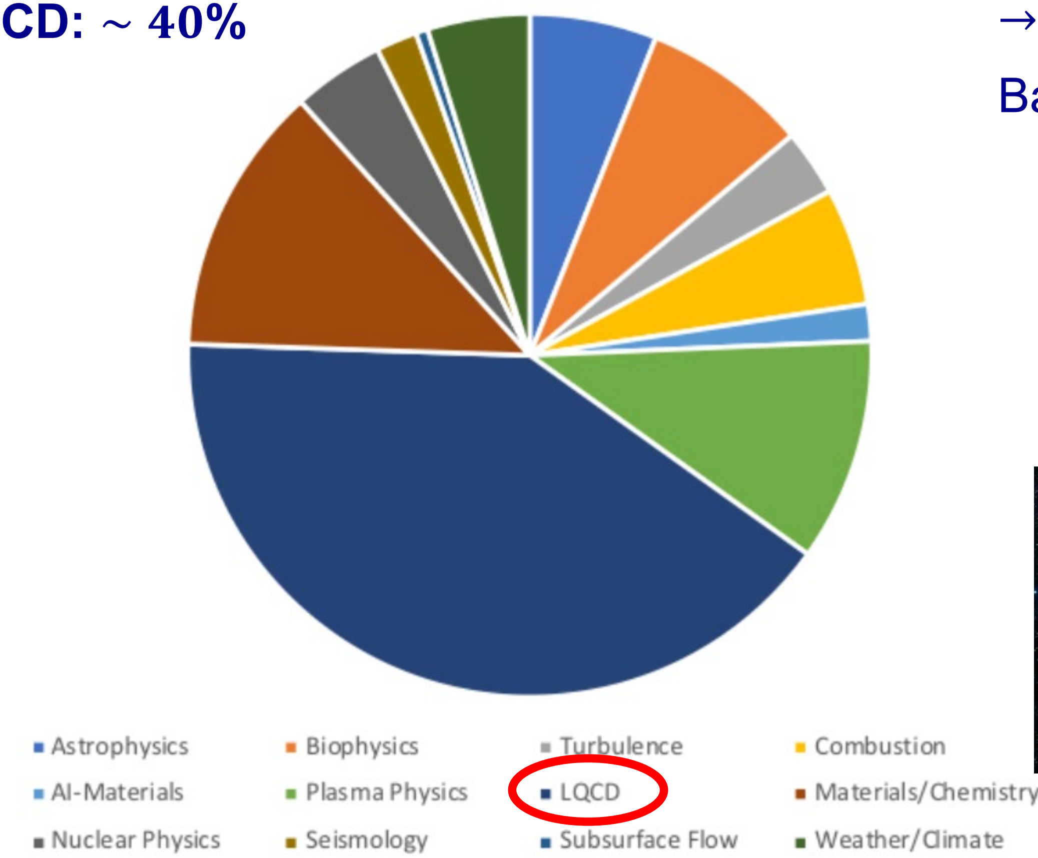

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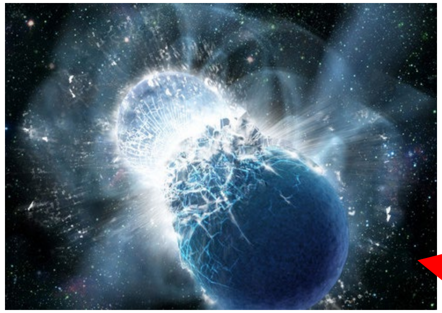

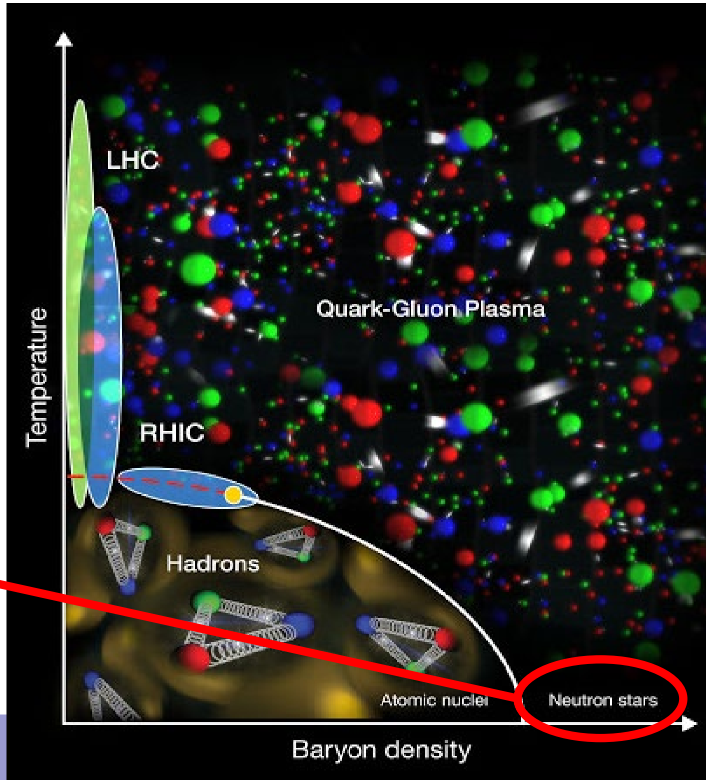

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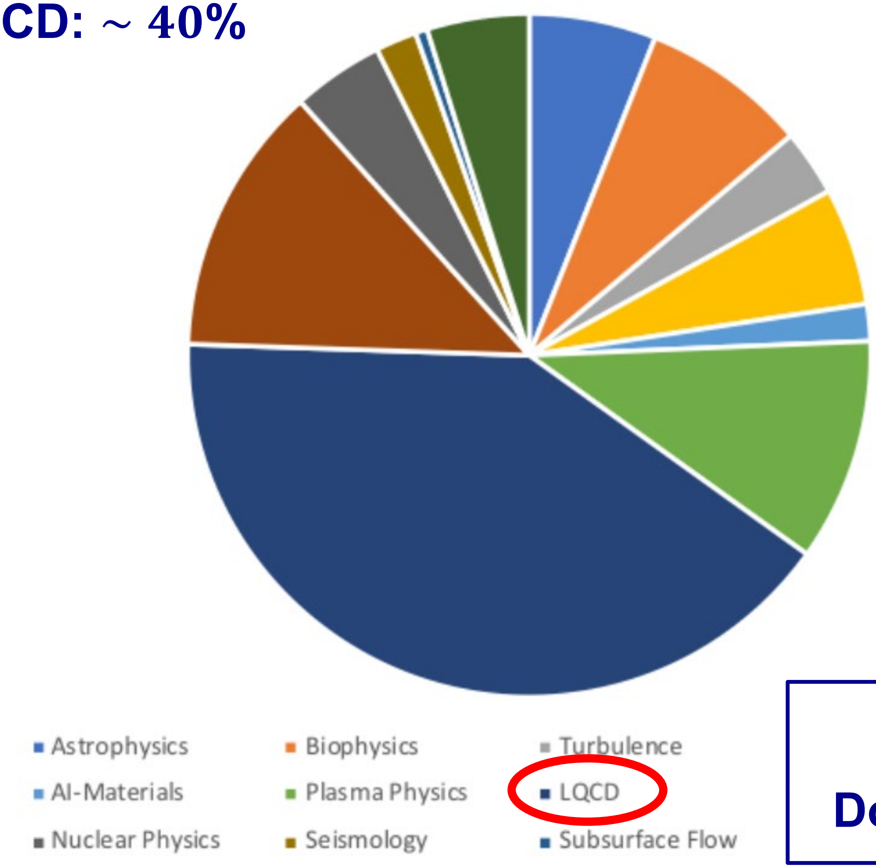


Figure credit:
Jack Wells,
Kate Clark

## Computational challenges of lattice field theory

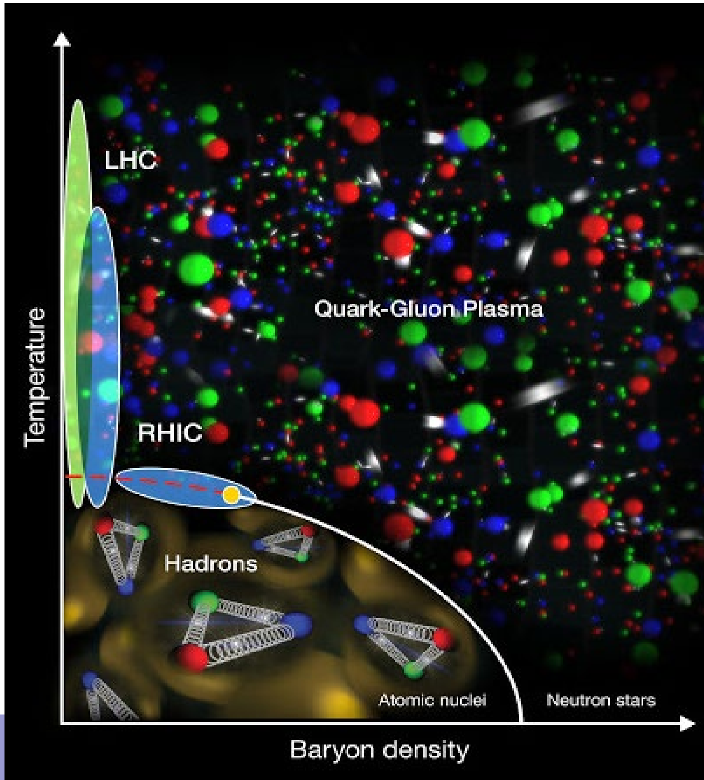
No direct computation of thermodynamic observables, …

→ Machine learning (lecture tomorrow)

Baryon chemical potential, $\theta$-term, real-time evolution, …

→ Tensor networks

→ Quantum computing
(lecture on Monday)



**Thanks for listening!**

**Do you have any questions?**