# Fritzsch-like textures from flavor symmetry: precision tests

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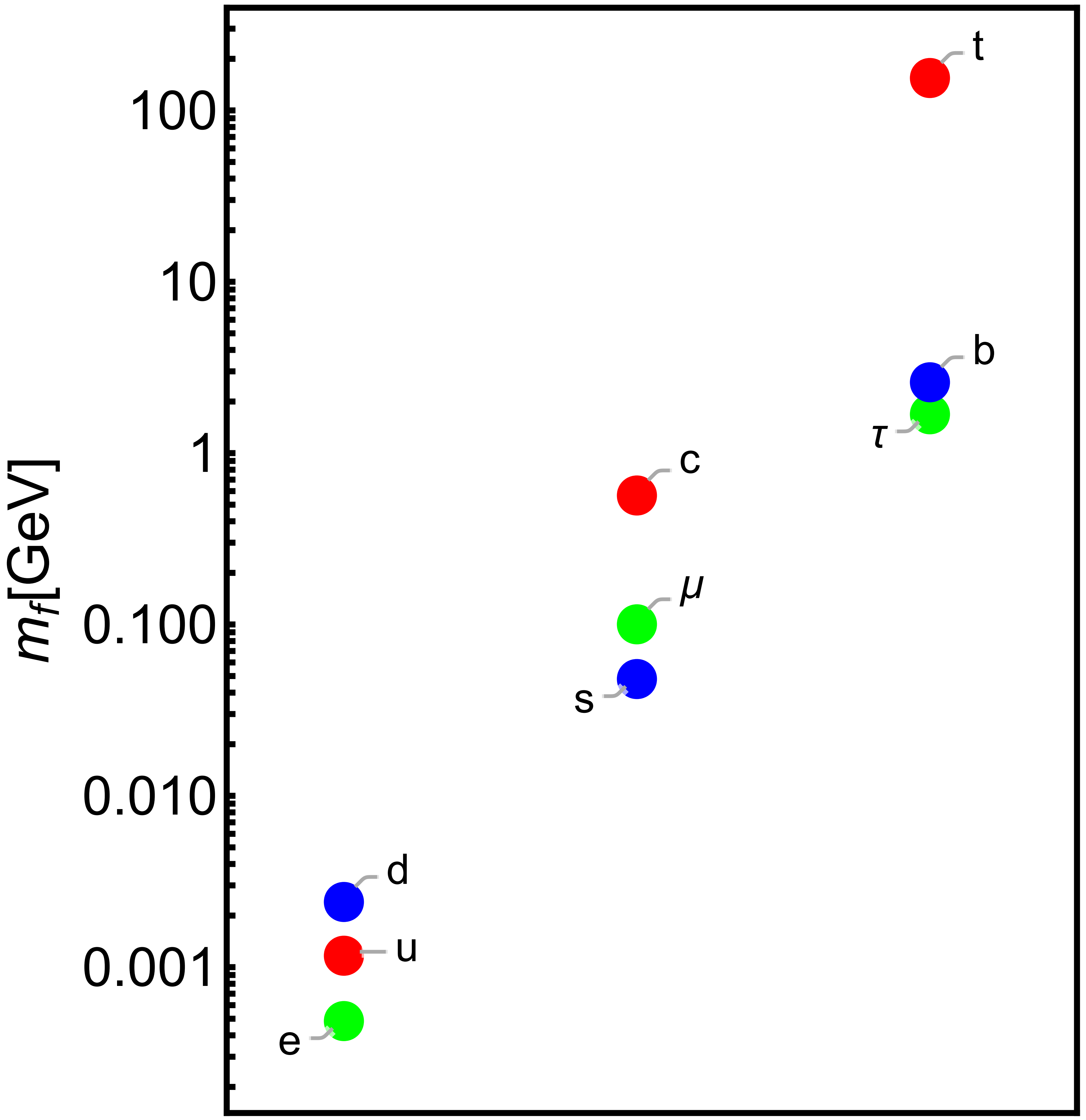
# The Standard Model

$$-\mathcal{L}_{\text{Yuk}} = Y_u^{ij}\phi\, u_i^c q_j + Y_d^{ij}\tilde{\phi}\, d_i^c q_j + Y_e^{ij}\tilde{\phi}\, e_i^c \ell_j + \text{h.c.}$$

- Three families of chiral fermions in identical representations under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry:

  $q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3,2)_{1/6}\,,\; u_R \sim (3,1)_{2/3}\,,\; d_R \sim (3,1)_{-1/3}\,,\; l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1,2)_{-1/2}\,,\; e_R \sim (1,1)_{-1}$
- Fermions and gauge bosons get masses through couplings to one Higgs doublet $\phi \sim (1,2)_{1/2}$.
- Yukawa matrices are completely **arbitrary**, not diagonal in weak basis:
- $m^{(u,d,e)ij} = Y_{u,d,e}^{ij} v_w\,, \qquad \mathbf{m}_{\text{diag}}^{u,d,e} = V'^{(u,d,e)\dagger}\mathbf{m}^{(u,d,e)}V^{(u,d,e)}$
- Quark charged currents in mass basis: $\frac{g}{\sqrt{2}} \begin{pmatrix} u & c & t \end{pmatrix}_L \gamma^\mu V^{(u)\dagger}V^{(d)} \begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu^\dagger$
- $V_{\text{CKM}} = V^{(u)\dagger}V^{(d)} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$. Unitary; 4 parmaters, three angles $\theta_{ij}$ and one phase $\delta$.
- Yukawa couplings and photon/Z couplings (for unitarity of $V^{(u)}$ and $V^{(d)}$) are diagonal in mass basis.
- **NO flavour changing neutral currents** at tree level.

# Fermion families, masses and mixing angles




- $m^{(u,d,e)ij} = Y^{ij}_{u,d,e} v_w$
- replication of fermion families;
- inter-family mass hierarchy (Yukawa hierarchy);
- weak mixing pattern: small angles for quarks, large angles for neutrinos;
- neutrino masses: very small (seesaw?), mass hierarchy yet unknown.
- Technically natural: SM tolerates Yukawa hierarchy but cannot explain it.

# Fermion families, masses and mixing angles

- The inter-family hierarchy between quark masses scales with small parameters:

$$m_d : m_s : m_b \sim \epsilon_d^2 : \epsilon_d : 1 \qquad m_u : m_c : m_t \sim \epsilon_u^2 : \epsilon_u : 1$$

with $\epsilon_d \sim 1/30$, $\epsilon_u \sim 1/300$, and for charged leptons $\epsilon_e \sim \epsilon_d$, $\epsilon'_e \sim \epsilon_u$:

$$m_e : m_\mu : m_\tau \sim \epsilon'_e \epsilon_e : \epsilon_e : 1$$

- CKM mixing angles show the hierarchy:

$$\sin\theta_{12} \sim \sqrt{\epsilon_d}\,, \quad \sin\theta_{23} \sim \epsilon_d\,, \quad \sin\theta_{13} \sim \epsilon_d^2$$

- Intriguing relations between masses and mixing angles. Formula for the **Cabibbo angle**:

$$\boxed{\sin\theta_{12} = \sqrt{m_d/m_s}}$$

- Relations between masses and mixing angles may not be accidental.
- Underlying theory may provide **predictive textures** to explain the pattern of Yukawa matrices.

# Fritzsch texture for mass matrices

- Flavour pattern from Yukawa matrices with reduced number of parameters;
- Yukawa matrices with zero elements: zero-textures originally thought to obtain the Cabibbo angle in two families.
- **Fritzsch texture** for six quarks:

$$M_f = \begin{pmatrix} 0 & A'_f & 0 \\ A_f & 0 & B'_f \\ 0 & B_f & C_f \end{pmatrix}$$

- Condition $|\mathbf{A_f}| = |\mathbf{A'_f}|$, $|\mathbf{B_f}| = |\mathbf{B'_f}|$ originally obtained in left-right symmetric models.
- Need for several scalars with discrete flavour symmetries, danger with flavour changing effects.
- More naturally, Fritzsch matrices can be obtained in the context of $SU(3)_H$ **gauge symmetry** between the three families.

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- $|\mathbf{A_f}| = |\mathbf{A'_f}|$, $|\mathbf{B_f}| = |\mathbf{B'_f}|$
- By phase rotations of quark fields the mass matrices can be made real.
- Three real parameters $A_f$, $B_f$, $C_f$ can be expressed in terms of the three eigenvalues

$$A_d = \sqrt{m_d\, m_s}\,, \qquad B_d = \sqrt{m_s\, m_b}\,, \qquad C_d = m_b - m_s$$

- $V_{\rm CKM} = O_u^T\, F\, O_d$, $O_d$ and $O_u$are orthogonal matrices and $F = {\rm diag}(e^{i(\tilde{\beta}+\tilde{\delta}}, e^{i\tilde{\beta}}, 1)$.
- Relations between quark mixing angles and mass ratios follow from Fritzsch mass matrices:

$$s_{12} = |(m_d/m_s)^{1/2} - (m_u/m_c)^{1/2}\, e^{-i\tilde{\delta}}|\,, \qquad s_{23} = |(m_s/m_b)^{1/2} - (m_c/m_t)^{1/2}\, e^{-i\tilde{\beta}}|\,,$$

$$s_{13} = |-(m_s/m_b)(m_d/m_b)^{1/2} - (m_u/m_c)^{1/2}(m_s/m_b)^{1/2}e^{-i\tilde{\delta}} + (m_u/m_c)^{1/2}(m_c/m_t)^{1/2}e^{-i\tilde{\alpha}}|$$

# Symmetric Fritzsch texture for mass matrices

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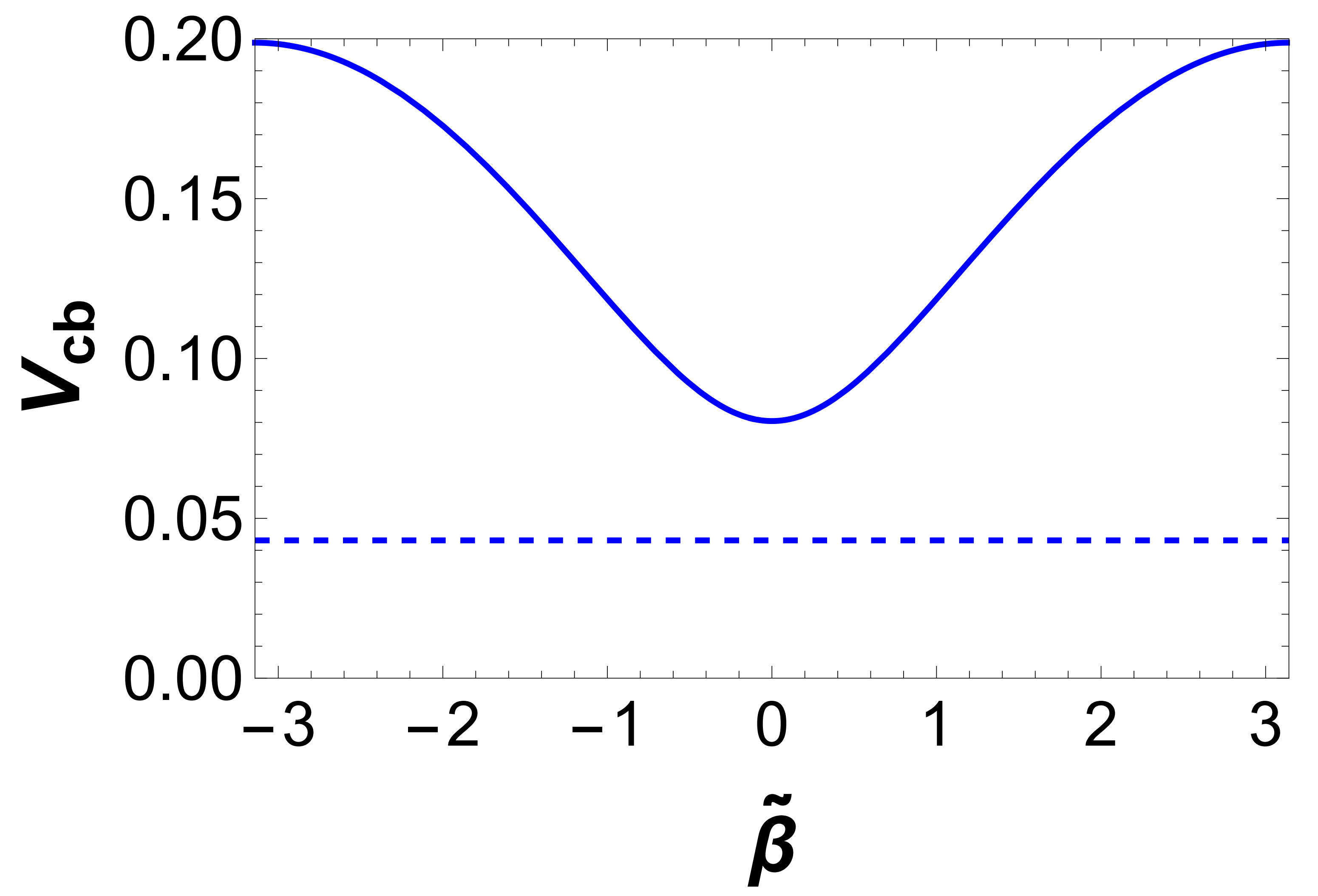
# Symmetric Fritzsch texture for mass matrices

- $\tilde{\delta} = -1.52 \quad \longrightarrow \quad |V_{us}| = s_{12} = |(m_d/m_s)^{1/2} - (m_u/m_c)^{1/2}\, e^{-i\tilde{\delta}}|$

- $|V_{cb}| = s_{23} \neq |(m_s/m_b)^{1/2} - (m_c/m_t)^{1/2}\, e^{-i\tilde{\beta}}|$

for every possible choice of the phase $\tilde{\beta}$ ...



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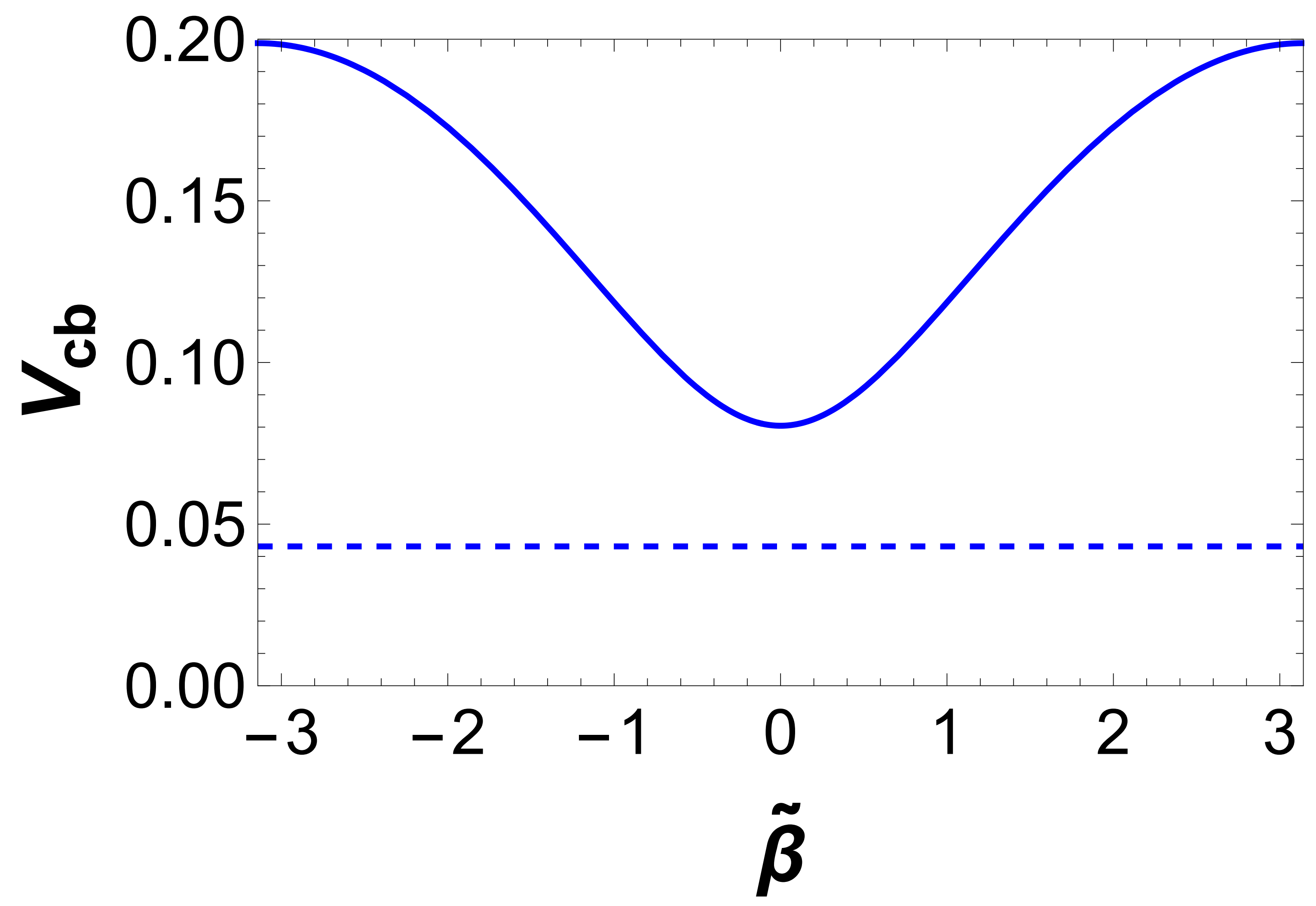


(Symmetric) Fritzsch texture is ruled out. Fritzsch-like texture?

# Fritzsch texture for mass matrices

- A feature of Fritzsch matrices: decoupling hypothesis.
- Suppose only the first two families would be present.
  In the limit $m_u = m_d = 0$, quarks c and s are infinitely heavy (**decoupling hypothesis**).
- One might expect that in this limit physics of u,d decouples from c,s: Cabibbo angle should vanish.

  $m_u, m_d \to 0\,,\ \theta_{12} \to 0$
- For six quarks decoupling hypothesis implies that all mixing angles $\theta_{ij}$ vanish as $m_u, m_d \to 0$, $m_c, m_s \to 0$.

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- Without reducing the number of zero entries one can introduce an **asymmetry** in the 23 block of $Y$:
  $|B_d| \neq |B'_d|$, and still conserving the decoupling feature.
- Fritzsch texture can be obtained (and asymmetrized) in the context of interfamily gauge $\mathbf{SU(3)_H}$ symmetry.

# Family symmetries

- In the limit of vanishing Yukawa couplings, $Y_f \to 0$, the SM acquires maximal global symmetry:
$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$$
- Yukawa interactions break the $SU(3)^5$ symmetry.
- Fermions cannot get mass if the symmetry is unbroken: LH and RH particles transform in different representations, as triplets and anti-triplets respectively.
- The fermion mass hierarchy can be related to the breaking pattern of a family symmetry;
- In the context of $SU(5)$ grand unified theory (GUT) the maximal symmetry reduces to $U(3)_\ell \times U(3)_e$

# Fritzsch texture in $\mathbf{SU(5) \times SU(3)_H}$

- Introduce **horizontal gauge symmetry $\mathbf{SU(3)_H}$** between three families.
- Chiral global $U(1)_H$ can be associated with Peccei-Quinn symmetry if it is not broken by the Lagrangian of flavon fields.
- Family symmetry should be chiral, so fermions cannot acquire masses without $U(3)_H$ breaking.
- In the context of $SU(5)$ GUT the fermions form the multiplets:

$$\bar{F}_i = (d^c, \ell)_i \sim (\bar{5}, 3)\,, \qquad T_i = (u^c, q, e^c)_i \sim (10, 3)$$

  where $f_L^c = C\overline{f_R}^T$ are the LH C-conjugates to the RH components $f_R$.

- Fermion mass hierarchy and mixing pattern is related to $U(3)_H$ breaking pattern and hierarchy of VEVs inducing the breaking $U(3)_H \to U(2)_H \to U(1)_H \to$ nothing.
- Natural realization of decoupling hypothesis: $U(3)_H \to U(2)_H$ third family massive, ... $U(1)$ broken first family become massive and mixing emerge.
- In the context of $SO(10) \times SU(3)_H$ all fermions together with RH neutrino into one multiplet in spinor representation of $SO(10)$, $\Psi_i = (\bar{F}, T, \nu^c)_i \sim (16, 3)$.

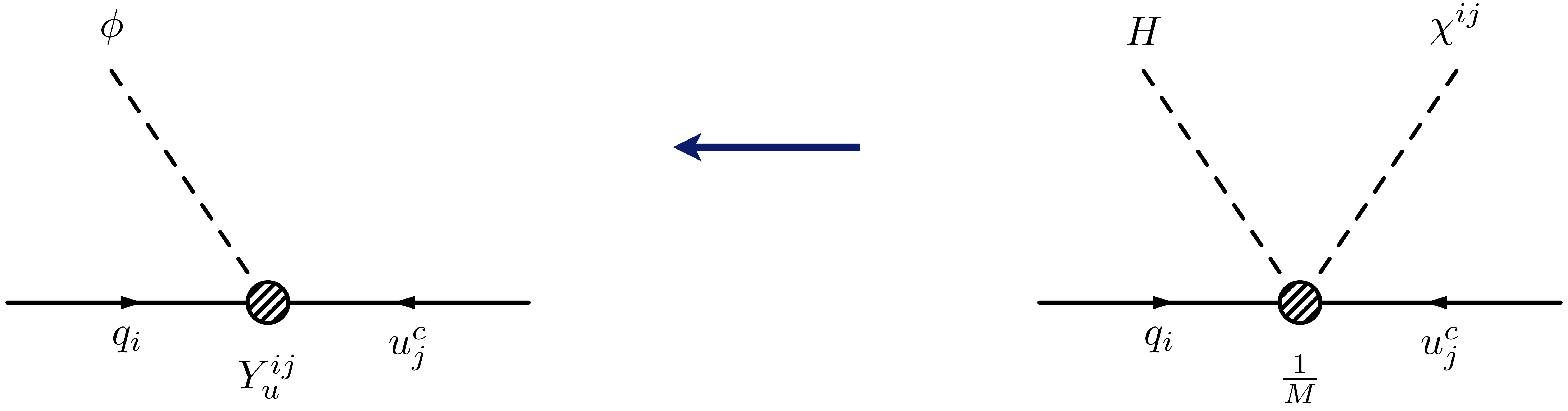
# Effective operators for fermion masses

- Fermion masses can be induced only via the higher order operators

$$\frac{\chi^{ij}}{M} u_i^c q_j \phi + \frac{\chi^{ij}}{M} d_i^c q_j \tilde{\phi} + \frac{\chi^{ij}}{M} e_i^c \ell_j \tilde{\phi} + \text{h.c.}$$

or in $SU(5) \times SU(3)_H$: $\chi^{ij}/M T_i T_j H + \chi^{ij}/M \bar{F}_i T_j H^*$.

- SM Yukawa couplings are induced by non zero VEVs and will reflect the breaking pattern.




- Fermion bilinears transform in representation $3 \times 3 = 6 + \bar{3}$.
- $\chi$ are horizontal scalars, anti-sextets $\chi^{\{ij\}} \sim \bar{6}$ or triplets $\chi^{[ij]} = \epsilon^{ijk}\chi_k \sim 3$.

# Fritzsch texture in $\mathbf{SU(5)} \times \mathbf{SU(3)_H}$

- Two scalar triplets $\chi_1$, $\chi_2$ and one anti-sextet $\chi_3$ with VEVs:

$$\langle \chi_3^{\{ij\}} \rangle = \mathrm{diag}(0,0,V_3) \qquad \langle \chi_{2i} \rangle = \begin{pmatrix} V_2 \\ 0 \\ 0 \end{pmatrix} \qquad \langle \chi_{1i} \rangle = \begin{pmatrix} 0 \\ 0 \\ V_1 \end{pmatrix}$$

$$U(3)_H \xrightarrow{V_3} U(2)_H \xrightarrow{V_2} U(1)_H \xrightarrow{V_1} \text{nothing}$$

$$\langle \chi^{ij} \rangle = \langle \chi_1^{ij} + \chi_2^{ij} + \chi_3^{ij} \rangle = \begin{pmatrix} 0 & V_1 & 0 \\ -V_1 & 0 & V_2 \\ 0 & -V_2 & V_3 \end{pmatrix}, \qquad Y_u\,, Y_d \propto \frac{\langle \chi \rangle}{M}$$

$$V_3 \gg V_2 \gg V_1$$

- After this breaking, the theory reduces to the SM with a Higgs doublet and flavour is conserved in neutral currents.

# Fritzsch texture in $\mathbf{SU(5)\times SU(3)_H}$

- Effective operators can be induced via renormalizable interactions after integrating out **heavy vector-like fermions**.

$$Q^i\,,U^{ci}\sim\bar{3}\,,\qquad Q_i^c\,,U_i\sim 3\qquad D_i\sim 3\,,D^{ci}\sim\bar{3}$$

- They are allowed to have invariant mass terms. Mass terms transform as $\bar{3}\times 3=1+8$ and can emerge from Yukawa couplings with scalar singlet or octet.

- Yukawa couplings with $U$ and $D$ vector-like quarks are:

$$g_U^{(n)}\chi_n^{ij}u_i^cU_j+g_D^{(n)}\chi_n^{ij}d_i^cD_j+h_UU^{ci}q_i\phi+h_dD^{ci}q_i\tilde{\phi}+g_U^{(n)}\chi_n^{ij}q_iQ_j^c+h_Uu_i^cQ^i\phi+h_Dd_i^cQ^i\tilde{\phi}$$

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- Mass matrices of fermions look like:

$$\begin{pmatrix} u^c & Q^c & U^c \end{pmatrix} \begin{pmatrix} 0 & h_U H & \chi \\ \chi^T & M_Q & 0 \\ h_U H & 0 & M_U \end{pmatrix} \begin{pmatrix} q \\ Q \\ U \end{pmatrix}, \qquad \begin{pmatrix} d^c & D^c & Q^c \end{pmatrix} \begin{pmatrix} 0 & \chi & h_D \bar{H} \\ h_d \bar{H} & M_D & 0 \\ \chi^T & 0 & M_Q \end{pmatrix} \begin{pmatrix} q \\ D \\ Q \end{pmatrix}$$

- Yukawa couplings are induced in **seesaw approximation**:

$$Y_u = M_U^{-1} \chi_U h_U + \chi_U^T M_Q^{-1} h_U\,, \qquad Y_d = M_D^{-1} \chi_D h_d + \chi_D^T M_Q^{-1} h_D\,; \qquad \chi_{U,D}^{ij} = g_{U,D}^{(n)} \langle \chi_n^{ij} \rangle$$

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- If heavy fermions are degenerate in masses between families, Yukawa matrices $Y_{u,d}$ have **Fritzsch texture**.

$$\langle \chi_1^{ij} + \chi_2^{ij} + \chi_3^{ij} \rangle = \begin{pmatrix} 0 & V_1 & 0 \\ -V_1 & 0 & V_2 \\ 0 & -V_2 & V_3 \end{pmatrix},$$

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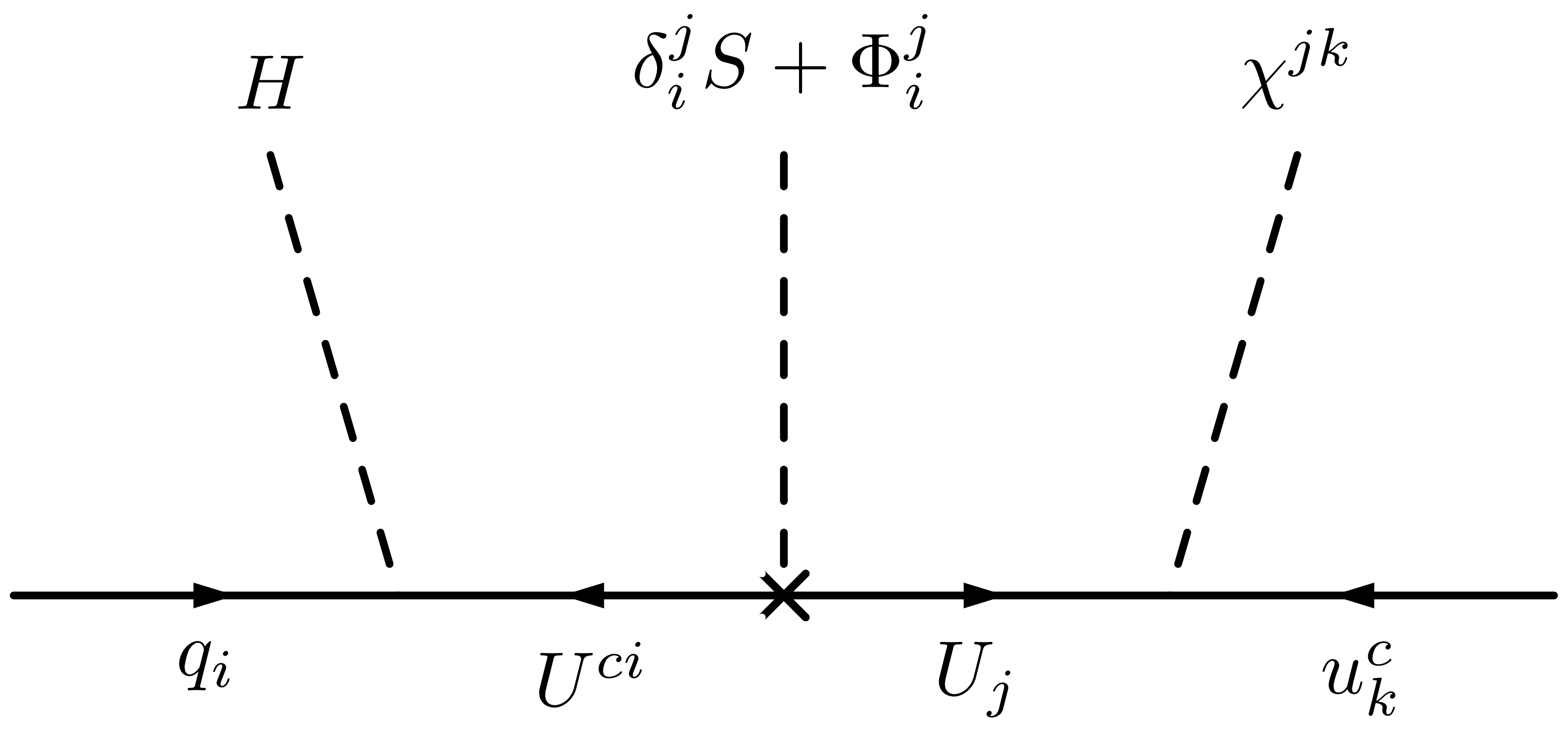
However symmetric Fritzsch texture is excluded.

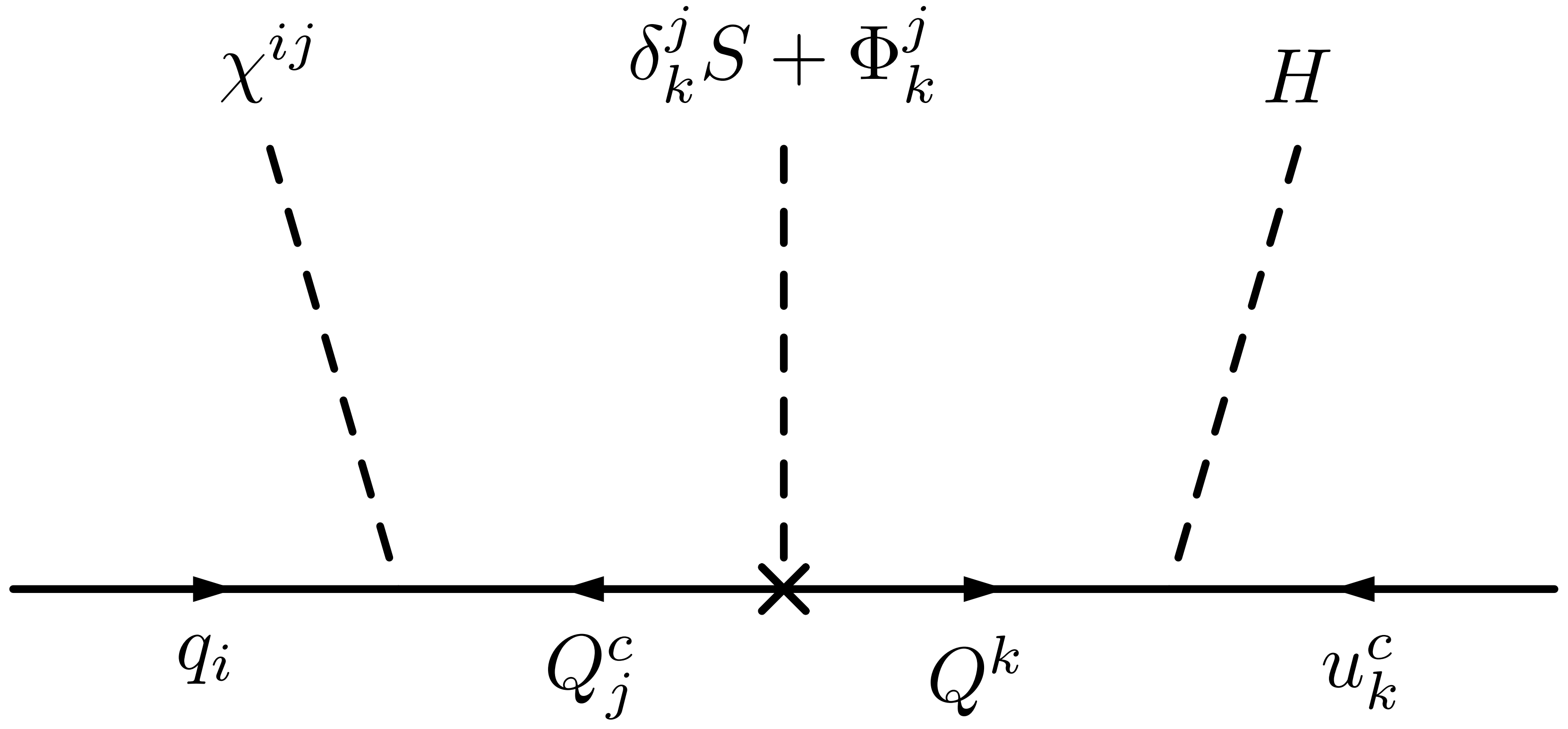
# Fritzsch texture in $\mathbf{SU(5)\times SU(3)_H}$

- Introduce the octet scalar in **adjoint representation** of $SU(3)_H$
- Assume that it has a VEV directed towards the $\lambda_8$ generator of $SU(3)_H$:

$$\langle\Phi\rangle = \mathrm{diag}(V, V, -2V)$$

- This field can contribute to the mass matrices of heavy fermions via the Yukawa couplings $G_U \Phi_i^j U^i U_j^c$ and $G_D \Phi_i^j D^i D_j^c$;
- Along with the $SU(3)_H$ invariant mass terms, the mass matrices of the heavy fermions will acquire the forms $M_U \propto (1, 1, X_U)$ and $M_D \propto (1, 1, X_D)$, where $X_U, X_D$ are generically complex.



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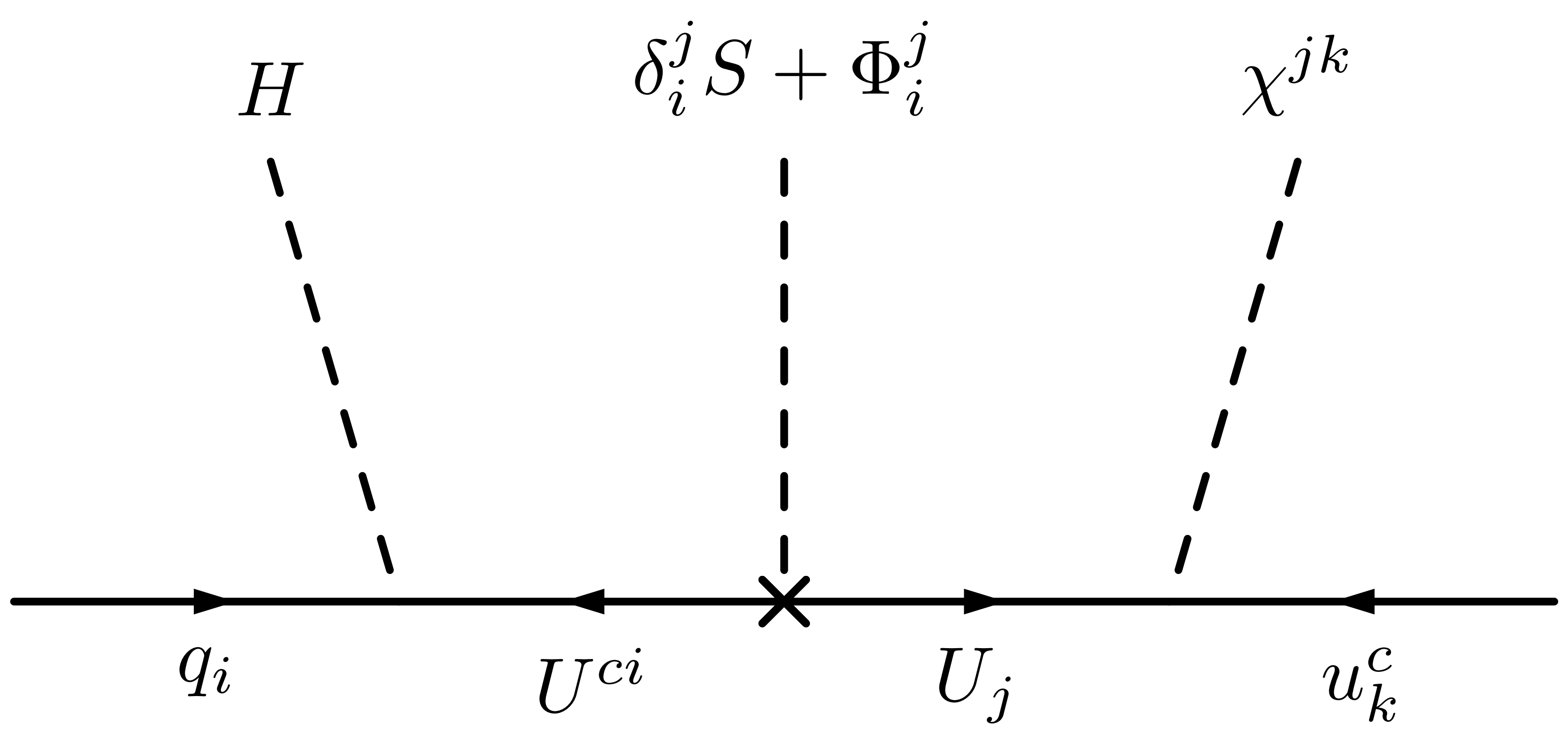


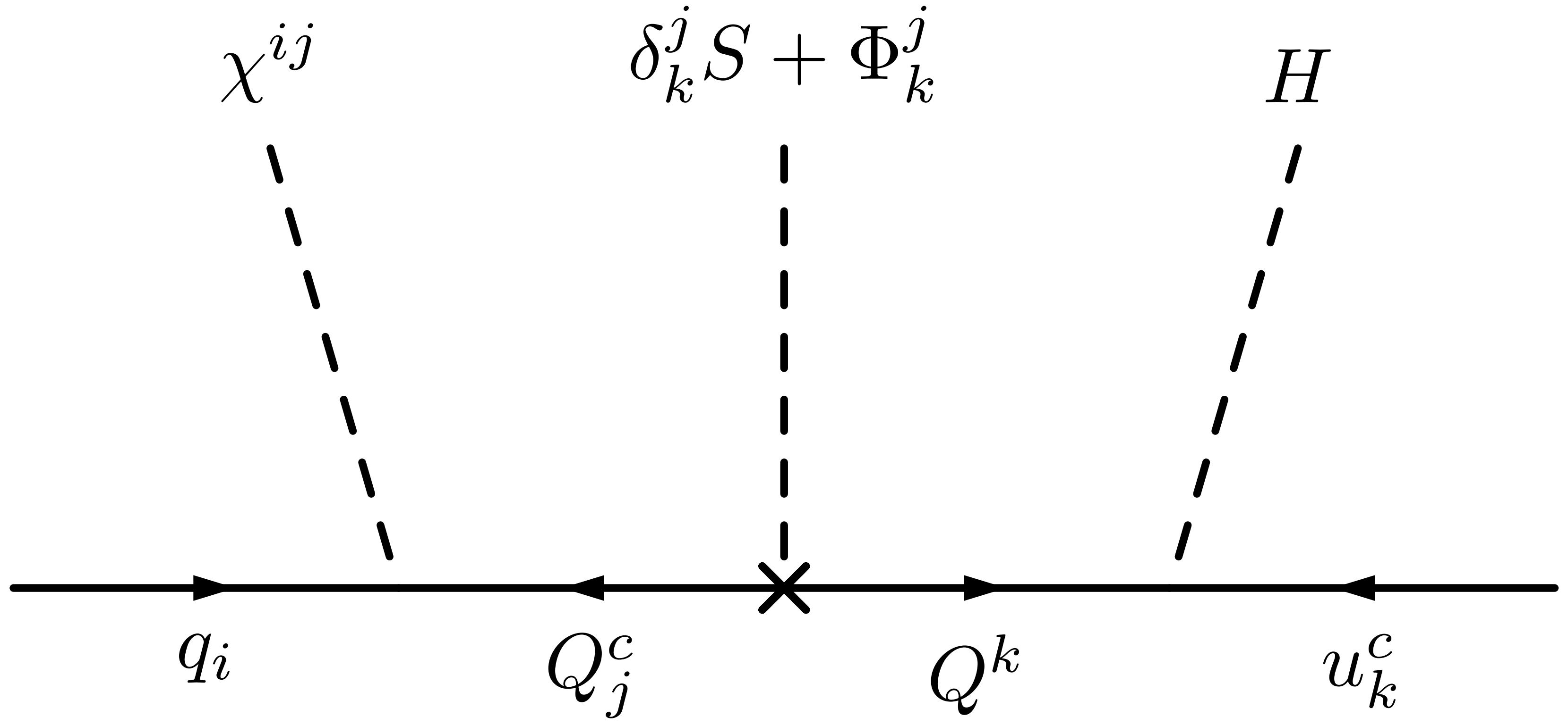


- Therefore the Yukawa constant matrices of quarks will have the forms:

$$Y_d = \begin{pmatrix} 0 & A_d e^{i\alpha'_d} & 0 \\ A_d e^{i\alpha_d} & 0 & x_d B_d e^{i\beta'_d} \\ 0 & x_d^{-1} B_d e^{i\beta_d} & C_d \end{pmatrix}, \qquad Y_u = \begin{pmatrix} 0 & A_u e^{i\alpha'_u} & 0 \\ A_u e^{i\alpha_u} & 0 & x_u B_u e^{i\beta'_u} \\ 0 & x_u^{-1} B_u e^{i\beta_u} & C_u \end{pmatrix}$$

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$$Y_d = \begin{pmatrix} 0 & A_d e^{i\alpha'_d} & 0 \\ A_d e^{i\alpha_d} & 0 & x_d B_d e^{i\beta'_d} \\ 0 & \frac{B_d}{x_d} e^{i\beta_d} & C_d \end{pmatrix}, \qquad Y_u = \begin{pmatrix} 0 & A_u e^{i\alpha'_u} & 0 \\ A_u e^{i\alpha_u} & 0 & x_u B_u e^{i\beta'_u} \\ 0 & \frac{B_u}{x_u} e^{i\beta_u} & C_u \end{pmatrix}$$

- Yukawa matrices can be made real after phase transformation

$$\tilde{Y}_d = F_d'^{\dagger} Y_d F_d = \begin{pmatrix} 0 & -A_d & 0 \\ -A_d & 0 & B_d\, x_d \\ 0 & B_d/x_d & C_d \end{pmatrix}$$

- Then they can be diagonalized by a bi-orthogonal transformation

$$O_d'^T \tilde{Y}_d O_d = \mathrm{diag}(y_d, -y_s, y_b)\,, \qquad O_u'^T \tilde{Y}_u O_u = \mathrm{diag}(y_u, -y_c, y_t)$$

- Then the **CKM matrix** will be:

$$V_{\rm CKM} = O_u^T\, F_u^*\, F_d\, O_d = O_u^T \begin{pmatrix} e^{i(\tilde{\beta}+\tilde{\delta})} & 0 & 0 \\ 0 & e^{i\tilde{\beta}} & 0 \\ 0 & 0 & 1 \end{pmatrix} O_d$$

- 10 **parameters** $A_{u/d}, B_{u/d}, C_{u/d}, x_d, x_u, \tilde{\beta}, \tilde{\delta}$, which need to satisfy the **6 constraints** from the values of **quark masses** and the **4 constraints** from the independent quantities of the **CKM matrix**.
- However, we can fix $\mathbf{x_u = 1}$ so that the up-quark Yukawa matrix assumes the original symmetric Fritzsch texture.

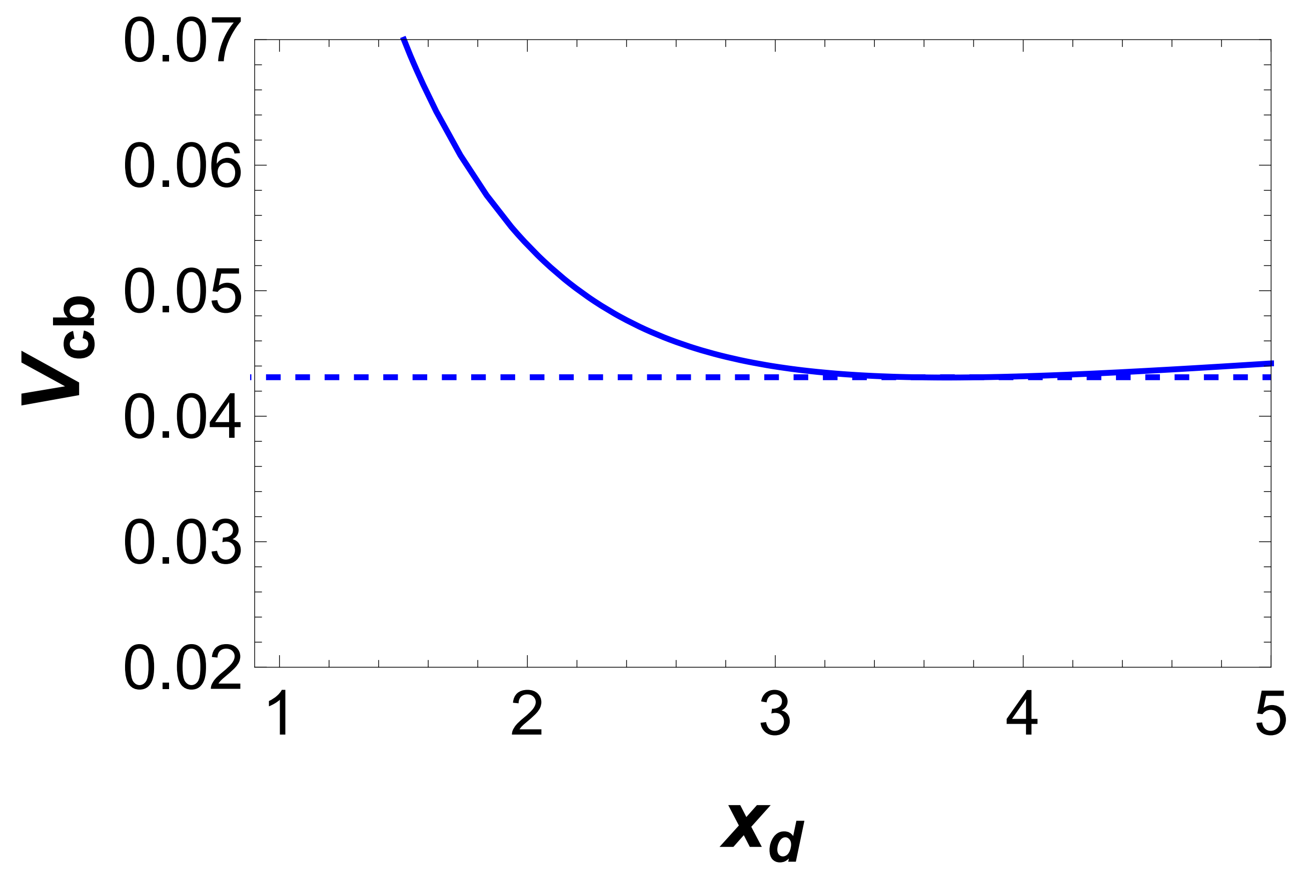
# Fritzsch texture in $\mathbf{SU(5)\times SU(3)_H}$

- For instance, up to corrections $O(\epsilon_d^2)$, $\epsilon_d \sim y_d/y_s$, we have:

$$V_{us} = \left(s^d_{12} - s^u_{12}\, e^{-i\tilde{\delta}}\right) e^{-i\gamma_1}$$
$$V_{cb} = \left(s^d_{23} - s^u_{23}\, e^{-i\tilde{\beta}}\right) e^{-i\gamma_2}$$
$$V_{ub} = (s^d_{13} - s^u_{12}s^d_{23}e^{-i\tilde{\delta}} + s^u_{12}s^u_{23}e^{-i\tilde{\alpha}}\,)\, e^{-i(\gamma_1+\gamma_2)}$$
$$\arctan(\eta/\rho) \approx \delta_{\rm SM} = -\arg[V_{ub}]$$

$$s^d_{23} \approx \frac{1}{x_d}\sqrt{\frac{y_s}{y_b}}\,, \qquad s^d_{13} \approx -x_d\frac{y_s}{y_b}\sqrt{\frac{y_d}{y_b}}\,, \qquad s^d_{12} \approx \sqrt{\frac{c'^d_{23}}{c^d_{23}}}\sqrt{\frac{y_d}{y_s}}\,,$$
$$s^u_{23} \approx \frac{1}{x_u}\sqrt{\frac{y_c}{y_t}}\,, \qquad s^u_{13} \approx -x_u\frac{y_c}{y_t}\sqrt{\frac{y_u}{y_t}}\,, \qquad s^u_{12} \approx \sqrt{\frac{y_u}{y_c}}$$

- one solution can be found for $x_d = 3.5$, $x_u = 1.0$, with $\tilde{\delta} \approx -1.86$, $\tilde{\beta} \approx -0.85$.



# Fritzsch texture in $\mathbf{SU(5)} \times \mathbf{SU(3)_H}$

- For the Yukawa matrices to be a viable texture, we must obtain **moduli and phases of CKM matrix** and of the quark masses, more precisely of the **ratios of Yukawa couplings** $y_d/y_s$, $y_s/y_b$, $y_d/y_b$, $y_u/y_c$, $y_c/y_t$, $y_u/y_t$ (taking into account that mass ratios and mixig elements involving the 3rd generation evolve according to renormalization group equation)

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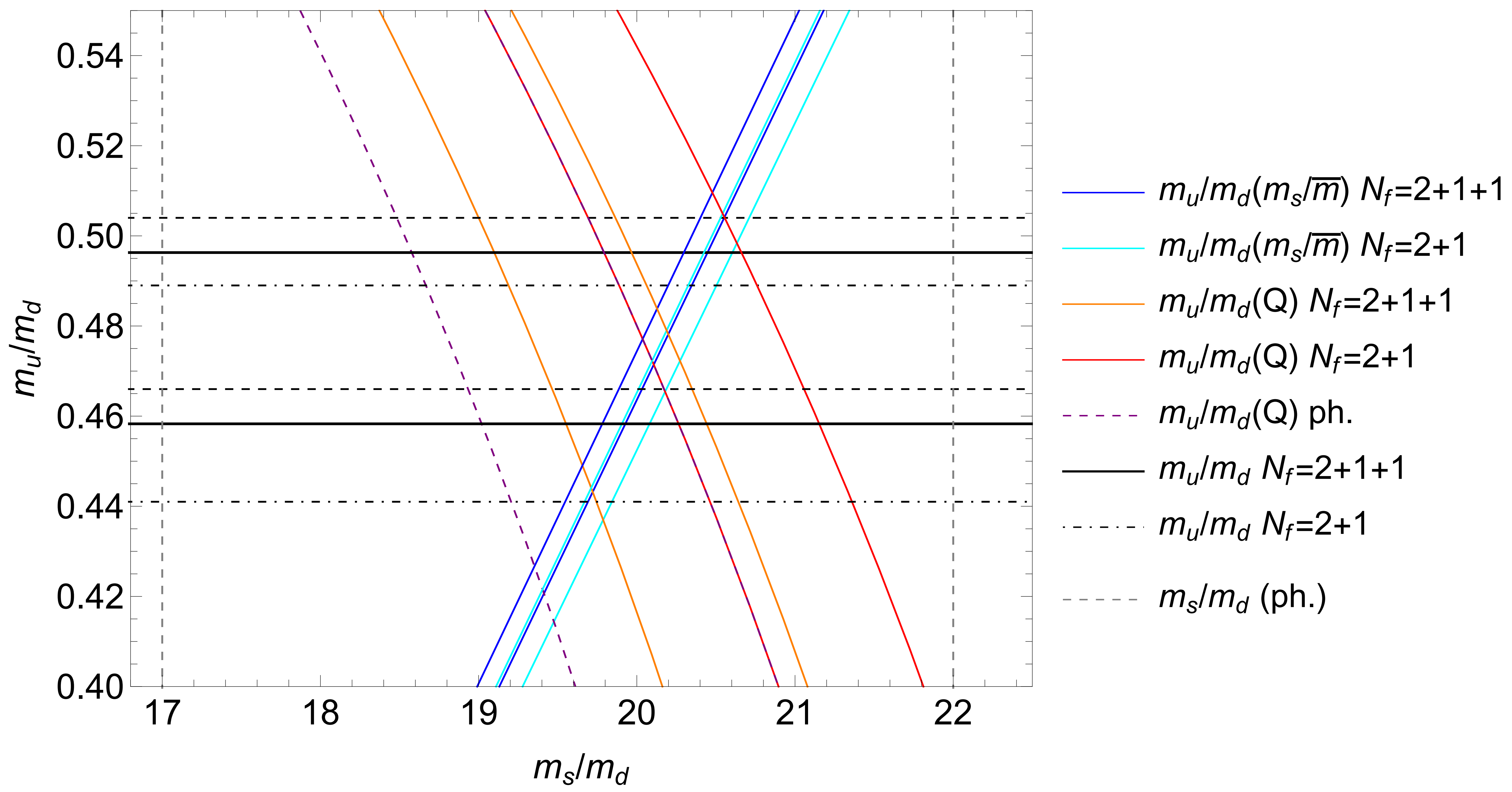

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- The matrix elements $A_d, B_d, C_d$ and $A_u, B_u, C_u$ of the (real) matrices can be expressed as functions of the mass ratios ($a = c'_{23}/c_{23}$):

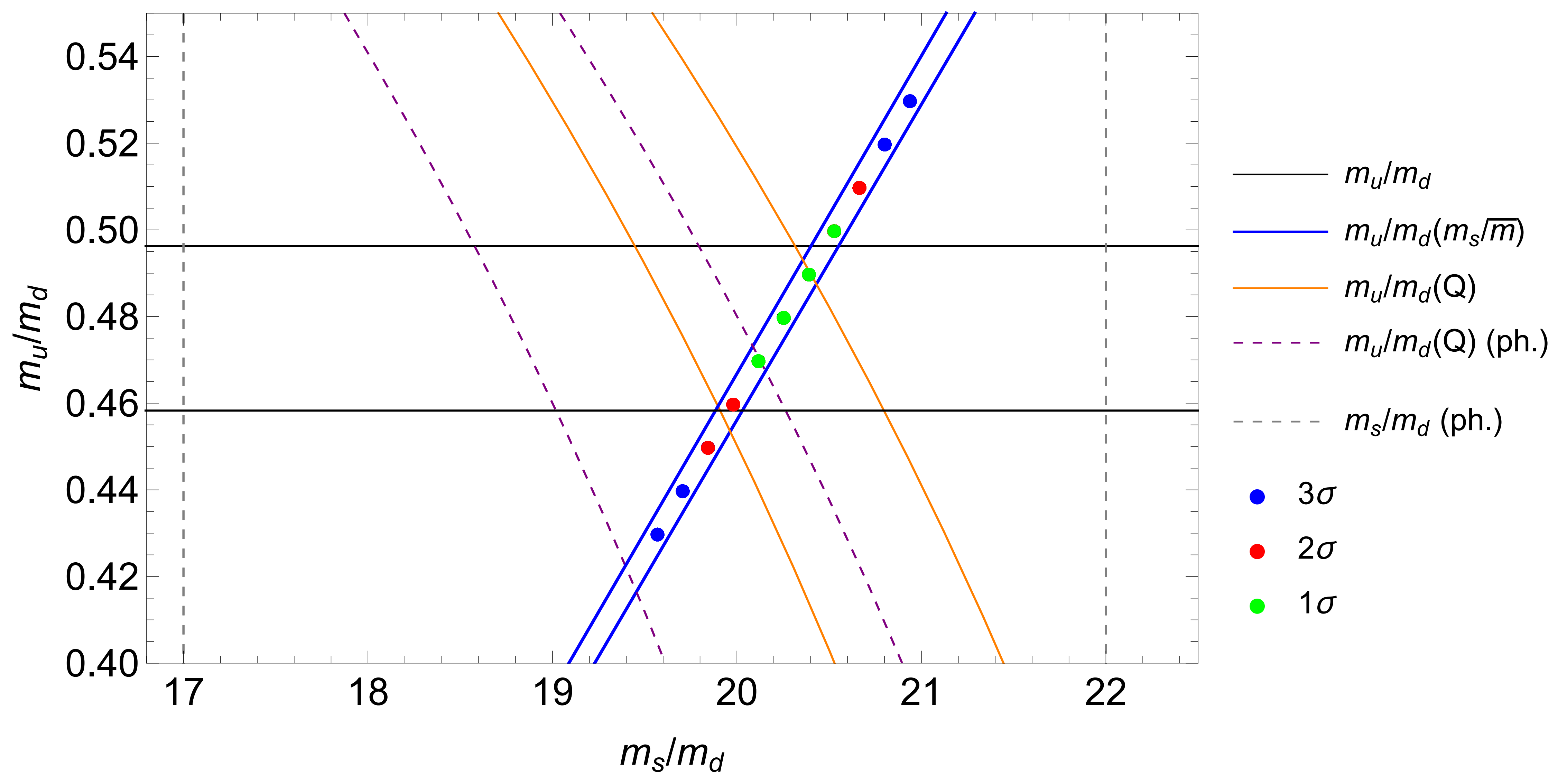
$$\frac{A_d}{y_b} = \sqrt{\frac{y_d\, y_s/y_b^2}{c_{23d}c'_{23d}}}\,, \qquad \frac{B_d}{y_b} = \sqrt{\frac{y_s}{y_b}\left(1 - \frac{a_d + a_d^{-1}}{2}\frac{y_d}{y_s} + \frac{1}{2}\frac{y_d^2}{y_s^2}\right)}\,, \qquad \frac{C_d}{y_b} = \sqrt{1 - (x_d^{-2} + x_d^2)\frac{B_d^2}{y_b^2} + \frac{y_s^2}{y_b^2} - A_d^2(c_{23d}^2 + c'^2_{23d} + 1)}$$

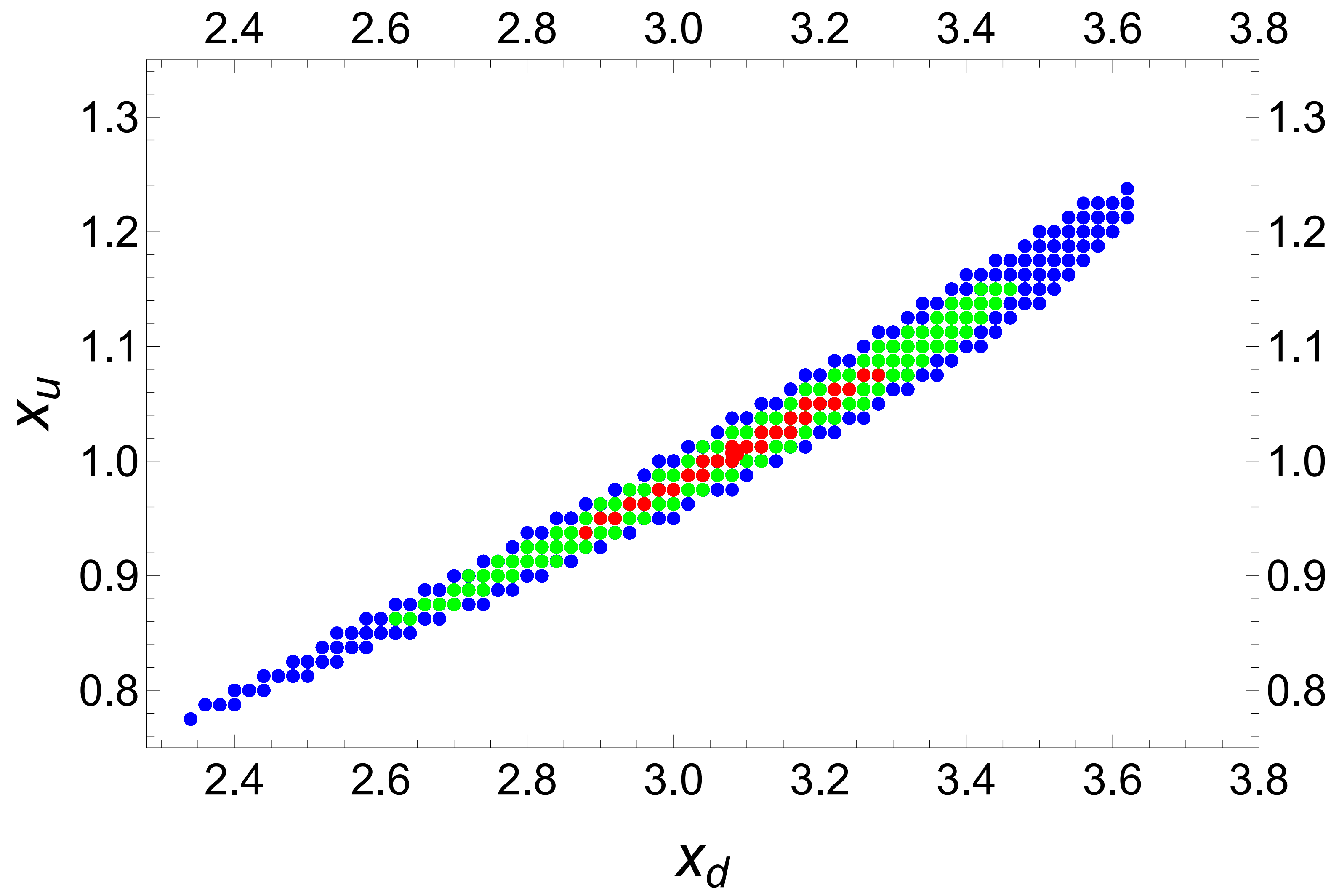
$$\frac{A_u}{y_t} = \sqrt{\frac{y_u\, y_c/y_t^2}{c_{23u}c'_{23u}}}\,, \qquad \frac{B_u}{y_t} = \sqrt{\left(y_c - \frac{a_u + a_u^{-1}}{2}y_u\right)/y_t}\,, \qquad \frac{C_u}{y_t} = \sqrt{1 - (x_u^{-2} + x_u^2)(y_c - y_u)/y_t + y_c^2/y_t^2}$$

$$V_{\rm CKM}(m_u/m_d, x_d, x_u, \tilde\beta, \tilde\delta) = O_u^T\, F\, O_d \qquad\qquad F = \begin{pmatrix} e^{i(\tilde\beta+\tilde\delta)} & 0 & 0 \\ 0 & e^{i\tilde\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

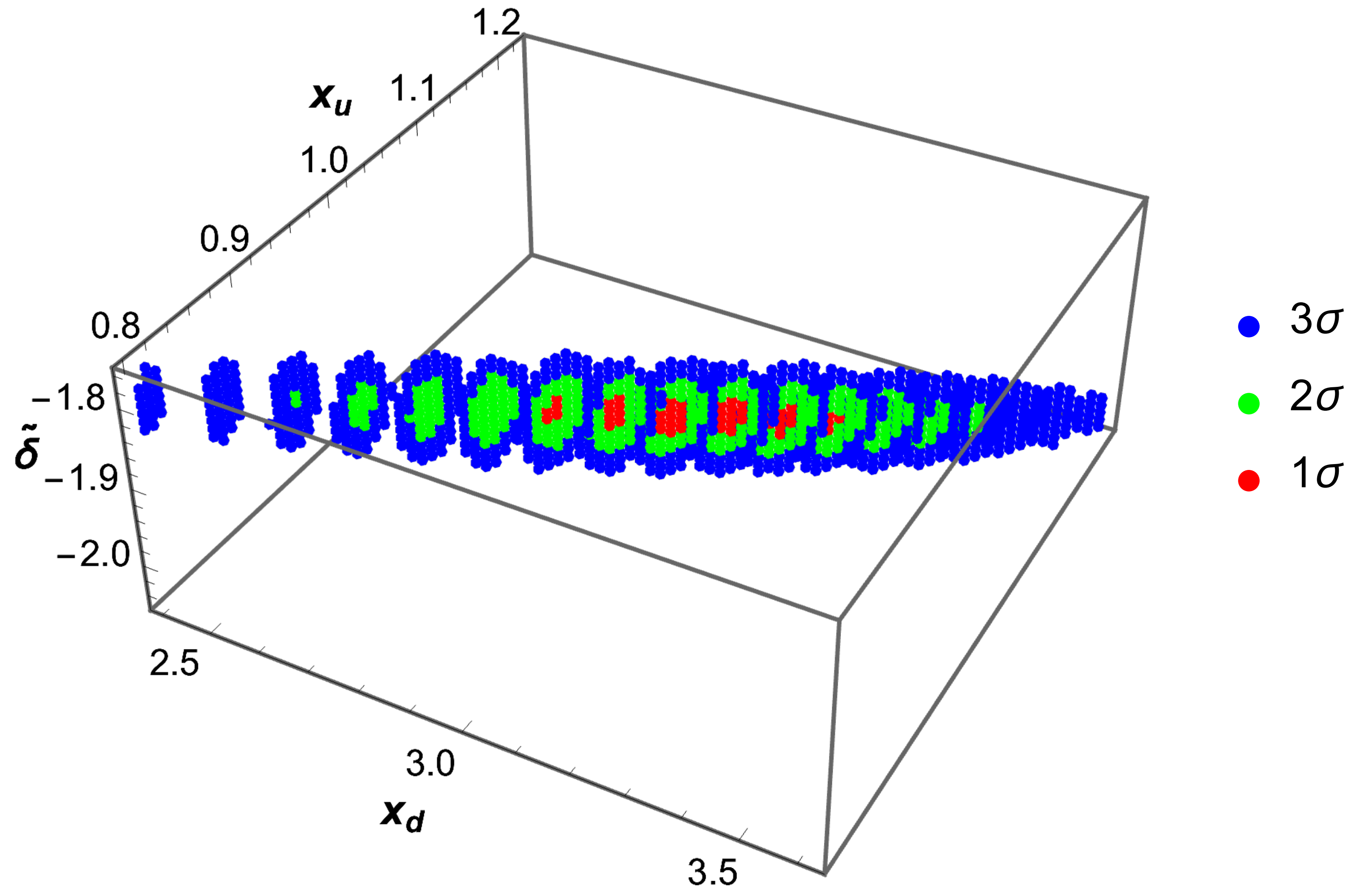
- Fit all independent measures of $V_{\rm CKM}$ moduli and phases with five parameters.

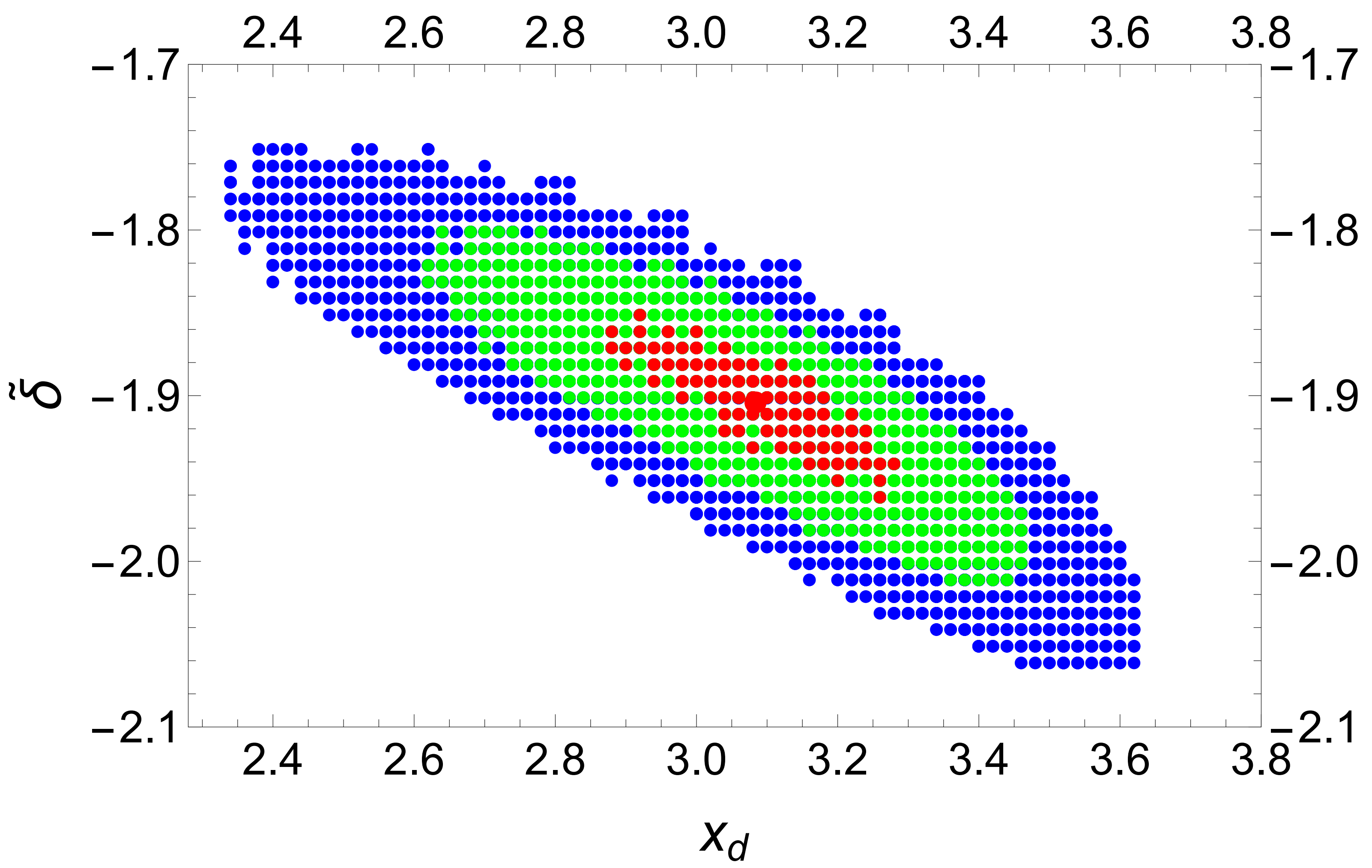
# Fritzsch-like textures from flavor symmetry: precision tests




$$
\begin{aligned}
m_u/m_d &= 0.48 \pm 0.03\,,\\
x_d &= 3.04 \pm 0.42\,,\\
x_u &= 1.00 \pm 0.14\,,\\
\tilde{\beta} &= -0.80 \pm 0.15\,,\\
\tilde{\delta} &= -1.91 \pm 0.11
\end{aligned}
$$



# Fritzsch-like textures from flavor symmetry: precision tests

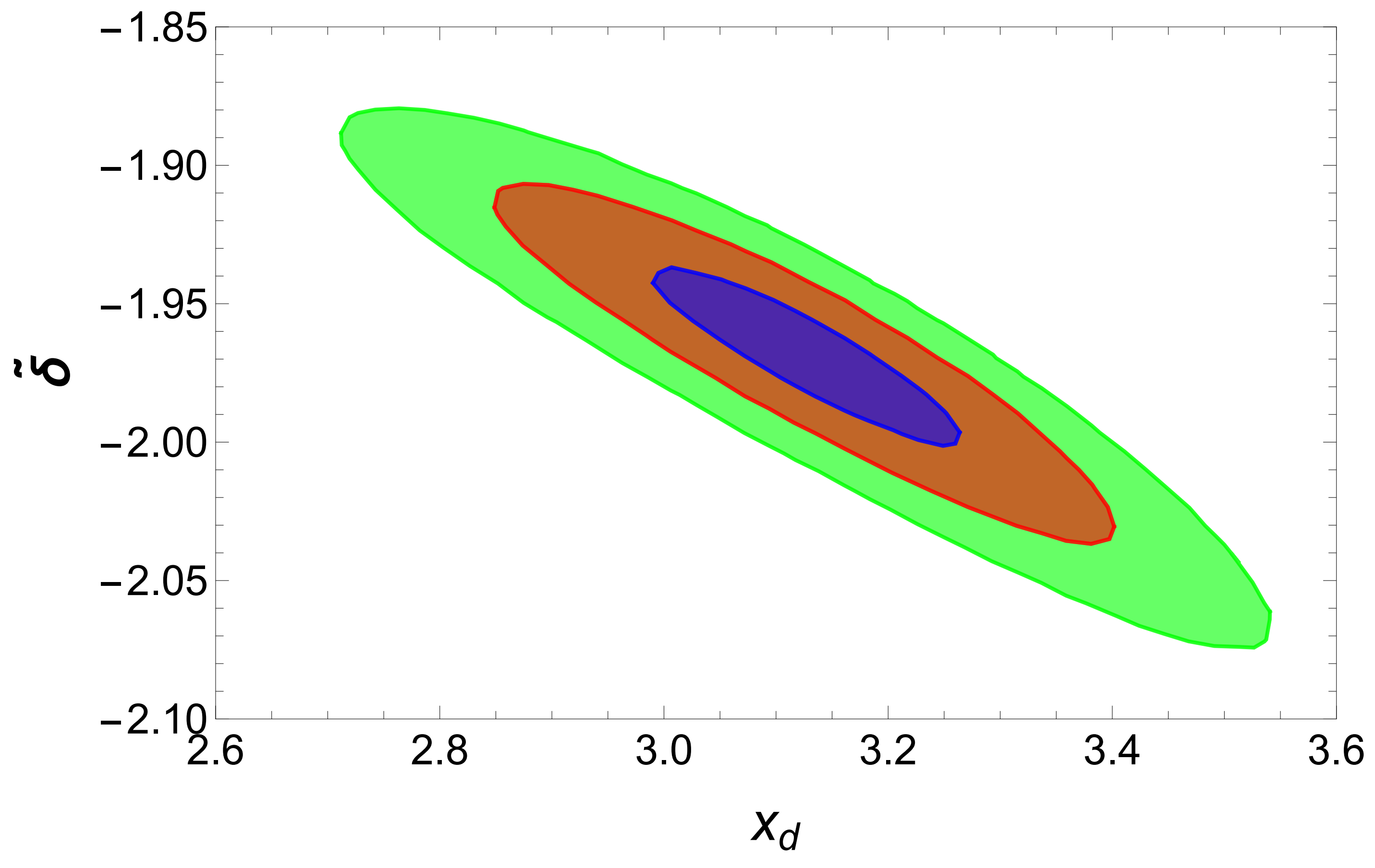
| Quantity | SM determination | Obt. value | Pull ($\sigma_i$) |
|---|---|---|---|
| $\lvert V_{us}\rvert$ | 0.2245(8) | 0.2242 | −0.4 |
| $\lvert V_{cd}\rvert$ | 0.221(4) | 0.224 | 0.8 |
| $\lvert V_{cb}\rvert$ | 0.0410(14) | 0.0397 | −0.9 |
| $\lvert V_{ub}\rvert$ | 0.00382(24) | 0.00355 | −1.1 |
| $\lvert V_{td}\rvert$ | 0.0080(3) | 0.0083 | 1.1 |
| $\lvert V_{ts}\rvert$ | 0.0388(11) | 0.0390 | 0.2 |
| $\lvert V_{td}/V_{ts}\rvert$ | 0.205(6) | 0.214 | 1.5 |
| $\alpha$ | $(84.9^{+5.1}_{-4.5})^\circ$ | $87.8^\circ$ | 0.6 |
| $\sin 2\beta$ | 0.699(17) | 0.694 | −0.3 |
| $\gamma$ | $(72.1^{+4.1}_{-4.5})^\circ$ | $69.3^\circ$ | −0.6 |
| $\lvert V_{ub}/V_{cb}\rvert$ | 0.079(6) | 0.089 | 1.7 |
| $\lvert V_{ts}/V_{cb}\rvert$ | 0.98(4) | 0.98 | 0.0 |

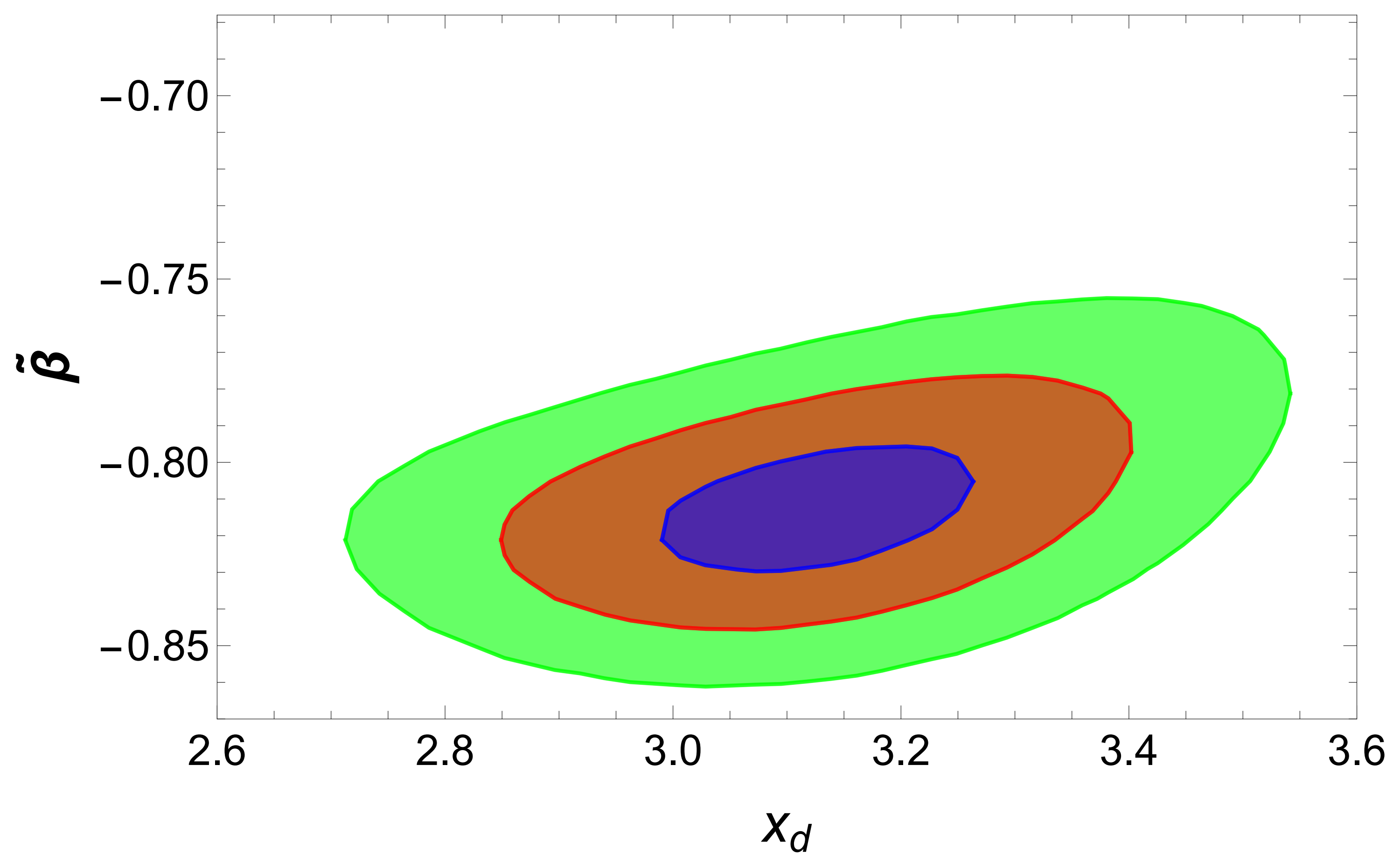
Values computed in $m_u/m_d = 0.49$, $m_s/m_d = 20.39$, $x_d = 3.06$, $x_u = 1.0$, $\tilde{\beta} = -0.79$, $\tilde{\delta} = -1.91$. Fritzsch-like form at $10^6$ GeV.

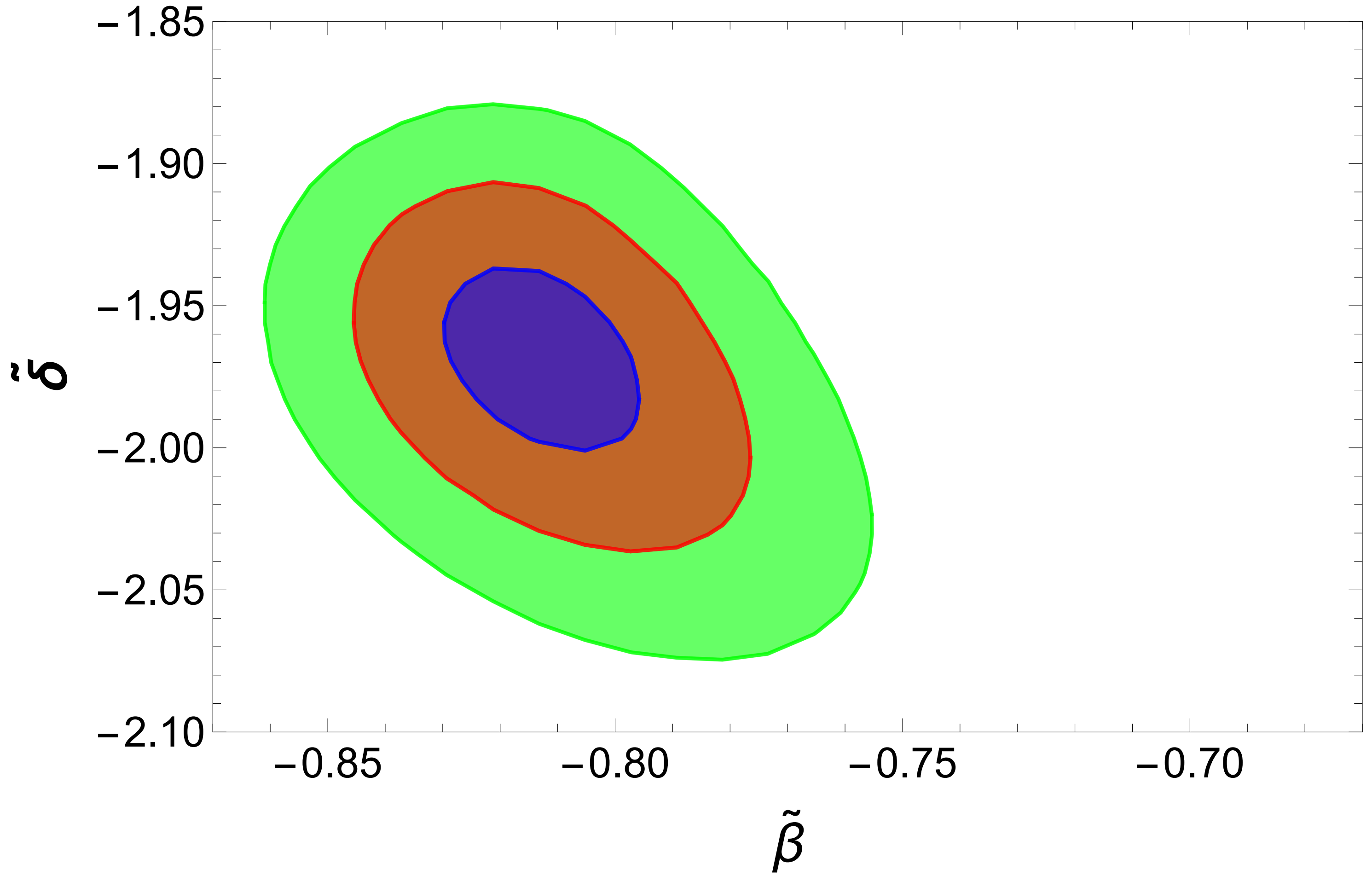
Using the given single determinations the ratios would be $|V_{ub}/V_{cb}| = 0.093(7)$, $|V_{ts}/V_{cb}| = 0.95(4)$.

# Fritzsch-like textures from flavor symmetry: precision tests

- Fit the **4** independent parameters of the standard parameterization of $V_{\mathrm{CKM}}$ with **3** parameters: asymmetry $x_d$ and two phases $\tilde{\delta}$, $\tilde{\beta}$, using $m_u/m_d = 0.477$ (central value), $x_u = 1$.



# Conclusions

- The Standard Model does not constrain the structure of the Yukawa matrices and thus the origin of fermion mass hierarchies and mixing pattern remains unexplained.
- Relations between fermion masses and mixing angles can suggest Yukawa matrices with some zero elements, such as the texture proposed by Fritszch, which can emerge naturally in the context of grand unified theories.
- The original (symmetric) Fritzsch texture is excluded since predicts a too large value of $V_{cb}$ (and a too small value of $V_{ub}$).
- **Fritzsch-like texture** is still viable if an **asymmetry** is introduced only between 23 and 32 entries in down-type quarks mass matrix.
- This modification mantains the decoupling feature.
- The asymmetry can naturally emerge in the context of **horizontal gauge $\mathbf{SU(3)_H}$ interfamily symmetry**.
- Quark mass hierarchy and mixing pattern can emerge from the spontaneous breaking pattern of the symmetry.

# Backup

# Fritzsch texture in $\mathbf{SU(5)} \times \mathbf{SU(3)_H}$

- Up to corrections of order $O(\epsilon^2)$, $\epsilon \sim Y_1/Y_2 \sim Y_2/Y_3$, with $Y_{1,2,3} = y_{d,s,b}$ or $Y_{1,2,3} = y_{u,c,t}$, the rotation angles are expressed in terms of mass ratios as:

$$\tan 2\theta_{23} = \frac{2}{x}\sqrt{\frac{Y_2 - Y_1}{Y_3}}\,\frac{\sqrt{1-(x^{-2}+x^2)\frac{Y_2-Y_1}{Y_3}}}{1-\frac{2}{x^2}\frac{Y_2-Y_1}{Y_3}}$$

$$\tan 2\theta_{12} = 2\sqrt{\frac{Y_1\, c'_{23}}{Y_2\, c_{23}}}\,\frac{\sqrt{1-(\frac{c_{23}}{c'_{23}}+\frac{c'_{23}}{c_{23}})\frac{Y_1}{Y_2}}}{1-2\frac{c'_{23}}{c_{23}}\frac{Y_1}{Y_2}}$$

$$\tan 2\theta_{13} = -\frac{2As'_{23}}{Y_3} = -2\frac{s'_{23}}{\sqrt{c'_{23}c_{23}}}\frac{\sqrt{Y_1Y_2}}{Y_3}$$

$$c_{23d} = \cos\left[\frac{1}{2}\arctan\left[\frac{2C_dB_d/x_d}{C_d^2+B_d^2(x_d^2-x_d^{-2})-A_d^2}\right]\right],$$

$$c'_{23d} = \cos\left[\frac{1}{2}\arctan\left[\frac{2C_dx_dB_d}{C_d^2+B_d^2(x_d^{-2}-x_d^2)-A_d^2}\right]\right]$$

$$V_{Rd}^\dagger Y_d V_{Ld} = \mathrm{diag}(y_d, y_s, y_b)\,, \qquad V_{Ru}^\dagger Y_u V_{Lu} = \mathrm{diag}(y_u, y_c, y_t)$$

$$V_{Ld} = F_d O_d\,, \qquad V_{Rd} = F_d' O_d'$$

$$F_d = \begin{pmatrix} e^{i(\pi-\alpha_d+\beta_d')} & 0 & 0 \\ 0 & e^{-i\beta_d} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad F_d' = \begin{pmatrix} e^{-i(\pi-\alpha_d'+\beta_d)} & 0 & 0 \\ 0 & e^{i\beta_d'} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

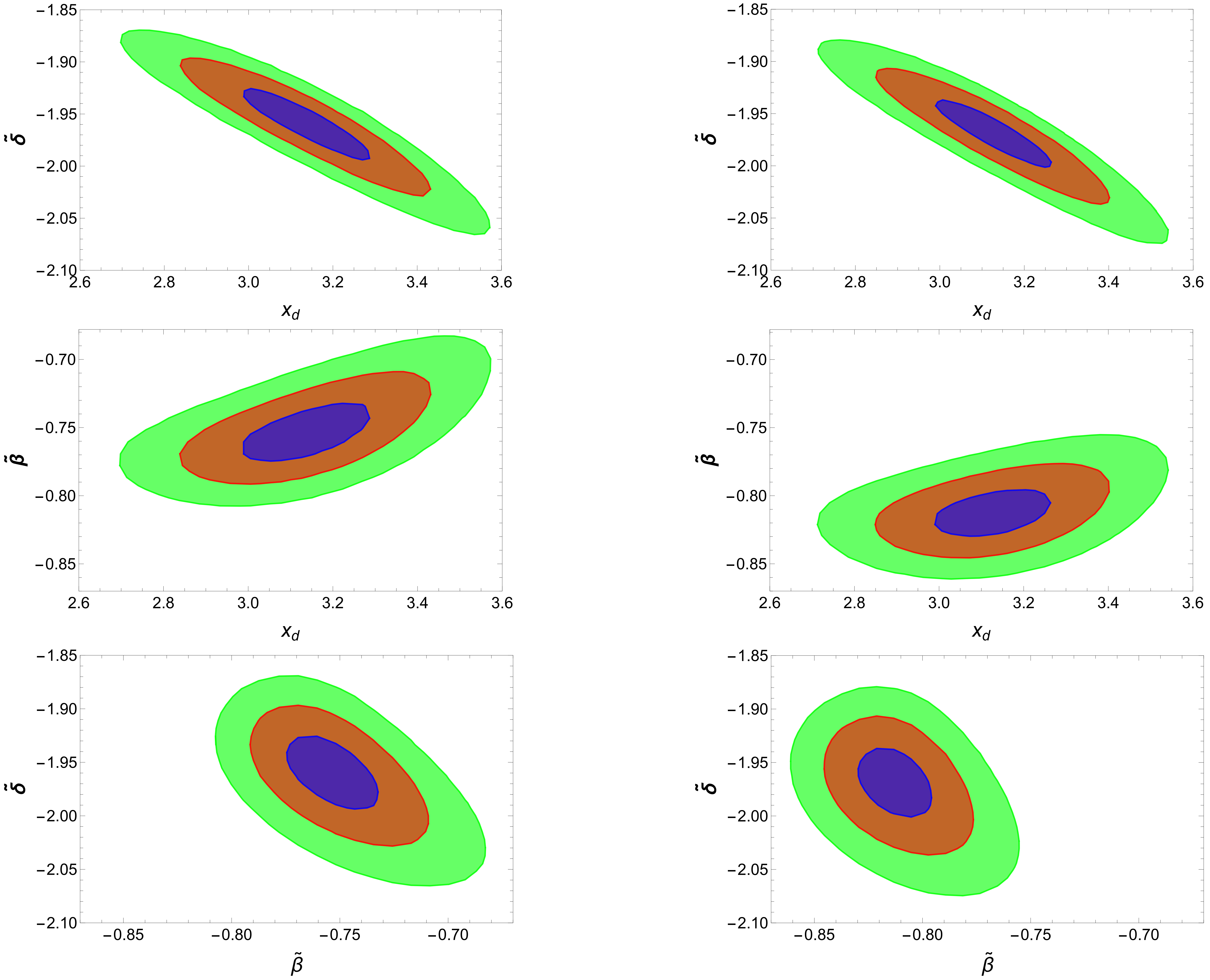
$$\tilde{Y}_d = F_d'^{\,\dagger} Y_d F_d = \begin{pmatrix} 0 & -A_d & 0 \\ -A_d & 0 & B_d\, x_d \\ 0 & B_d/x_d & C_d \end{pmatrix}$$

$$O_d'^T \tilde{Y}_d O_d = \mathrm{diag}(y_d, -y_s, y_b)\,, \qquad O_u'^T \tilde{Y}_u O_u = \mathrm{diag}(y_u, -y_c, y_t)$$

$$V_{\mathrm{CKM}} = F_1\, O_u^T\, F_u^*\, F_d\, O_d\, F_2 \;=\; \mathrm{diag}\big(e^{-i(\tilde\beta+\tilde\delta+\gamma_1)}, e^{i\tilde\beta}, e^{i\gamma_2}\big)\, O_u^T\, \mathrm{diag}\big(e^{i(\tilde\beta+\tilde\delta)}, e^{i\tilde\beta}, 1\big)\, O_d\, \mathrm{diag}\big(e^{i\gamma_1}, 1, e^{-i\gamma_2}\big)$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- Fit the **4** independent parameters of the standard parameterization of $V_{\text{CKM}}$ with **3** parameters: asymmetry $x_d$ and two phases $\tilde{\delta}$, $\tilde{\beta}$, using $m_u/m_d = 0.477$ (central value), $x_u = 1$.




$10^3$ GeV $10^6$ GeV $10^{15}$ GeV

$$\frac{A_d}{y_b} = \sqrt{\frac{y_d\, y_s/y_b^2}{c_{23d}c'_{23d}}}\,, \qquad \frac{B_d}{y_b} = \sqrt{\frac{y_s}{y_b}\left(1 - \frac{a_d + a_d^{-1}}{2}\frac{y_d}{y_s} + \frac{1}{2}\frac{y_d^2}{y_s^2}\right)}$$

$$\frac{C_d}{y_b} = \sqrt{1 - (x_d^{-2} + x_d^2)\frac{B_d^2}{y_b^2} + \frac{y_s^2}{y_b^2} - A_d^2(c_{23d}^2 + c'^2_{23d} + 1)}$$

$$\frac{A_u}{y_t} = \sqrt{\frac{y_u\, y_c/y_t^2}{c_{23u}c'_{23u}}}\,, \qquad \frac{B_u}{y_t} = \sqrt{\left(y_c - \frac{a_u + a_u^{-1}}{2}y_u\right)/y_t}$$

$$\frac{C_u}{y_t} = \sqrt{1 - (x_u^{-2} + x_u^2)(y_c - y_u)/y_t + y_c^2/y_t^2}$$

- The values at low energies of the mixing elements and of mass ratios are taken from Particle Data Group.
- Yukawa matrices evolve according to the renormalization group equations (RGE).
- Since the light generations evolve in the same way with gauge couplings and trace of Yukawa matrices, the ratios $y_d/y_s$, $y_u/y_c$ and the mixing angles between the first two generations remain invariant.
- The third generation receives additional Yukawa contributions. The ratios $y_c/y_t$, $y_s/y_b$ and the mixings involving the third generation evolve according to RGEs.

# FCNC in SU(5)

- In the context of GUT theories new gauge interactions require the mass scale of gauge bosons to be much higher than few TeV, also in $SU(5)$ scenario.
- Tree level contribution to K mesons decays $K^- \to \pi^- \mu^- e^+$, $K_L^0 \to e^\pm \mu^\mp$.
- Mass scales of gauge bosons are constrained to be higher than 100 TeV and 300 TeV respectively.

# Fritzsch texture in $\mathbf{SU(5)} \times \mathbf{SU(3)_H}$

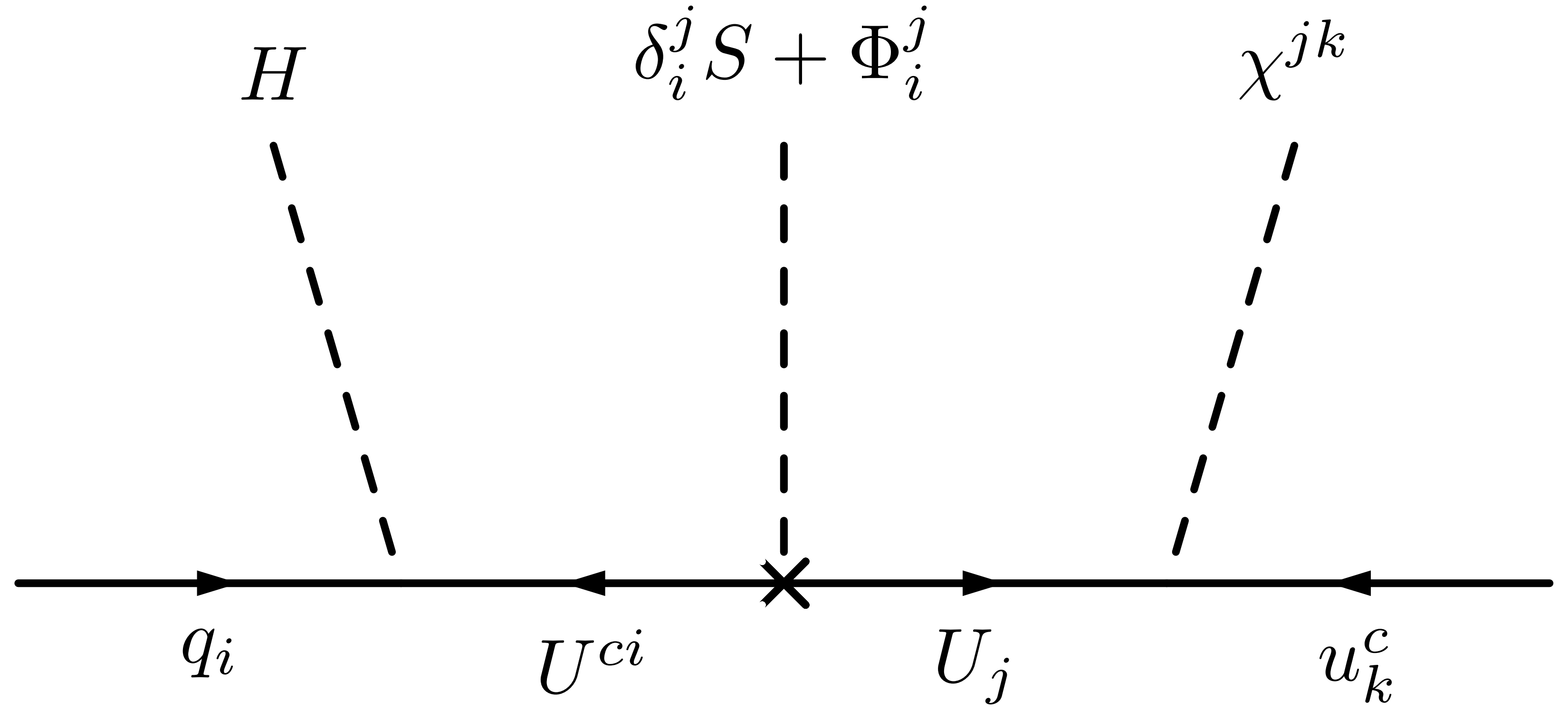
- Effective operators can be induced via renormalizable interactions after integrating out **heavy vector-like fermions**.

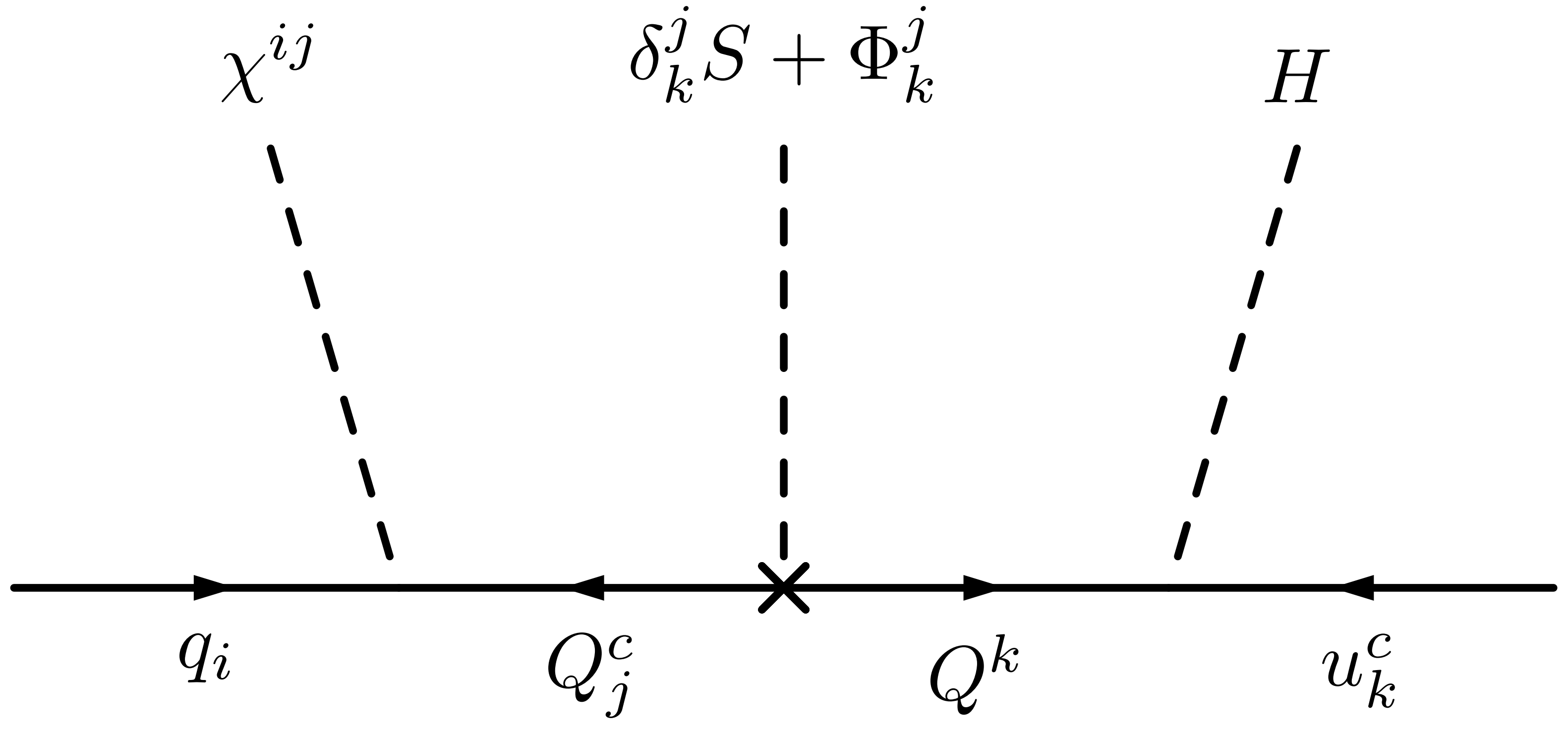
$$Q^i\,, U^{ci} \sim \bar{3}\,, \qquad Q^c_i\,, U_i \sim 3 \qquad D_i \sim 3\,, D^{ci} \sim \bar{3}$$

- They are allowed to have invariant mass terms. Mass terms transform as $\bar{3} \times 3 = 1 + 8$ and can emerge from Yukawa couplings with scalar singlet or octet.

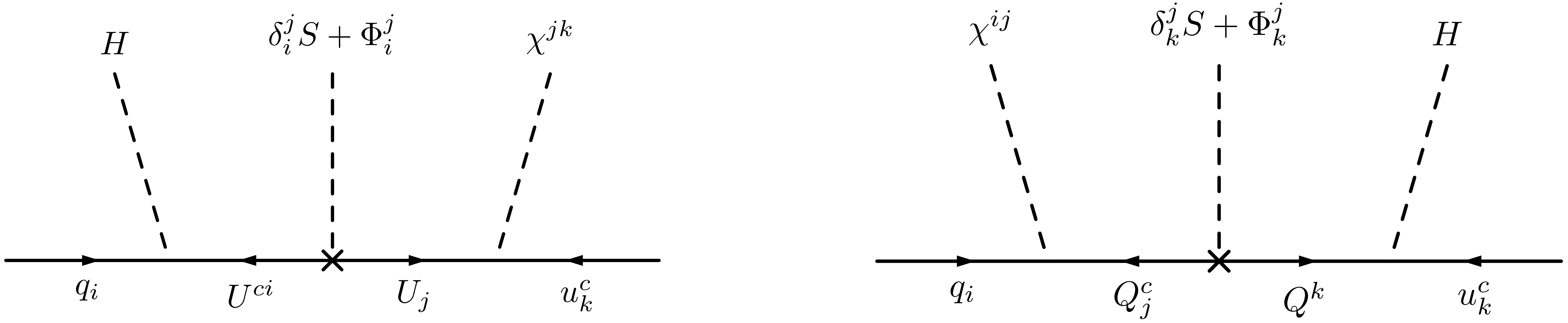
- Yukawa couplings with $U$ and $D$ vector-like quarks are:

$$g_U^{(n)} \chi_n^{ij} u_i^c U_j + g_D^{(n)} \chi_n^{ij} d_i^c D_j + h_U U^{ci} q_i \phi + h_d D^{ci} q_i \tilde{\phi} + g_U^{(n)} \chi_n^{ij} q_i Q_j^c + h_U u_i^c Q^i \phi + h_D d_i^c Q^i \tilde{\phi}$$



# Fritzsch texture in $\mathbf{SU(5)} \times \mathbf{SU(3)_H}$




- Mass matrices of fermions look like:

$$\begin{pmatrix} u^c & Q^c & U^c \end{pmatrix} \begin{pmatrix} 0 & h_U H & \chi \\ \chi^T & M_Q & 0 \\ h_U H & 0 & M_U \end{pmatrix} \begin{pmatrix} q \\ Q \\ U \end{pmatrix}, \qquad \begin{pmatrix} d^c & D^c & Q^c \end{pmatrix} \begin{pmatrix} 0 & \chi & h_D \bar{H} \\ h_d \bar{H} & M_D & 0 \\ \chi^T & 0 & M_Q \end{pmatrix} \begin{pmatrix} q \\ D \\ Q \end{pmatrix}$$

- Yukawa couplings are induced in **seesaw approximation**:

$$Y_u = M_U^{-1} \chi_U h_U + \chi_U^T M_Q^{-1} h_U \,, \qquad Y_d = M_D^{-1} \chi_D h_d + \chi_D^T M_Q^{-1} h_D \,; \qquad \chi_{U,D}^{ij} = g_{U,D}^{(n)} \langle \chi_n^{ij} \rangle$$

- If heavy fermions are degenerate in masses between families, Yukawa matrices $Y_{u,d}$ have **Fritzsch texture**.

However symmetric Fritzsch texture is excluded.