# CNB assisted dark energy

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# Outline

- Parameter fine-tuning problem in quintessence models for addressing coincidence problem.
- Quintessence coupled with neutrinos.
- Fine-tuning problem of initial conditions in quintessence coupled with neutrinos.
- Inflationary quintessence model.
- Neutrino lumps.

# Runaway model based on Ratra-Peebles potential.

$$U(\phi) = V \left(M_P/\phi\right)^{\alpha} \ .$$

In this model the dark energy is a slow roll phenomenon.

$$\frac{M_P^2}{16\pi}\left(\frac{U'}{U}\right)^2 = \frac{M_P^2}{16\pi}\left(\frac{\alpha}{\phi}\right)^2 \ , \qquad \frac{M_P^2}{8\pi}\,\frac{U''}{U} = \frac{M_P^2}{8\pi}\,\frac{\alpha(1+\alpha)}{\phi^2} \ .$$

$\phi \gtrsim M_P$ marks the onset of DE.

$V$ should be taken to be of the same order of magnitude as $\rho_{DE}^0$.

B. Ratra & P. J. E. Peebles, "Cosmological Consequences of a Rolling Homogeneous Scalar Field," Phys. Rev. D37 (1988) 3406.

# Mass varying neutrino model.

This sort of models are based on the coupling between quintessence and neutrinos and intend to relate the present dark energy density to the neutrino mass-scale, 0.05 eV, in order to mitigate the parameter fine-tuning problem of quintessence dark energy.

The late-time predominance of DE is caused by the back-reaction on the quinton field due to neutrinos after they enter the non-relativistic regime: $T_\nu \simeq 0.05\,\mathrm{eV}$.

R. Fardon, A. E. Nelson & N. Weiner, "Dark energy from mass varying neutrinos," JCAP 10 (2004) 005.

# The equations of motion.

Model is based on action functional

$$\int \mathrm{d}^4 x \sqrt{-g} \left( \frac{g^{\alpha\beta}\partial_\alpha \phi \partial_\beta \phi}{2} - U(\phi) - \frac{M_P^2 R}{16\pi} + \frac{i}{2} \left[ \bar{\psi}_\nu \gamma^\alpha(x) \mathfrak{D}_\alpha \psi_\nu - \left( \mathfrak{D}_\alpha \bar{\psi}_\nu \right) \gamma^\alpha(x) \psi_\nu \right] - m_\nu(\phi) \bar{\psi}_\nu \psi_\nu \right) ,$$

in a flat FLRW metric

$$\mathrm{ds}^2 = \mathrm{d}t^2 - a^2(t) \mathrm{dx}^2 .$$

# The equations of motion.

The variation with respect to $\phi$ results in $-m'_\nu \bar{\psi}_\nu \psi_\nu$ from the fermion sector, which can be expressed as

$$-m'_\nu \bar{\psi}_\nu \psi_\nu = \frac{m'_\nu}{m_\nu} T^{\alpha}_{\ \alpha} = \frac{m'_\nu}{m_\nu}(\rho_\nu - 3p_\nu) \ .$$

As a next step one takes a finite-temperature expression in order to describe CNB.

$$\dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) = \frac{\mathrm{d}\ln m_\nu}{\mathrm{d}\phi}(\rho_\nu - 3p_\nu)\dot{\phi} \ ,$$

$$\ddot{\phi} + 3H\dot{\phi} + U'(\phi) = -\frac{\mathrm{d}\ln m_\nu}{\mathrm{d}\phi}(\rho_\nu - 3p_\nu) \ ,$$

$$H^2 = \frac{8\pi}{3M_P^2}(\rho_\nu + \rho_\phi + \rho_r + \rho_m) \ .$$

# Free streaming neutrinos.

$$\rho_\nu = \frac{\mathrm{g}}{a^3}\int \frac{\mathrm{d}^3 k}{(2\pi)^3}\,\frac{\varepsilon_\nu(\mathsf{k})}{\mathrm{e}^{k/aT_\nu}+1}\ ,$$

$$p_\nu = \frac{\mathrm{g}}{3a^5}\int \frac{\mathrm{d}^3 k}{(2\pi)^3}\,\frac{k^2}{\varepsilon_\nu(\mathsf{k})\Big(\mathrm{e}^{k/aT_\nu}+1\Big)}\ ,$$

$$\varepsilon_\nu(\mathsf{k}) = \sqrt{\frac{\mathsf{k}^2}{a^2}+m_\nu^2}\ ,$$

$$\frac{m_\nu'}{m_\nu}\left[\rho_\nu - 3p_\nu\right] = \frac{\partial \rho_\nu}{\partial \phi} \quad \Rightarrow \quad \ddot{\phi} + 3H\dot{\phi} = -\frac{\partial\big[U(\phi)+\rho_\nu(\phi, T_\nu)\big]}{\partial \phi}\ .$$

g counts two helicity states per flavor: $\mathrm{g} = 2\times$number of neutrino species.

# Non-relativistic regime for CNB.

We expect two mass eigenstates to be non-relativistic at present since present CNB temperature $T_\nu^0 \simeq 1.67 \times 10^{-4}$ eV and neutrino masses (in normal hierarchical spectrum) are

$$m_1 \ll m_2 < m_3 \ , \quad m_2 \sim 8.6 \times 10^{-3}\,\text{eV} \quad m_3 \sim 0.05\,\text{eV} \ .$$

Correspondingly $m_2/T_\nu^0 \sim 51.5$ and $m_3/T_\nu^0 \sim 300$.

In non-relativistic approximation

$$\rho_\nu = \frac{3 g m_\nu \zeta(3) T_\nu^3}{4\pi^2} \equiv n_\nu m_\nu \ ,$$

and one arrives at the system of equations

$$\dot{n}_\nu + 3Hn_\nu = 0 \ \Rightarrow \ n_\nu \propto a^{-3} \ , \quad \ddot{\phi} + 3H\dot{\phi} + U'(\phi) = -m'_\nu(\phi) n_\nu \ .$$

# Approximate analytical solution.

The minimum of effective potential

$$U'\big(\phi_+(t)\big) + m'_\nu\big(\phi_+(t)\big) n_\nu\big(t\big) = 0 \ , \qquad (1)$$

gives an approximate solution. The system of equations we obtained can be put in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{\phi}^2}{2} + U + m_\nu n_\nu\right) = -3H\left(\dot{\phi}^2 + m_\nu n_\nu\right) \ . \qquad (2)$$

If the slow roll conditions

$$\dot{\phi}_+^2 \ll m_\nu(\phi_+)n_\nu \ , \qquad |\ddot{\phi}_+\dot{\phi}_+| \ll 3Hm_\nu(\phi_+)n_\nu \ .$$

are satisfied, then Eq.(2) is reduced to Eq.(1).

# Dark energy.

Some preliminary constraints on the model are as follows.

$$\omega(t_0) = -\frac{U(\phi_+(t_0))}{U(\phi_+(t_0)) + \rho_\nu} = -0.9 \quad \Rightarrow$$

$$U(\phi_+(t_0)) = 9\rho_\nu^0 \;, \qquad \rho_{DE}^0 = 10\rho_\nu^0 \;, \qquad m_\nu\big(\phi_+(t_0)\big) = \frac{\rho_{DE}^0}{10 n_\nu^0} \;.$$

The condition for accelerating expansion

$$2U(\phi_+) > \rho_r^0(1+z)^4 + \Big(\rho_m^0 + m_\nu(\phi_+)n_\nu^0\Big)(1+z)^3 \;.$$

## Runaway model: $U(\phi) = V\mathrm{e}^{-\alpha\phi/M_P}$ , $m_\nu(\phi) = \mu_\nu \mathrm{e}^{\beta\phi/\mu_\nu}$.

Here the parameters $\alpha, \beta \sim 1$.

$$U_{eff} = V\mathrm{e}^{-\alpha\phi/M_P} + n_\nu(t)\mu_\nu \mathrm{e}^{\beta\phi/\mu_\nu} \ \Rightarrow \ \phi_+(t) = \frac{M_P}{\alpha + \beta\frac{M_P}{\mu_\nu}} \ln \frac{\alpha V}{\beta n_\nu(t) M_P}$$

Since $H_0^2 \propto \rho_{DE}^0/M_P^2$, the slow-roll condition (in the present epoch)

$$\frac{\dot{\phi}_+^2}{2} = \frac{9H^2M_P^2}{2(\alpha + \beta\frac{M_P}{\mu_\nu})^2} \ll U_{eff} \equiv \rho_{DE} \ ,$$

requires that $V/\mu_\nu$ is large enough. The growth of neutrino mass is unlimited

$$m_\nu(\phi_+) = \mu_\nu \frac{\alpha V}{\beta n_\nu(t) M_P} \ .$$

One can set $V$ by demanding $m_\nu^0 = \mu_\nu$.

The cosmological equations can be written in phase-space variables

$$\zeta_1 = \sqrt{\frac{8\pi}{6}}\frac{\dot{\phi}}{M_P H}\ ,\ \zeta_2 = \sqrt{\frac{8\pi}{3}}\frac{\sqrt{U}}{M_P H}\ ,\ \zeta_3 = \sqrt{\frac{8\pi}{3}}\frac{\sqrt{\rho_m}}{M_P H}\ ,$$

as (here $'$ denotes the derivative with respect to $N = \ln(a/a_0)$)

$$\begin{aligned}\zeta_1' &= -3\zeta_1 + \sqrt{\frac{3}{16\pi}}\alpha\zeta_2^2 + \frac{3}{2}\zeta_1\left(1+\zeta_1^2-\zeta_2^2\right) - \\ &\qquad \sqrt{\frac{3}{16\pi}}\frac{M_P\beta}{\mu_\nu}\left(1-\zeta_1^2-\zeta_2^2-\zeta_3^2\right)\ ,\\ \zeta_2' &= -\sqrt{\frac{3}{16\pi}}\alpha\zeta_1\zeta_2 + \frac{3}{2}\zeta_2\left(1+\zeta_1^2-\zeta_2^2\right)\ ,\\ \zeta_3' &= -\frac{3}{2}\zeta_3 + \frac{3}{2}\zeta_3\left(1+\zeta_1^2-\zeta_2^2\right)\ .\end{aligned}$$

This system has a stable critical point $(\zeta_1^\star, \zeta_2^\star, \zeta_3^\star)$ where

$$\dot{\phi}_+(t) = \frac{3HM_P}{\alpha + \beta\frac{M_P}{\mu_\nu}} \quad \Rightarrow \quad \zeta_1^\star = \frac{\sqrt{12\pi}}{\alpha + \beta\frac{M_P}{\mu_\nu}} \ ,$$

$$\zeta_3^\star = 0 \ , \qquad \zeta_2^\star = \left(1 + \left(\zeta_1^\star\right)^2 - \frac{\alpha\zeta_1^\star}{\sqrt{12\pi}}\right)^{1/2} \quad .$$

C. Wetterich, "The Cosmon model for an asymptotically vanishing time dependent cosmological 'constant'," Astron. Astrophys. 301, 321–328 (1995), arXiv:hep- th/9408025.

L. Amendola, "Coupled quintessence," Phys. Rev. D 62, 043511 (2000), arXiv:astro-ph/9908023.

C. Wetterich, "Growing neutrinos and cosmological selection," Phys. Lett. B 655, 201–208 (2007), arXiv:0706.4427 [hep-ph].

# Inflationary quintessence.

We assume cold inflation throughout of our discussion. Warm inflation is conceptually different. It assumes that for sizable couplings of inflaton with the matter fields, the particle production becomes appreciable at early stages and the equation of motion for the inflaton gets modified by the presence of an additional friction term

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} = -U'(\phi) \ , \qquad \dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2 \ .$$

Of course, in all cold inflation models there are couplings of inflaton with the matter fields to produce an ordinary matter in the universe but it is assumed that particle production becomes appreciable only after the exit of inflation when field enters the oscillation regime near the minimum.

# Inflationary quintessence: $\mathcal{Z}_2$ symmetric potential and $\phi$-$\nu$ coupling

$$U(\phi) = V\left(1 - \mathrm{e}^{-\alpha\phi^2/M_P^2}\right) , \qquad m_\nu(\phi) = \mu_\nu \mathrm{e}^{-\beta\phi^2/M_P^2} .$$

The potential we have chosen has an (infinitely) long plateau for $\phi^2 \gtrsim M_P^2/\alpha$ and provides perfect conditions for the inflation as the field rolling down will arrive at the attractor from a very wide range of initial conditions. The slow-roll parameters are defined as

$$\epsilon = \frac{M_P^2}{16\pi}\left(\frac{U'}{U}\right)^2 , \quad \eta = \frac{M_P^2}{8\pi}\frac{U''}{U} , \quad \xi = \frac{M_P^4}{(8\pi)^2}\frac{U'U'''}{U^2} .$$

The end of inflation occurs for $\phi_f$ at which $\epsilon(\phi_f) \simeq 1$.

$$\phi_f \simeq \begin{cases} 0.27 \times M_P & \text{for} \quad \alpha = 1 , \\ 0.25 \times M_P & \text{for} \quad \alpha = 4 , \\ 0.22 \times M_P & \text{for} \quad \alpha = 9 , \\ 0.2 \times M_P & \text{for} \quad \alpha = 16 . \end{cases}$$

# Inflationary quintessence.

Demanding the e-folding number to be around 60, one finds

$$\phi_i \simeq \begin{cases} 1.35 \times M_P & \text{for} \quad \alpha = 1 \ , \\ 1.1 \times M_P & \text{for} \quad \alpha = 4 \ , \\ 0.81 \times M_P & \text{for} \quad \alpha = 9 \ , \\ 0.65 \times M_P & \text{for} \quad \alpha = 16 \ . \end{cases}$$

The slow-roll parameters can be used to express the spectral index, its derivative and tensor-to-scalar ratio as

$$n_s = 1 - 6\epsilon + 2\eta \ , \quad \frac{\mathrm{d} n_s}{\mathrm{d} \ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi \ , \quad r = 16\epsilon \ .$$

To fit the present observational data

$$n_s = 0.9649 \pm 0.0042 \ , \quad r < 0.06 \ , \quad \frac{\mathrm{d} n_s}{\mathrm{d} \ln k} = -0.0045 \pm 0.0067 \ ,$$

the parameter $\alpha$ should satisfy $\alpha \gtrsim 6.4$.

# Inflationary quintessence.

As to the the energy scale of inflation, $V$, it is commonly expected to lie approximately between the TeV and Planck scales. It is related to the amplitude of tensor modes

$$V^{1/4} \simeq 3.3 \times 10^{16} r^{1/4} \mathrm{GeV} \ ,$$

indicating that the "detectable" gravitational waves require

$$V^{1/4} \simeq 10^{16} \mathrm{GeV} \ ,$$

In general, such a big value is not typical for the existing models of inflation. In what follows we admit the whole "possible" range of parameter $V$ but for the discussion of nuggets it is favorable to take this parameter near the lower bound.

# Inflationary quintessence.

Instant preheating, which is inevitable for the runaway type potentials of quintessential inflation having no oscillation regime

C.-Q. Geng, Md. Wali Hossain, R. Myrzakulov, M. Sami, E. N. Saridakis, "Quintessential inflation with canonical and noncanonical scalar fields and Planck 2015 results," Phys.Rev.D 92 (2015) 2, 023522, arXiv:1502.03597 [gr-qc].

C.-Q. Geng, C.-C. Lee, M. Sami, E. N. Saridakis, and A. A. Starobinsky, "Observational constraints on successful model of quintessential Inflation," JCAP 06 (2017) 011, arXiv:1705.01329 [gr-qc].

is not required in the present case.

# Inflationary quintessence.

The role of neutrinos is that they activate inflaton at the early stage of Big-Bang (after the termalization of the early universe) leading to the dynamical braking of $\mathcal{Z}_2$ symmetry.

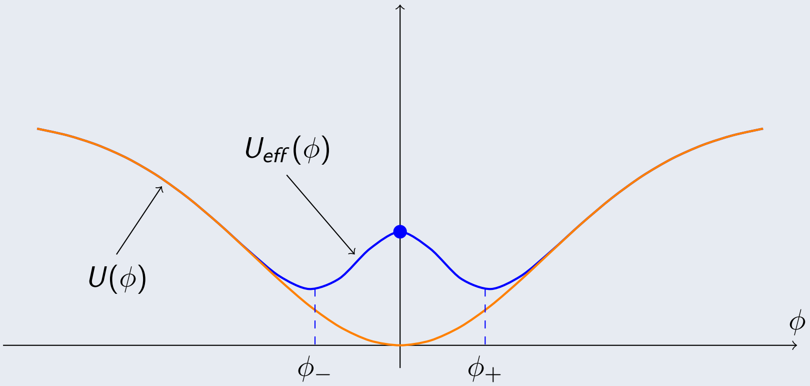


Figure: The symmetry breaking effective potential.

# Inflationary quintessence.

Above the decoupling temperature one has to use the equilibrium distribution for neutrinos, which for the effective potential gives

$$U_{eff}(\phi, T_\nu) = U(\phi) - p_\nu(\phi, T_\nu) \ .$$

Expanding $p_\nu(\phi, T_\nu)$ in a power series in $m_\nu^2/T_\nu^2$, one obtains

$$p_\nu(\phi, T_\nu) = \frac{\mathrm{g}\, T_\nu^4}{3}\left[\frac{7\pi^2}{240} - \frac{m_\nu^2}{16\, T_\nu^2} + O\left(\frac{m_\nu^4}{T_\nu^4}\right)\right] \ .$$

The $\phi$ dependent expression of the effective potential looks like

$$U(\phi) + \frac{\mathrm{g}\, m_\nu^2(\phi)\, T_\nu^2}{48} \ .$$

# Inflationary quintessence.

The onset of dark energy begins when $T_\nu \simeq m_\nu$. The effective potential

$$U_{eff}(\phi) = V\left(1 - \mathrm{e}^{-\alpha\phi^2/M_P^2}\right) + n_\nu \mu_\nu \mathrm{e}^{-\beta\phi^2/M_P^2} \ ,$$

has degenerate minima

$$\frac{\phi_\pm}{M_P} = \pm\sqrt{\frac{1}{(\beta-\alpha)} \ln \frac{\beta n_\nu \mu_\nu}{\alpha V}} \ .$$

When

$$\frac{\beta n_\nu \mu_\nu}{\alpha V} = 1 \ ,$$

the symmetry restoration takes place.

# Inflationary quintessence.

The neutrino masses increase in such a way

$$m_\nu(\phi_+) = \mu_\nu \mathrm{e}^{-\beta\phi_+^2/M_P^2} \approx \frac{\alpha V}{\beta n_\nu} \ ,$$

that the neutrino energy density

$$\rho_\nu = n_\nu m_\nu \approx \frac{\alpha V}{\beta} = \text{const.} \ .$$

The condition $\rho_{DE}^0 = 10\rho_\nu^0$ implies that $\beta \simeq 10^{58}$. It is curious to note that for this value of $\beta$ - the mass scale $M_P^2/\beta$ appearing in

$$m_\nu(\phi) = \mu_\nu \mathrm{e}^{-\beta\phi^2/M_P^2} \ ,$$

is of the order of $\mu_\nu^2$. That is, a natural reparametrization of coupling function is tantamount to the replacement $M_P \to \mu_\nu$ and therefore $\mu_\nu$ appears as a sole dimensional parameter in the coupling function.

# Inflationary quintessence.

Interestingly enough, the broad class of inflationary potentials derived in

R. Kallosh and A. Linde, "Universality Class in Conformal Inflation," JCAP 07 (2013) 002, arXiv:1306.5220 [hep-th].

as a result of spontaneously broken conformal symmetry, can be straightforwardly used in the above discussion with the same $\phi$-$\nu$ coupling term (which is certainly taken by hand).

The construction similar to what we have discussed may work without demanding $\mathcal{Z}_2$ symmetry for the inflaton potential. For instance, one may consider Starobinsky-like models.

Z. Kepuladze and M. Maziashvili, "New take on the inflationary quintessence," Phys. Rev. D103 (2021) 6, 063540; arXiv:2102.09203 [astro-ph.CO].

# Neutrino lumps.

Forgetting about gravitation, let us consider perturbations

$$\phi \to \phi_+ + \chi \ , \qquad \bar{\psi}_\nu \psi_\nu \to n_\nu + \bar{\psi}_\nu \psi_\nu \ .$$

The Lagrangian takes the form

$$\frac{\partial_\alpha \chi \partial^\alpha \chi}{2} - \frac{\Big(U''(\phi_+) + m_\nu''(\phi_+) n_\nu\Big)\chi^2}{2} + i\bar{\psi}\gamma^\alpha \partial_\alpha \psi \ - m_\nu'(\phi_+)\chi\bar{\psi}\psi + \text{C.C.} + \text{H.T.} \ ,$$

where C.C. denotes complex conjugate and H.T. stands for the higher order terms.

# Neutrino lumps.

One sees that there is an attractive force between the neutrinos mediated by the exchange of $\chi$ quanta. Its Yukawa potential

$$-\frac{\left(m'_\nu(\phi_+)\right)^2 \exp\left(-\sqrt{U''(\phi_+)+m''_\nu(\phi_+)n_\nu}\, r\right)}{4\pi r},$$

where $r$ stands for the physical distance, is characterized with the screening length

$$\frac{1}{\sqrt{U''(\phi_+)+m''_\nu(\phi_+)n_\nu}}=\frac{1}{\sqrt{U''_{eff}(\phi_+)}}\equiv\frac{1}{m_{eff}(\phi_+)}.$$

This force leads to the growth of neutrino density perturbations.

N. Afshordi, M. Zaldarriaga and K. Kohri, "On the stability of dark energy with mass-varying neutrinos," Phys. Rev. D72 (2005) 065024; arXiv:astro-ph/0506663.

# Top issues to be addressed.

How does model look after the neutrino clumping? In other words, what's the back-reaction of these climps on the cosmological evolution?

The stability of mass-varying neutrino models against the radiative corrections. As most of the models are non-renormalizable, it's hard to evaluate the size of one-loop corrections "properly".

What's the proper picture of above described inflationary quintessence model at early stages (prior to the Big-Bang)?

Thanking you for your kind interest!