# Semileptonic decay of $\Xi_{c c}^{++} \rightarrow \Xi_{c}^{+} \overline{\boldsymbol{l}} \boldsymbol{\nu}_{\boldsymbol{l}}$ in QCD 

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#### Abstract

We investigate the semileptonic decay of the doubly heavy baryon $\Xi_{c c}^{++}$into the single heavy baryon $\Xi_{c}^{+}$in the framework of the QCD sum rule method. We calculate the weak form factors entering the amplitude of this transition. Numerical results of these form factors will be used to predict the decay width and branching ratio of $\Xi_{c c}^{++} \rightarrow \Xi_{c}^{+} \bar{l} v_{l}$. Our results may be checked via future experiments like


 LHCb at CERN.
## Introduction

In 2017, the LHCb collaboration announced the observation of the doubly charmed baryon $\Xi_{c c}^{++}$via the decay $\Xi_{c c}^{++} \rightarrow \Lambda_{c}^{+} \mathrm{K}^{-} \pi^{+} \pi^{+}$[1]. The $\Xi_{c c}^{++}$mass is measured to be $3620.6 \pm 1.5$ (stat) $\pm 0.4$ (syst) $\pm 0.3$ ( $\left.\Xi_{c}^{+}\right) \mathrm{MeV} / \mathrm{c}^{2}$. They confirmed the finding by another decay mode $\Xi_{c c}^{++} \rightarrow \Xi_{c}^{+} \pi^{+}$in 2018 [2]. Finally in 2022, the $\Xi_{c c}^{++} \rightarrow \Xi_{c}^{\prime+} \pi^{+}$ decay is observed using proton-proton collisions collected by the LHCb experiment [3]. The QCD sum rule method is a powerful tool in studying of the doubly heavy baryons [4,5]. The spectroscopic parameters of the doubly heavy baryons have been calculated in Refs. [6,7] using this method.

## Transition form factors

The $\Xi_{c c}^{++} \rightarrow \Xi_{c}^{+} \bar{l} v_{l}$ decay channel proceeds via $\mathrm{c} \rightarrow \mathrm{s} \bar{l} v_{l}$ at quark level


FIG. 1: The $\Xi_{c c}^{++} \rightarrow \Xi_{c}^{+} \bar{l} v_{l}$ decay channel.
The low-energy effective Hamiltonian of this transition can be written as

$$
\begin{equation*}
\mathcal{F}_{\text {eff }}=\frac{G_{F}}{\sqrt{v}} V_{s c} \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) \text { c } \bar{l} \gamma^{\mu}\left(1-\gamma_{5}\right) v_{l}, \tag{1}
\end{equation*}
$$

where $G_{F}$ is the Fermi coupling constant and $V_{s c}$ is one of the elements of the CKM matrix. The amplitude of this channel is

$$
\begin{equation*}
\mathrm{M}=\left\langle\Xi_{c}^{+}\right| \mathcal{F}_{\mathrm{eff}}\left|\Xi_{c c}^{++}\right\rangle \tag{2}
\end{equation*}
$$

The point-like particles immediately go out of the matrix element and remaining parts are parameterized in terms of six form factors the $F_{1,2,3}\left(\mathrm{q}^{2}\right)$ and $\mathrm{G}_{1,2,3}\left(\mathrm{q}^{2}\right)$

$$
\begin{align*}
& \left\langle\Xi_{c}^{+}\left(p^{\prime}, \mathrm{s}^{\prime}\right)\right| V^{\mu}\left|\Xi_{c c}^{++}(p, s)\right\rangle=\bar{u}_{\Xi_{c}^{+}}\left(p^{\prime}, \mathrm{s}^{\prime}\right)\left[\mathrm{F}_{1}\left(\mathrm{q}^{2}\right) \gamma^{\mu}+\mathrm{F}_{2}\left(\mathrm{q}^{2}\right) \frac{p_{\mu}}{M_{\Xi_{c c}^{++}}^{+}}+\mathrm{F}_{s}\left(\mathrm{q}^{2}\right) \frac{p^{\prime} \mu}{{ }^{M} \Xi_{c}^{+}}\right] \\
& u_{\Xi_{c c}^{++}}(p, s) \tag{3}
\end{align*}
$$

$\left\langle\Xi_{c}^{+}\left(p^{\prime}, \mathrm{s}^{\prime}\right)\right| A^{\mu}\left|\Xi_{c c}^{++}(p, s)\right\rangle=\bar{u}_{\Xi_{c}^{+}}\left(p^{\prime}, \mathrm{s}^{\prime}\right)\left[\mathrm{G}_{1}\left(\mathrm{q}^{2}\right) \gamma^{\mu}+\mathrm{G}_{2}\left(\mathrm{q}^{2}\right) \frac{p_{\mu}}{{ }^{M} \Xi_{c c}^{++}}+\mathrm{G}_{s}\left(\mathrm{q}^{2}\right) \frac{p^{\prime} \mu}{{ }^{M} \Xi_{c}^{+}}\right]$ $\gamma_{5} u_{\Xi_{c c}^{+c}}(p, s)$,
where $V^{\mu}=\bar{s} \gamma_{\mu} \mathrm{c}, A^{\mu}=\bar{s} \gamma_{\mu} \gamma_{5} \mathrm{c}$; and $u_{\Xi_{c c}^{++}}(p, s)$ and $\bar{u}_{\Xi_{c}^{+}}\left(p^{\prime}, \mathrm{s}^{\prime}\right)$ are Dirac spinors of the initial and final baryonic states. The main porpuse is to calculate the six transition form factors using the three-point QCD sum rule method. We consider an appropriate correlation function of interpolating and transition currents. The sum rules for transition form factors are obtained by equating the physical side to the QCD side of this function that is obtained using the operator product expansion (OPE). The three-point correlation function for this transition is

$$
\begin{equation*}
\left.\Pi_{\mu}\left(p, p^{\prime}, q\right)=\mathrm{i}^{2} \int \mathrm{~d}^{4} \mathrm{x} \mathrm{e}^{-\mathrm{i} p \cdot \mathrm{x}} \int \mathrm{~d}^{4} \mathrm{ye}^{\mathrm{i} p^{\prime} \cdot \mathrm{y}}\langle 0| \mathcal{T}\left|J^{\Xi_{c}^{+}}(\mathrm{y}) \mathcal{J}_{\mu}^{\operatorname{tr} A(V)}(0) \mathrm{J}^{\Xi_{c c}^{++}}(\mathrm{x})\right| 0\right\rangle \tag{5}
\end{equation*}
$$

where $\mathcal{T}$ is the time-ordereing operator and $\mathcal{J}^{\Xi_{c}^{+}}$and $\mathcal{J}^{\Xi_{c c}^{++}}$are the interpolating currents of the $\Xi_{c}^{+}$and $\Xi_{c c}^{++}$baryons. The forms of the interpolating currents of the $\Xi_{c}^{+}$and $\Xi_{c c}^{++}$baryons in their antisymmetric and symmetric forms can be written as


FIG. 2: The Spin $\frac{1}{2}$ single charmed and doubly charmed baryons that represent $\operatorname{SU}(3)$ antitriplets $\overline{3}$ (antisymmetric flavour) and triplets 3 (symmetric flavour).
$\jmath^{\Xi_{c}^{+}}(y)=\frac{1}{\sqrt{6}} \epsilon_{a b c}\left\{2\left(u^{a T}(y) C s^{b}(y)\right) \gamma_{5} c^{c}(y)+\left(u^{a T}(y) C c^{b}(y)\right) \gamma_{5} s^{c}(y)+\right.$
$\left(c^{a T}(y) C s^{b}(y)\right) \gamma_{5} u^{c}(y)+2 \beta\left(u^{a T}(y) C \gamma_{5} s^{b}(y)\right) c^{c}(y)+\beta\left(u^{a T}(y) C \gamma_{5} c^{b}(y)\right) s^{c}(y)+\beta\left(c^{a T}(y) C \gamma_{5}\right.$ $\left.\left.s^{b}(y)\right) u^{c}(y)\right\}$,
$J^{\Xi_{c c}^{++}}(x)=\sqrt{2} \epsilon_{a b c}\left\{\left(c^{a T}(x) C u^{b}(x)\right) \gamma_{5} c^{c}(x)+\beta^{\prime}\left(c^{a T}(x) C \gamma_{5} u^{b}(x)\right) c^{c}(x)\right\}$.
The physical side is found by inserting complete sets of the initial and final baryonic states with the same quantum numbers as the interpolating currents into the correlation function,
$\Pi_{\mu}^{\text {Phys. }}\left(p, p^{\prime}, q\right)=\frac{\langle 0| \Xi_{c}^{+}(0)\left|\Xi_{c}^{+}\left(p^{\prime}\right)\right\rangle\left\langle\Xi_{c}^{+}\left(p^{\prime}\right)\right| j_{\mu}^{\text {tr } A(V)}(0)\left|\Xi_{c c}^{++}(p)\right\rangle\left\langle\Xi_{c c}^{++}(p)\right| j \Xi_{c c}^{++}(0)|0\rangle}{\left(p^{\prime 2}-m^{-} \Xi_{c}^{+}\right)\left(p^{2}-m_{\Xi_{c c}^{+}}^{+}\right)}+\cdots$.
We define the matrix elements in terms of the residues of the initial and final states

$$
\begin{align*}
& \langle 0| J^{\Xi_{c}^{+}}(0)\left|\Xi_{c}^{+}\left(p^{\prime}\right)\right\rangle=\lambda_{\Xi_{c}^{+}} u_{\Xi_{c}^{+}}\left(p^{\prime}, s^{\prime}\right), \\
& \left\langle\Xi_{c c}^{++}(p)\right| J^{\Xi_{c c}^{+}}(0)|0\rangle=\lambda_{\Xi_{c c}^{++}} u_{\Xi_{c c}^{++}}(p, s) . \tag{9}
\end{align*}
$$

Finally we put all the matrix elements into Eq. (8) and use the summation over Dirac spinors

$$
\begin{align*}
& \sum_{s^{\prime}} u_{\Xi_{c}^{+}}\left(p^{\prime}, s^{\prime}\right) \bar{u}_{\Xi_{c}^{+}}\left(p^{\prime}, s^{\prime}\right)=p^{\prime}+m_{\Xi_{c}^{+}} \\
& \sum_{s} u_{\Xi_{c c}^{+}}(p, s) \bar{u}_{\Xi_{c c}^{++}}(p, s)=p+m_{\Xi_{c c}^{++}} \tag{10}
\end{align*}
$$

We obtain the following expression for the final form of the physical side in terms of the structures used in the calculations in Borel transformed form that is applied to suppress the contributions of the higher states and continuum:

$$
\begin{align*}
& \widehat{\mathbf{B}} \Pi_{\mu}^{\text {Phys. }}\left(\mathrm{p}, \mathrm{p}^{\prime}, \mathrm{q}\right)=\left[\mathrm{m}_{\Xi_{c c}^{++}} \mathrm{F}_{1} \not p^{\prime} \gamma^{\mu}+\frac{1}{\mathrm{~m}_{\Xi_{c c}^{++}}} \mathrm{F}_{2} p_{\mu} \not p^{\prime} \not p+\frac{1}{m_{\Xi_{c}^{+}}} \mathrm{F}_{3} p^{\prime}{ }_{\mu} \not p^{\prime} p \boldsymbol{p}\right. \\
& +\mathrm{m}_{\Xi_{\mathrm{cc}}^{++}} \mathrm{m}_{\Xi_{c}^{+}} \mathrm{G}_{1} \gamma^{\mu} \gamma_{5}-\frac{1}{\mathrm{~m}_{\Xi_{c c}^{++}}} \mathrm{G}_{2} p_{\mu} \not p^{\prime} \not p \gamma_{5}-\frac{1}{\left.\mathrm{~m}_{\Xi_{c c}^{++}} \mathrm{G}_{3} p^{\prime}{ }_{\mu} \not p^{\prime} \not p \gamma_{5}\right] .} . \tag{11}
\end{align*}
$$

To find the correlation function on QCD side, we insert the interpolating currents into the correlation function that leads to an expression in terms of propagators of the light and heavy quarks using the Wick's theorem. After performing the regular calculations, we find the function $\Pi_{\mu}^{\text {QCD }}\left(p, p^{\prime}, q\right)$ which includes twenty-four different structures that not all of them are written here:

$$
\begin{equation*}
\Pi_{\mu}^{\mathrm{QCD}}\left(p, p^{\prime}, q\right)=\Pi_{p^{\prime} \gamma_{\mu}}^{\mathrm{QCD}}\left(p^{2}, p^{\prime 2}, q^{2}\right) p^{\prime} \gamma_{\mu}+\Pi_{p_{\mu} \not \phi^{\prime} p}^{\mathrm{QCD}}\left(p^{2}, p^{\prime 2}, q^{2}\right) p_{\mu} \not \phi^{\prime} p p+\ldots . \tag{12}
\end{equation*}
$$

where, the invariant functions $\Pi_{\mathrm{i}}^{\mathrm{QCD}}\left(p, p^{\prime}, q\right)$, with i representing different structures, are represented in terms of a double dispersion integral as

$$
\begin{equation*}
\Pi_{i}^{\mathrm{QCD}}\left(p^{2}, p^{\prime 2}, q^{2}\right)=\int_{s_{m i n}}^{\infty} d s \int_{s_{m i n}^{\prime}}^{\infty} d s^{\prime} \frac{\rho_{i}^{\mathrm{QCD}}\left(s, s^{\prime}, q^{2}\right)}{\left(s-p^{2}\right)\left(s^{\prime}-p^{\prime 2}\right)}+\text { subtracted terms } \tag{13}
\end{equation*}
$$

where $\rho_{\mathrm{i}}^{\mathrm{QCD}}\left(s, s^{\prime}, q^{2}\right)$ are the spectral densities, which are obtained by taking the imaginary parts of the $\Pi_{\mathrm{i}}^{\mathrm{QCD}}\left(p^{2}, p^{\prime 2}, q^{2}\right)$ functions according to the standard prescriptions of the method used, include two different parts and can be classified as

$$
\begin{equation*}
\rho_{i}^{\mathrm{QCD}}\left(s, s^{\prime}, q^{2}\right)=\rho_{i}^{\text {Pert. }}\left(s, s^{\prime}, q^{2}\right)+\sum_{n=3}^{5} \rho_{i}^{n}\left(s, s^{\prime}, q^{2}\right) \tag{14}
\end{equation*}
$$

where by $\rho_{i}^{n}\left(s, s^{\prime}, q^{2}\right)$, we denote the nonperturbative contributions to $\rho_{\mathrm{i}}^{\mathrm{QCD}}\left(s, s^{\prime}, q^{2}\right)$ $\mathrm{n}=3,4,5$, stand for the quark, gluon and mixed condensates, respectively. After applying the double Borel transformation on the variables $p^{2}$ and $p^{\prime 2}$ in QCD side and subtracting the contributions of the higher resonances and continuum states supported by the quark-hadron duality assumption and matching the coefficients of different structures from the physical and QCD sides of the correlation function, we will find the required sum rules for the form factors that will be used in numerical calculations.

## References

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