COSMIC STRINGS AND BLACK HOLES

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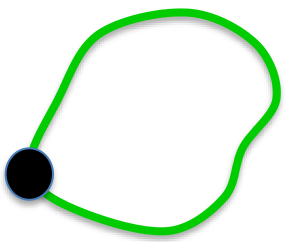
Cosmic strings and black holes

String loops can be captured by black holes and can interact with them in interesting ways.



Work with:

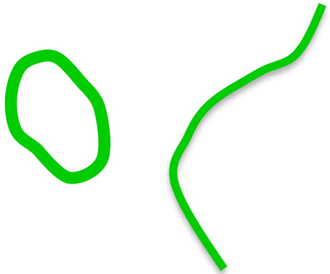
Yuri Levin, Andrei Gruzinov , Hengrui Xing, Heling Deng

Strings could be formed at a symmetry breaking phase transition in the early universe.

Nielsen & Olesen (1973)
Kibble (1976)

Predicted in a wide variety of particle physics models.

Can be either infinite or closed.



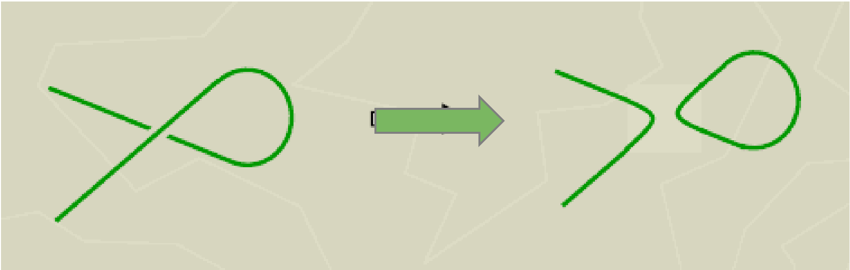
$\mu \sim \eta^2$ – mass per unit length

Symmetry breaking scale (→ η)

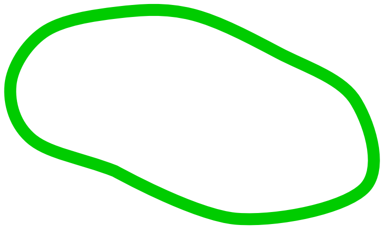
$$10^{-34} \lesssim G\mu \lesssim 10^{-10}$$

Tension = μ ⇒ relativistic motion.

String reconnection



Loop dynamics



Worldsheet area

Nambu-Goto action: $S = -\mu \mathcal{A}$

Solution of NG eqs of motion:

$$\mathbf{x}(\sigma, t) = \frac{1}{2}\left[\mathbf{a}(\sigma - t) + \mathbf{b}(\sigma + t)\right] \qquad \mathbf{a}'^2 = \mathbf{b}'^2 = 1$$

$$\mathbf{x}(\sigma + L, t) = \mathbf{x}(\sigma, t), \qquad L = m/\mu$$

Invariant length

Loops oscillate with a period $T = L/2.$

Loop captured by a black hole

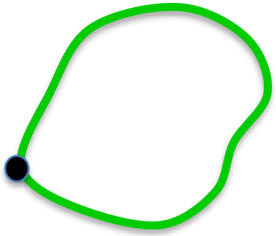
H. Xing, Y. Levin,
A. Gruzinov, & A.V. (2020)

It is like a loop pinned at one point.

Boundary conditions: $\mathbf{x}(0,t) = \mathbf{x}(L,t) = 0.$

$$\mathbf{x}(\sigma,t) = \frac{1}{2}\left[\mathbf{a}(\sigma - t) - \mathbf{a}(-\sigma - t)\right]$$

The loop oscillates with a period *2L*.



$R = GM$

$$M \gg \mu L$$

$$R \ll L$$

$$m = \mu L$$

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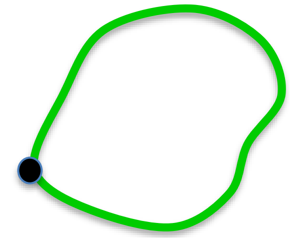

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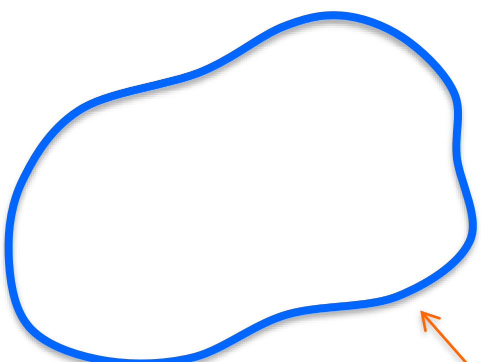
Fixed auxiliary curve

$\mathbf{a}(\sigma)$



$\mathbf{a}'^2 = 1$

$0 < \sigma < 2L$

Not the shape of the physical loop!

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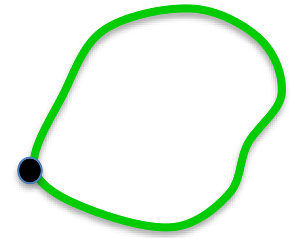

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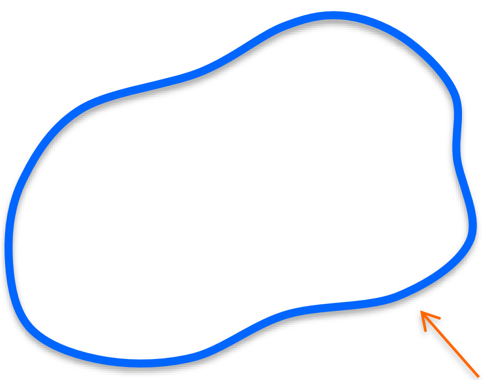


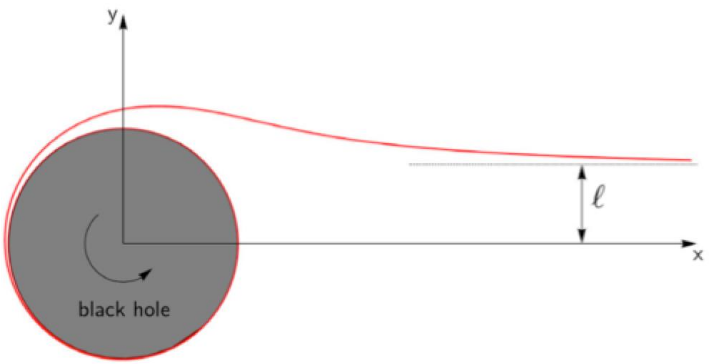
$\mathbf{a}'^2 = 1$

$0 < \sigma < 2L$

Not the shape of the physical loop!

In the next approximation in *R / L* ,
The loop can exchange energy and angular momentum with the BH.
⇨ The auxiliary curve gradually evolves.

Rotating black hole



If the string is in the equatorial plane, $\ell = 4R^2\Omega$.

The torque is $Q = \mu\ell = 4\mu R^2\Omega$.

Frolov et al (!989)

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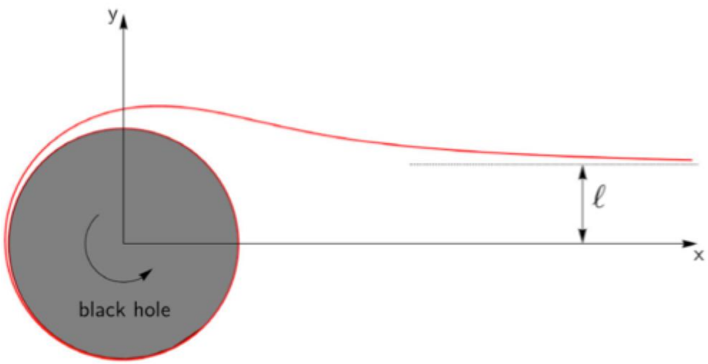

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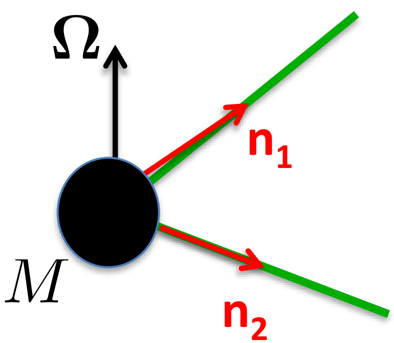
Equal and opposite torque acts on the string:

$$\mathbf{Q} = 4\mu R^2[\mathbf{\Omega} - (\mathbf{n}\cdot\mathbf{\Omega})\mathbf{n} - \mathbf{n}\times\dot{\mathbf{n}}]$$



$\omega = \mathbf{n}\times\dot{\mathbf{n}}$ – angular velocity of the string.

n varies on a time scale ~ *L* >> *R* ⇒ *quasistationary*

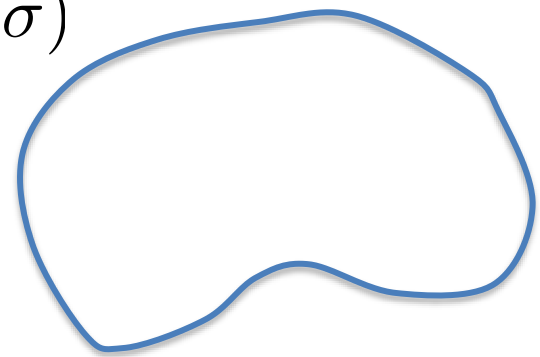
Rate of energy change:

$$\dot{E} = \mathbf{Q}_1\cdot\omega_1 + \mathbf{Q}_2\cdot\omega_2$$

Loop orbit evolves slowly compared to the oscillation period.

This can be described as *slow deformation of the auxiliary curve*.

$\mathbf{a}(\sigma)$



Averaged over oscillation period:

$$\mathbf{v}(\sigma) = \frac{8R^2}{L}[\mathbf{a}'(\sigma) \times \mathbf{\Omega} + \mathbf{a}''(\sigma)]$$

Xing, Levin, Gruzinov, & A.V. (2020)

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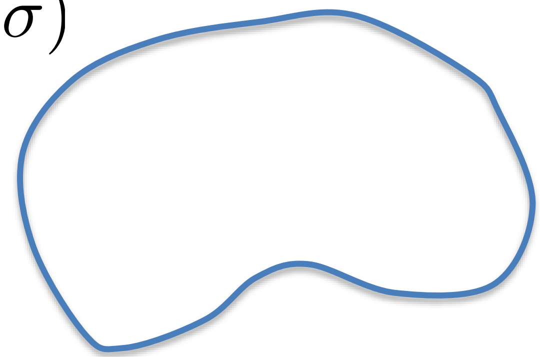

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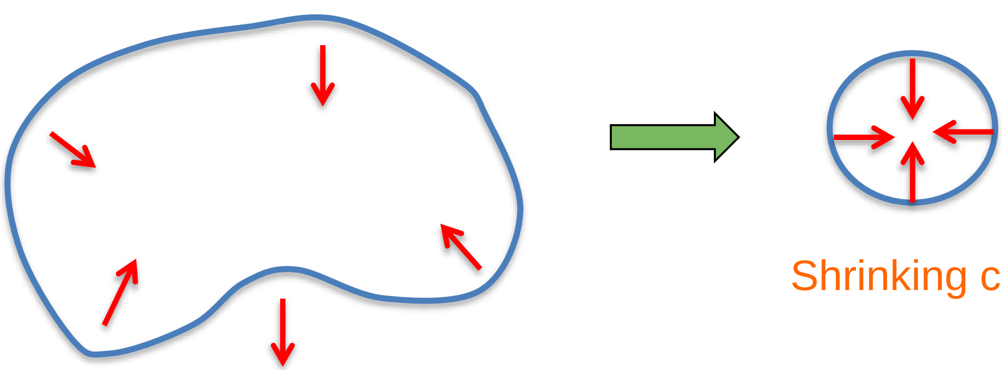
For $\boldsymbol{\Omega} = 0$: $\mathbf{v}(\sigma) \propto \mathbf{a}''(\sigma)$

Curve shortening flow

Gage (!984), Gage & Hamilton (1986), …



Shrinking circle

In the end the loop is swallowed by the BH.

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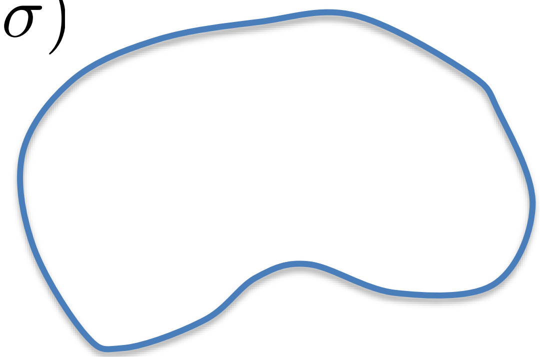

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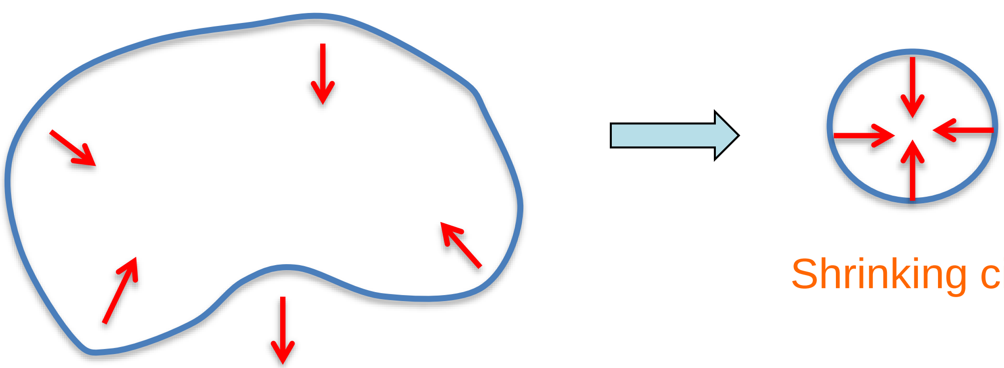

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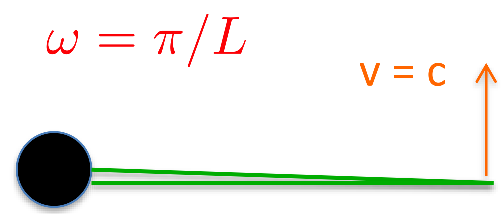


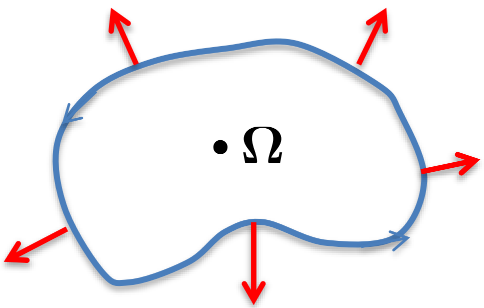
The physical loop is a rotating double line.

A strong emitter of gravitational waves.

Now consider $\mathbf{\Omega} \neq 0$

$$\mathbf{v}(\sigma) = \frac{8R^2}{L}[\mathbf{a}'(\sigma) \times \mathbf{\Omega} + \mathbf{a}''(\sigma)]$$

For a maximally rotating BH: $\Omega \sim 1/R$



The 1st term dominates if $\Omega L \gg 1$.

Auxiliary curve expands, approaching a circle.

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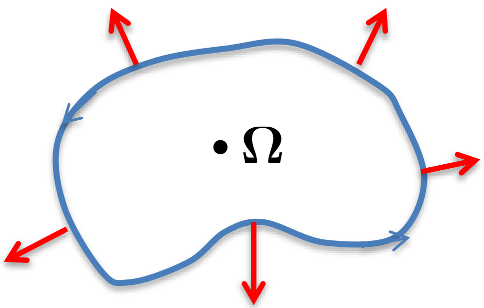

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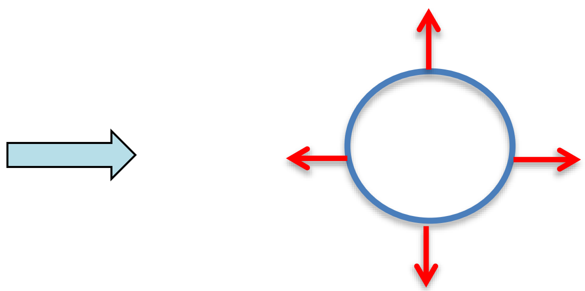
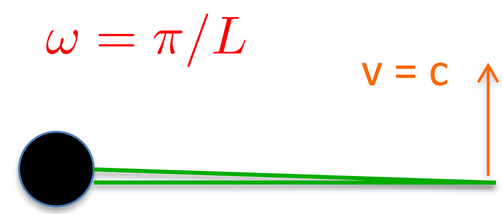




$$L \propto \sqrt{t}$$

The loop grows by extracting rotational energy from the BH.

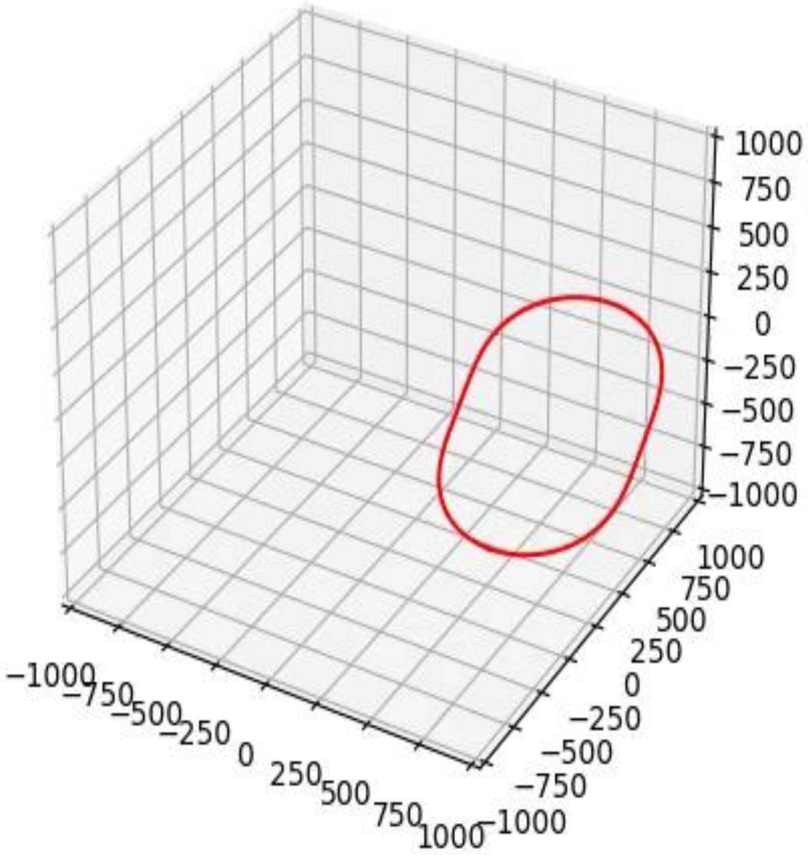
Complete spin down of a supermassive BH in 10^{10} yrs for $G\mu \gtrsim 10^{-15}$.

Numerical simulations

Heling Deng

Loop in Schwarzschild spacetime

$L_i/R \sim 100$

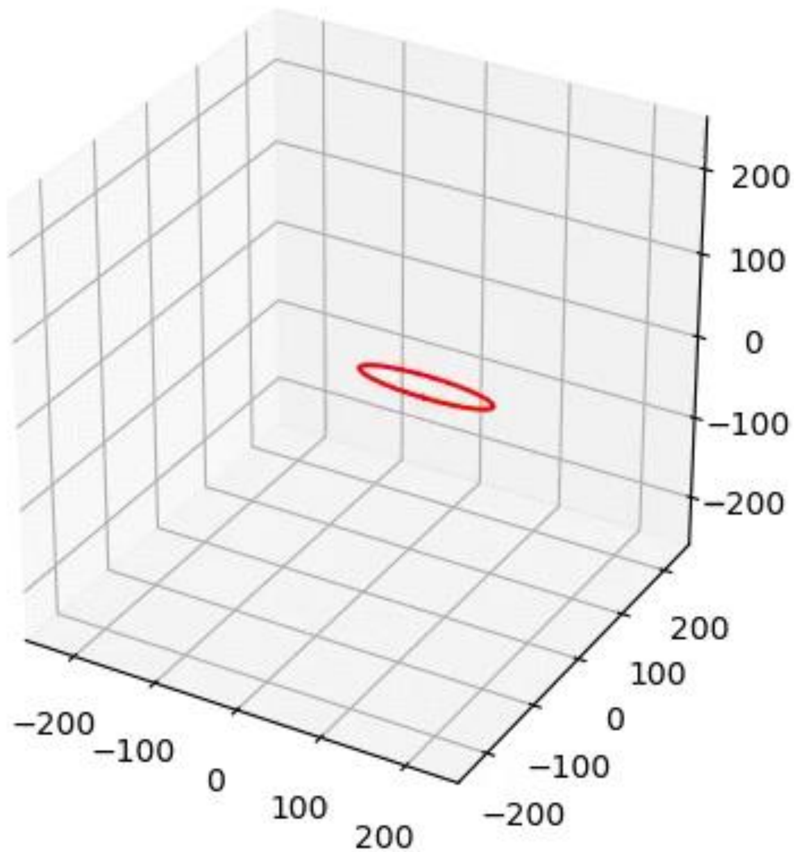


Numerical simulations

Heling Deng

Loop in Kerr spacetime

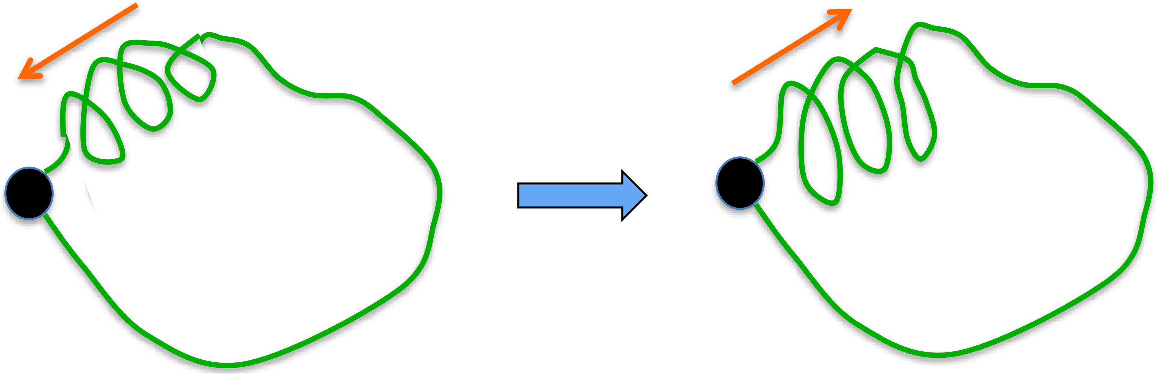
$L_i/R \sim 50$



Superradiance

Helical wave amplifies upon reflection.

Zel'dovich



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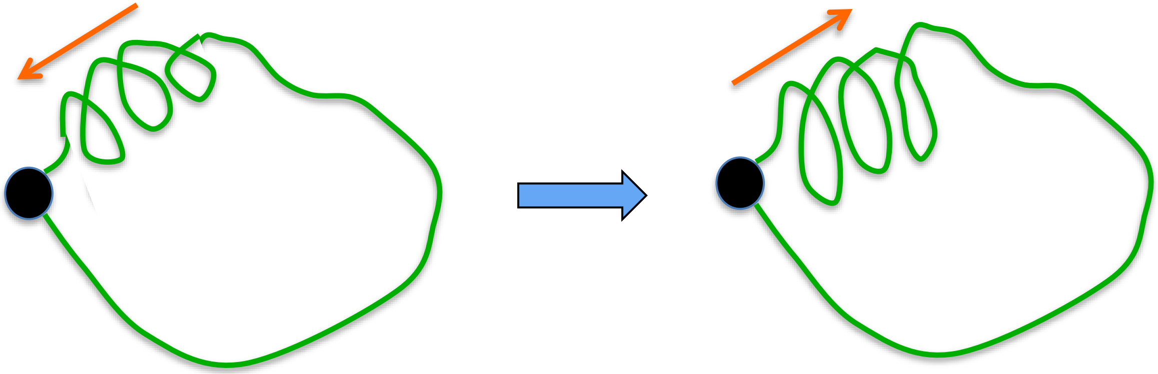
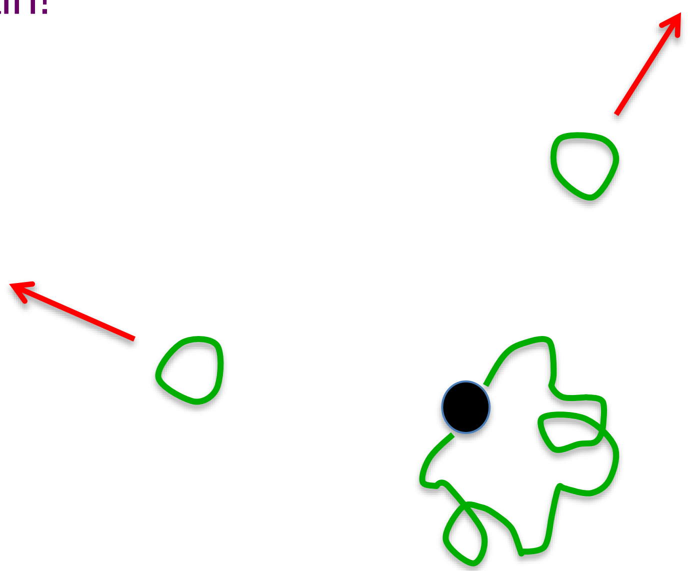


Then travels to the other side – and amplifies again!

Perturbations become nonlinear.

May lead to continuous loop production.



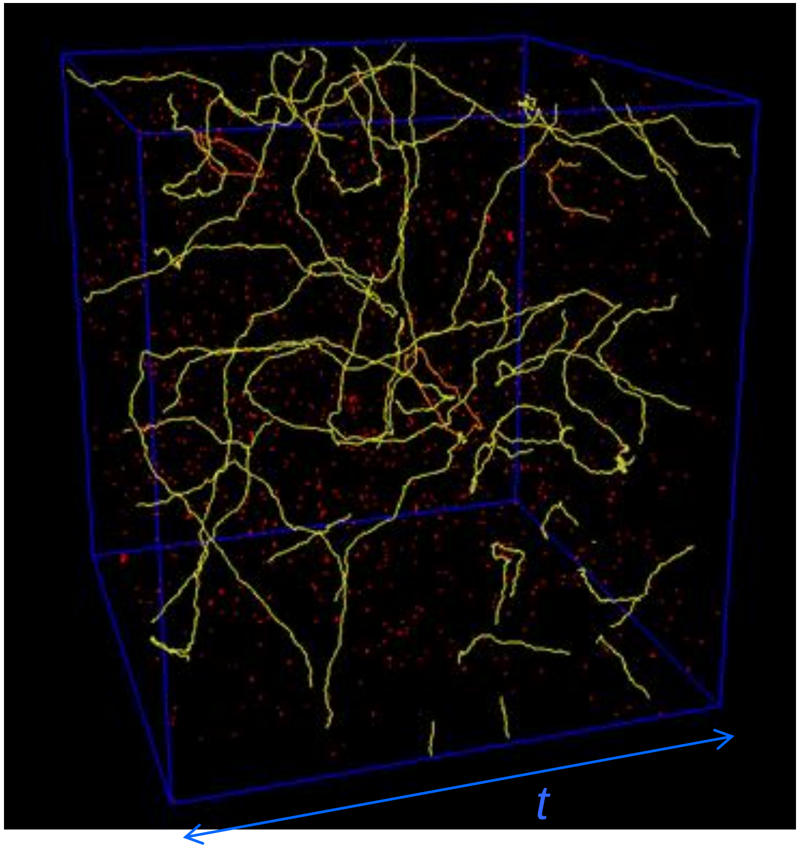
STRING EVOLUTION AND CAPTURE

Self-similar evolution

- Each horizon volume contains several long strings and a large number of loops with a wide distribution of sizes.

- Loops oscillate and decay by emitting gravitational waves.

- Loop density: $n \propto (G\mu)^{-3/2}$



Bennett & Bouchet (1990)

Allen & Shellard (1990)

Ringeval, Sakellariadou & Bouchet (2005)

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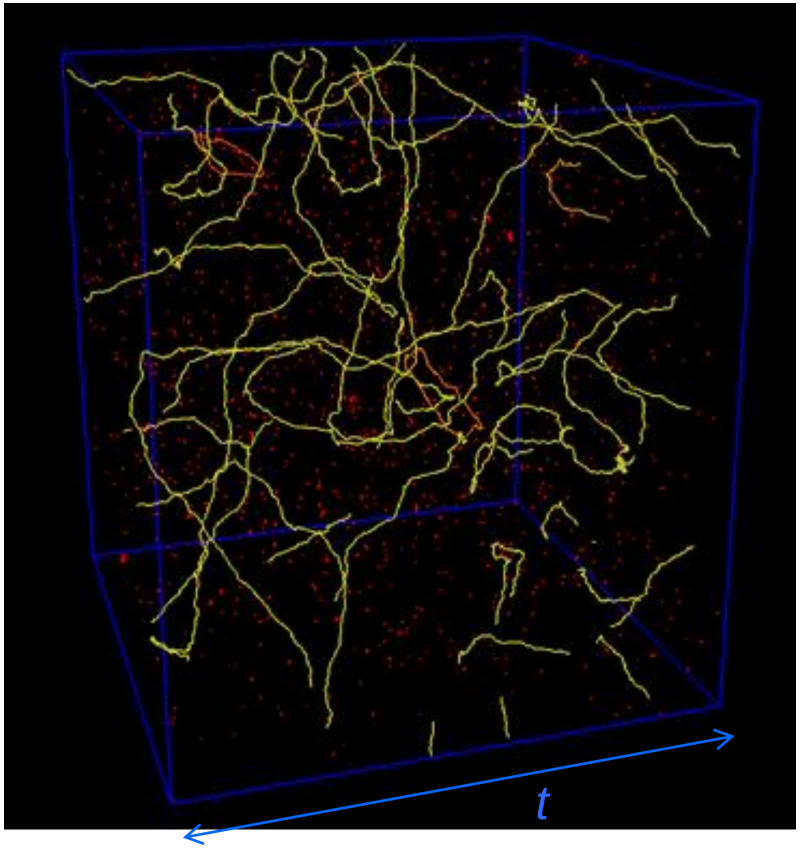

Loop capture

The probability of capture by a SMBH

is *P ~ 1* for $G\mu \lesssim 10^{-17}$.

H. Xing, Y. Levin,
A. Gruzinov, & A.V. (2020)

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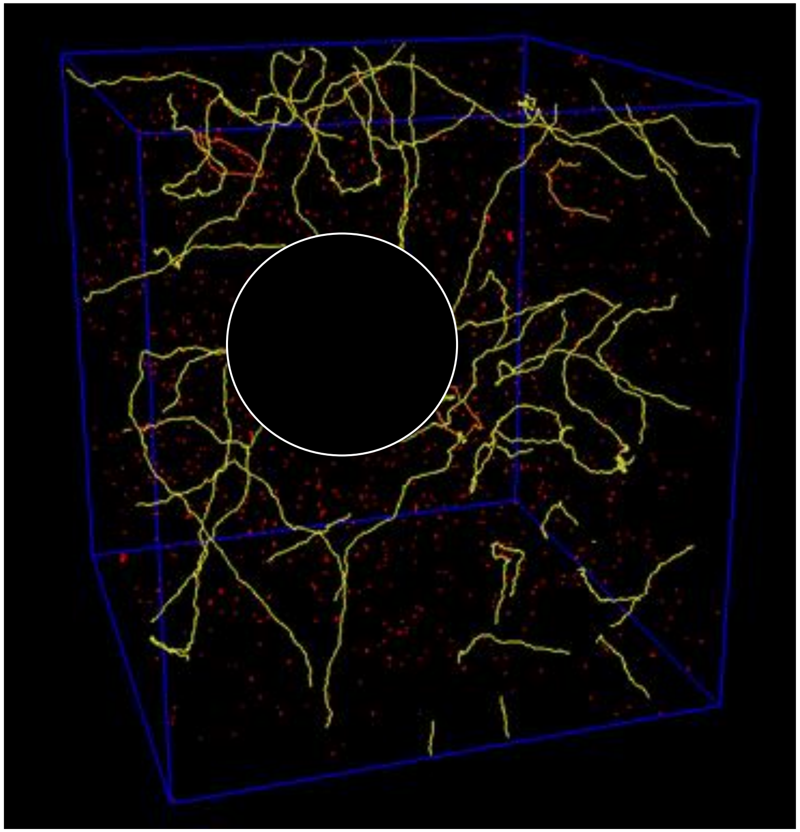
Strings and primordial BHs

A. Gruzinov, Y. Levin & A.V. (2020)

- BHs have size ~ horizon at formation.
- A few strings are captured by each BH.

→ BH-string network.



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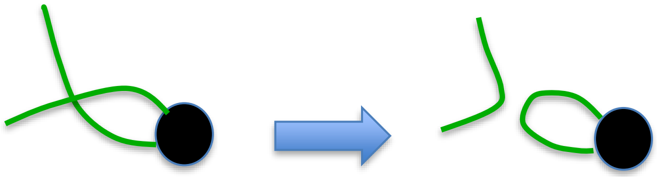
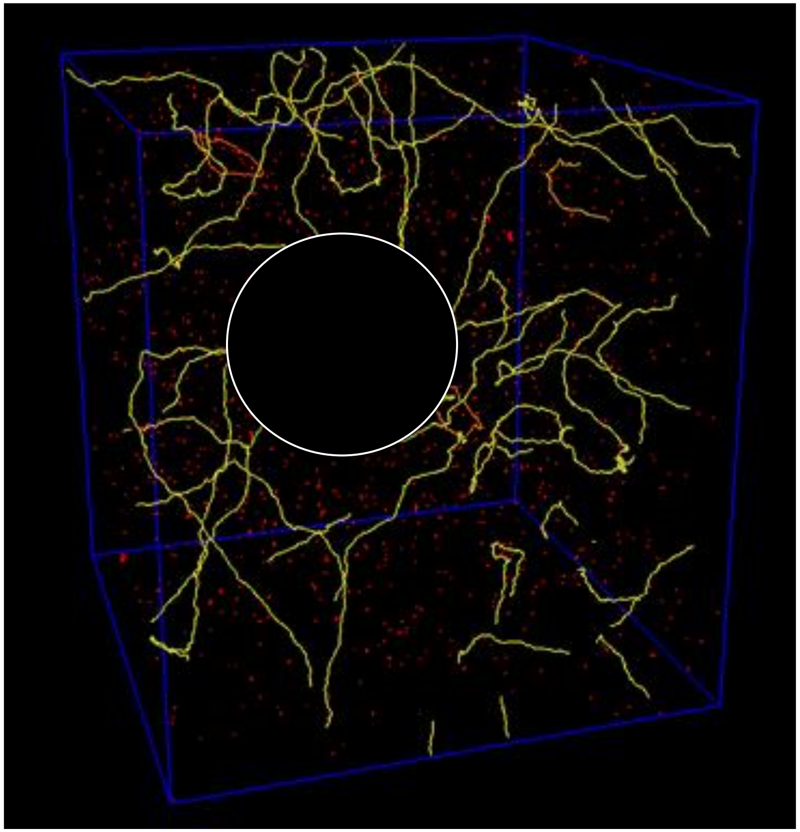




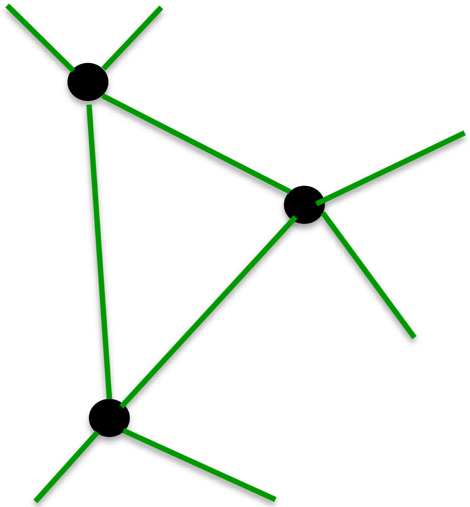
BHs can disconnect from the network, but only with loops attached.

Numerical simulations of BH-string networks

A. Lopez, K. Olum & A.V. (in progress)

- During the radiation era, BH disconnection is very efficient. BH separation in the network grows faster than the horizon.

- In the matter era disconnection is much less efficient. BH separation may become smaller than the horizon. The strings are then stretched by the expansion and a frozen network is formed.



Conclusions

String loops are likely to be captured by SMBH in galactic centers (for sufficiently small $G\mu$) and by primordial BHs (for any $G\mu$).

A variety of physical effects:

- *BH spin down*
- *Superradiance*
- *GW emission*

Work in progress: reconnections, evolution of BH-string networks, etc.