Lessons from Gravity about Axion

Gia Dvali

LMU – MPI, Munich

Based on: G.D., 0507215 [hep-th]

G.D, Folkerts, Franca 1312.7273 [hep-th]

G.D., Gomez, Zell, 1811.03079 [hep-th]

G.D, 2012.02133 [hep-th]

See also: Otari Sakhelashvili, 2110.03386

Summary:

- $\mathcal{S}$-matrix formulation of quantum gravity excludes de Sitter vacua.

- This excludes massless 3-forms

- Θ - vacua must be unphysical

- Axion (for each gauge) group is necessity

- Dual formulation

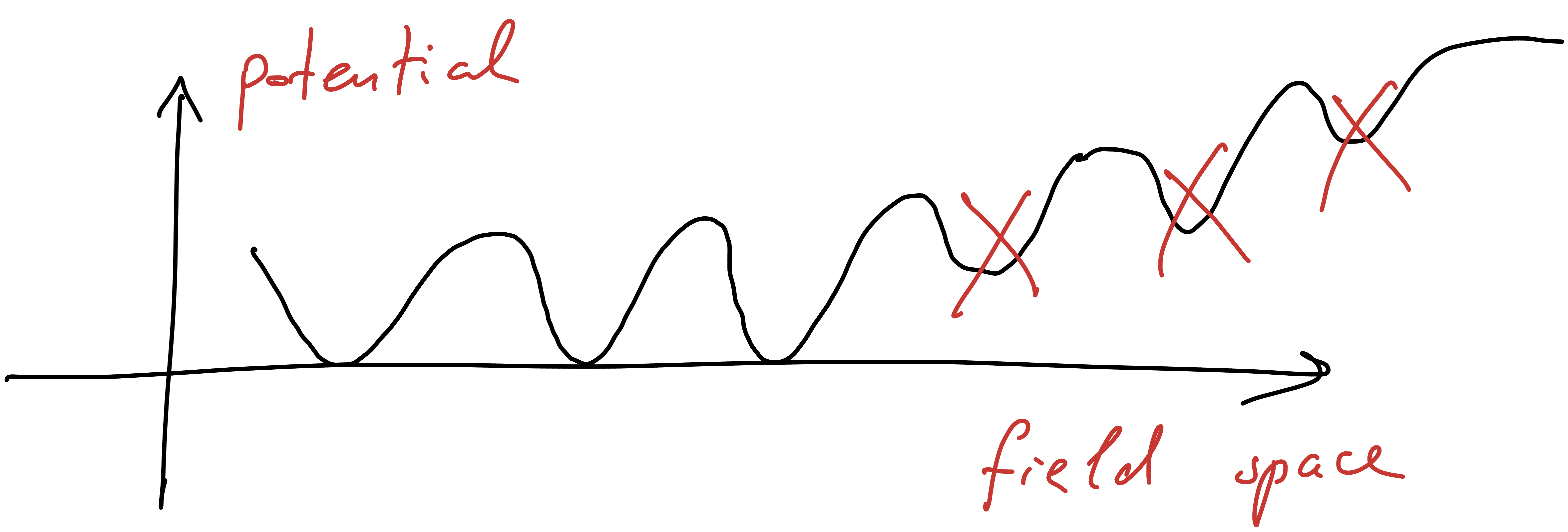
$$a \to a + \text{const} \longleftrightarrow B_{\mu\nu} \to B_{\mu\nu} + \Omega_{\mu\nu}$$

- Hints $f_a \longrightarrow$ scale of gravity?

S-matrix is currently the only existing formulation of quantum gravity (organic to string theory).

For "no-go" scaling arguments see:
G.D., 2012.02133 [hep-th]

Our framework: 3+1 dimensional EFT of S-matrix theory defined on asymptotic S-matrix vacuum of Minkowski



This fact severely constrains the field content.

In particular, massless 3-form fields are excluded.

$$C_{\alpha\beta\gamma} \to C_{\alpha\beta\gamma} + \partial_\alpha \Omega_{\beta\gamma}$$

Lagrangian:

$$\mathcal{L} = E^2 \qquad E \equiv \varepsilon^{\alpha\beta\gamma\delta} \partial_\alpha C_{\beta\gamma\delta}$$

Equation

$$\partial^\mu E = 0$$

Vacua

$$E = E_0 = \text{const}$$

arbitrary constant "electric" field.

The vacuum landscape with energy density

$$\rho_{vac} = E_0^2$$

The same holds for

$$\mathcal{L} = \mathcal{K}(E)$$

$$\partial_\mu \frac{\partial K(E)}{\partial E} = 0$$

vacua

$$E = E_0 = \text{const}$$

Superselection sectors.

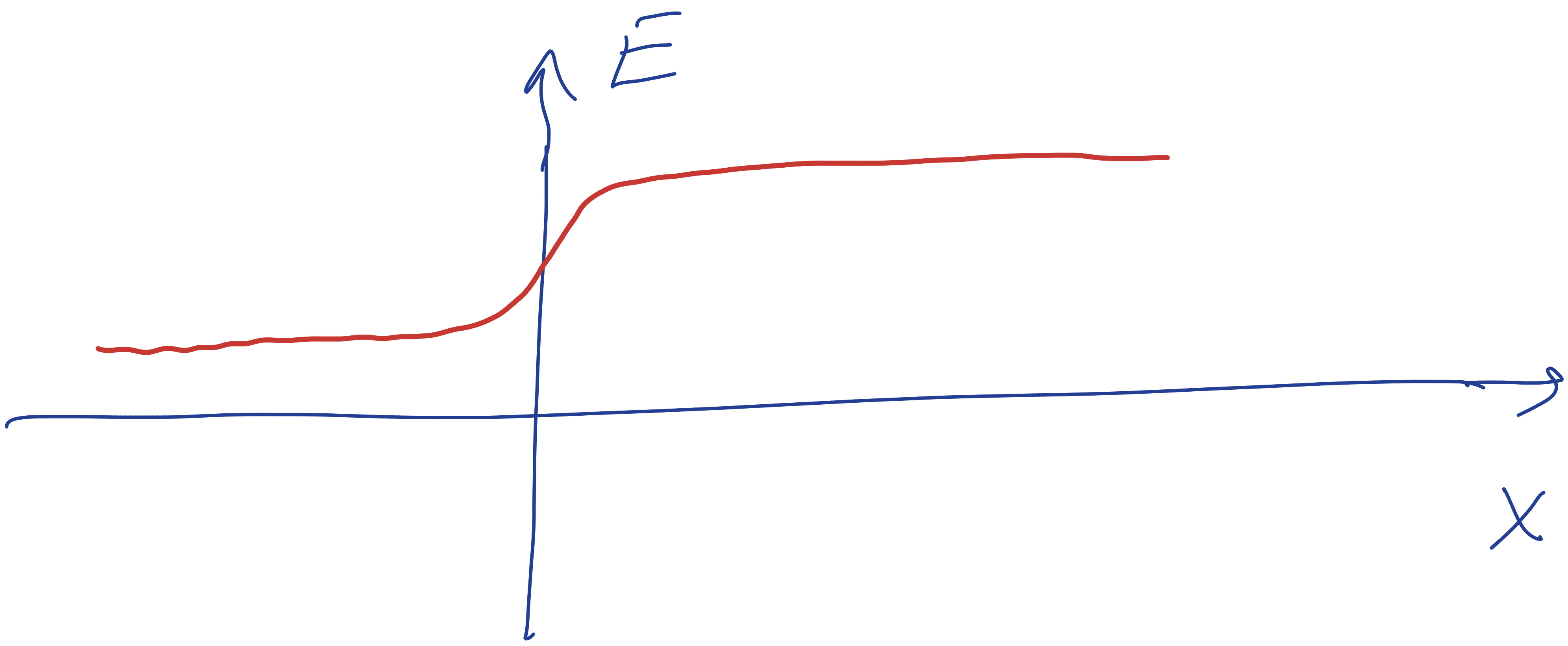
Sources allow some transitions

$$\mathcal{L} = E^2 + C_{\alpha\beta\gamma} J^{\alpha\beta\gamma}$$

$$J_{\alpha\beta\gamma} = Q \int dx_\alpha \wedge dx_\beta \wedge dx_\gamma$$

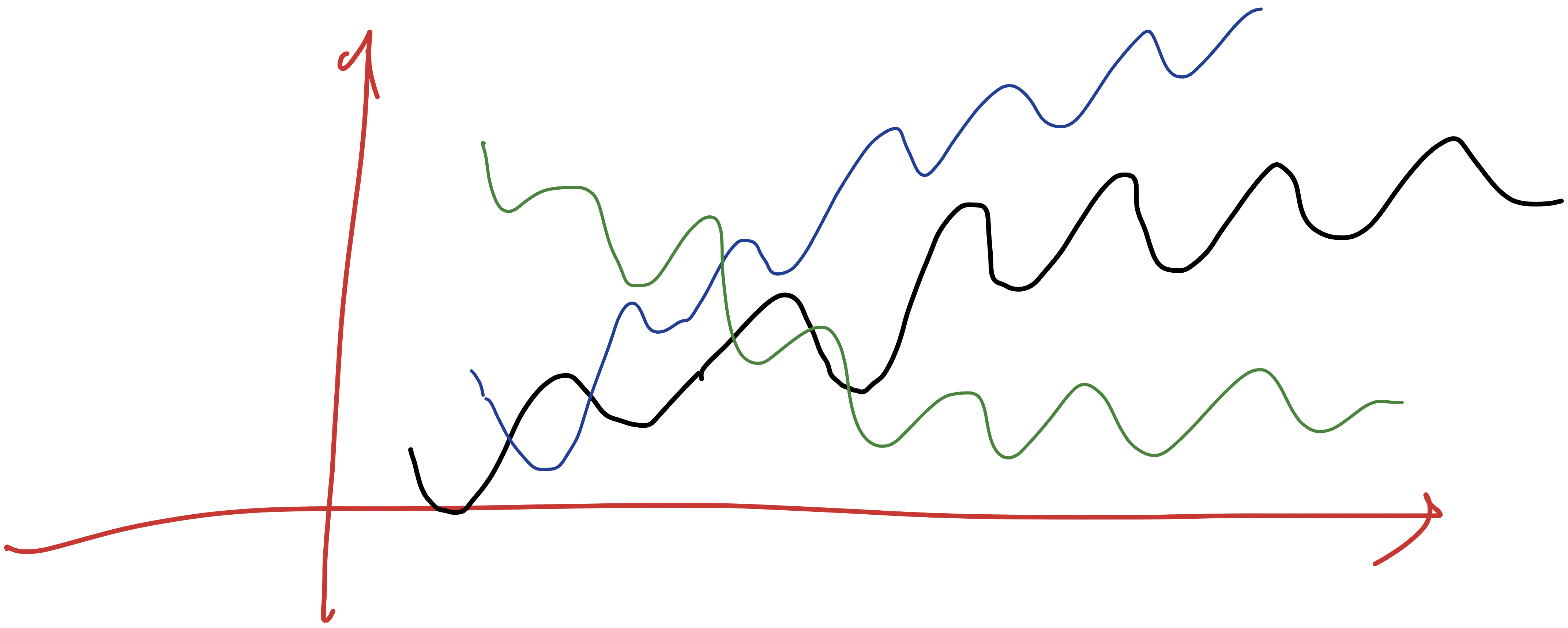
2-brane, axionic domain wall

$$\partial^\mu E = \partial^\mu {}^* J$$



$$\Delta E = Q$$

This allows creation of connected sets of vacua



Excluded by S-matrix.

3-forms must be massive!

This requires axions.

$$\mathcal{L} = E^2 + \frac{a}{f_a} E + (\partial_\mu a)^2$$

Three form Higgs effect:

const

$$\partial_\mu E = \partial_\mu \frac{a}{f_a} \longrightarrow E = \frac{1}{f_a}(a - a_0)$$

$$\Box a + \frac{1}{f_a} E = 0$$

$$\left(\Box + \frac{1}{f_a^2}\right)(a - a_0) = 0$$

$$m_a^2 = \frac{1}{f_a^2}$$

Vacuum: $a = a_0 \longrightarrow E = 0$

This has direct relevance for θ- vacua in gauge theories:

θ - vacuum = 3 - form vacuum.

Chern-Simons 3-form:

$$C_{\alpha\beta\gamma} \equiv A_{[\alpha} A_\beta A_{\gamma]} - \frac{3}{2} A_{[\alpha} \partial_\beta A_{\gamma]}$$

gluon matrix $A_\alpha \equiv A^a_\alpha T^a$

under gauge transformation:

$$A \to e^{i\omega^a T^a} A \, e^{-i\omega^a T^a}$$

$$C_{\alpha\beta\gamma} \to C_{\alpha\beta\gamma} + \partial_{\alpha} \Omega_{[\beta\gamma]}$$

$$\Omega_{\beta\gamma} = A^a_{[\alpha} \partial_{\beta]} \omega^a$$

1

$$E = \varepsilon^{\alpha\beta\gamma\delta}\partial_\alpha C_{\beta\gamma\delta} = F\tilde{F}$$

in common notations

topological vacuum susceptibility

$$\langle E, E\rangle_{q\to 0} \equiv \lim_{q\to 0} \int d^4x\, e^{iqx} \langle T\{E(x)\, E(0)\}\rangle =$$

$$= \mathrm{const}(\theta) \neq 0$$

Implies

$$\langle C\, C\rangle = \frac{1}{q^2} + \sum_{m\neq 0} \frac{c_m}{q^2 - m^2}$$

$$C = \text{massless 3-form} + \text{massive modes}$$

Theta vacua:

$$\langle E \rangle \neq 0 \longleftrightarrow \theta \neq 0$$

This is excluded by S-matrix formulation of gravity.

Thus, gauge theory must come with axion, by consistency!

Massless quark is equivalent to axion, since for $M_{quark} = 0$, axion is η'-meson (or the analog).

The shift symmetry

$$a \to a + \text{const}$$

must not be broken by extra source.
otherwise, θ-vacua will be reintroduced.

What about gravity?

Can gravity be controlled?

Dual (Gauge) formulation of the axion solution

G. D., 0507215 [hep-th]

$$\mathcal{L} = K(E) + m_a^2 \underline{(C - dB)^2}$$

gauge symmetry: $\underline{\begin{aligned} C &\to C + d\Omega, \\ B &\to B + \Omega, \end{aligned}}$

$$\Omega_{\mu\nu} = A^a_{[\mu}\partial_{\nu]}\omega^a$$
$$A_\mu \to e^{i\omega^a T^a} A_\mu e^{-i\omega^a T^a}$$

Exact dual

$$m_a = \frac{1}{f_a}$$

$$\mathcal{L} = K(E) + (\partial_\mu a)^2 - m_a a E$$

$$\mathcal{L} = (\partial_\mu a)^2 - V(a)$$

$$V(z) = m \int dz \; \mathrm{inv} \, \underline{\frac{\partial K(mz)}{\partial z}}$$

At the level of EFT there is exact duality

$$a \longleftrightarrow B_{\mu\nu}$$

global symmetry $\longleftrightarrow$ Gauge symmetry

$a \to a + \text{const}$

$B_{\mu\nu} \to B_{\mu\nu} + \Omega_{\mu\nu}$

"broken" by 3-form

Higgsed by 3-form

By power of gauge redundancy, only way to create θ-vacua is by introducing additional 3-form, unpaired with additional axion.

$$\mathcal{L} = K(E) + m_a^2 (C + \tilde{C} - dB)^2$$
$$+ \tilde{K}(\tilde{E})$$

$\hookrightarrow \tilde{E} \equiv d\tilde{C}$

A candidate in gravity, gravitational Chern-Simons

$$\tilde{C} \equiv \Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma$$
$$\tilde{E} \equiv d\tilde{C} = R\tilde{R} = \epsilon^{\alpha\beta\mu\nu} R^i_{j\alpha\beta} R^j_{i\mu\nu}$$

If in pure gravity $\langle R\tilde{R}, R\tilde{R} \rangle_{q \to 0} \neq 0$,

gravity must provide additional axion.

OR

anomalous chiral symmetry of neutrino

$$\nu \to e^{i\alpha\gamma_5}\,\nu$$

can neutralize TVS of gravity, resulting into $m_\nu \neq 0$.

G.D., Folkerts, Franca 1312.7273 [hep-th]
G.D., Funcke, 1602.03191 [hep-ph]

Each 3-form provided by gravity must be accompanied either by axion or a chiral fermion.

QCD axion is safe by consistency of S-matrix gravity.

QCD vacuum must be exact CP-conserving: θ unphysical.

What can we say about the scale f_a ?

UV-completion of $B_{\mu\nu}$ formulation is not possible by complex scalar A la Peccei-Quinn.

This favors quantum gravitational UV-completion (A la string theory). This would hint that f_a is connected with the scale of gravity.

In general, since QCD axion is required by consistency of gravity, it is natural that the origin of the scale f_a is gravitational.

$$\left. \begin{array}{l} M_p \to \infty \\ \\ f_a \to \infty \end{array} \right\} ?$$