Reciprocal of CPT theorem: a short communication

Masud Chaichian

Department of Physics
University of Helsinki

Workshop on Recent Advances in Fundamental Physics,
Tbilisi, 27 September 2022

CPT Theorem in Relativistic QFT

CPT Theorem

Any local relativistic quantum field theory is CPT-invariant, i.e.

$$\mathcal{L}_{free}^{CPT}(x) = \mathcal{L}_{free}(-x) \ , \quad \mathcal{L}_{int}^{CPT}(x) = \mathcal{L}_{int}(-x) \ ,$$
$$\mathcal{H}_{free}^{CPT}(x) = \mathcal{H}_{free}(-x), \quad \mathcal{H}_{int}^{CPT}(x) = \mathcal{H}_{int}(-x).$$

Lüders (1954), Pauli (1955)

CPT theorem is one of the few general results which can be proven in axiomatic quantum field theory, without reference to any concrete Lagrangian or Hamiltonian model.

Jost (1957)

Consequences of the CPT theorem in Quantum Field Theory

For (elementary or composite) particle and antiparticle,

- MASSES are equal: $m = \bar{m}$;
- LIFETIMES (decay widths) are equal: $\tau = \bar{\tau}$;
- MAGNETIC MOMENTS are equal: $\mu = \bar{\mu}$.

CPT theorem is one of the few general results which can be proven in Axiomatic Quantum Field Theory:

Jost (1957)

- Axioms of relativistic invariance, locality and spectrality;
- Wightman functions: vacuum expectation values of products of relativistic fields (representations of the Lorentz group):

$$W_n(x_1, x_2, \ldots, x_n) = \langle 0|\phi(x_1)\phi(x_2)\ldots\phi(x_n)|0\rangle$$

- CPT theorem states that the CPT invariance condition

$$W_n(x_1, x_2, \dots, x_n) = W_n(-x_n, \dots, -x_2, -x_1), \quad \text{for any } x_1, x_2, \dots, x_n$$

is equivalent to the weak local commutativity (WLC) condition

$$W_n(x_1, x_2, \dots, x_n) = W_n(x_n, \dots, x_2, x_1)$$

where the points $x_1, x_2, \dots, x_n$ are spatially separated (Jost point), i.e.

$$\left(\sum_{j=1}^{n-1} \lambda_j (x_j - x_{j+1}) \right)^2 < 0, \quad \text{for all } \lambda_j \geq 0 \text{ with } \sum_{j=1}^{n-1} \lambda_j > 0.$$

What is the fate of CPT invariance if Lorentz invariance is violated?

General proof of the CPT theorem for noncommutative QFT
(in both Lagrangian and Axiomatic QFT)

Chaichian, Nishijima, Tureanu (2002), (2004)

Thus, CPT invariance can be conserved and Lorentz invariance violated.

Lorentz-invariant but CPT-violating models

Consider the Hamiltonian of interaction:

$$\mathcal{H}_{int}(x) = \lambda \int d^4y\ \phi^*(x)\phi(x)\phi^*(x)\theta(x_0 - y_0)\theta((x-y)^2)\phi(y) + h.c.$$

Chaichian, Dolgov, Novikov, Tureanu (2011)

- Nonlocal model.
- The combination $\theta(x_0 - y_0)\theta((x-y)^2)$ ensures the Lorentz invariance, i.e. invariance under the proper orthochronous Lorentz transformations.
- C and P invariance are trivially satisfied.
- T invariance is broken due to the presence of $\theta(x_0 - y_0)$ in the integrand.

A model with splitting of particle-antiparticle masses:

- Action with nonlocal additional quadratic ("mass") term:

$$S = \int d^4x \{\bar{\psi}(x) i\gamma^\mu \partial_\mu \psi(x) - m\bar{\psi}(x)\psi(x)$$
$$- \int d^4y [\theta(x^0 - y^0) - \theta(y^0 - x^0)] \delta((x-y)^2 - l^2)[i\mu\bar{\psi}(x)\psi(y)]\},$$

Chaichian, Fujikawa, Tureanu (2012)

- Quantization formally by path integral formalism.
- The manifestly Lorentz covariant (off-shell) propagator is defined by

$$\int d^4x\, d^4y\, e^{ip(x-y)} \langle T^\star \psi(x)\bar{\psi}(y)\rangle = \frac{i}{\not{p} - m + i\epsilon - i\mu[f_+(p) - f_-(p)]},$$

with form factors

$$f_\pm(p) = \int d^4z_1\, e^{\pm ipz_1} \theta(z_1^0)\delta((z_1)^2 - l^2),$$

- Dirac equation in momentum space becomes:

$$\not{p}U(p) = mU(p) + i\mu[f_+(p) - f_-(p)]U(p), \quad \psi(x) = e^{-ipx}U(p)$$

- Describe the particle and antiparticle propagation with definite masses by pole approximation.
- Propagator has poles only for timelike momentum, therefore we take frame where $\vec{p} = 0$, resulting in the eigenvalue equation:

$$p_0 = \gamma_0 \left[m - 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0\sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}} \right],$$

- Take above $p_0 \to -p_0$ and sandwich by γ_5 (CPT-operation):

$$p_0 = \gamma_0 \left[m + 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0\sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}} \right],$$

- The mass eigenvalues resulting from the original equation are not identical with the ones resulting from the CPT-inverted equation, except for $\mu = 0$!

Conclusion

What is responsible for particle-antiparticle masses NOT to be equal?

- Only the **CPT breaking in the mass term** (which means that the particle has a "form-factor"/ is not point-like) **breaks the equality of masses**.
- The **CPT breaking only in the interaction term** of Lagranigian **does NOT split the masses** of an elementary particle and its antiparticle.
- **However, it breaks the equality of the masses of bound states**, their **lifetimes**, . . . , and their **anti bound-states**.

Discussion

Is there any relation between the violation of CPT and the Spin-Statistics?

Some general arguments give a positive answer:

- Axiomatic QFT proof of **CPT theorem** requires that the Wightman functions should be invariant ONLY under cyclic permutations of fields at Jost spacetime points.
- Axiomatic QFT proof of **Spin-Statistics theorem** requires that the Wightman functions should be invariant for ALL permutations of fields at Jost spacetime points.

Thus, the **requirements for the validity of Spin-Statistics theorem are stronger than for the CPT theorem**.

Implying **CPT violation ——> Spin-Statistics violation**.

Question:

- How, e.g. in the proposed nonlocal (effective?) theory, to derive the (anti-)commutation relations from the Lagrangian?

Instability of the Universe, Lieb and Thirring works . . . , very strong bound, . . . !