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Isospin symmetry of ω meson in the AdS/QCD soft-wall model at finite temperature

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Nasibova Narmin

Ministry of Science and Education Republic of Azerbaijan, Institute of Physics.

The coupling constants of ρ - and ω- meson-nucleon are related to each other with the isospin relation. Our goal is to study the violation (if it exists) of isospin symmetry by using the soft-wall model of AdS/QCD at finite temperature.

Context

- Introduction.
- Thermal Soft wall model.
- Meson and nucleon profile functions at finite temperatures.
- ω meson coupling constant at finite temperature.
- Numerical results.
- Conclusion.

Introduction

- The influence of the hot hadronic medium on interactions between hadrons is one of the topical questions of elementary particle physics in the hot nuclear medium.
- AdS/QCD models which based on AdS/CFT duality are useful for understanding hadron features. These models do not impose any restrictions on calculations.

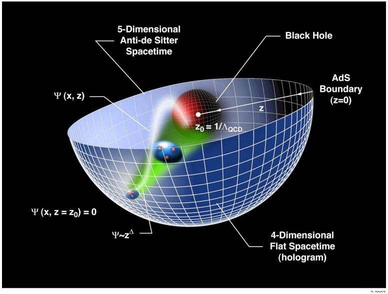

Figure 1: AdS/CFT duality.

The AdS/CFT conjecture relates quantum gravity on Anti-de Sitter (AdS) space to a conformal field theory (CFT) which is defined on the spacetime boundary.

AdS/QCD models

Hard-wall model

AdS geometry is cutted by two branes UV ($r = \epsilon \to 0$) and $IR(r = r_{IR})$

Soft-wall model

Soft cuttoff AdS space by dilaton field $e^{-\varphi(r)}$

Soft wall model at finite temperature

- The action for the soft-wall model, which we use for our finite-temperature study, contains a dilaton field ϕ depending on temperature:

$$S = \int d^4xdr\sqrt{g}e^{-\varphi(r,T)}L(x,r,T), \qquad (1)$$

where g denotes $g = |detg_{MN}|$, $(M, N = 0, 1, 2, 3, 5)$ and the extra dimension r varies in the range $0 \leq r < \infty$.

Thermal dilaton field

- The thermal version of the usual quadratic dilaton $(\varphi(r) = k^2 r^2)$
$$\varphi(r, T) = K^2(T) r^2 \tag{2}$$
The $K^2(T)$ is the parameter of spontaneous breaking of chiral symmetry
$$K^2(T) = k^2[1 + \rho(T) + O(T^6)]. \tag{3}$$

The $\rho(T)$ function encodes the T dependence of the dilaton field

$$\rho(T) = \frac{9\alpha\pi^2}{16}\frac{T^2}{12F^2} - \frac{N_f^2-1}{N_f}\frac{T^2}{12F^2} - \frac{N_f^2-1}{2N_f^2}\left(\frac{T^2}{12F^2}\right)^2 + O\left(T^6\right). \tag{4}$$

Here N_f is the number of quark flavors. This result is valid for an adjustable number of quark flavors with $N_f \geq 2$ and the pion decay constant is $F = \frac{k\sqrt{6}}{8}$. The parameter α parametrize the thermal correction proportional to the r^2 dependence. It encodes the contribution of gravity to the restoration of chiral symmetry at the critical temperature $T_c = 0.2$ GeV and accepts small values.

Meson profile function at finite temperature

- The action for the scalar and vector fields in the AdS-Schwarzschild space-time in the general reads as

$$S_M = -\frac{1}{2}\int d^4x dr \sqrt{g} e^{-\varphi(r,T)}[\partial_N M_N(x,r,T)\partial^N M^N(x,r,T) \\ -(\mu^2(r,T) + V(r,T))M_N(x,r,T)M^N(x,r,T)]. \quad (5)$$

Here $V(r,T)$ is the thermal dilaton potential.

The temperature dependent bulk "mass" $\mu(r, T)$ of the boson field M_N is related to that at zero temperature as follows:

$$\mu^2(r, T) = \frac{\mu^2}{f^{\frac{3}{5}}(r, T)}. \tag{6}$$

$$\mu^2 R^2 = (\Delta - 1)(\Delta - 3). \tag{7}$$

The five-dimensional mass μ^2 is expressed by means of the conformal dimension $\Delta = N + L$ of the interpolating operator dual to the meson.

The axial gauge $M_z(x, r, T) = 0$ is chosen for the vector field M_N, and the Klauza - Klein (KK) expansion is performed as follows:

$$M_\mu(x, r, T) = \sum_n M_{\mu n}(x)\Phi_n(r, T). \tag{8}$$

Here $M_{\mu n}(x)$ are KK modes wave functions corresponding to meson states, $\Phi_n(r, T)$ are their temperature-dependent profile functions, and n is the radial quantum number.

E.o.m. for the vector field will be reduced to the Schrödinger-type equation with the following replacement: $\phi_n(r,T) = e^{-\frac{B_T(r)}{2}}\Phi_n(r,T)$ with $B_T(r) = \varphi(r,T) - A(r)$. In the rest frame of the vector field the equation of motion (e.o.m.) will give us the following equation for the $\phi_n(r,T)$ profiles:

$$\left[-\frac{d^2}{dr^2} + U(r,T)\right]\phi_n(r,T) = M_n^2(T)\phi_n(r,T). \qquad (9)$$

Here $U(r, T)$ is the effective potential and is the sum of the temperature-dependent and nondependent parts:

$$U(r, T) = U(r) + \Delta U(r, T). \tag{10}$$

Explicit forms of the $U(r)$ and $\Delta U(r, T)$ terms were given as

$$U(r) = k^4 r^2 + \frac{(4m^2 - 1)}{4r^2}, \tag{11}$$

$$\Delta U(r, T) = 2\rho(T) k^4 r^2. \tag{12}$$

In the low temperature case the meson mass spectrum M_n^2 is written as the following sum of zero and finite temperature parts:

$$M_n^2(T) = M_n^2(0) + \Delta M_n^2(T), \tag{13}$$

$$\Delta M_n^2(T) = \rho(T) M_n^2(0) + \frac{R\pi^4 T^4}{k^2}, \tag{14}$$

$$M_n^2(0) = 4k^2 \left(n + \frac{m+1}{2} \right), R = (6n-1)(m+1). \tag{15}$$

Finally, the solution of E.O.M for the bulk meson profile $\phi_n(r, T)$ was given in the following form:

$$\phi_n(r, T) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+m+1)}} K^{m+1} r^{m+\frac{1}{2}} e^{-\frac{K^2 r^2}{2}} L_n^m(K^2 r^2). \tag{16}$$

This solution coincides with the one at zero temperature case with the replacements $z \to r$, $K(T) \to k$ in it.

For ω meson by taking $m = 1$ profile function is as follows:

$$M_0(r, T) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+2)}} K^2 r^{\frac{3}{2}} L_n^1(K^2 r^2). \tag{17}$$

Nucleon profile functions at finite temperature

- Action for the bulk fermion field in this model is as follows:

$$S = \int d^4x dr e^{-\varphi(r,T)} \sqrt{g} \bar{\Psi}(x,r,T) D(r) \Psi(x,r,T). \quad (18)$$

Here $D(r)$ is covariant derivative

E.o.m. obtained from the action (18) is the 5D Dirac equation at finite temperature. Substituting $\Phi_n^{L/R}(r,T)$ in the EOM the following equation is obtained for the $F_n^{L/R}(r,T)$ profiles in the nucleon's rest frame:

$$\left[\partial_r^2 + U_{L/R}(r,T)\right] F_n^{L/R}(r,T) = M_n^2(T) F_n^{L/R}(r,T). \tag{19}$$

Effective potentials are written in the sum of the zero- and finite-temperature terms:

$$U_{L/R}(r,T) = U_{L/R}(r) + \Delta U_{L/R}(r,T) \tag{20}$$

with the explicit forms

$$U(r) = k^4 r^2 + \frac{(4m^2-1)}{4r^2},\ \Delta U(r,T) = 2\rho(T) k^4 r^2. \tag{21}$$

$$U_{L,R}(r,T) = U_{L,R}(r) + \Delta U_{L,R}(r,T),$$
$$\Delta U_{L,R}(r,T) = 2\rho(T)k^2\left(k^2r^2 + m \mp \frac{1}{2}\right). \qquad (22)$$

Here

$$m = N + L - \frac{3}{2}. \qquad (23)$$

Profile functions for baryons:

$$F_n^L(r,T) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+m_L+1)}} K^{m_L+1} r^{m_L+\frac{1}{2}} e^{-\frac{K^2r^2}{2}} L_n^{m_L}\left(K^2r^2\right),$$
$$F_n^R(r,T) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+m_R+1)}} K^{m_R+1} r^{m_R+\frac{1}{2}} e^{-\frac{K^2r^2}{2}} L_n^{m_R}\left(K^2r^2\right), \qquad (24)$$

where for nucleons $m_{L,R} = m \pm \frac{1}{2}$.
$m_L = N + L - 1 = 2$ $m_R = N + L - 2 = 1$
The profile functions obey normalization condition.

A minimal term

- $L^{(0)}_{MNN}(T) = \bar{N}_1 e^M_A \Gamma^A M_M N_1 + \bar{N}_2 e^M_A \Gamma^A M_M N_2.(25)$ Next, terms are connected with the bulk spinor field's five-dimensional "magnetic moments,"

$$L^{(1)}_{MNN}(T) = ik_1 e^M_A e^N_B \Big[\bar{N}_1 \Gamma^{AB} (F_L)_{MN} N_1 - \bar{N}_2 \Gamma^{AB} (F_R)_{MN} N_1 + h.c. \\ = ik_1 e^M_A e^N_B \Big[\bar{N}_1 \Gamma^{AB} F_{MN} N_1 - \bar{N}_2 \Gamma^{AB} F_{MN} N_1 + h.c. + axial\ \ vector\ \ term, \tag{26}$$

where $F_{MN} = \partial_M M_N - \partial_N M_M$ is the field strength tensor of the M_N vector field.

The second such magnetic moment terms has the form

$$L^{(2)}_{MNN}(T) = \frac{i}{2} k_2 e^M_A e^N_B \left[\bar{N}_1 X \Gamma^{AB} (F_R)_{MN} N_2 + \right.$$
$$\bar{N}_2 X^+ \Gamma^{AB} (F_L)_{MN} N_1 - h.c. = \frac{i}{2} k_2 e^M_A e^N_B \left[\bar{N}_1 X \Gamma^{AB} F_{MN} N_2 + \right.$$
$$\bar{N}_2 X^+ \Gamma^{AB} F_{MN} N_1 + axial \;\; vector \;\; term. \tag{27}$$

Thus, the total "magnetic" -type Lagrangian is the following sum of these two terms:

$$L'_{MNN}(T) = L^{(1)}_{MNN}(T) + L^{(2)}_{MNN}(T). \tag{28}$$

Coupling constants

$$g^{(0)nm}_{\omega NN}(T) = 3\int_0^\infty \frac{dr}{r^4} e^{-K^2r^2} M_0(r,T)\left[F^{*(n)}_{1L}(r,T)F^{(m)}_{1L}(r,T)\right.$$
$$\left.+F^{*(n)}_{2L}(r,T)F^{(m)}_{2L}(r,T)\right].(29)$$

Here we have used relations between the profile functions of the bulk fermion fields as $F^{(s)}_{1L} = F^{(s)}_{2R}$, $F^{(s)}_{1R} = -F^{(s)}_{2L}$, which are correct for parity even states of the nucleons. . $M_0(r,T)$ is the profile function of a vector meson in the ground state.

- In the total Lagrangian $L'_{MNN}(T)$ the $\Gamma^{5\nu}F_{5\nu}$ terms contribute to the $g^{(1)}_{\omega NN}(T)$ constant, and the contribution of this term is expressed as follows:

$$g^{(1)nm}_{\omega NN}(T) = -6\int_0^\infty \frac{dr}{r^3} e^{-K^2r^2} M_0'(r,T)$$
$$\Big[k_1\left(F^{*(n)}_{1L}(r,T)F^{(m)}_{1L}(r,T) \; - F^{*(n)}_{2L}(r,T)F^{(m)}_{2L}(r,T)\right)$$
$$+k_2 v(r,T)\left(F^{*(n)}_{1L}(r,T)F^{(m)}_{2L}(r,T) - F^{*(n)}_{2L}(r,T)F^{(m)}_{1L}(r,T)\right). \qquad (30$$

Here the prime on M_n denotes the derivative over r. The $\Gamma^{\mu\nu}F_{\mu\nu}$ term makes the following contrubution to the expression:

$$f_{\omega NN}^{nm}(T) = -12m_N \int_0^\infty \frac{dr}{r^3} e^{-K^2 r^2} M_0(r, T)$$
$$\left[k_1 \left(F_{1L}^{*(n)}(r,T)F_{1R}^{(m)}(r,T) - F_{2L}^{*(n)}(r,T)F_{2R}^{(m)}(r,T)\right)\right.$$
$$\left. +k_2 v(r,T)\left(F_{1L}^{*(n)}(r,T)F_{2R}^{(m)}(r,T) - F_{2L}^{*(n)}(r,T)F_{1R}^{(m)}(r,T)\right)\right]. \quad (31)$$

where m_N is the mass of the nucleon. $f_{\omega NN}^{nm}(T)$ is interpreted as the contribution of the nucleon-ω meson interaction by means of the magnetic moment of the nucleon.

The total coupling constant $g^{s.w.}_{\omega NN}(T)$ is the following sum of the previous coupling constants:

$$g^{s.w.}_{\omega NN}(T) = g^{(0)nm}_{\omega NN}(T) + g^{(1)nm}_{\omega NN}(T). \tag{32}$$

The $g^{(0)nm}_{\omega NN}(T)$ coupling constant is interpreted as the "strong charge" of this interaction.

Numerical result

We present our numerical results for the choice of parameters $N_f = 2$ case with the pseudoscalar decay constant in the chiral limit $F = 0.087\ GeV$; $N_f = 3$ case with $F = 0.1\ GeV$; and $k = 0.383\ GeV$. $k_1 = -0.78\ GeV^3$, $k_2 = 0.5\ GeV^3$. $\Sigma = (0.368)^3\ GeV^3$ and $m_q = 0.00145\ GeV$. To have an idea of relative contributions of different terms of Lagrangian, we present results for the temperature dependencies of the coupling constants separately. In the figures below, the blue graph curve represents the $g^{(0)nm}_{\omega NN}$, the orange curve shows the $g^{(1)nm}_{\omega NN}(T)$, the green curve shows the $g^{(s.w.)}_{\omega NN}(T)$, and the red one shows the $f^{nm}_{\omega}(T)$ at finite temperature. We have considered these dependencies for the ground state $(n = 0)$ and first excited state $n = 1$ of the nucleon.

We studied violation of isospin symmetry of ω meson by compairing the difference between $g^{(0)nm}_{\omega NN}(z, T)$ and $g^{(0)nm}_{\rho NN}(z, T)$ coupling constant at finite temperature.In addition, ρ and ω vector mesons have the same quantum numbers. To check violation of ω meson isospin symmetry, we calculate the ratio of $g^{(s.w.)}_{\omega NN}(z, T)/g^{(s.w.)}_{\rho NN}(z, T)$ for different values of temperature. The ratio of these mesons couplings at zero temperature case is equal $g^{(s.w.)}_{\omega NN}/g^{(s.w.)}_{\rho NN} \approx N_c$ (N_c is quark colour number according to isotopic invariance). We have used *MATEMATICA* package for numerical calculation.

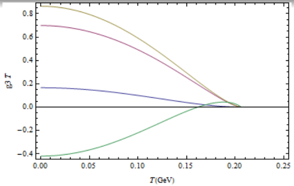

Figure 2: Comparison of $g^{(0)nm}_{\omega NN}$, $g^{(1)nm}_{\omega NN}(T)$,$g^{(s.w.)}_{\omega NN}(T)$ and $f^{nm}_{\omega}(T)$ coupling constants at finite temperature for $N_f = 2$, $F = 0.087 GeV$ at groun state ($n = 0$).

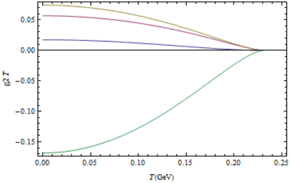

Figure 3: Comparison of $g^{(0)nm}_{\omega NN}$, $g^{(1)nm}_{\omega NN}(T)$, $g^{(s.w.)}_{\omega NN}(T)$ and $f^{nm}_{0mega}(T)$ at the parameter values $N_f = 2$, $F = 0.1 GEV$ at excited state ($n = 1$).

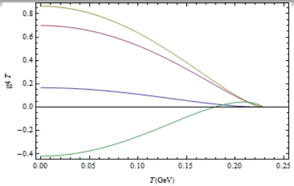

Figure 4: Temperature dependence of the $g^{(0)nm}_{\omega NN}$, $g^{(1)nm}_{\omega NN}(T)$, $g^{(s.w.)}_{\omega NN}(T)$ and $f^{nm}_{\omega}(T)$ at $N_f = 3$, $F = 0.1\ GeV$ at groun state $(n = 0)$.

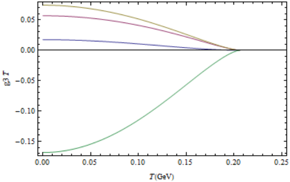

Figure 5: The $g^{(0)nm}_{\omega NN}$, $g^{(1)nm}_{\omega NN}(T)$, $g^{(s.w.)}_{\omega NN}(T)$ and $f^{nm}_{\omega}(T)$ for the first excited nucleons at the parameter values $N_f = 3$, $F = 1\ GeV$ at excited state $(n = 1)$.

Conclusion

We have plotted this dependence for each term in the coupling constant and have observed that all terms become zero at the same point near the Hawking temperature. (This point shifts slightly from case to case, which we think is related to the calculation accuracy.) The result here is reasonable from a physical interpretation point of view. Since the confinement-deconfinement phase transition occurs at the Hawking temperature and there are no hadrons after this temperature, we have obtained a zero value for the coupling constant between the hadrons below this temperature. This result here is reasonable from a physical interpretation point of view and maybe of use for the understanding processes of the early Universe. In addition, we have studied that there is no violation of isospin symmetry at nonzero temperatures.

- There are differences between the expression of the coupling constant of ρ and ω mesons. We have studied the violation of isospin symmetry at finite temperature by comparing the coupling of ω and ρ mesons (ρ meson coupling studied at finite temperature in Ref. [4]. As a result of our calculation, we obtain that for ρ and ω meson coupling given ratio also has the same ($g_{\omega NN}^{(s.w.)}(z,T)/g_{\rho NN}^{(s.w.)}(z,T) = Nc$), at the temperatures different from zero. It means that there are no any violation at finite temperature for ω meson.

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