A two-potential formalism for the pion VFF [1]

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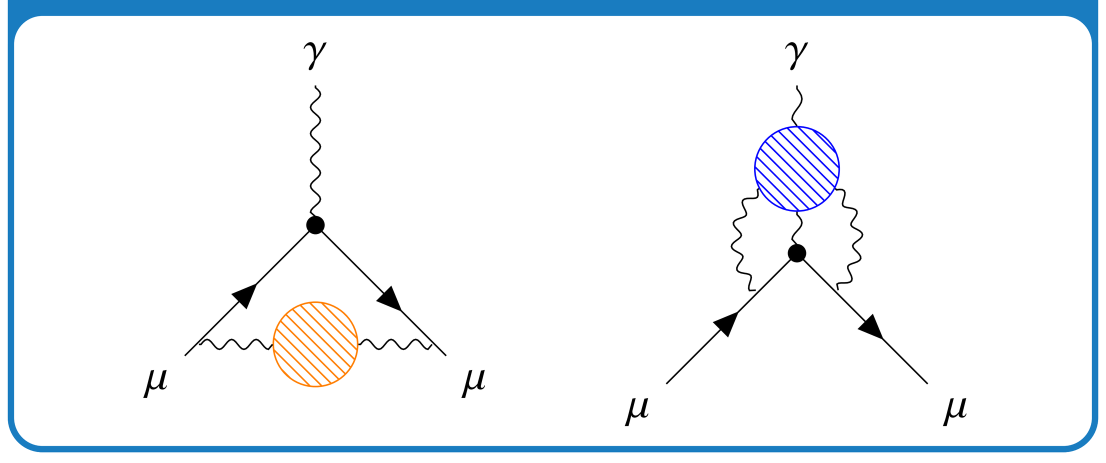
Motivation

Leading contributions to a_μ



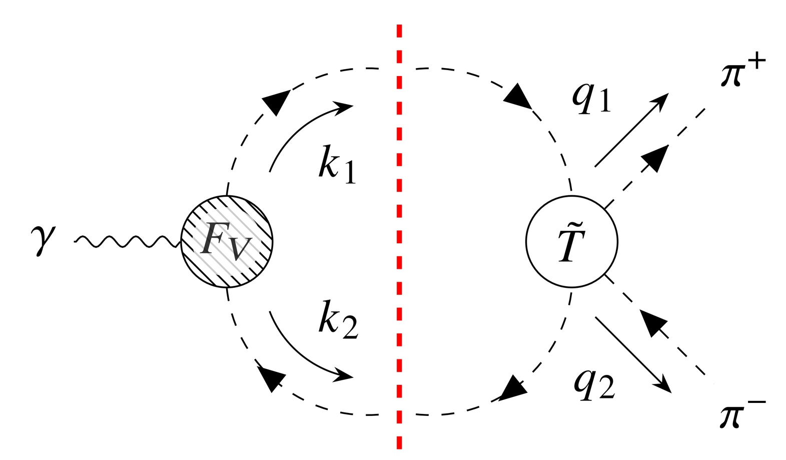
The leading contributions to the anomalous magnetic moment of the muon a_μ from the hadronic sector are the hadronic vacuum polarization (HVP) and the hadronic light-by-light scattering (HLbL) [2]. Pions, being the lightest mesons, contribute substantially to both HVP and HLbL.

Definition and elastic solution

The pion vector form factor (VFF) is defined as

$$\langle \pi^+(q_1)\pi^-(q_2)|\, j_\mu(0)\, |0\rangle = (q_1-q_2)_\mu F^V_\pi(s),$$

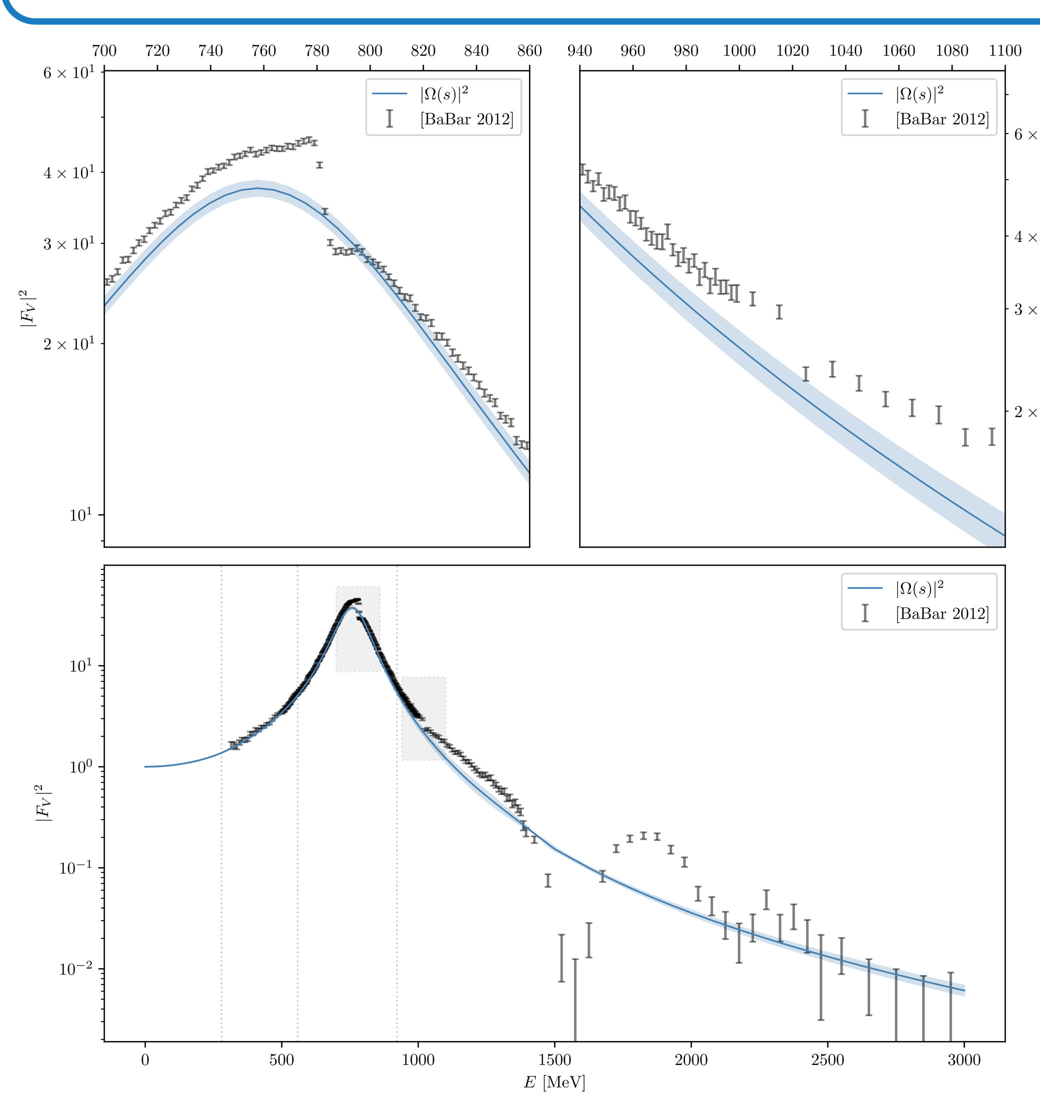
where $s=(q_1+q_2)^2$.



$$\text{disc}\,[F_V(s)] = 2i\,\text{Im}\,[F_V(s)] = 2i\tilde{t}_1^*(s)\sigma_{\pi\pi}(s)F_V(s),$$
$$\sigma_{\pi\pi}(s) = \frac{1}{16\pi}\sqrt{1-4m_\pi^2/s},$$
$$\tilde{t}_1(s) = \sin(\tilde{\delta}_1(s))e^{i\tilde{\delta}_1(s)}/\sigma_{\pi\pi}(s).$$

Omnès–Muskhelishvili solution [3]

$$F_V(s) = P_A(s)\underbrace{\exp\left\{\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\tilde{\delta}_1(s')}{s'-s-i\epsilon}\right\}}_{\Omega[\tilde{\delta}_1](s)}.$$



We need a model that:

- preserves analyticity and unitarity;
- maps to $\Omega[\tilde{\delta}_1](s)$ at low energies;
- provides an accurate high-energy description;
- includes contributions from coupled channels;
- includes isospin-violating effects.

Formalism [1, 4]

$$V_{ij}(s) = \tilde{V}_{ij}(s) + \underbrace{V_{Rij}(s) + V_{Cij}(s)}_{V_{RCij}(s)},$$
$$V_{Rij}(s) = -\sum_{l,l'}^{n_R} g_i^{(l)}\left(G^{(l,l')}(s) - G^{(l,l')}(0)\right)g_j^{(l')},$$
$$V_{Cij}(s) = g_{Cij}, \qquad G^{(l,l')}(s) = \frac{\delta_{l,l'}}{s-m_l^2},$$
$$M_i(s) = c_i - \sum_{l,l'}^{n_R} g_i^{(l)}G^{(l,l')}(s)\alpha^{(l')}s,$$
$$\Sigma_k(s) = \frac{s}{\pi}\int_{s_{\text{thr},k}}^{\infty}\frac{ds'}{s'}\frac{\sigma_k(s')\xi_k^2(s')|\Gamma_k(s')|^2}{s'-s-i\epsilon},$$
$$t_{RC} = [1 - V_{RC}\Sigma]^{-1}V_{RC},$$

Multi-channel solution [4]

$$F_i = \Gamma_{\text{out},i}\,[1 - V_{RC}\Sigma]^{-1}_{ij}M_j.$$

$$\xi_i(s)\,\text{disc}\,[F_i(s)] = 2i\sum_{k=1}^{n_C}T^*_{ik}(s)\sigma_k(s)\xi_k(s)F_k(s).$$

Isospin-violating effects

Photon-resonance mixing:

$$G \mapsto \hat{G} = \left[1 - Gs\alpha\alpha^T\right]^{-1}G,$$
$$g_i^{(l)} \mapsto \hat{g}_i^{(l)} = g_i^{(l)} - e^2\alpha^{(l)}c_i.$$

$\rho-\omega$ and $\rho-\phi$ mixing:

$$c_1 \mapsto c_1\left(1 + \kappa_\omega\frac{s}{s-m_\omega^2+im_\omega\Gamma_\omega} + \kappa_\phi\frac{s}{s-m_\phi^2+im_\phi\Gamma_\phi}\right).$$

Channels included

$\pi\pi$ channel:

$$\sigma_1(s) = \frac{1}{16\pi}\sqrt{1-\frac{4m_\pi^2}{s}} \equiv \sigma_{\pi\pi}(s),$$
$$\xi_1(s) = \frac{1}{\sqrt{3}}\sqrt{s-4m_\pi^2};$$

4π channel:

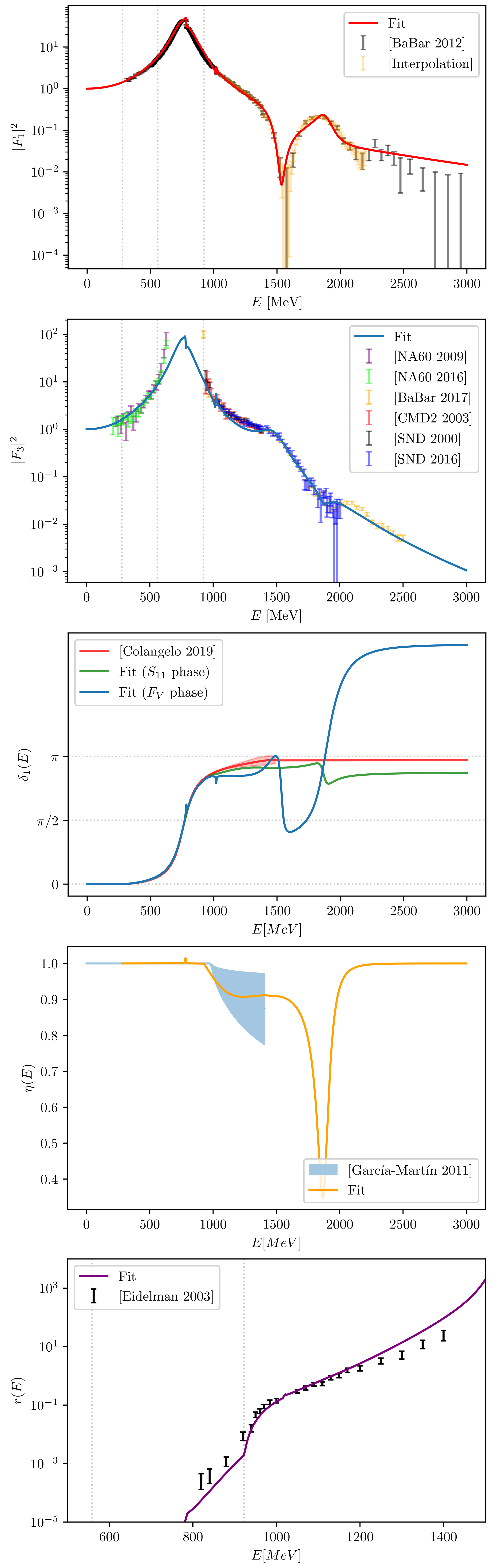
$$\sigma_2(s) = \frac{1}{16\pi}s^2\sqrt{1-\frac{16m_\pi^2}{s}}^{\,7},$$
$$\xi_2(s) = \frac{1}{\sqrt{3}}\sqrt{s-16m_\pi^2}.$$

$\pi^0\omega$ channel:

$$\sigma_3(s) = \frac{1}{16\pi}\frac{\lambda^{1/2}_{\pi^0\omega}(s)}{s},$$
$$\xi_3(s) = \frac{2\sqrt{\tilde{C}_\omega}}{\sqrt{3}}\lambda^{1/2}_{\pi^0\omega}(s),$$

where $\lambda_{ab}(s) = \left(s-(m_a+m_b)^2\right)\left(s-(m_a-m_b)^2\right)$.

Preliminary results



References

[1] George Chanturia. "A two-potential formalism for the pion vector form factor". In: *PoS* (2022). DOI: 10.22323/1.412.0030.

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