Intrinsic coherence and particle oscillations

Anca Tureanu

University of Helsinki

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Eur.Phys.J.C 81 (2021) 12, 1092, arXiv: 2109.02139.

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- 2015 Nobel Prize in Physics to T. Kajita and A. McDonald

  "for the discovery of **neutrino oscillations**, which shows that **neutrinos have mass**."

Outline

- Standard approach to neutrino oscillations and the theoretical challenge
- Oscillations and coherence in Quantum Mechanics
  - two-level systems
  - coherent states in quantum optics
- Intrinsically coherent oscillating particle states
- Conclusions and outlook

Standard theory of neutrino oscillations

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- Lagrangian with flavour violation (induced by Yukawa terms in SM Lagrangian) and Dirac neutrino masses:

$$\mathcal{L} = \overline{\Psi}_{\nu_e} i\partial\!\!\!/ \Psi_{\nu_e} + \overline{\Psi}_{\nu_\mu} i\partial\!\!\!/ \Psi_{\nu_\mu} - \begin{pmatrix} \overline{\Psi}_{\nu_e} & \overline{\Psi}_{\nu_\mu} \end{pmatrix} \begin{pmatrix} m_{ee} & m_{e\mu} \\ m_{e\mu} & m_{\mu\mu} \end{pmatrix} \begin{pmatrix} \Psi_{\nu_e} \\ \Psi_{\nu_\mu} \end{pmatrix}$$

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- Diagonalization in terms of massive neutrino fields Ψ_1, Ψ_2 of masses m_1, m_2:

$$\begin{pmatrix} \Psi_{\nu_e}(x) \\ \Psi_{\nu_\mu}(x) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \end{pmatrix}, \quad tan^2\theta = \frac{2m_{e\mu}}{m_{\mu\mu} - m_{ee}}$$

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- **Conjecture**: There exist **flavour neutrino states** $|\nu_e\rangle, |\nu_\mu\rangle$ defined as **COHERENT** superpositions of **massive neutrino states** $|\nu_1\rangle, |\nu_2\rangle$ with different masses (m_1, m_2), by replicating the mixing formula for the fields:

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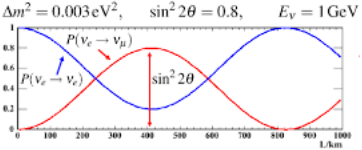
- Then oscillations can take place:

$$\mathcal{P}_{\nu_e \to \nu_\mu} = |\langle \nu_\mu(\mathbf{p})| e^{-iHt} |\nu_e(\mathbf{p})\rangle|^2$$
$$= \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4E}L\right),$$
$$\Delta m^2 = m_2^2 - m_1^2, \quad \frac{m_i}{E} \ll 1.$$



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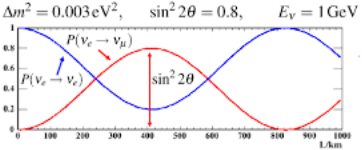

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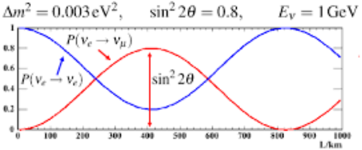

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- Recall QFT: particles with different masses are always *incoherently* produced and absorbed!

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Oscillations of states and coherence in Quantum Mechanics

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$$\mathcal{P}_{|0\rangle\to|1\rangle} = \langle 1|e^{-iH\Delta t}|0\rangle \sim \sin^2\left(\frac{\Delta E}{2}\Delta t\right)$$

Note:

- because of Stone–von Neumann theorem, all the representations of the canonical algebra for a given quantum mechanical system are equivalent, implying *unitary change of basis*:

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Can this simple quantum mechanical picture be extended straightforwardly to particle oscillations?

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The quantum mechanical interpretation of neutrino oscillation as two-level system oscillation is conceptually untenable!

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$$|\alpha\rangle = e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}|0\rangle = e^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle,$$

i.e. the coherent state is a superposition of an infinite number of particle states (or Fock states), all belonging to the same Fock space.

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$$|\alpha\rangle = e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}|0\rangle = e^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle,$$

i.e. the coherent state is a superposition of an infinite number of particle states (or Fock states), all belonging to the same Fock space.

- In QFT, the notion of coherent state appears as vacuum condensate.

How to define coherent oscillating states in quantum field theory, as superposition of finite number of particle states belonging to different Fock spaces?

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- Procedure reminiscent of the Nambu–Jona-Lasinio model for dynamical generation of nucleon masses

Nambu and Jona-Lasinio (1961),
see also Umezawa, Takahashi and Kamefuchi (1964)

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inspired by Bardeen–Cooper–Schrieffer theory of superconductivity in Bogoliubov's formulation

Bogoliubov (1958)

The technique: Quantum Hamiltonian diagonalization

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- Flavour number-violating **Hamiltonian**

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\begin{aligned}
H &= \int d^3x \Big[-\overline{\Psi}_{\nu_e} i\gamma^i \partial_i \Psi_{\nu_e} - \overline{\Psi}_{\nu_\mu} i\gamma^i \partial_i \Psi_{\nu_\mu} \Big] \\
&+ \int d^3x \Big[m_{ee} \overline{\Psi}_{\nu_e} \Psi_{\nu_e} + m_{\mu\mu} \overline{\Psi}_{\nu_\mu} \Psi_{\nu_\mu} + m_{e\mu} \left(\overline{\Psi}_{\nu_e} \Psi_{\nu_\mu} + \overline{\Psi}_{\nu_\mu} \Psi_{\nu_e} \right) \Big] = H_0 + H_{mass}.
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$$H = \int d^3x \Big[- \overline{\Psi}_{\nu_e} i\gamma^i \partial_i \Psi_{\nu_e} - \overline{\Psi}_{\nu_\mu} i\gamma^i \partial_i \Psi_{\nu_\mu} \Big] + \int d^3x \Big[m_{ee} \overline{\Psi}_{\nu_e} \Psi_{\nu_e} + m_{\mu\mu} \overline{\Psi}_{\nu_\mu} \Psi_{\nu_\mu} + m_{e\mu} \left(\overline{\Psi}_{\nu_e} \Psi_{\nu_\mu} + \overline{\Psi}_{\nu_\mu} \Psi_{\nu_e} \right) \Big] = H_0 + H_{mass}.$$

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$$\psi_{\nu_l}(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2p}} \sum_\lambda \left(a_{l\lambda}(\mathbf{p}) u_\lambda(\mathbf{p}) e^{-ipx} + b^\dagger_{l\lambda}(\mathbf{p}) v_\lambda(\mathbf{p}) e^{ipx} \right), \quad l = e, \mu$$

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$$

- Treat H_{mass} as an interaction term for massless SM flavour fields.

- Nondiagonal Hamiltonian in terms of massless (bare) particles' operators :

$$
\begin{aligned}
H &= \int d^3p \sum_{\lambda} \Big\{ \mathrm{p} \left(a^{\dagger}_{e\lambda}(\mathbf{p}) a_{e\lambda}(\mathbf{p}) + b^{\dagger}_{e\lambda}(\mathbf{p}) b_{e\lambda}(\mathbf{p}) + a^{\dagger}_{\mu\lambda}(\mathbf{p}) a_{\mu\lambda}(\mathbf{p}) + b^{\dagger}_{\mu\lambda}(\mathbf{p}) b_{\mu\lambda}(\mathbf{p}) \right) \\
&+ \operatorname{sgn}\lambda \Big[m_{ee} \left(a^{\dagger}_{e\lambda}(\mathbf{p}) b^{\dagger}_{e\lambda}(-\mathbf{p}) + b_{e\lambda}(\mathbf{p}) a_{e\lambda}(-\mathbf{p}) \right) + m_{\mu\mu} \left(a^{\dagger}_{\mu\lambda}(\mathbf{p}) b^{\dagger}_{\mu\lambda}(-\mathbf{p}) + b_{\mu\lambda}(\mathbf{p}) a_{\mu\lambda}(\right. \\
&+ m_{e\mu} \left(a^{\dagger}_{e\lambda}(\mathbf{p}) b^{\dagger}_{\mu\lambda}(-\mathbf{p}) + b_{\mu\lambda}(\mathbf{p}) a_{e\lambda}(-\mathbf{p}) + a^{\dagger}_{\mu\lambda}(\mathbf{p}) b^{\dagger}_{e\lambda}(-\mathbf{p}) + b_{e\lambda}(\mathbf{p}) a_{\mu\lambda}(-\mathbf{p}) \right) \Big] \Big\}.
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- Diagonal form:

$$H = \int d^3p \sum_{\lambda, i=1,2} E_{i\mathrm{p}} \Big[A^\dagger_{i\lambda}(\mathbf{p}) A_{i\lambda}(\mathbf{p}) + B^\dagger_{i\lambda}(\mathbf{p}) B_{i\lambda}(\mathbf{p}) \Big], \qquad E_{i\mathrm{p}} = \sqrt{\mathrm{p}^2 + m_i^2}$$

- The eigenstates of the diagonal Hamiltonian are the physical particle states (Bogoliubov quasiparticles).

Three sets of canonical fields:

$\psi_{\nu_l}(x)$, $l = e, \mu$ massless,
$a_{l\lambda}(\mathbf{p}), b_{l\lambda}(\mathbf{p})$

$\psi_{\nu_i}(x)$, $i = 1, 2$ massless,
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Two (orthogonal) vacua:

$|0\rangle$ non-physical
$a_{l\lambda}(\mathbf{p})|0\rangle = b_{l\lambda}(\mathbf{p})|0\rangle = 0$

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- Unitary transformation (rotation) between the operators of the massless fields:

$$\begin{pmatrix} a_{e\lambda}(\mathbf{p}) \\ a_{\mu\lambda}(\mathbf{p}) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a_{1\lambda}(\mathbf{p}) \\ a_{2\lambda}(\mathbf{p}) \end{pmatrix}$$

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- Bogoliubov transformations between the "massless" and "massive" operators:

$$\begin{aligned} A_{i\lambda}(\mathbf{p}) &= \alpha_{i\mathrm{p}} a_{i\lambda}(\mathbf{p}) + \beta_{i\mathrm{p}} b^{\dagger}_{i\lambda}(-\mathbf{p}), \qquad i = 1, 2, \\ B_{i\lambda}(\mathbf{p}) &= \alpha_{i\mathrm{p}} b_{i\lambda}(\mathbf{p}) - \beta_{i\mathrm{p}} a^{\dagger}_{i\lambda}(-\mathbf{p}), \quad \alpha_{i\mathrm{p}} = \sqrt{\frac{1}{2}\left(1 + \frac{\mathrm{p}}{E_{i\mathrm{p}}}\right)}, \beta_{i\mathrm{p}} = \operatorname{sgn}\lambda \sqrt{\frac{1}{2}\left(1 - \frac{\mathrm{p}}{E_{i\mathrm{p}}}\right)} \end{aligned}$$

- Physical vacuum is a condensate of "Cooper-like pairs" of massless neutrino-antineutrino – coherent state!

$$|\Phi_0\rangle = \Pi_{i,\mathbf{p},\lambda} \left(\alpha_{i\mathrm{p}} - \beta_{i\mathrm{p}}\, a^{\dagger}_{i\lambda}(\mathbf{p}) b^{\dagger}_{i\lambda}(-\mathbf{p})\right) |0\rangle,$$

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- Massive neutrino states interpreted as Bogoliubov quasiparticles.

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\end{aligned}
$$

- Oscillation amplitude is never zero!

$$
\mathcal{A}_{\nu_e\to\nu_\mu}(t) = \frac{1}{2}\sin 2\theta e^{-i\mathrm{p}t}\left[-\left(1-\frac{1}{4}\frac{m_1^2}{\mathrm{p}^2}\right)^2 e^{-i\frac{m_1^2}{2\mathrm{p}}t} + \left(1-\frac{1}{4}\frac{m_2^2}{\mathrm{p}^2}\right)^2 e^{-i\frac{m_2^2}{2\mathrm{p}}t}\right].
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- There is always a portion of muon neutrino in the electron neutrino and vice-versa.
- In the ultrarelativistic limit, one recovers Pontecorvo's oscillation probability:

$$
P_{\nu_e\to\nu_\mu} = \sin^2 2\theta\sin^2\left(\frac{\Delta m^2}{4\mathrm{p}}t\right), \quad \Delta m^2 = m_2^2 - m_1^2.
$$

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- To be elucidated:

  the mechanism of interaction (production and absorbtion) of oscillating particle states.