The Standard Model of Particle Physics

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24-26 Sep 2022

Plan:

Lecture 1: Conceptual foundations

Lecture 2: Theory & phenomenology

Lecture 3: Observations & problems

Assumed knowledge: basics of relativistic QFT

Units & dimensions: $\boxed{\hbar = c = 1}$ (natural units)

$[E] = [p] = [m] = [t]^{-1} = [\ell]^{-1}$ (check, e.g., $\Delta p \Delta x \sim \hbar = 1$)

Action: $S = \int d^4x \, \mathcal{L}$, $[S] = [E \Delta t] = 1$ (dimensionless)

$$[\mathcal{L}] = [m]^4$$

Dimensions of various fields:

Fermions $\quad [f] = [m]^{3/2}$ (check, $\mathcal{L}_{f,m} = m\bar{f}f$)

Bosons $\quad [b] = [m]^1$ (check $\mathcal{L}_{b,m} = m^2 b^2$)
(scalar, vector)

Check: $[J_\mu] = [m]^3$; $[G_F] = [m]^{-2}$ ($\mathcal{L}_{Fermi} \propto G_F J_\mu J^\mu$)

The Standard Model is the most complete & theoretically consistent description of strong, weak & electromagnetic interactions of fundamental quarks & leptons, which is verified empirically with unprecedented accuracy at distance scales as small as $\sim 10^{-17}$ cm

Concept of Unification – the tradition of thought which has been a major driver of scientific breakthroughs in the past,

Reductionism & symmetries: at smaller distance scales (high energies) more fundamental structures emerge. Those structures may reveal commonalities throug symmetries and thus allow a unified description of phenomena ~~which occur~~ in terms of fewer elements and parameters which at larger scales look completely unrelated

The unification paradigm is relevant, not only for modern science:

Ancient Greek Comologies:

- Parmenides' static substances
- Heraclitus' flux of becoming
- Empedocle's four elements
- Democritus' atoms
- Pythagoras' numbers

Religion: e.g., Christian monotheism

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etc.

Unification in physics:

- Newtonian mechanics: Terrestrial & celestial laws are universal (Galileo's Pendulum Law vs Kepler's Laws)

- Maxwell's electromagnetism: a unified description of electric & magnetic phenomena

- Quantum Field Theory: a consistent framework for Quantum mechanics & special relativity

- The Standard Model: common gauge theory desription of fundamental forces; unification of weak & electro-magnetic interactions

Why do we bother?

Unification leads to predictions!

New phenomena, deeper knowledge...

'Symmetry as wide or as narrow as you may define its meaning, is an old idea by which man through the ages has tried to comprehend and create order, beauty and perfection... As far as I can see, all a priori statements in physics have their origin in symmetry'

— Herman Weyl

'The universe is an enormous direct product, of representations of symmetry groups.'

— Steven Weinberg

What is symmetry?

Intuitively, we associate a symmetry with geometric objets. A symmetry of a geometric object is a transformation of the object whose realisation (effect) is impossible to detect,

More fancy definition: A symmetry or symmetry transformation of a geometric object in space is an isometry which maps the object onto itself. If an object admits a certain symmetry, it is said to have this <u>invariance</u>

Examples:

Spatial rotations, translations, reflections

— Eucledian space

Relativistic transformations

— Minkowski space-time

Symmetries of the laws of nature

Definition: A symmetry transformation of a physical law is a change of variables &/or space-time coordinates (in terms of which it is formulated) such that the equations describing the law have the same form in terms of new variables and coordinates as they had in terms of old ones. One says that the equations preserve their form or that they are covariant with respect to symmetry transformations.

Mathematical description of symmetries

The set of all ~~symmetryes~~ symmetry transformations of geometric objects ~~or~~ or physical laws represents a group (the group multiplication being the consecutive application of transformations), called the symmetry group

Hence, the math language to study symmetries is given by Group Theory

Formal definition: A group G is a finite or infinite set of elements together with an operation of multiplication which satisfy the four fundamental properties (group axioms):

1. Closure: $A, B \in G \Rightarrow A \cdot B \in G$
2. Associativity: $(AB)C = A(BC), \ \forall A, B, C \in G$
3. Identity: $AE = EA, \ E \in G$ for $\forall A \in G$
4. Inverse: $A^{-1} \in G$ such that ~~$A A^{-1} = E$~~

   $A \cdot A^{-1} = E$ for $\forall A \in G$.

Lie group – a set of unitary operators parametrized by continuous parameters $\{\theta^a\}$ with the multiplication rule that depends smoothly on the parameters

Lie group element

$$A(\theta_1, \ldots \theta_n) = \exp\left[i\theta_a X_a\right]$$

$$= A(0) + i\theta_a X_a + \ldots$$

θ_a $(a = 1, \ldots, N)$ – N real parameters

$A(0) \equiv E$ – the identity element

Compactness – the group is compact if each θ_a resumes values in a finite interval

$$X_a = -i \frac{\partial A}{\partial \theta_a}\bigg|_{\theta_a = 0}$$

are N linearly independent, Hermitian $[X_a^+ = X_a]$ operators called the group generators

- N-dimensional vector space: $\vec{\theta} = \theta_a X_a$, $\{X_a\}$ – basis
- X_a act on another vector space – the Hilbert space of states

Compactness => finite dimensional Hilbert space of states

Lie algebra

From the closure & identity properties one can obtain [Q: convince yourself]:

$$[X_a, X_b] = i\, C_{abc} X_c$$

structure constants (antisymmetric in a, b, c: $C_{[abc]}$)

From the associativity property [Q - convince yourself]

$$[X_a, [X_b, X_c]] + \text{cyclic perm. of } a, b, c = 0$$

Jacoby identity

- Two (or more) sets of commuting operators form a group which is a direct product of two (or more) groups

$$\begin{tikzcd} & {[X_a, Y_\alpha] = 0} \arrow[dl] \arrow[dr] & \\ G & & H \end{tikzcd}$$

$$\{X_a, Y_\alpha\} \in G \times H$$

- If all the generators commute,

$$[X_a, X_b] = [Y_\alpha, Y_\beta] = [X_a, Y_\alpha] = 0,$$

the group is called Abelian

Otherwise, the group is non-Abelian

Group representations

Group action leaves a system unchanged, but, in general, transforms states

The relation between abstract group elements & transformations is studied within the group representation theory

Definition: A representation $D(A)$ of a group G is a map of abstract group elements to operators (matrices), $A \to D(A)$, such that

(i) $A = BC \Rightarrow D(A) = D(B)\,D(C)$

(ii) $D(E) = \mathbb{1}$

$$D(A(\theta)) = \exp\left[i\, \theta_a T_a \right]$$

$T(X_a) = T_a$ - matrix representation of generators

$$\left[T_a, T_b \right] = i\, C_{abc} T_c$$

Example: $(T_a)_{bc} = -i\, C_{abc}$

Jacoby identity $\Rightarrow$ the structure constants generate a representation called the adjoint representation

[Q. - verify this]

Physical significance of generators

Generators T_a of symmetry transformations parameterized by constant θ_a's are time-independent

$$\frac{dT_a}{dt} = 0$$

That is, $T_a |\psi_a\rangle = q_a |\psi_a\rangle$

classical conserved 'charges', $\frac{dq_a}{dt} = 0$

[Q. Prove this!]

Noether's Theorem

For any continuous N-parametric transformations under which a physical system is invariant, there are corresponding N conserved charges

Examples:

Energy conservation — time translation invariance

Linear momentum conservation — space translation invariance

Angular momentum conservation - spatial rotations

Charge (electric) conservation - phase invariance

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etc

Casimir operators

Rank of the group = # of diagonal generators

= # of Casimir operators

Casimir operator is a combination of generators which commute with all the generators

Example: Quadratic Casimir op.:

$$\hat{C}_2 = \sum_{a=1}^{N} T_a T_a$$

$$[T_b, \hat{C}_2] = \left[T_b, \sum_{a=1}^{N} T_a T_a\right] = \sum_{a=1}^{N} [T_b, T_a] T_a + \sum_{a=1}^{N} T_a [T_b, T_a]$$

$$= \sum_{a=1}^{N} \sum_{c=1}^{N} i\, C_{b\underbrace{ac}_{\text{antisymmetric}}} T_c T_a + \sum_{a=1}^{N} \sum_{c=1}^{N} i\, C_{b\underbrace{ac}_{S}} T_a T_c = 0$$

- Eigenvalues of Casimir operators are basis independent [Q. - show this!], hence they can be used to clasify representations ("states").

$$\hat{C}_{\bullet}|\psi\rangle = c_{\bullet}|\psi\rangle$$

$$|\psi'\rangle = U|\psi\rangle, \quad U = e^{i\theta_a T_a}$$

$$\Downarrow$$

$$\hat{C}_{\bullet}|\psi'\rangle = c_{\bullet}|\psi'\rangle$$

Example: Relativistic (Poincaré) group

$$T_4 \otimes SO(1,3) \equiv \bar{I}SO(1,3)$$

Generators: $P_\mu = -i\partial_\mu$, $M_{\mu\nu} = L_{\mu\nu} + S_{\mu\nu}$ $[L_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial$

Poincaré group is 10-parametric group with rank = 2

- Quadratic Casimir $C_2 = P_\mu P^\mu$

$$C_2 |\psi\rangle = m^2 |\psi\rangle$$

defines intrinsic property of particles — <u>mass</u>

- Quartic Casimir $C_4 = W_\mu W^\mu$, $W_\mu = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma} S^{\nu\rho} P^\sigma$

$$C_4 |\psi\rangle = m^2 s(s+1), \; s = 0, 1/2, 1, 3/2, \dots$$

defines another intrinsic property of particles — <u>spin</u>

- Q. - Why is spin 'quantized', while mass not?

- Q. - 10 generators, but only 7 conserved charges (energy, 3 momentum, 3 spin-orbital momentum)?

- Relativistic invariance is an example of the so-called geometric (space-time) symmetry, i.e. the symmetry which operates on space-time coordinates

- Discrete symmetries

(i) Spatial inversion, $\vec{x} \to -\vec{x}$ – parity (broken in weak interactions)

(ii) Charge conjugation, particle $\leftrightarrow$ antiparticle (supported by relativistic invariance; broken in weak interactions)

(iii) Combined parity, CP – broken in weak interactions

(iv) Time inversion, T, $t \to -t$, – broken in weak interactions (anti-unitary)

(v) CPT – exact! The CPT theorem

- More continoous space-time symmetries

Conformal invariance – SO(2,4) group

15-parametric group = 10-Poincaré generators +

+ 4 special conformal transf.

+ 1 scaling (dilatation) transf.

$$x' = e^{\lambda} x \implies ds'^2 = e^{2\lambda} ds^2$$

This symmetry is broken (e.g., particle masses)

Not compatible with quantisation (anomaly)

• Non-geometric, internal symmetries, do not affect space-time coordinates:

E.g., U(1) phase invariance $|\psi'\rangle = e^{i\alpha}|\psi\rangle$

Realisation: conserved electric charge

B, baryon number (anomalous)

L, lepton number (anomalous)

B-L - exact in the Standard Model

Further classification of symmetries

- θ_a's are constant – global symmetry – Noether's theorem $\Rightarrow$ conserved charges

- $\theta_a = \theta_a(\vec{x},t)$ – local (gauge) symmetry

$$|\psi'\rangle = e^{i\theta_a(x)T_a}|\psi\rangle \equiv U|\psi\rangle$$

$$\left(P_\mu|\psi\rangle\right)' = P_\mu|\psi'\rangle = P_\mu U|\psi\rangle$$

$$= U\left(P_\mu|\psi\rangle\right) + \underline{[P_\mu, U]|\psi\rangle}$$

– transforms non-covariantly!

$$P_\mu \longrightarrow \tilde{P}_\mu \equiv P_\mu + g A_\mu(x) \text{ - covariant derivative}$$

$$\left(\tilde{P}_\mu |\psi\rangle\right)' = \left(P_\mu + g A'_\mu\right)|\psi'\rangle = U\left(\tilde{P}_\mu |\psi\rangle\right)$$

if $\quad \underbrace{A'_\mu}_{\text{gauge field}} = U \underbrace{A_\mu}_{\text{gauge field}} U^+ - \frac{1}{\underbrace{g}_{\text{gauge coupling constant}}} [P_\mu, U] U^+$

- Gauge symmetry: (i) introduces gauge fields

  (ii) sets a specific form for interactions!

Examples: (i) Gauged phase invariance, $U = e^{i\alpha(x)}$ — Maxwell's electromagnetism

(ii) Gauge Poincaré invariance, $ISO(1,3)$, — Einstein's General Relativity

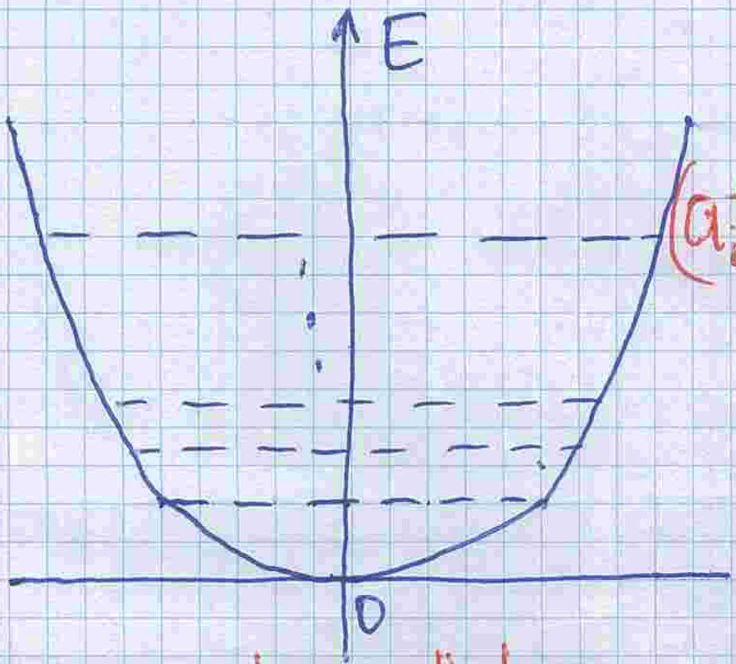
(iii) Strong & weak interactions are also described by gauge theories

This universality of gauge theories has a deep theoretical root — consistency with quantum mechanical description of relativistic spin ≥ 1 fields (particles) require gauge invariance!

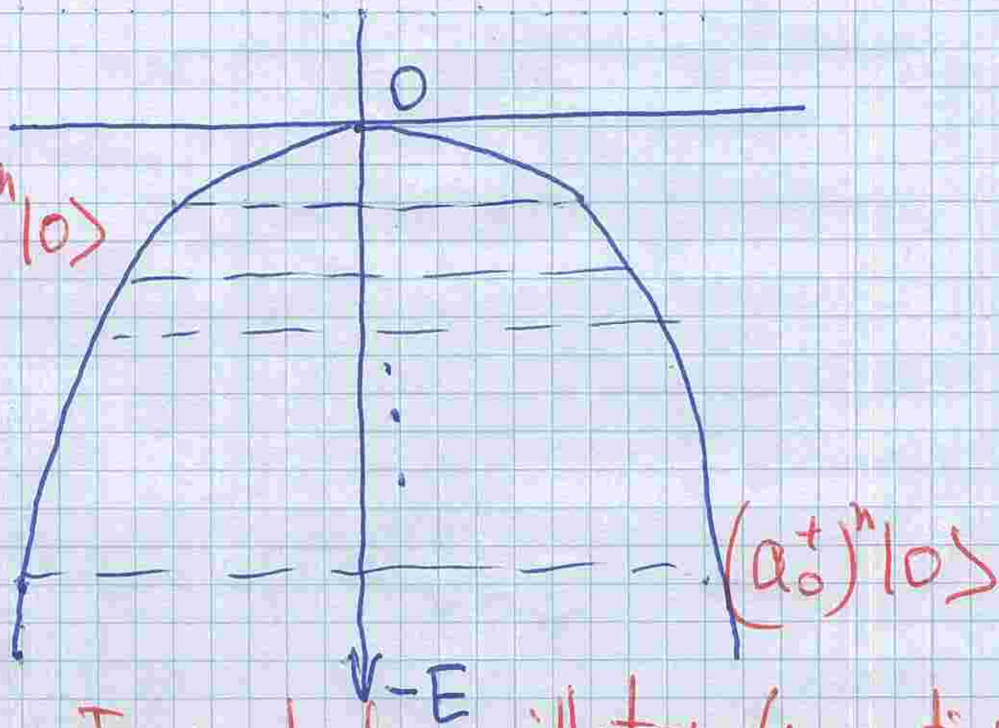
Free quantum fields ≡ infinite # of quantum harmonic oscillators

$$[a_\mu(\vec{k}), a_\nu^\dagger(\vec{k}')] = -\eta_{\mu\nu}\delta^3(\vec{k}-\vec{k}')$$

$\eta_{\mu\nu} = \mathrm{diag}[1,-1,-1,-1]$



Normal oscillator



Inverted oscillator (negative energy states)

Unphysical states are 'removed' by gauge invariance

Broken symmetries

- Explicit symmetry breaking

$$\hat{H} = \hat{H}_{sym} + \int d^3x \, \lambda \, \hat{Q}_{br}$$

$$\frac{dT_a}{dt} = i \left[\hat{H}, T_a \right]$$

$$= i \overset{\to 0}{\left[\hat{H}_{sym}, T_a \right]} + i \left[\int d^3x \, \lambda \, \hat{Q}_{br} , T_a \right]$$

$$\neq 0 \quad \text{(No conserved charges)}$$

- Soft explicit breaking

$$d \left[\hat{Q}_{br} \right] < 4$$

- If explicit breaking is soft, than the wave function renormalisation is symmetric, i.e.,

$$|\psi'_{\lambda=0}\rangle = U|\psi_{\lambda=0}\rangle$$

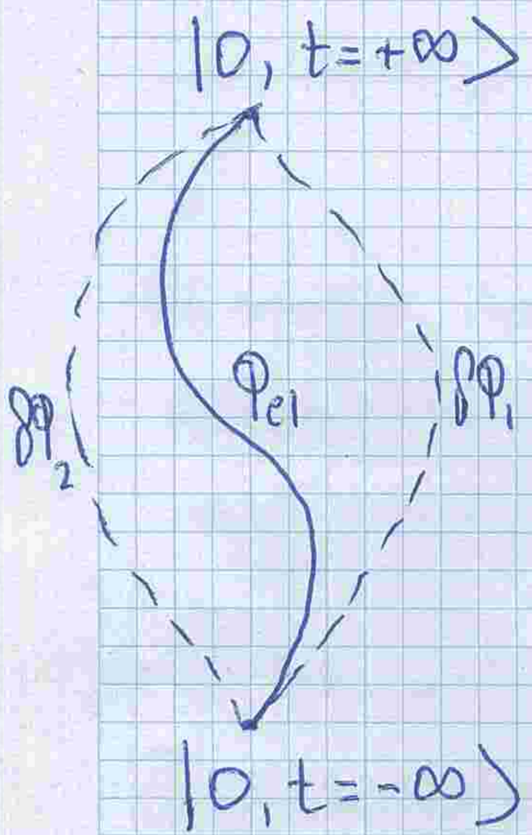
$$|\psi'_{\lambda}\rangle = U|\psi_{\lambda}\rangle$$

Hence, we can still classify particle states according to representations of softly broken symmetry groop

- The soft breaking terms are renormalisable. Moreover, quantum-induced symmetry breaking terms are finite!

• Anomalous symmetry breaking

Charges conserve only in the classical limit, $\hbar \to 0$



$$\left.\frac{dT_a}{dt}\right|_{\hbar \to 0} = 0$$

$$\langle 0|0\rangle_J = N \int D\varphi \; e^{i S_{cl}(\varphi) + \int d^4x \, J\varphi}$$

$D\varphi$: asymmetric $\qquad$ $e^{i S_{cl}(\varphi)}$: symmetric

Example: Anomalous breaking of scaling invariance – effective running parameters:

$$g_0 \to g(\mu), \quad \frac{dg}{d\ln\mu} = \beta(g(\mu))$$

- Spontaneous symmetry breaking

Hamiltonian (Lagrangian) is symmetric:

$$H' = U H U^{\dagger} = H \quad , \quad U = e^{i\theta_a T_a}$$

$$\Downarrow$$

$$[H, \theta_a T_a] = 0$$ [Q. check this]

The vacuum state [$|0\rangle$, $H|0\rangle = 0$]
is asymmetric:

$$U|0\rangle \neq |0\rangle \;=\; T_a|0\rangle \neq 0$$

[Q. check this]

There are degenerate vacuum states due to the underline symmetry (a 'vacuum manifold')

$$|0\rangle_\theta = e^{i\theta_a T_a}|0\rangle = U_\theta|0\rangle$$

$${}_\theta\langle 0|H|0\rangle_\theta = \langle 0|U_\theta^+ \, H \, U_\theta|0\rangle = \langle 0|U_\theta^+ \, U_\theta H U_\theta^+ U_\theta|0\rangle$$

$$= \langle 0|H|0\rangle = 0$$

- Relativistic invariance also implies that the vacuum states are translationally invariant:

$$|0\rangle_\theta = T|0\rangle_\theta = e^{ix^\mu P_\mu} e^{i\theta_a T_a}|0\rangle = e^{i(x^\mu P_\mu + \theta_a T_a)}|0\rangle$$

$$[x^\mu P_\mu, \theta_a T_a] = 0$$

Let's consider a scalar fields $\theta_a(x)$ & Taylor-expand them around $x^\mu = 0$:

$$\theta_a(x) = \theta_a(0) + \partial_\mu \theta_a\Big|_{x=0} x^\mu + \frac{1}{2!}\partial_\mu \partial_\nu \theta_a\Big|_{x=0} x^\mu x^\nu + \ldots$$

$$\equiv \theta_a + \sum_{n\geq 1}^{\infty} \frac{1}{n!}\partial_{\mu_1}\ldots\partial_{\mu_n}\theta_a \, x^{\mu_1}\ldots x^{\mu_n}$$

Hence,

$$|0\rangle_\theta = e^{i x^\mu P_\mu} e^{i\left[\theta_a(x) T_a - \sum_{n\geq 1}^{\infty}\frac{1}{n!}\partial_{\mu_1}\ldots\partial_{\mu_n}\theta_a \, x^{\mu_1}\ldots x^{\mu_n} T_a\right]}|0\rangle$$

Use Baker-Campbell-Hausdorff formulae

$$e^X e^Y = e^{X+Y+\frac{1}{2}[X,Y]+\frac{1}{12}[X,[X,Y]]-\frac{1}{12}[Y,[X,Y]]+\ldots}$$

The desired result requires:

$$\left[P_\mu , \left[P_\mu , \theta_a(x) \right] \right] = 0$$

$$\Downarrow$$

$$\partial_\mu \partial^\mu \theta_a = 0$$

Hence, the theory contains N-massless scalar fields – the Goldstone theorem

More generally: $G \longrightarrow H$

# of Goldstone bosons $\dim G - \dim H$

- What about breaking of local gauge invariance?

- Local gauge invariance can not be broken spontaneously (but the Higgs mechanism)

- Consistency: no explicit breaking, no anomalies

- Q. Is there spontaneous symmetry breaking in QM?