QCD Exotica

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Outline

1. **Inroduction**

2. **Hadron resonances discovered since 2003**
   - Open-flavor heavy mesons
   - XYZ states
   - Pentaquark candidates

3. **Theory ideas and applications**
   - Approximate symmetries of QCD and applications to exotics
   - Compositeness and hadronic molecules
   - Threshold cusps and triangle singularties

Introduction

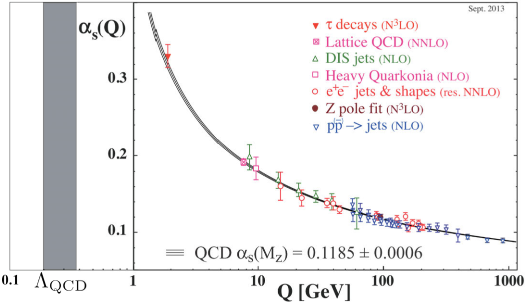
Two recent reviews:

- S. L. Olsen, T. Skwarnicki, *Nonstandard heavy mesons and baryons: Experimental evidence*, Rev. Mod. Phys. 90 (2018) 015003 [arXiv:1708.04012]
  experimental facts and interpretations
- F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.-S. Zou, *Hadronic molecules*, Rev. Mod. Phys. 90 (2018) 015004 [arXiv:1705.00141]
  theoretical formalisms

Many more in the last few years, the latest one:

N. Brambilla et al., *The XYZ states: experimental and theoretical status and perspectives*, arXiv:1907.07583

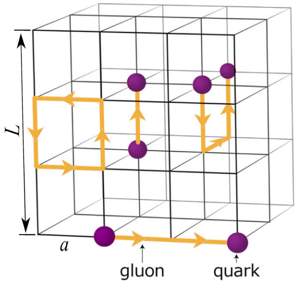
- Running of the coupling constant $\alpha_s = g_s^2/(4\pi)$

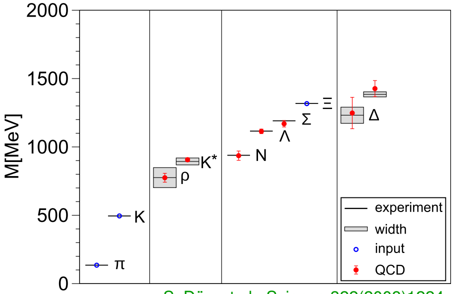


- High energies
  - ☞ asymptotic freedom, perturbative
  - ☞ degrees of freedom: quarks and gluons
- Low energies
  - ☞ nonperturbative, $\Lambda_{\rm QCD} \sim 250$ MeV $= \mathcal{O}\left(1 \text{ fm}^{-1}\right)$
  - ☞ color confinement, detected particles: mesons and baryons

    $\Rightarrow$ challenge: how do hadrons emerge/how is QCD spectrum organized?

- Lattice QCD: in discretized Euclidean space-time — lectures by Tom Luu
  - finite volume (L should be large)
  - finite lattice spacing (a should be small)
  - often using $m_{u,d}$ larger than the physical values $\Rightarrow$ chiral extrapolation





S. Dürr et al., Science 322(2008)1224

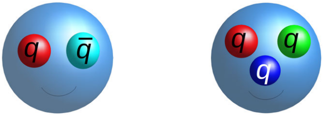
- Phenomenological models, such as quark model, QCD sum rules , ...
- Low-energy EFT: — lecture by Ulf-G. Meißner

  mesons and baryons as effective degrees of freedom

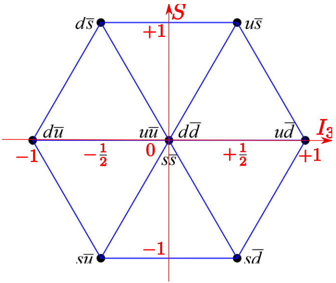
  most important ingredients: symmetries (see later), power counting

u	d	s		c	b	t
~2 MeV	~5 MeV	~100 MeV	$\ll \Lambda_{\rm QCD} \ll$	~1.3 GeV	~4.2 GeV	~173 GeV
Light quarks				Heavy quarks		

☞ Mesons and baryons in quark model



Light meson SU(3) [u, d, s] multiplets (octet + singlet):



- Vector mesons

meson	quark content	mass (MeV)
ρ^+/ρ^-	$u\bar{d}\,/\,d\bar{u}$	775
ρ^0	$(u\bar{u} - d\bar{d})/\sqrt{2}$	775
K^{*+}/K^{*-}	$u\bar{s}\,/\,s\bar{u}$	892
$K^{*0}/\bar{K}^{*0}$	$d\bar{s}\,/\,s\bar{d}$	896
ω	$(u\bar{u} + d\bar{d})/\sqrt{2}$	783
ϕ	$s\bar{s}$	1019

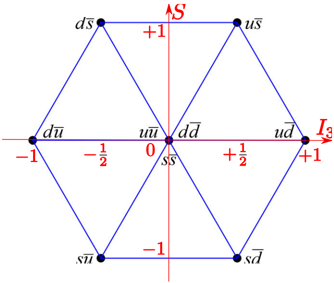
☞ approximate SU(3) symmetry

$$m_\rho \simeq m_\omega, \qquad m_\phi - m_{K^*} \simeq m_{K^*} - m_\rho$$

☞ very good isospin SU(2) symmetry

$$m_{\rho^0} - m_{\rho^\pm} = (-0.7 \pm 0.8)\ \text{MeV}, \quad m_{K^{*0}} - m_{K^{*\pm}} = (6.7 \pm 1.2)\ \text{MeV}$$

Light meson SU(3) $[u, d, s]$ multiplets (octet + singlet):



- Pseudoscalar mesons

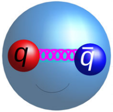
meson	quark content	mass (MeV)
π^+/π^-	$u\bar{d}\,/\,d\bar{u}$	140
π^0	$(u\bar{u} - d\bar{d})/\sqrt{2}$	135
K^+/K^-	$u\bar{s}\,/\,s\bar{u}$	494
$K^0/\bar{K}^0$	$d\bar{s}\,/\,s\bar{d}$	498
η	$\sim (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$	548
η'	$\sim (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$	958

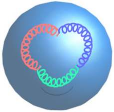
☞ very good isospin SU(2) symmetry

$$m_{\pi^\pm} - m_{\pi^0} = (4.5936 \pm 0.0005)\ \text{MeV}, \quad m_{K^0} - m_{K^\pm} = (3.937 \pm 0.028)\ \text{MeV}$$

☞ Q: Why are the pions so light?

- Quark model notation:
  any hadron resonances beyond picture of $q\bar{q}$ for a meson and qqq for a baryon
  - ☞ Gluoinc excitations: hybrids and glueballs

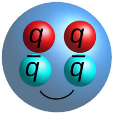


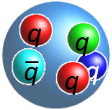


  - ☞ Multiquark states

A Schematic Model of Baryons and Mesons

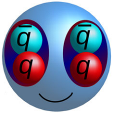
M.Gell-Mann, Phys.Lett.**8**(1964)214-215





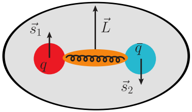
We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest baryon configuration (qqq) gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\bar{q})$ similarly gives just **1** and **8**.

- Hadronic molecules:
  bound states of two or more hadrons,
  analogues of nuclei



J^{PC} and exotic quantum numbers

- J^{PC} of regular $q\bar{q}$ mesons



  L: orbital angular momentum
  $S = (0, 1)$: total spin of q and $\bar{q}$
  $P = (-1)^{L+1}$ $\quad [Y_{Lm}(\theta - \pi, \phi + \pi) = (-1)^L Y_{Lm}(\theta, \phi)]$
  $C = (-1)^{L+S} = (-1)^{L+1+S+1}$ for flavor-neutral mesons

  $$S = 0\text{: } \frac{1}{\sqrt{2}} \left|\uparrow_q \downarrow_{\bar{q}} - \downarrow_q \uparrow_{\bar{q}}\right\rangle; \quad S = 1\text{: } \left\{ \left|\uparrow_q \uparrow_{\bar{q}}\right\rangle, \frac{1}{\sqrt{2}} \left|\uparrow_q \downarrow_{\bar{q}} + \downarrow_q \uparrow_{\bar{q}}\right\rangle, \left|\downarrow_q \downarrow_{\bar{q}}\right\rangle \right\}$$

  ☞ For $S = 0$, the meson spin $J = L$, one has $P = (-1)^{J+1}$ and $C = (-1)^J$. Hence,

  $$J^{PC} = \text{even}^{-+} \text{ and odd}^{+-}$$

  ☞ For $S = 1$, one has $P = C = (-1)^{L+1}$. Hence,

  $$J^{PC} = 1^{--}, \{0, 1, 2\}^{++}, \{1, 2, 3\}^{--}, \ldots$$

- Exotic J^{PC} for mesons:

  $$\boxed{J^{PC} = 0^{--}, \text{even}^{+-} \text{ and odd}^{-+}}$$

Mesons with exotic quantum numbers

Both listed as extablished particles by the PDG

$\pi_1(1400)$ $I^G(J^{PC}) = 1^-(1^{-+})$

See also the mini-review under non- $q\bar{q}$ candidates in PDG 2006 , Journal of Physics G33 1 (2006).

$\pi_1(1400)$ MASS	1354 ± 25 MeV (S = 1.8)
$\pi_1(1400)$ WIDTH	330 ± 35 MeV

Decay Modes

Mode		Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level	P (MeV/c)
Γ_1	$\eta\pi^0$	seen		557
Γ_2	$\eta\pi^-$	seen		556
Γ_3	$\eta'\pi$			318
Γ_4	$\rho(770)\pi$	not seen		442

$\pi_1(1600)$ $I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1600)$ MASS	1660^{+15}_{-11} MeV (S = 1.2)
$\pi_1(1600)$ WIDTH	257 ± 60 MeV (S = 1.9)

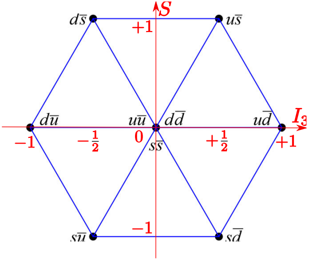
Decay Modes

Mode		Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level	P (MeV/c)
Γ_1	$\pi\pi\pi$	seen		802
Γ_2	$\rho^0\pi^-$	seen		640
Γ_3	$f_2(1270)\pi^-$	not seen		316
Γ_4	$b_1(1235)\pi$	seen		355
Γ_5	$\eta'(958)\pi^-$	seen		542
Γ_6	$f_1(1285)\pi$	seen		312

It is unclear what they are: hybrids? hadronic molecules? or sth. else?

Some trivial facts about additive quantum numbers of regular mesons

- Light-flavor mesons (here S =strangeness)
  - Nonstrange mesons: $S = 0$, $I = 0, 1$
  - Strange mesons: $S = \pm 1$, $I = \frac{1}{2}$
- Open-flavor heavy mesons
  - $Q\bar{q}(q = u, d)$: $S = 0$, $I = 1/2$
  - $Q\bar{s}$: $S = 1$, $I = 0$
- Heavy quarkonia ($Q\bar{Q}$): $S = 0$, $I = 0$, neutral



Charge, isospin, strangeness etc. which cannot be achieved in the $q\bar{q}$ and qqq scheme would be a smoking gun for an exotic nature

more subtleties later...

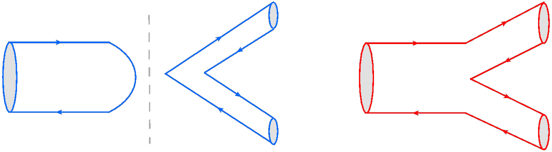
Decay patterns: regular hadrons

- SU(3) flavor symmetry is usually satisfied to 30%

Example

$$\frac{\Gamma(K^{*+})}{\Gamma(\rho^+)} = \frac{50\text{ MeV}}{150\text{ MeV}} = 0.33\text{ [exp]}, \quad \frac{3}{4}\left(\frac{M_\rho}{M_{K^*}}\right)^2\left(\frac{q_{K\pi}}{q_{\pi\pi}}\right)^3 = 0.29\text{ [SU(3)]}$$

- Okubo–Zweig–Iizuka (OZI) rule:



drawing only quark lines, the disconnected diagrams are strongly suppressed relative to the connected ones [explanation from large N_c (number of color)]

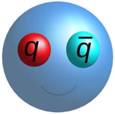
Example

$\psi(3770)$: ∼40 MeV above the $D\bar{D}$ threshold

$$\mathcal{B}(D\bar{D}) = (93^{+8}_{-9})\% \gg \mathcal{B}(\text{sum of all other modes})$$

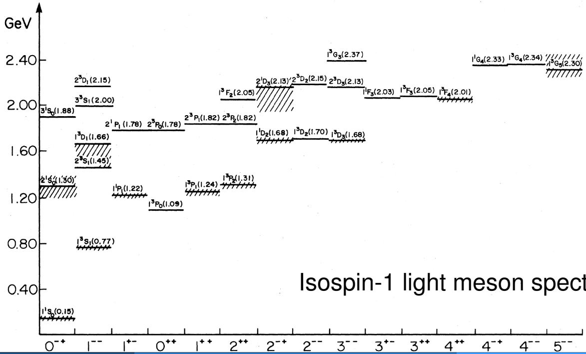
Mesons in a Relativized Quark Model with Chromodynamics

S. Godfrey, Nathan Isgur (Toronto U.). 1985. 43 pp.
Published in **Phys.Rev. D32 (1985) 189-231**
DOI: 10.1103/PhysRevD.32.189
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
OSTI.gov Server
Detailed record - Cited by 2488 records 1000+



$$\left(\sqrt{m_1^2+p^2}+\sqrt{m_2^2+p^2}+V\right)|\Psi\rangle = E|\Psi\rangle$$

Potential V: One-gluon exchange + linear confinement + relativistic effects



Isospin-1 light meson spectroscopy

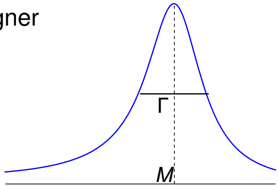
New discoveries since 2003

Many new hadron resonances observed in experiments since 2003

- Inactive: BaBar, Belle, CDF, CLEO-c, D0, ...
- Running: Belle-II, BESIII, COMPASS, LHCb, ...
- Under construction/discussion: PANDA, EIC, EicC, ...

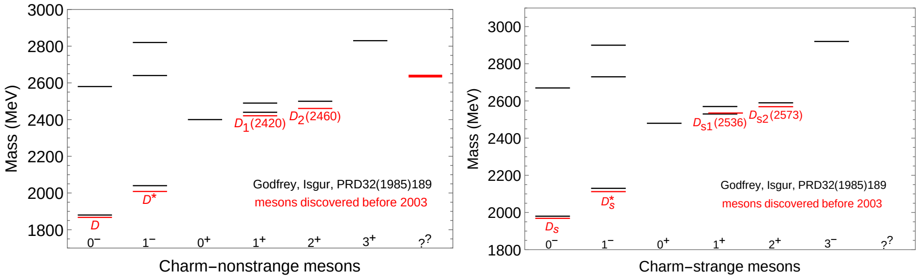
Common strategy: search for peaks, fit with Breit–Wigner

$$\propto \frac{1}{(s-M^2)^2 + s\,\Gamma^2(s)}$$



Lots of mysteries right now ...

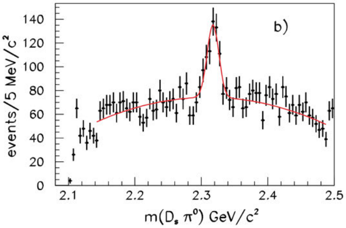
Open-flavor heavy mesons

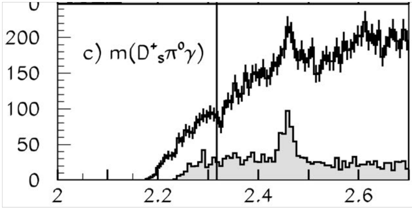


Most quark-model predicted states were still missing before 2003

Discoveries in 2003 (both Belle and BaBar started data taking in 1999):

- $D^*_{s0}(2317)$: discovered in $e^+e^- \to D_s^+\pi^0 X$ BaBar, PRL90(2003)242001 [hep-ex/0304021]





$J^P = 0^+$, $M = (2317.7 \pm 0.6)$ MeV, $\Gamma < 3.8$ MeV

$I = 0$, $\to D_s\pi^0$: breaks isospin symmetry

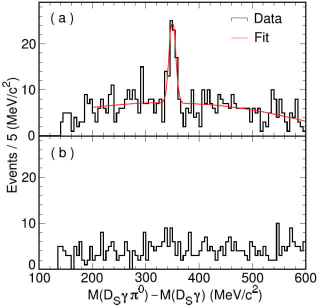
- $D_{s1}(2460)$: discovered in $e^+e^- \to D_s^{*+}\pi^0 X$

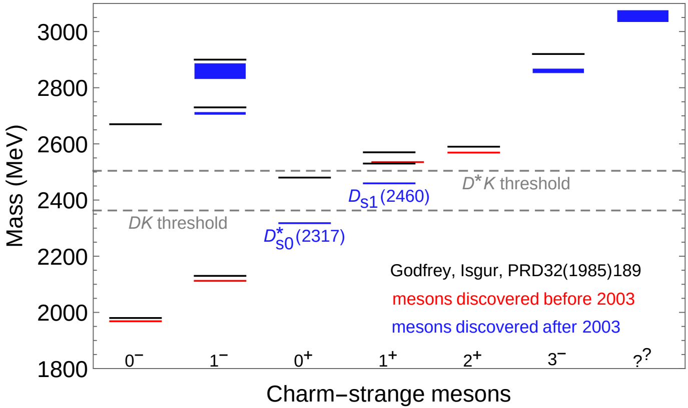
CLEO, PRD68(2003)032002 [hep-ex/0305100]

$J^P = 1^+$, $M = (2459.5 \pm 0.6)$ MeV, $\Gamma < 3.5$ MeV

$I = 0$, $\to D_s^*\pi^0$: breaks isospin symmetry

other decays: $D_s^+\gamma$, $D_s^+\pi^+\pi^-$, $D^*_{s0}(2317)\gamma$





$D_{s0}^*(2317)$ and $D_{s1}(2460)$: the first established new hadrons

- Puzzle 1: Why are $D_{s0}^*(2317)$ and $D_{s1}(2460)$ so light?
- Puzzle 2: Why $\underbrace{M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)}}_{=(141.8\pm0.8)\ \text{MeV}} \simeq \underbrace{M_{D^{*\pm}} - M_{D^\pm}}_{=(140.67\pm0.08)\ \text{MeV}}$?

Obervations of charm-nonstrange excited mesons in 2003

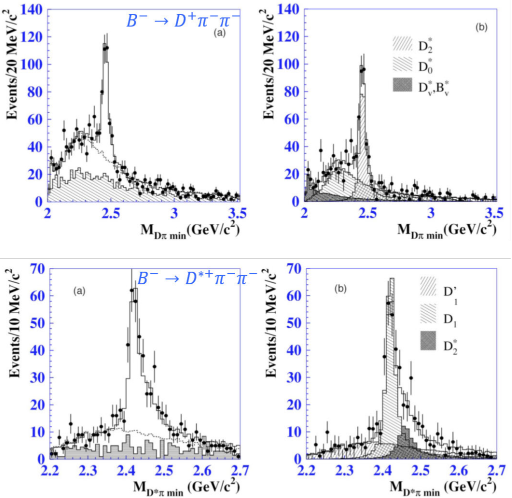
$B^- \to D^{(*)+}\pi^-\pi^-$ Belle, PRD69(2004)112002 [hep-ex/0307021]

- $D_0^*(2300)$ [was called $D_0^*(2400)$]: $J^P = 0^+$

  $\Gamma = (274 \pm 40)$ MeV

  Mass (MeV):

2300 ± 19	PDG19	
2297 ± 22	BaBar	B decays
2308 ± 36	Belle	B decays
2401 ± 41	FOCUS	γA
2360 ± 34	LHCb	B decays

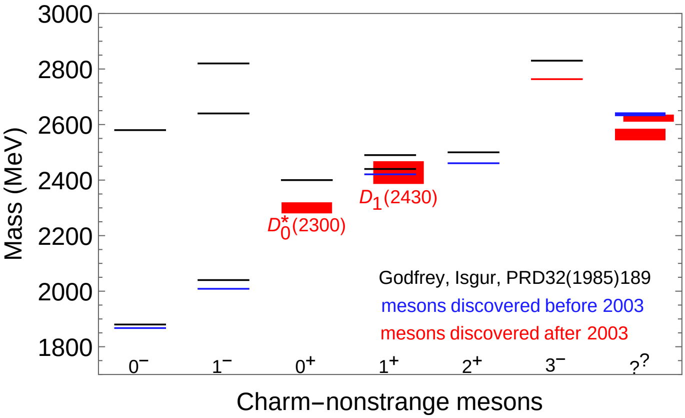


- $D_1(2430)$: $J^P = 1^+$

  $\Gamma = 384^{+130}_{-110}$ MeV

  $M = (2427 \pm 36)$ MeV

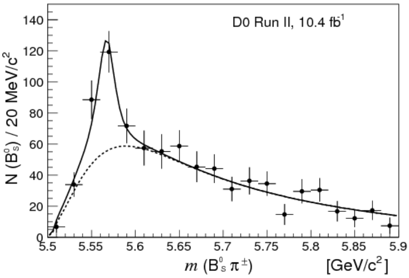
Charm-nonstrange mesons (2)



- Puzzle 3: Why $M_{D_0^*(2300)} \gtrsim M_{D_{s0}^*(2317)}$ and $M_{D_1(2430)} \sim M_{D_{s1}(2460)}$?

Most exotic: $X(5568)$

- $X(5568)$ by D0 Collaboration ($p\bar{p}$ collisions) PRL117(2016)022003; PRD97(2018)092004



$$M = \left(5567.8 \pm 2.9^{+0.9}_{-1.9}\right) \text{ MeV}$$

$$\Gamma = \left(21.9 \pm 6.4^{+5.0}_{-2.5}\right) \text{ MeV}$$

- Observed in $B_s^{(*)0}\pi^+$, sizeable width $\Rightarrow I = 1$:

  minimal quark contents is $\bar{b}s\bar{d}u$!
- a favorite mulqituark candidate:

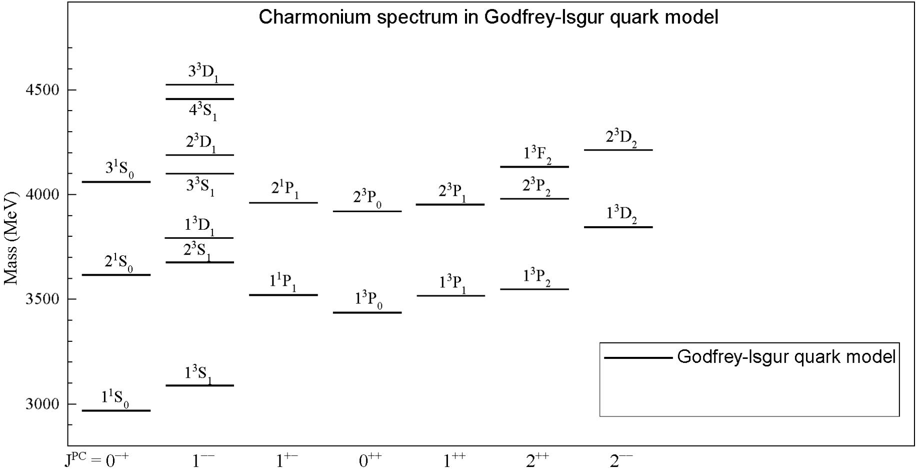
  explicitly flavor exotic, minimal number of quarks ≥ 4

Estimate of isospin breaking decay width:

$$\Gamma_{I\!\!/} \sim \left(\frac{\left(\frac{m_d - m_u}{\Lambda_{\rm QCD}}\right)^2}{\alpha^2} \right) \times \mathcal{O}\left(100\text{ MeV}\right)$$

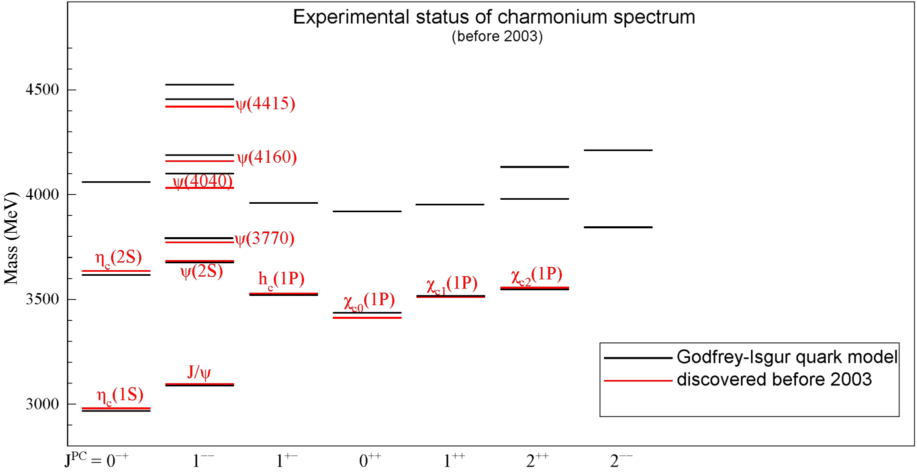
$$= \mathcal{O}\left(10\text{ keV}\right)$$

XYZ states



Charmonium spectrum in Godfrey-Isgur quark model
Mass (MeV)
4500
4000
3500
3000
3^3D_1
4^3S_1
2^3D_1
3^1S_0
3^3S_1
2^1P_1
2^3P_0
2^3P_1
1^3F_2
2^3P_2
2^3D_2
1^3D_2
1^3D_1
2^3S_1
2^1S_0
1^1P_1
1^3P_0
1^3P_1
1^3P_2
1^3S_1
1^1S_0
Godfrey-Isgur quark model
J^PC = 0^-+
1^--
1^+-
0^++
1^++
2^++
2^--

XYZ states


Experimental status of charmonium spectrum
(before 2003)
Mass (MeV)
4500
4000
3500
3000
ψ(4415)
ψ(4160)
ψ(4040)
ψ(3770)
η_c(2S)
ψ(2S)
h_c(1P)
χ_c0(1P)
χ_c1(1P)
χ_c2(1P)
J/ψ
η_c(1S)
Godfrey-Isgur quark model
discovered before 2003
J^PC = 0^−+
1^−−
1^+−
0^++
1^++
2^++
2^−−

Naming convention

For states with properties in conflict with naive quark model (normally):

- X: $I=0$, J^{PC} other than 1^{--} or unknown
- Y: $I=0$, $J^{PC}=1^{--}$
- Z: $I=1$

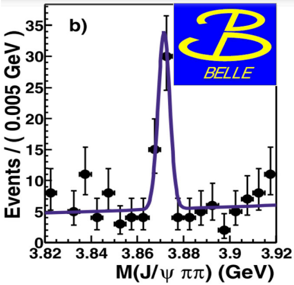
PDG2018 naming scheme:

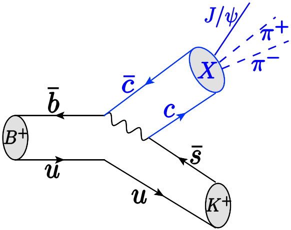
$J^{PC} =$	$0^{-+}, 2^{-+}, \ldots$	$1^{+-}, 3^{+-}, \ldots$	$1^{--}, 2^{--}, \ldots$	$0^{++}, 1^{++}, \ldots$
Minimal quark content				
$u\bar{d}, u\bar{u}-d\bar{d}, d\bar{u}$ $(I=1)$	π	b	ρ	a
$d\bar{d}+u\bar{u}$ and/or $s\bar{s}$ $(I=0)$	η, η'	h, h'	ω, ϕ	f, f'
$c\bar{c}$	η_c	h_c	$\psi^\dagger$	χ_c
$b\bar{b}$	η_b	h_b	Υ	χ_b
$I=1$ with $c\bar{c}$	(Π_c)	Z_c	R_c	(W_c)
$I=1$ with $b\bar{b}$	(Π_b)	Z_b	(R_b)	(W_b)

†The J/ψ remains the J/ψ.

"Young man, if I could remember the names of these particles, I would have been a botanist." — *Enrico Fermi*

Belle, PRL91(2003)262001 [hep-ex/0309032]





- Named as $\chi_{c1}(3872)$ since PDG18
- The beginning of the XYZ story, discovered in $B^{\pm} \to K^{\pm} J/\psi\pi\pi$

  $M_X = (3871.69 \pm 0.17)$ MeV
- $\Gamma < 1.2$ MeV Belle, PRD84(2011)052004
- Confirmed in many experiments: Belle, BaBar, BESIII, CDF, CMS, D0, LHCb, ...
- 10 years later, $J^{PC} = 1^{++}$

  LHCb, PRL110(2013)222001

  $\Rightarrow$ S-wave coupling to $D\bar{D}^*$

Mysterious properties:

- Mass coincides with the $D^0\bar{D}^{*0}$ threshold:

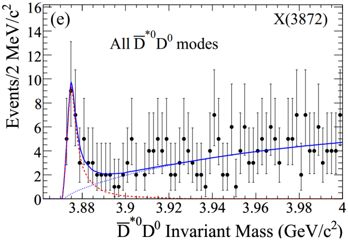
  $M_{D^0} + M_{D^{*0}} - M_X = (0.00 \pm 0.18)$ MeV

Mysterious properties (cont.):

- Large coupling to $D^0\bar{D}^{*0}$:

  $\mathcal{B}(X \to D^0\bar{D}^{*0}) > 30\%$ Belle, PRD81(2010)031103

  $\mathcal{B}(X \to D^0\bar{D}^0\pi^0) > 40\%$ Belle, PRL97(2006)162002



BaBar, PRD77(2008)011102

- No isospin partner observed $\Rightarrow I = 0$

  but, large isospin breaking:

$$\frac{\mathcal{B}(X \to \omega J/\psi)}{\mathcal{B}(X \to \pi^+\pi^- J/\psi)} = 0.8 \pm 0.3$$

$$C(X) = +, C(J/\psi) = - \quad \Rightarrow C(\pi^+\pi^-) = - \quad \Rightarrow I(\pi^+\pi^-) = 1$$

- Radiative decays:

$$\frac{\mathcal{B}(X \to \gamma\psi')}{\mathcal{B}(X \to \gamma J/\psi)} = 2.6 \pm 0.6$$

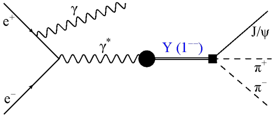
PDG18 average of BaBar(2009) and LHCb(2014) measurements

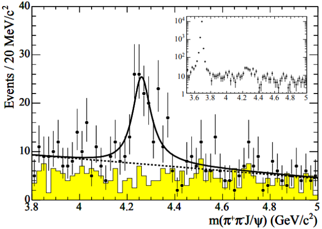
Exercise:

1) Why is the isospin of the negative C-parity $\pi^+\pi^-$ system equal to 1?

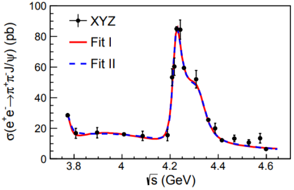
2) Is $\Upsilon\pi^+\pi^-$ a good choice of final states for the search of X_b, the $J^{PC} = 1^{++}$ bottom analogue of the $X(3872)$?

- Discovered by BaBar in 2005 PRL95(2005)142001

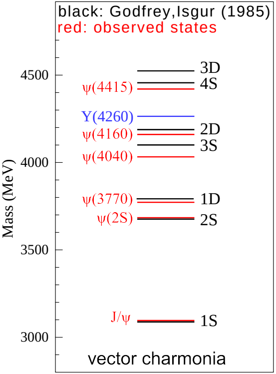




$J^{PC} = 1^{--}$, confirmed by Belle, CLEO, BESIII



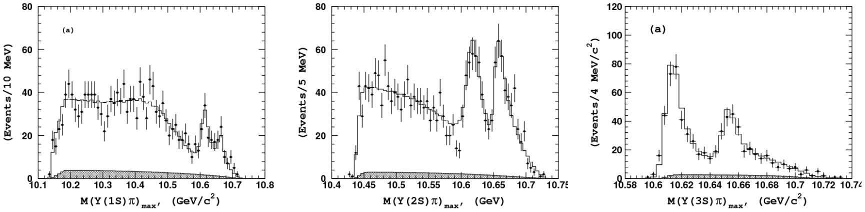
BESIII, PRL118(2017)092001



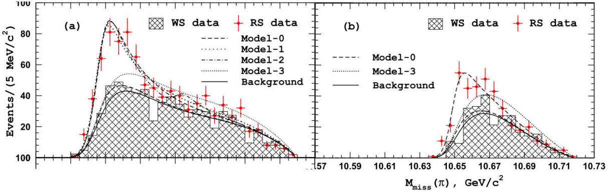
- $M = (4230 \pm 8)$ MeV, $\Gamma = (55 \pm 19)$ MeV PDG2019
- Puzzles:
  - ☞ no slot in the quark model
  - ☞ well above $D\bar{D}$ threshold, but not seen in $D\bar{D}$ (recall the OZI rule)

$Z_c^\pm$ and $Z_b^\pm$ (1)

- $Z_c^\pm$, $Z_b^\pm$: charged structures in heavy quarkonium mass region, excellent tetraquark candidates: $Q\bar{Q}\bar{d}u, Q\bar{Q}\bar{u}d$
- $Z_b(10610)^\pm$ and $Z_b(10650)^\pm$: Belle, arXiv:1105.4583; PRL108(2012)122001
  observed in $\Upsilon(10860) \to \pi^\mp[\pi^\pm\Upsilon(1S,2S,3S)/h_b(1P,2P)]$

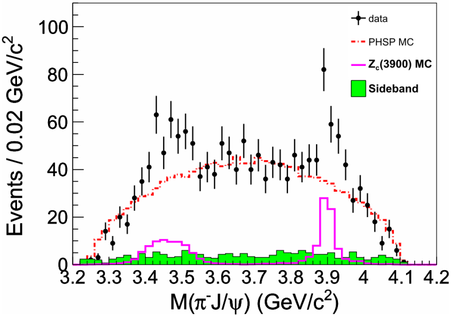


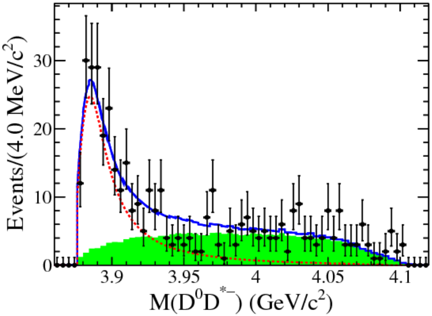
also in $\Upsilon(10860) \to \pi^\mp[B^{(*)}\bar{B}^*]^\pm$ Belle, arXiv:1209.6450; PRL116(2016)212001



- $Z_b(10610)^\pm$ and $Z_b(10650)^\pm$ very close to $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds

- $Z_c(3900)^{\pm}$: structure around 3.9 GeV seen in $J/\psi\pi^{\pm}$ by BESIII and Belle in $Y(4260) \to J/\psi\pi^+\pi^-$, BESIII, PRL110(2013)252001; Belle, PRL110(2013)252002
  and in $D\bar{D}^*$ by BESIII in $Y(4260) \to \pi^{\pm}(D\bar{D}^*)^{\mp}$ BESIII, PRD92(2015)092006



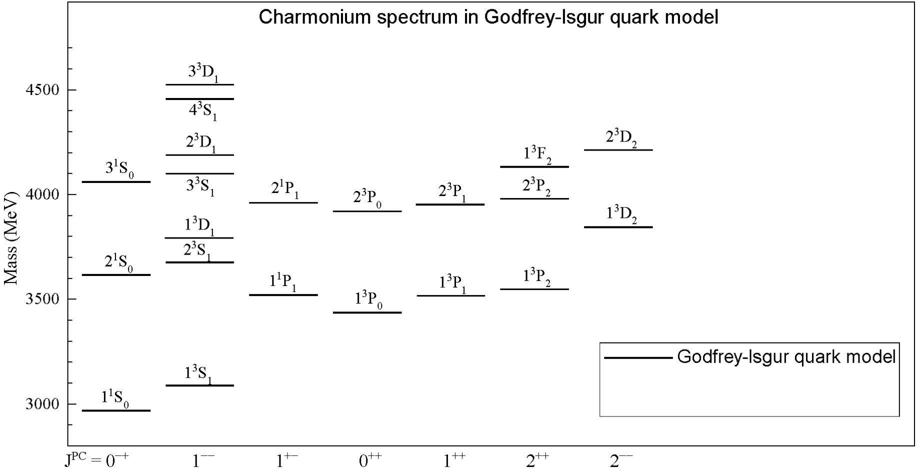


- $Z_c(4020)^{\pm}$ observed in $h_c\pi^{\pm}$ and $(\bar{D}^*D^*)^{\pm}$ distributions

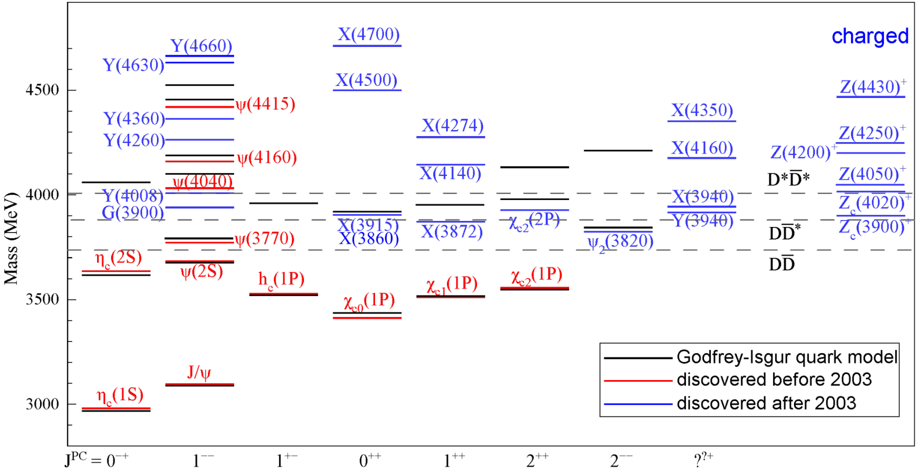
  BESIII, PRL111(2013)242001; PRL112(2014)132001

- $Z_c(3900)^{\pm}$ and $Z_c(4020)^{\pm}$ very close to $D\bar{D}^*$ and $D^*\bar{D}^*$ thresholds

Charmonium spectrum: current status

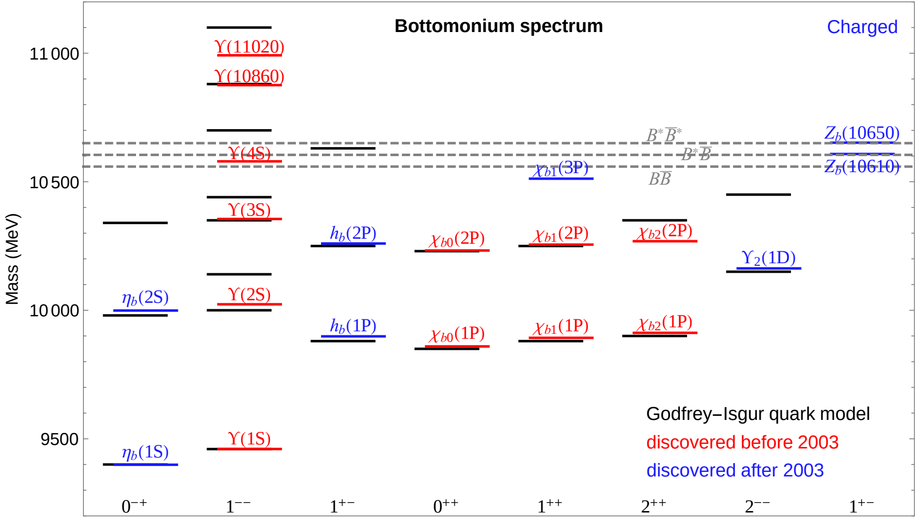


Charmonium spectrum: current status



Note:

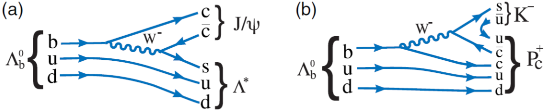
- $X(3915)$ is probably just the $\chi_{c2}(2P)$ with 2^{++} Z.-Y. Zhou et al., PRL115(2015)022001
- $\psi_3(3^{--})$ recently discovered: $M = 3842.71 \pm 0.20$ MeV LHCb, JHEP1907(2019)035

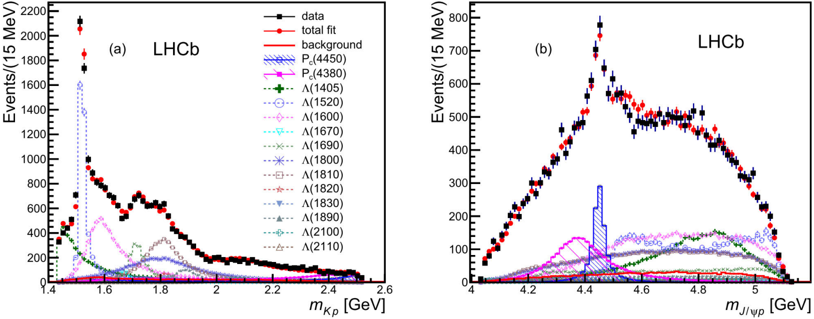

Bottomonium spectrum
Charged
Mass (MeV)
11 000
10 500
10 000
9500
Υ(11020)
Υ(10860)
Υ(4S)
Υ(3S)
Υ(2S)
Υ(1S)
η_b(2S)
η_b(1S)
h_b(2P)
h_b(1P)
χ_{b0}(2P)
χ_{b0}(1P)
χ_{b1}(3P)
χ_{b1}(2P)
χ_{b1}(1P)
χ_{b2}(2P)
χ_{b2}(1P)
Υ_2(1D)
Z_b(10650)
Z_b(10610)
$B^*\bar{B}^*$
$B^*\bar{B}$
$B\bar{B}$
Godfrey–Isgur quark model
discovered before 2003
discovered after 2003
0^{-+}
1^{--}
1^{+-}
0^{++}
1^{++}
2^{++}
2^{--}
1^{+-}

Pentaquark candidates

Discovered in $\Lambda_b^0 \to J/\psi p K^-$

LHCb, PRL115(2015)072001 [arXiv:1507.03414]





Two Breit–Wigner resonances needed:

$$M_1 = (4380 \pm 8 \pm 29)\ \text{MeV}, \qquad \Gamma_1 = (205 \pm 18 \pm 86)\ \text{MeV},$$

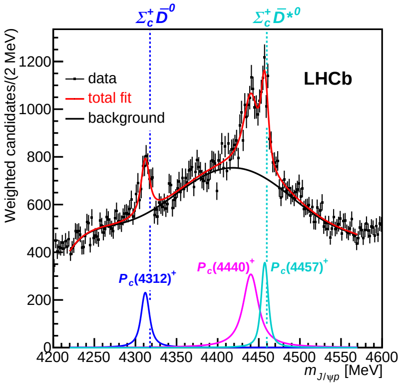
$$M_2 = (4449.8 \pm 1.7 \pm 2.5)\ \text{MeV}, \qquad \Gamma_2 = (39 \pm 5 \pm 19)\ \text{MeV}.$$

- In $J/\psi\, p$ invariant mass distribution, with hidden charm
  $\Rightarrow$ pentaquarks if they are hadron resonances
- Quantum numbers not fully determined, for ($P_c(4380), P_c(4450)$):
  $(3/2^-, 5/2^+)$, $(3/2^+, 5/2^-)$, $(5/2^+, 3/2^-)$, ... LHCb, PRL115(2015)072001

  From a reanalysis using an extended Λ^* model: N. Jurik, CERN-THESIS-2016-086

		$P_c(4380)$		$P_c(4450)$	
$J^p(4380, 4450)$	$(\sqrt{\Delta(-2\ln\mathcal{L})})^2$	M_0	Γ_0	M_0	Γ_0
	$(3/2-, 5/2^+)$ solution				
$3/2^-, 5/2^+$	$--$	4359	151	4450.1	49
	Δ from $(3/2-, 5/2^+)$ solution				
$5/2^+, 3/2^-$	-3.6^2	10	-7	-1.6	-6
$5/2^-, 3/2^+$	-2.7^2	-4	-9	-3.6	-2
$3/2^-, 5/2^+$	–	–	–	–	–

- Early prediction:
  Prediction of narrow N^* and Λ^* resonances with hidden charm above 4 GeV,
  J.-J. Wu, R. Molina, E. Oset, B.-S. Zou, PRL105(2010)232001



State	M [MeV]	Γ [MeV]	(95% CL)	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+\ 3.7}_{-\ 4.5}$	(< 27)	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+\ 8.7}_{-10.1}$	(< 49)	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+\ 5.7}_{-\ 1.9}$	(< 20)	$0.53 \pm 0.16^{+0.15}_{-0.13}$

$$\mathcal{R} \equiv \mathcal{B}(\Lambda_b^0 \to P_c^+ K^-)\mathcal{B}(P_c^+ \to J/\psi p)/\mathcal{B}(\Lambda_b^0 \to J/\psi p K^-)$$

Recent reviews on new hadrons (incomplete list)

- H.-X. Chen et al., *The hidden-charm pentaquark and tetraquark states*, Phys. Rept. 639 (2016) 1 [arXiv:1601.02092]
- A. Hosaka et al., *Exotic hadrons with heavy flavors — X, Y, Z and related states*, Prog. Theor. Exp. Phys. 2016, 062C01 [arXiv:1603.09229]
- R. F. Lebed, R. E. Mitchell, E. Swanson, *Heavy-quark QCD exotica*, Prog. Part. Nucl. Phys. 93 (2017) 143, arXiv:1610.04528 [hep-ph]
- A. Esposito, A. Pilloni, A. D. Polosa, *Multiquark resonances*, Phys. Rept. 668 (2017) 1 [arXiv:1611.07920]
- F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.-S. Zou, *Hadronic molecules*, Rev. Mod. Phys. 90 (2018) 015004 [arXiv:1705.00141]
- S. L. Olsen, T. Skwarnicki, *Nonstandard heavy mesons and baryons: Experimental evidence*, Rev. Mod. Phys. 90 (2018) 015003 [arXiv:1708.04012]
- M. Karliner, J. L. Rosner, T. Skwarnicki, *Multiquark states*, Ann. Rev. Nucl. Part. Sci. 68 (2018) 17 [arXiv:1711.10626]
- C.-Z. Yuan, *The XYZ states revisited*, Int. J. Mod. Phys. A 33 (2018) 1830018 [arXiv:1808.01570]
- N. Brambilla et al., *The XYZ states: experimental and theoretical status and perspectives*, Phys. Rept. [arXiv:1907.07583]

Approximate symmetries of QCD: chiral and heavy quark

Useful monographs:

- H. Georgi, *Weak Interactions and Modern Particle Physics* (2009)
- J.F. Donoghue, E. Golowich, B.R. Holstein, *Dynamics of the Standard Model* (1992)
- S. Scherer, M.R. Schindler, *A Primer for Chiral Perturbation Theory* (2012)
- A.V. Manohar, M.B. Wise, *Heavy Quark Physics* (2000)

Symmetries for different sectors

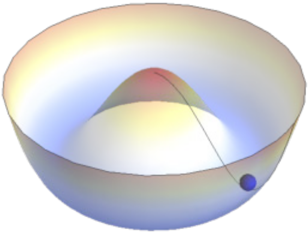
- Different quark flavors:

u	d	s		c	b	t
~2 MeV	~5 MeV	~100 MeV	$\ll \Lambda_{\rm QCD} \ll$	~1.3 GeV	~4.2 GeV	~173 GeV

Light quarks (u, d, s) — Heavy quarks (c, b, t)

☞ Spontaneously broken chiral symmetry: π, K and η as the pseudo-Goldstone bosons

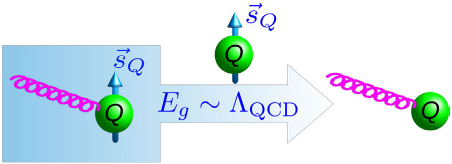
lectures by Ulf



☞ Heavy quark spin symmetry

☞ Heavy quark flavor symmetry

☞ Heavy antiquark-diquark symmetry

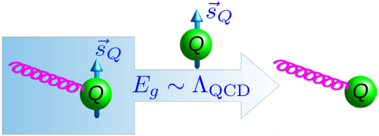


- For heavy quarks (charm, bottom) in a hadron, typical momentum transfer Λ_{QCD}

  ☞ heavy quark spin symmetry (HQSS):

  chromomag. interaction $\propto \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{m_Q}$

  spin of the heavy quark decouples



  Let total angular momentum $\boldsymbol{J} = \boldsymbol{s}_Q + \boldsymbol{s}_\ell$,

  $\boldsymbol{s}_Q$: heavy quark spin,

  $\boldsymbol{s}_\ell$: spin of the light degrees of freedom (including orbital angular momentum)

  ✓ HQSS:

  s_ℓ and s_Q are conserved separately in the heavy quark limit!

  ✓ spin multiplets:

  for singly heavy mesons, e.g. $\{D, D^*\}$, $\{B, B^*\}$ with $s_\ell^P = \frac{1}{2}^-$;

  for heavy quarkonia, e.g. S-wave: $\{\eta_c, J/\psi\}$, $\{\eta_b, \Upsilon\}$;

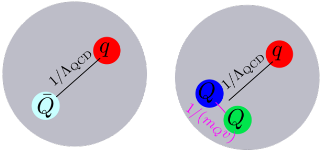
  P-wave: $\{h_c, \chi_{c0,c1,c2}\}$, $\{h_b, \chi_{b0,b1,b2}\}$

Heavy quark symmetries (2)

- For heavy quarks (charm, bottom) in a hadron, typical momentum transfer Λ_{QCD}
  - ☞ heavy quark flavor symmetry (HQFS) for any hadron containing one heavy quark:
    velocity remains unchanged in the limit $m_Q \to \infty$: $\Delta v = \dfrac{\Delta p}{m_Q} = \dfrac{\Lambda_{\text{QCD}}}{m_Q}$
    $\Rightarrow$ heavy quark is like a static color triplet source, m_Q is irrelevant
  - ☞ heavy anti-quark–diquark symmetry

    Savage, Wise (1990)

    $m_Q v \gg \Lambda_{\text{QCD}}$,
    the diquark serves as a point-like color-$\bar{3}$ source, like a heavy anti-quark.
    It relates doubly-heavy baryons to anti-heavy mesons



Intermediate summary

- Many new hadrons observed (in particular in the charm sector), lots of mysteries
- Symmetries of QCD:
  - spontaneously broken chiral symmetry for light flavors
  - heavy quark spin and flavor symmetries for heavy flavors

$\Rightarrow$ next, applications of symmetries to the new hadrons

HQS for open-flavor heavy hadrons

Examples of HQSS phenomenology:

- In the Review of Particle Physics (RPP) by the Particle Data Group (PDG), there are two D_1 ($J^P = 1^+$) mesons with very different widths
  - ☞ $\Gamma[D_1(2420)] = (27.4 \pm 2.5)$ MeV $\ll$ $\Gamma[D_1(2430)] = (384^{+130}_{-110})$ MeV
  - ☞ $\boldsymbol{s}_\ell = \boldsymbol{s}_q + \boldsymbol{L} \ \ \Rightarrow$ for P-wave charmed mesons: $s_\ell^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$
  - ☞ for decays $D_1 \to D^*\pi$:

    $\frac{1}{2}^+ \to \frac{1}{2}^- + 0^-$ in S-wave $\Rightarrow$ large width

    $\frac{3}{2}^+ \to \frac{1}{2}^- + 0^-$ in D-wave $\Rightarrow$ small width
  - ☞ thus, dominant components: $D_1(2420)$: $s_\ell = \frac{3}{2}$, $\quad D_1(2430)$: $s_\ell = \frac{1}{2}$
- Suppression of the S-wave production of $\frac{3}{2}^+ + \frac{1}{2}^-$ heavy meson pairs in e^+e^- annihilation Table VI in E.Eichten et al., PRD**17**(1978)3090; X. Li, M. Voloshin, PRD**88**(2013)034012

> Exercise: Try to understand this statement as a consequence of HQSS.
> Hint: in e^+e^- collisions, the leading production mechanism of heavy meson pairs is from the vector current $\bar{Q}\gamma^\mu Q$ which couples to the virtual photon, *i.e.*, $e^+e^- \to \gamma^* \to \bar{Q}Q$ with the $Q\bar{Q}$ pair in an S-wave.

Applications of HQS: $D_{s0}^*(2317)$ and $D_{s1}(2460)$ (1)

- HQFS: for a singly-heavy hadron, $M_{H_Q} = m_Q + A + \mathcal{O}\left(m_Q^{-1}\right)$
- rough estimates of bottom analogues whatever the D_{sJ} states are

$$M_{B_{s0}^*} = M_{D_{s0}^*(2317)} + \Delta_{b-c} + \mathcal{O}\left(\Lambda_{\text{QCD}}^2\left(\frac{1}{m_c}-\frac{1}{m_b}\right)\right) \simeq (5.65\pm0.15)\ \text{GeV}$$

$$M_{B_{s1}} = M_{D_{s1}(2460)} + \Delta_{b-c} + \mathcal{O}\left(\Lambda_{\text{QCD}}^2\left(\frac{1}{m_c}-\frac{1}{m_b}\right)\right) \simeq (5.79\pm0.15)\ \text{GeV}$$

here $\Delta_{b-c} \equiv m_b - m_c \simeq \overline{M}_{B_s} - \overline{M}_{D_s} \simeq 3.33$ GeV, where $\overline{M}_{B_s} = 5.403$ GeV, $\overline{M}_{D_s} = 2.076$ GeV: spin-averaged g.s. $Q\bar{s}$ meson masses

☞ comparing with the lattice QCD results: Lang et al., PLB750(2015)17

$$M_{B_{s0}^*}^{\text{lat.}} = (5.711\pm0.013\pm0.019)\ \text{GeV}$$
$$M_{B_{s1}}^{\text{lat.}} = (5.750\pm0.017\pm0.019)\ \text{GeV}$$

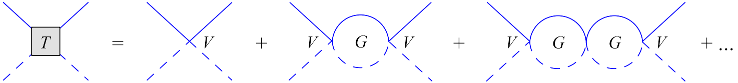
☞ both to be discovered [1]

- more precise predictions can be made in a given model, e.g. hadronic molecules

[1] The established meson $B_{s1}(5830)$ is probably the bottom partner of $D_{s1}(2536)$.

- in had. mol. model: $D^*_{s0}(2317)[\simeq DK(I=0)]$, $D_{s1}(2460)[\simeq D^*K(I=0)]$

  Barnes, Close, Lipkin (2003); van Beveren, Rupp (2003); Kolomeitsev, Lutz (2004); FKG et al. (2006); FKG, Hanhart, Meißner (2009); . . .

  $D^{(*)}K$ bound states: poles of the T-matrix

- HQSS $\Rightarrow$ similar binding energies $M_D + M_K - M_{D^*_{s0}} \simeq 45$ MeV

  $M_{D_{s1}(2460)} - M_{D^*_{s0}(2317)} \simeq M_{D^*} - M_D$ is natural

- HQFS $\Rightarrow$ predicting the 0^+ and 1^+ bottom-partner masses

$$M_{B^*_{s0}} \simeq M_B + M_K - 45\,\text{MeV} \simeq 5.730 \text{ GeV}$$
$$M_{B_{s1}} \simeq M_{B^*} + M_K - 45\,\text{MeV} \simeq 5.776 \text{ GeV}$$

Recall the lattice QCD results: Lang et al., PLB750(2015)17

$$M^{\text{lat.}}_{B^*_{s0}} = (5.711 \pm 0.013 \pm 0.019) \text{ GeV}$$
$$M^{\text{lat.}}_{B_{s1}} = (5.750 \pm 0.017 \pm 0.019) \text{ GeV}$$

Applications of HQS: $X(5568)$

FKG, Meißner, Zou, *How the X(5568) challenges our understanding of QCD*, Commun.Theor.Phys. 65 (2016) 593

- mass too low for $X(5568)$ to be a $\bar{b}s\bar{u}d$: $M \simeq M_{B_s} + 200$ MeV
  - ☞ $M_\pi \simeq 140$ MeV because pions are pseudo-Goldstone bosons
  - ☞ Gell-Mann–Oakes–Renner: $M_\pi^2 \propto m_q$
  - ☞ For any matter field: $M_R \gg M_\pi$; we expect $M_{\bar{u}d} \sim M_R \gtrsim M_\sigma$

$$M_{\bar{b}s\bar{u}d} \gtrsim M_{B_s} + 500 \text{ MeV} \sim 5.9 \text{ GeV}$$

- HQFS predicts an isovector X_c:

$$M_{X_c} = M_{X(5568)} - \Delta_{b-c} + \mathcal{O}\left(\Lambda^2_{\text{QCD}}\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\right) \simeq (2.24 \pm 0.15) \text{ GeV}$$

  but in $D_s\pi$, only the isoscalar $D^*_{s0}(2317)$ was observed! BaBar (2003)

- negative results reported by LHCb, LHCb, PRL117(2016)152003
  by CMS, CMS, PRL120(2018)202005
  by CDF, CDF, PRL120(2018)202006
  by ATLAS ATLAS, PRL120(2018)202007

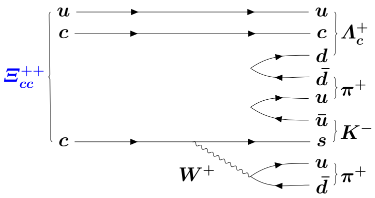
Development inspired by the LHCb discovery of the $\Xi_{cc}(3620)^{++}$

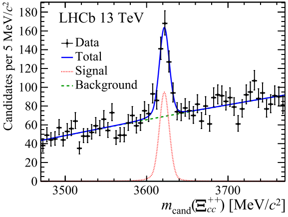
- Heavy antiquark-diquark symmetry (HADS):
  repalcing $\bar{Q}$ in $\bar{Q}q$ by $QQ\,[\bar{3}_{\text{color}}]$ $\Rightarrow$ QQq;
  repalcing $\bar{Q}$ in $\bar{Q}\bar{q}\bar{q}$ by $QQ\,[\bar{3}_{\text{color}}]$ $\Rightarrow$ $QQ\bar{q}\bar{q}$;

$$\begin{array}{lccccccc} & \bar{Q}q & \Rightarrow & QQq, & \bar{Q}\bar{q}\bar{q} & \Rightarrow & QQ\bar{q}\bar{q} \\ \text{mass} \approx & m_Q + A & \Rightarrow & m_{QQ} + A, & m_Q + B & \Rightarrow & m_{QQ} + B \end{array}$$

  Prediction: $M_{QQ\bar{q}\bar{q}} - M_{\bar{Q}\bar{q}\bar{q}} \simeq M_{QQq} - M_{\bar{Q}q}$

- Doubly-charmed baryon discovered by LHCb PRL119(2017)112001 [arXiv:1707.01621]





$M_{\Xi_{cc}^{++}} = (3621.40 \pm 0.78)$ MeV can be used as input

Applications from heavy baryons to doubly-heavy tetraquarks (2)

TABLE II. Expectations for the ground-state tetraquark masses, in MeV.[a] The column labeled "HQS relation" is the result of our heavy-quark symmetry relations and is explicitly given by the sum of the right-hand side of Eq. (1) and the kinetic-energy mass shifts of Eq. (7). Here q denotes an up or down quark. For stable tetraquark states the $\mathcal{Q}$ value is highlighted in a box.

State	J^P	j_ℓ	$m(Q_iQ_jq_m)$ (c.g.)	HQS relation	$m(Q_iQ_j\bar{q}_k\bar{q}_l)$	Decay channel	$\mathcal{Q}$ (MeV)
$\{cc\}[\bar{u}\,\bar{d}]$	1^+	0	3663[b]	$m(\{cc\}u)+315$	3978	D^+D^{*0} 3876	102
$\{cc\}[\bar{q}_k\bar{s}]$	1^+	0	3764[c]	$m(\{cc\}s)+392$	4156	$D^+D_s^{*-}$ 3977	179
$\{cc\}\{\bar{q}_k\bar{q}_l\}$	$0^+,1^+,2^+$	1	3663	$m(\{cc\}u)+526$	4146,4167,4210	D^+D^0, D^+D^{*0} 3734,3876	412,292,476
$[bc][\bar{u}\,\bar{d}]$	0^+	0	6914	$m([bc]u)+315$	7229	B^-D^+/B^0D^0 7146	83
$[bc][\bar{q}_k\bar{s}]$	0^+	0	7010[d]	$m([bc]s)+392$	7406	B_sD 7236	170
$[bc]\{\bar{q}_k\bar{q}_l\}$	1^+	1	6914	$m([bc]u)+526$	7439	B^*D/BD^* 7190/7290	249
$\{bc\}[\bar{u}\,\bar{d}]$	1^+	0	6957	$m(\{bc\}u)+315$	7272	B^*D/BD^* 7190/7290	82
$\{bc\}[\bar{q}_k\bar{s}]$	1^+	0	7053[d]	$m(\{bc\}s)+392$	7445	DB_s^* 7282	163
$\{bc\}\{\bar{q}_k\bar{q}_l\}$	$0^+,1^+,2^+$	1	6957	$m(\{bc\}u)+526$	7461,7472,7493	BD/B^*D 7146/7190	317,282,349
$\{bb\}[\bar{u}\,\bar{d}]$	1^+	0	10 176	$m(\{bb\}u)+306$	10 482	$B^-\bar{B}^{*0}$ 10 603	$\boxed{-121}$
$\{bb\}[\bar{q}_k\bar{s}]$	1^+	0	10 252[c]	$m(\{bb\}s)+391$	10 643	$\bar{B}\bar{B}_s^*/\bar{B}_s\bar{B}^*$ 10 695/10 691	$\boxed{-48}$
$\{bb\}\{\bar{q}_k\bar{q}_l\}$	$0^+,1^+,2^+$	1	10 176	$m(\{bb\}u)+512$	10 674,10 681,10 695	B^-B^0, B^-B^{*0} 10 559,10 603	115,78,136

[a]Masses of the unobserved doubly heavy baryons are taken from Ref. [14]; for lattice evaluations of b-baryon masses, see Ref. [15].
[b]Based on the mass of the LHCb Ξ_{cc}^{++} candidate, 3621.40 MeV, Ref. [10].
[c]Using the s/d mass differences of the corresponding heavy-light mesons.
[d]Evaluated as $\frac{1}{2}[m(c\bar{s})-m(c\bar{d})+m(b\bar{s})-m(b\bar{d})]+m(bcd)$.

Eichten, Quigg, PRL119(2017)202002

- HADS $\Rightarrow$ stable doubly-bottom tetraquarks (only decay weakly) are likely to exist

  see also Carlson, Heller, Tjon, PRD37(1988)744; Manohar, Wise, NPB399(1993)17; Karliner, Rosner, PRL119(2017)202001; Czarnecki, Leng, Voloshin, PLB778(2018)233; ...

- support from lattice QCD — Francis, Hudspith, Lewis, Maltman, PRL118(2017)142001
- Detecting method: T. Gershon, A. Poluektov, *Displaced B_c mesons as an inclusive signature of weakly decaying double beauty hadrons*, JHEP 01 (2019) 019

HQS for XYZ states

- Hadronic molecular model: $X(3872)$: $D\bar{D}^*$; $Z_b(10610, 10650)$: $B\bar{B}^*$ and $B^*\bar{B}^*$
- Consider S-wave interaction between a pair of $s_\ell^P = \frac{1}{2}^-$ (anti-)heavy mesons:

$$\begin{aligned}
0^{++}: &\quad D\bar{D}, \quad D^*\bar{D}^* \\
1^{+-}: &\quad \frac{1}{\sqrt{2}}\left(D\bar{D}^* + D^*\bar{D}\right), \quad D^*\bar{D}^* \\
1^{++}: &\quad \frac{1}{\sqrt{2}}\left(D\bar{D}^* - D^*\bar{D}\right); \qquad 2^{++}: \qquad D^*\bar{D}^*
\end{aligned}$$

here, phase convention: $D \xrightarrow{C} +\bar{D},\ D^* \xrightarrow{C} -\bar{D}^*$

- Heavy quark spin irrelevant ⇒ interaction matrix elements:

$$\left\langle s_{1c}, s_{2c}, s_{c\bar{c}}; s_{1\,\ell}, s_{2\,\ell}, s_L; J \left|\hat{\mathcal{H}}\right| s'_{1c}, s'_{2c}, s'_{c\bar{c}}; s'_{1\,\ell}, s'_{2\,\ell}, s'_L; J' \right\rangle$$
$$= \left\langle s_{1\,\ell}, s_{2\,\ell}, s_L \left|\hat{\mathcal{H}}\right| s'_{1\,\ell}, s'_{2\,\ell}, s_L \right\rangle \delta_{s_{c\bar{c}}, s'_{c\bar{c}}} \delta_{s_L, s'_L} \delta_{JJ'}$$

For each isospin, 2 independent terms

$$\left\langle \frac{1}{2}, \frac{1}{2}, 0 \left|\hat{\mathcal{H}}\right| \frac{1}{2}, \frac{1}{2}, 0 \right\rangle, \qquad \left\langle \frac{1}{2}, \frac{1}{2}, 1 \left|\hat{\mathcal{H}}\right| \frac{1}{2}, \frac{1}{2}, 1 \right\rangle$$

⇒ 6 pairs grouped in 2 multiplets with $s_L = 0$ and 1, respectively

- For the HQSS consequences, convenient to use the basis of states: $s_L^{PC} \otimes s_{c\bar{c}}^{PC}$
  - ☞ S-wave: $s_L^{PC}, s_{c\bar{c}}^{PC} = 0^{-+}$ or 1^{--}
  - ☞ multiplet with $s_L = 0$:

$$0_L^{-+} \otimes 0_{c\bar{c}}^{-+} = 0^{++}, \qquad 0_L^{-+} \otimes 1_{c\bar{c}}^{--} = 1^{+-}$$

  - ☞ multiplet with $s_L = 1$:

$$1_L^{--} \otimes 0_{c\bar{c}}^{-+} = 1^{+-}, \qquad 1_L^{--} \otimes 1_{c\bar{c}}^{--} = 0^{++} \oplus \boxed{\mathbf{1^{++}}} \oplus 2^{++}$$

- Multiplets in strict heavy quark limit:
  - ☞ $X(3872)$ has three partners with 0^{++}, 2^{++} and 1^{+-}

Hidalgo-Duque et al., PLB727(2013)432; Baru et al., PLB763(2016)20

  - ☞ Z_b, Z_b' as $B^{(*)}\bar{B}^*$ molecules would imply 6 $I = 1$ hadronic molecules: $Z_b[1^{+-}], Z_b'[1^{+-}]$ and $W_{b0}[0^{++}], W_{b0}'[0^{++}], W_{b1}[1^{++}]$ and $W_{b2}[2^{++}]$

Bondar et al., PRD84(2011)054010; Voloshin, PRD84(2011)031502; Mehen, Powell, PRD84(2011)114013

- Recall the excercise in Lecture-1:

  Is $\Upsilon\pi^+\pi^-$ a good choice of final states for the search of X_b, the $J^{PC} = 1^{++}$ bottom analogue of the $X(3872)$?

  Answer: No. $X_b \to \Upsilon\pi\pi$ breaks isospin symmetry

  FKG, Hidalgo-Duque, Nieves, Valderrama, PRD88(2013)054007; Karliner, Rosner, PRD91(2015)014014

  $M_{B^0} - M_{B^\pm} = (0.31 \pm 0.06)$ MeV [$M_{D^\pm} - M_{D^0} = (4.822 \pm 0.015)$ MeV]

- Negative results:

  CMS, *Search for a new bottomonium state decaying to $\Upsilon(1S)\pi^+\pi^-$ in pp collisions at $\sqrt{s} = 8$ TeV*, PLB727(2013)57;

  ATLAS, *Search for the X_b and other hidden-beauty states in the $\pi^+\pi^-\Upsilon(1S)$ channel at ATLAS*, PLB740(2015)199

- The results can be reinterpreted as for the search of W_{bJ} ($I = 1, J^{++}$)

$$1_L^{--} \otimes 1_{c\bar c}^{--} = 0^{++} \oplus \boxed{\mathbf{1^{++}}} \oplus 2^{++}$$

- Heavy quark spin selection rule for $X(3872)$: Voloshin, PLB604(2004)69
  for $X(3872)$ being a $\mathbf{1^{++}}$ $D\bar D^*$ molecule, $s_L = 1$, $s_{c\bar c} = 1$
- spin structure of $Q\bar Q$:

	s_L	$s_{c\bar c}$	J^{PC}	$c\bar c$
S-wave	0	0	0^{-+}	η_c
	0	1	1^{--}	J/ψ
P-wave	1	0	1^{+-}	h_c
	1	1	$(0,1,2)^{++}$	$\chi_{c0}, \chi_{c1}, \chi_{c2}$

- allowed: $X(3872) \to J/\psi\pi\pi$, $X(3872) \to \chi_{cJ}\pi$, $X(3872) \to \chi_{cJ}\pi\pi$
  suppressed: $X(3872) \to \eta_c\pi\pi$, $X(3872) \to h_c\pi\pi$
- Interesting feature of $Z_b^{(\prime)}$: observed with similar rates in both $\Upsilon\pi\pi[s_{b\bar b} = 1]$ and $h_b\pi\pi[s_{b\bar b} = 0]$ Bondar, Garmash, Milstein, Mizuk, Voloshin, PRD84(2011)054010

  $Z_b \sim B\bar B^* \sim 0_{b\bar b}^- \otimes 1_{q\bar q}^- - 1_{b\bar b}^- \otimes 0_{q\bar q}^-$, $\quad Z_b' \sim B^*\bar B^* \sim 0_{b\bar b}^- \otimes 1_{q\bar q}^- + 1_{b\bar b}^- \otimes 0_{q\bar q}^-$

unitary transformation from two-meson basis to $\left|s_{1\,c}, s_{2\,c}, s_{c\bar c}; s_{1\,\ell}, s_{2\,\ell}, s_L; J\right\rangle$:

$$\left|s_{1\,c}, s_{1\,\ell}, j_1; s_{2\,c}, s_{2\,\ell}, j_2; J\right\rangle = \sum_{s_{c\bar c}, s_L} \sqrt{(2j_1+1)(2j_2+1)(2s_{c\bar c}+1)(2s_L+1)}$$

$$\times \begin{Bmatrix} s_{1\,c} & s_{2\,c} & s_{c\bar c} \\ s_{1\,\ell} & s_{2\,\ell} & s_L \\ j_1 & j_2 & J \end{Bmatrix} \left|s_{1\,c}, s_{2\,c}, s_{c\bar c}; s_{1\,\ell}, s_{2\,\ell}, s_L; J\right\rangle$$

$j_{1,2}$: meson spins;

J: the total angular momentum of the whole system

$s_{1\,c(2\,c)} = \frac{1}{2}$: spin of the heavy quark in meson 1 (2)

$s_{1\,\ell(2\,\ell)} = \frac{1}{2}$: angular momentum of the light quarks in meson 1 (2)

- $s_{c\bar c} = 0, 1$: total spin of $c\bar c$, conserved but decoupled
- $s_L = 0, 1$: total angular momentum of the light-quark system, conserved
- only two independent $\left\langle s_{\ell 1}, s_{\ell 2}, s_L \middle| \hat{\mathcal{H}} \middle| s'_{\ell 1}, s'_{\ell 2}, s_L \right\rangle_I$ terms for each isospin I:

$$F_{I0} = \left\langle \tfrac{1}{2}, \tfrac{1}{2}, 0 \middle| \hat{\mathcal{H}} \middle| \tfrac{1}{2}, \tfrac{1}{2}, 0 \right\rangle_I, \quad F_{I1} = \left\langle \tfrac{1}{2}, \tfrac{1}{2}, 1 \middle| \hat{\mathcal{H}} \middle| \tfrac{1}{2}, \tfrac{1}{2}, 1 \right\rangle_I$$

$$\begin{pmatrix} D\bar{D} \\ D^*\bar{D}^* \end{pmatrix}: \qquad V^{(0^{++})} = \begin{pmatrix} C_{IA} & \sqrt{3}C_{IB} \\ \sqrt{3}C_{IB} & C_{IA} - 2C_{IB} \end{pmatrix},$$

$$\begin{pmatrix} D\bar{D}^* \\ D^*\bar{D}^* \end{pmatrix}: \qquad V^{(1^{+-})} = \begin{pmatrix} C_{IA} - C_{IB} & 2C_{IB} \\ 2C_{IB} & C_{IA} - C_{IB} \end{pmatrix},$$

$$D\bar{D}^*: \qquad V^{(1^{++})} = C_{IA} + C_{IB},$$

$$D^*\bar{D}^*: \qquad V^{(2^{++})} = C_{IA} + C_{IB},$$

here, $C_{IA} = \frac{1}{4}(3F_{I1} + F_{I0})$, $C_{IB} = \frac{1}{4}(F_{I1} - F_{I0})$

- This would suggest spin multiplets. Good candidates:
  - ☞ $X(3872)$ and $X_2(4013)$ (not observed yet!); $Z_c(3900)$ and $Z_c(4020)$

    Nieves, Valderrama, PRD86(2012)056004; ...

    $$M_{X_2(4013)} - M_{X(3872)} \approx M_{Z_c(4020)} - M_{Z_c(3900)} \approx M_{D^*} - M_D$$

  - ☞ $Z_b(10610)$ and $Z_b(10650)$:

    Bondar et al., PRD84(2011)054010; ...

    $$M_{Z_b(10650)} - M_{Z_b(10610)} \approx M_{B^*} - M_B$$

  - ☞ Z_c and Z_b states need a suppression of coupled-channel effect (reason?)

The LHCb P_c states might be $\Sigma_c^{(*)}\bar{D}^{(*)}$ molecules predicted in Wu, Molina, Oset, Zou (2010)

$P_c(4312) \sim \Sigma_c\bar{D}$, $P_c(4440, 4457) \sim \Sigma_c\bar{D}^*$

Consider S-wave pairs of $\Sigma_c^{(*)}\bar{D}^{(*)}$ [$J_{\Sigma_c} = \frac{1}{2}, J_{\Sigma_c^*} = \frac{3}{2}$]:

$$J^P = \frac{1}{2}^- : \ \Sigma_c\bar{D},\ \Sigma_c\bar{D}^*,\ \Sigma_c^*\bar{D}^*$$
$$J^P = \frac{3}{2}^- : \ \Sigma_c^*\bar{D},\ \Sigma_c\bar{D}^*,\ \Sigma_c^*\bar{D}^*$$
$$J^P = \frac{5}{2}^- : \ \Sigma_c^*\bar{D}^*$$

Spin of the light degrees of freedom s_ℓ: $s_\ell(D^{(*)}) = \frac{1}{2}$, $s_\ell(\Sigma_c^{(*)}) = 1$. Thus, $s_L = \frac{1}{2}, \frac{3}{2}$

For each isospin, 2 independent terms

$$\left\langle 1, \frac{1}{2}, \frac{1}{2} \left| \hat{\mathcal{H}} \right| 1, \frac{1}{2}, \frac{1}{2} \right\rangle, \qquad \left\langle 1, \frac{1}{2}, \frac{3}{2} \left| \hat{\mathcal{H}} \right| 1, \frac{1}{2}, \frac{3}{2} \right\rangle$$

Thus, the 7 pairs are in two spin multiplets: 3 with $s_L = \frac{1}{2}$ and 4 with $s_L = \frac{3}{2}$

HQSS for P_c (2)

Seven P_c generally expected in this hadronic molecular model Xiao, Nieves, Oset (2013); Liu et al. (2018, 2019); Sakai et al. (2019);. . .

Predictions using the masses of $P_c(4440, 4457)$ as inputs Liu et al., PRL122(2019)242001

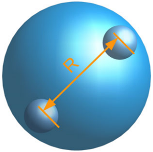
Scenario	Molecule	J^P	B (MeV)	M (MeV)
A	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	7.8 – 9.0	4311.8 – 4313.0
A	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	8.3 – 9.2	4376.1 – 4377.0
A	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	Input	4440.3
A	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	Input	4457.3
A	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	25.7 – 26.5	4500.2 – 4501.0
A	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	15.9 – 16.1	4510.6 – 4510.8
A	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	3.2 – 3.5	4523.3 – 4523.6
B	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	13.1 – 14.5	4306.3 – 4307.7
B	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	13.6 – 14.8	4370.5 – 4371.7
B	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	Input	4457.3
B	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	Input	4440.3
B	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	3.1 – 3.5	4523.2 – 4523.6
B	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	10.1 – 10.2	4516.5 – 4516.6
B	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	25.7 – 26.5	4500.2 – 4501.0

Compositeness and hadronic molecules

FKG, Hanhart, Meißner, Wang, Zhao, Zou, *Hadronic molecules*, Rev. Mod. Phys. **90** (2018) 015004

- Hadronic molecule:
  dominant component is a composite state of 2 or more hadrons
- Concept at large distances, so that can be approximated by system of multi-hadrons at low energies
  Consider a 2-body bound state with a mass $M = m_1 + m_2 - E_B$

size: $$R \sim \frac{1}{\sqrt{2\mu E_B}} \gg r_{\text{hadron}}$$



- scale separation $\Rightarrow$ power expansion in p/Λ, (nonrelativistic) EFT applicable!
- Only narrow hadrons can be considered as components of hadronic molecules, $\Gamma_h \ll 1/r$, r: range of forces

Filin *et al.*, PRL105(2010)019101; FKG, Meißner, PRD84(2011)014013

- Why are hadronic molecules interesting?
  - ☞ one realization of color-neutral objects, analogue of light nuclei
  - ☞ important information for hadron-hadron interaction
  - ☞ understanding the XYZ states
  - ☞ EFT applicable; model-independent statements can be made for S-wave, compositeness $(1-Z)$ related to measurable quantities compositeness: probability of the physical state being a 2-body bound state

Weinberg, PR137(1965); Baru *et al.*, PLB586(2004); Hyodo, IJMPA28(2013)1330045; . . .

see also, e.g., Weinberg's books: QFT Vol.I, Lectures on QM

$$|g_{\rm NR}|^2 \approx (1-Z)\frac{2\pi}{\mu^2}\sqrt{2\mu E_B} \leq \frac{2\pi}{\mu^2}\sqrt{2\mu E_B}$$

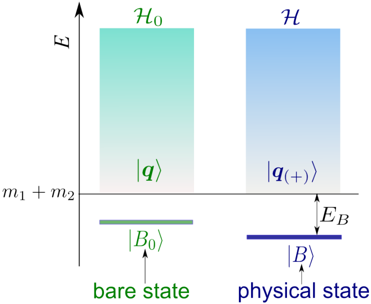
$$a \approx -\frac{2(1-Z)}{(2-Z)\sqrt{2\mu E_B}}, \quad r_e \approx \frac{Z}{(1-Z)\sqrt{2\mu E_B}}$$

Model-independent result for S-wave loosely bound composite states:

Consider a system with Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + V$$

$\mathcal{H}_0$: free Hamiltonian, V: interaction potential

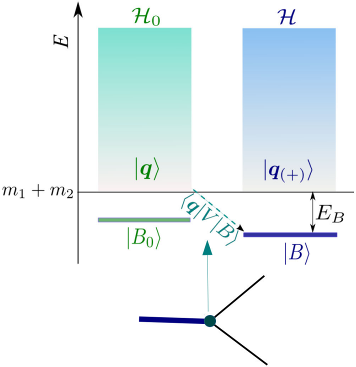


- Compositeness: the probability of finding the physical state $|B\rangle$ in the 2-body continuum $|\boldsymbol{q}\rangle$

$$1 - Z = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} \left|\langle \boldsymbol{q}|B\rangle\right|^2$$

- $Z = \left|\langle B_0|B\rangle\right|^2, \qquad 0 \le (1-Z) \le 1$
  - ☞ $Z = 0$: pure bound (composite) state
  - ☞ $Z = 1$: pure elementary state

Compositeness : $1 - Z = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} |\langle \boldsymbol{q}|B\rangle|^2$



- Schrödinger equation

$$(\mathcal{H}_0 + V)|B\rangle = -E_B|B\rangle$$

multiplying by $\langle \boldsymbol{q}|$ and using $\mathcal{H}_0|\boldsymbol{q}\rangle = \frac{\boldsymbol{q}^2}{2\mu}|\boldsymbol{q}\rangle$:
$\Rightarrow$ momentum-space wave function:

$$\langle \boldsymbol{q}|B\rangle = -\frac{\langle \boldsymbol{q}|V|B\rangle}{E_B + \boldsymbol{q}^2/(2\mu)}$$

- S-wave, small binding energy so that $R = 1/\sqrt{2\mu E_B} \gg r$, r: range of forces

$$\langle \boldsymbol{q}|V|B\rangle = g_{\rm NR}\left[1 + \mathcal{O}(r/R)\right]$$

- Compositeness:

$$1 - Z = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} \frac{|g_{\rm NR}|^2}{\left[E_B + \boldsymbol{q}^2/(2\mu)\right]^2} \left[1 + \mathcal{O}\left(\frac{r}{R}\right)\right] = \frac{\mu^2|g_{\rm NR}|^2}{2\pi\sqrt{2\mu E_B}} \left[1 + \mathcal{O}\left(\frac{r}{R}\right)\right]$$

Compositeness (3)

- Coupling constant measures the compositeness for an S-wave shallow bound state

$$|g_{\rm NR}|^2 \approx (1-Z)\frac{2\pi}{\mu^2}\sqrt{2\mu E_B} \leq \frac{2\pi}{\mu^2}\sqrt{2\mu E_B}$$

bounded from the above

Exercise:

Show that $|g_{\rm NR}|^2$ is the residue of the T-matrix element at the pole $E=-E_B$:

$$|g_{\rm NR}|^2 = \lim_{E\to -E_B}(E+E_B)\langle \boldsymbol{k}|T|\boldsymbol{k}\rangle$$

Hint: use the Lippmann–Schwinger equation $T = V + V\frac{1}{E-\mathcal{H}_0+i\epsilon}T$ and the completeness relation $|B\rangle\langle B| + \int\frac{d^3q}{(2\pi)^3}|\boldsymbol{q}_{(+)}\rangle\langle\boldsymbol{q}_{(+)}| = 1$ to derive the Low equation (noticing $T|\boldsymbol{q}\rangle = V|\boldsymbol{q}_{(+)}\rangle$):

$$\langle\boldsymbol{k}'|T|\boldsymbol{k}\rangle = \langle\boldsymbol{k}'|V|\boldsymbol{k}\rangle + \frac{\langle\boldsymbol{k}'|V|B\rangle\langle B|V|\boldsymbol{k}\rangle}{E+E_B+i\epsilon} + \int\frac{d^3q}{(2\pi)^3}\frac{\langle\boldsymbol{k}'|T|\boldsymbol{q}\,\rangle\langle\boldsymbol{q}\,|T^\dagger|\boldsymbol{k}\rangle}{E-\boldsymbol{q}^2/(2\mu)+i\epsilon}$$

- Z can be related to scattering length a and effective range r_e Weinberg (1965)

$$a = -\frac{2R(1-Z)}{2-Z}\left[1+\mathcal{O}\left(\frac{r}{R}\right)\right], \quad r_e = \frac{RZ}{1-Z}\left[1+\mathcal{O}\left(\frac{r}{R}\right)\right]$$

Effective range expansion: $f^{-1}(k) = 1/a + r_e k^2/2 - ik + \mathcal{O}\left(k^4\right)$

Derivation:

$$T(E) \equiv \langle k|T|k\rangle = -\frac{2\pi}{\mu} f(k) \quad \Rightarrow \quad \operatorname{Im} T^{-1}(E) = \frac{\mu}{2\pi}\sqrt{2\mu E}\,\theta(E)$$

Twice-subtracted dispersion relation for $T^{-1}(E)$

$$T^{-1}(E) = \frac{E+E_B}{|g_{\rm NR}|^2} + \frac{(E+E_B)^2}{\pi}\int_0^{+\infty} dw \frac{\operatorname{Im} T^{-1}(w)}{(w-E-i\epsilon)(w+E_B)^2}$$

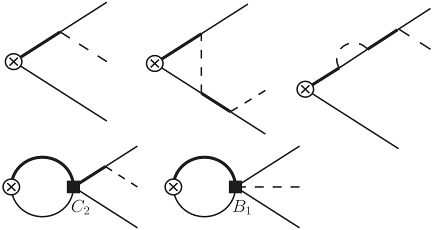
$$= \frac{E+E_B}{|g_{\rm NR}|^2} + \frac{\mu R}{4\pi}\left(\frac{1}{R} - \sqrt{-2\mu E - i\epsilon}\right)^2$$

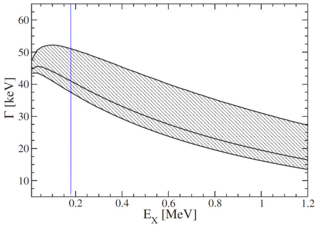
- Example: deuteron as pn bound state. Exp.: $E_B = 2.2$ MeV, $a_{^3S_1} = -5.4$ fm

$$a_{Z=1} = 0 \text{ fm}, \qquad a_{Z=0} = (-4.3 \pm 1.4) \text{ fm}$$

- Coupling constant fixed by binding energy, long-distance processes such as $X(3872) \to D^0 \bar{D}^0 \pi^0, D^0 \bar{D}^0 \gamma$ calculable

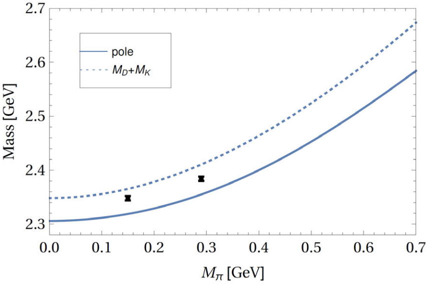
  E.g., XEFT prediction of $\Gamma(X \to D^0 \bar{D}^0 \pi^0)$ Fleming et al., PRD76(2007)034006





Compositeness from scattering length:
scattering lengths calculable using the Lüscher formalism in lattice QCD
E.g., from DK $I = 0$ scattering length $\Rightarrow$ the DK component of $D_{s0}^*(2317)$

- $(70 \pm 4)\%$ from DK isoscalar scattering length computed from unitarized CHPT with LECs fixed from fitting to scattering lengths of five other channels Liu, Orignos, FKG, Hanhart, Meißner, PRD86(2013)014508
- $(72 \pm 13 \pm 5)\%$ from the lattice energy levels in C. Lang et al., PRD90(2014)034510
  Martínez Torres, Oset, Prelovsek, Ramos, JHEP1505,153
- Latest lattice results in G. Bali et al., PRD96(2017)074501



$$1 - Z = 1.04(0.08)(+0.30)$$

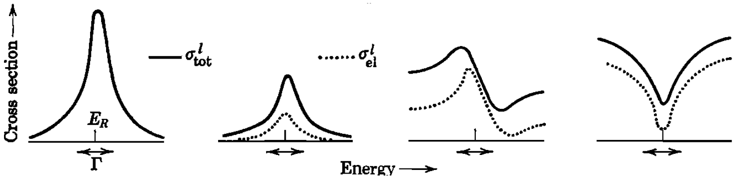
M_π [MeV]	150	290
$M_{D_{s0}^*(2317)}$ [MeV]	2348 ± 4	2384 ± 3
M_{D_s} [MeV]	1977 ± 1	1980 ± 1

strong M_π dependence!

curves: prediction in Du, FKG, Meißner, Yao, EPJC77(2017)728

Threshold cusps and triangle singularties

Resonances do not always appear as peaks:



J. R. Taylor, *Scattering Theory: The Quantum Theory on Nonrelativistic Collisions*

Peaks are not always due to resonances:

- Dynamics $\Rightarrow$ poles in the S-matrix (resonances): genuine physical states.
- Kinematic effects $\Rightarrow$ branching points of S-matrix
  - ☞ normal two-body threshold cusp
  - ☞ triangle singularity
  - ☞ . . .

tools/traps in hadron spectroscopy

Unitarity and threshold cusp

- Unitarity of the S-matrix: $S\,S^\dagger = S^\dagger S = \mathbb{1}$, $S_{fi} = \delta_{fi} - i(2\pi)^4\delta^4(p_f - p_i)T_{fi}$

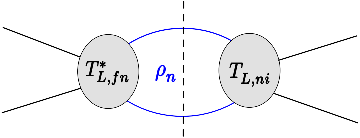
  T-matrix: $T_{fi} - T^\dagger_{fi} = -i(2\pi)^4 \underbrace{\sum_n}_{\text{all physically accesible states}} \delta(p_n - p_i) T^\dagger_{fn} T_{ni}$

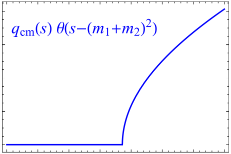
  assuming all intermediate states are two-body, partial-wave unitarity relation:

  $$\boxed{\text{Im}\, T_{L,fi}(s) = -\sum_n T^*_{L,fn}\,\rho_n(s)\,T_{L,ni}}$$

  2-body phase space factor: $\rho_n(s) = q_{\text{cm},n}(s)/(2\sqrt{s})\theta(\sqrt{s} - m_{n1} - m_{n2})$,

  $q_{\text{cm},n}(s) = \sqrt{[s - (m_{n1} + m_{n2})^2][s - (m_{n1} - m_{n2})^2]}/(2\sqrt{s})$



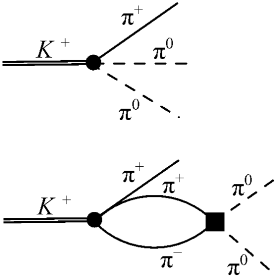


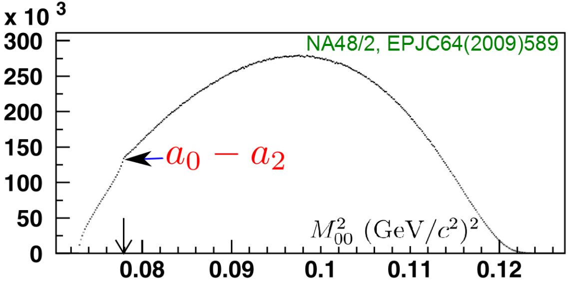
- There is always a cusp at an S-wave threshold

Threshold cusp: a well-known example

- Cusp effect as a useful tool for precise measurement:
  - ☞ example of the cusp in $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$
  - ☞ strength of the cusp measures the interaction strength!

  Budini, Fonda (1961); Meißner, Müller, Steininger (1997); Cabibbo (2004); Colangelo, Gasser, Kubis, Rusetsky (2006); . . .

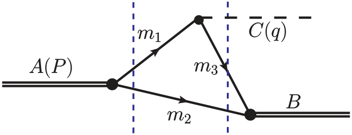




$\sim$ threshold, only sensitive to scattering length, $(a_0 - a_2)M_{\pi^+} = 0.2571 \pm 0.0056$

- Very prominent cusp $\Rightarrow$ large scattering length $\Rightarrow$ likely a nearby pole

  effective range expansion (ERE): $f(k) = \frac{1}{1/a + rk^2/2 - ik}$



$$\frac{1}{2m_A}\sqrt{\lambda(m_A^2,m_1^2,m_2^2)} \equiv \boxed{p_{2,\text{left}} = p_{2,\text{right}}} \equiv \gamma\left(\beta\, E_2^* - p_2^*\right)$$

on-shell momentum of m_2 at the left and right cuts in the A rest frame

$\beta = |\vec{p}_{23}|/E_{23}, \gamma = 1/\sqrt{1-\beta^2}$ — Bayar et al., PRD94(2016)074039

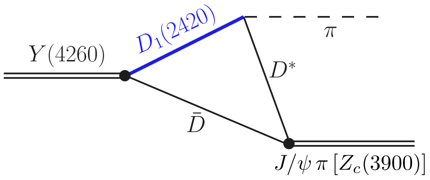
- $p_2 > 0,\, p_3 = \gamma\left(\beta\, E_3^* + p_2^*\right) > 0 \Rightarrow m_2$ and m_3 move in the same direction
- velocities in the A rest frame: $v_3 > \beta > v_2$

$$v_2 = \beta\,\frac{E_2^* - p_2^*/\beta}{E_2^* - \beta\, p_2^*} < \beta\,, \qquad v_3 = \beta\,\frac{E_3^* + p_2^*/\beta}{E_3^* + \beta\, p_2^*} > \beta$$

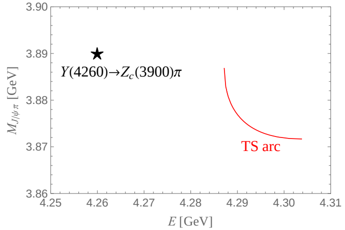
- Conditions (Coleman–Norton theorem): — Coleman, Norton (1965); Bronzan (1964)
  ☞ all three intermediate particles can go on shell simultaneously
  ☞ $\vec{p}_2 \parallel \vec{p}_3$, particle-3 can catch up with particle-2 (as a classical process)
- needs very special kinematics ⇒ process dependent! (contrary to pole position)

Applications: $Z_c(3900)$ (1)

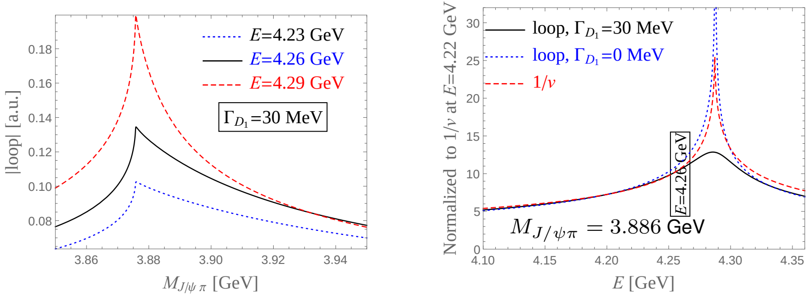
- Consider the triangle loop:



- For $E_{\rm cm} = 4.26$ GeV, TS in the unphysical region



- Enhancement very sensitive to the cm energy



Applications: $Z_c(3900)$ (2)

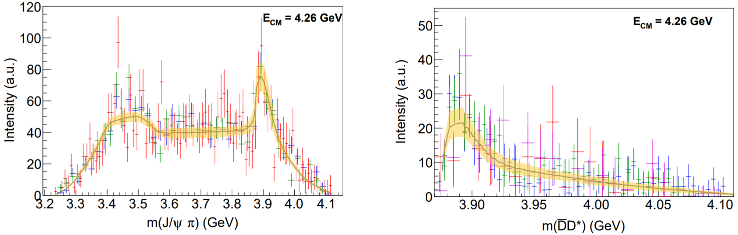
- Importance of TS in $Y(4260) \to Z_c\pi$ already noticed, but Z_c pole still needed

Q.Wang, Hanhart, Q.Zhao, PRL111(2013)132002; PLB725(2013)106

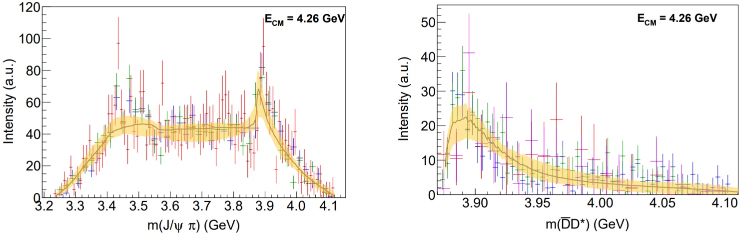
- however, debate continues:

Pilloni et al. (JPAC), PLB772(2017)200

Pole+TS:

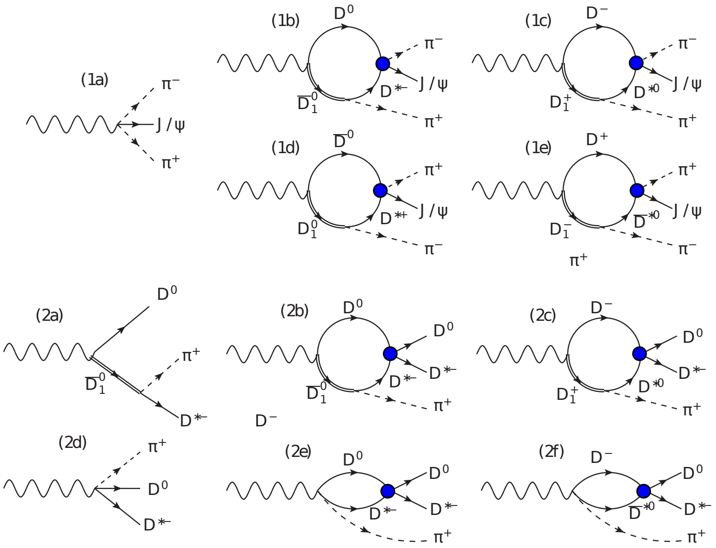


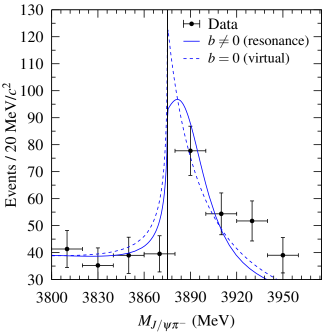
Only TS:

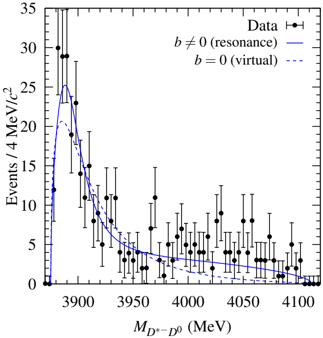


Applications: $Z_c(3900)$ (3)

Albaladejo, FKG, Hidalgo-Duque, Nieves, PLB755(2016)337







M_{Z_c} (MeV)	$\Gamma_{Z_c}/2$ (MeV)	Ref.	Final state
3899 ± 6	23 ± 11	[1] (BESIII)	$J/\psi\ \pi$
3895 ± 8	32 ± 18	[2] (Belle)	$J/\psi\ \pi$
3886 ± 5	19 ± 5	[3] (CLEO-c)	$J/\psi\ \pi$
3884 ± 5	12 ± 6	[4] (BESIII)	$\bar{D}^* D$
3882 ± 3	13 ± 5	[5] (BESIII)	$\bar{D}^* D$
$3894 \pm 6 \pm 1$	$30 \pm 12 \pm 6$	$\Lambda_2 = 1.0$ GeV	$J/\psi\ \pi,\ \bar{D}^* D$
$3886 \pm 4 \pm 1$	$22 \pm 6 \pm 4$	$\Lambda_2 = 0.5$ GeV	$J/\psi\ \pi,\ \bar{D}^* D$
$3831 \pm 26^{+7}_{-28}$	virtual state	$\Lambda_2 = 1.0$ GeV	$J/\psi\ \pi,\ \bar{D}^* D$
$3844 \pm 19^{+12}_{-21}$	virtual state	$\Lambda_2 = 0.5$ GeV	$J/\psi\ \pi,\ \bar{D}^* D$

resonance pole

or virtual state

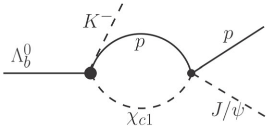
Applications: P_c (1)

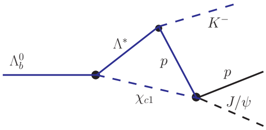
FKG, Meißner, Wang, Yang, PRD92(2015)071502

- Mass: $M_{P_c(4450)} = (4449.8 \pm 1.7 \pm 2.5)$ MeV
- Trivial observation: $P_c(4450)$ coincides with the $\chi_{c1}p$ threshold:

$$M_{P_c(4450)} - M_{\chi_{c1}} - M_p = (0.9 \pm 3.1)\ \text{MeV}$$

- A bit non-trivial observation: there is a triangle singularity at the same time!
  Solving the equation $\boxed{p_{2,\text{left}} = p_{2,\text{right}}}$
  $\Rightarrow$
  for $M_{\Lambda^*} \simeq 1.89$ GeV, a TS at $M_{J/\psi p} = M_{\chi_{c1}} + M_p$
  four-star baryon: $\Lambda(1890)$, $J^P = \frac{3}{2}^+$, $\Gamma : 60 - 200$ MeV
  On shell $\Rightarrow \Lambda^*$ must be unstable, the TS is then a finite peak

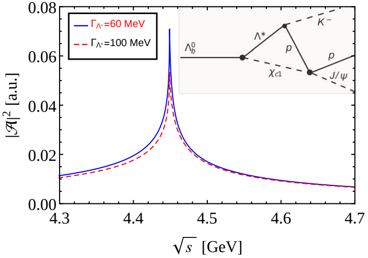


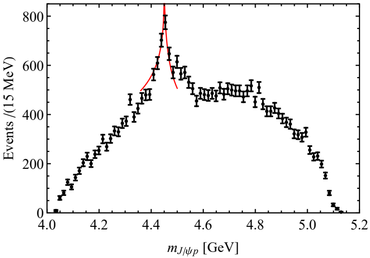


More possible relevant TSs, see X.-H. Liu, Q. Wang, Q. Zhao, PLB757(2015)231

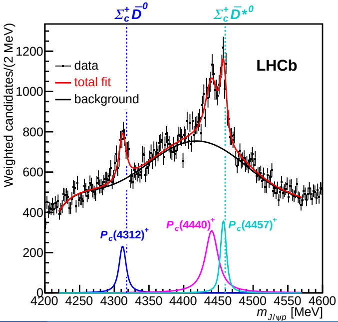
Applications: P_c (2)

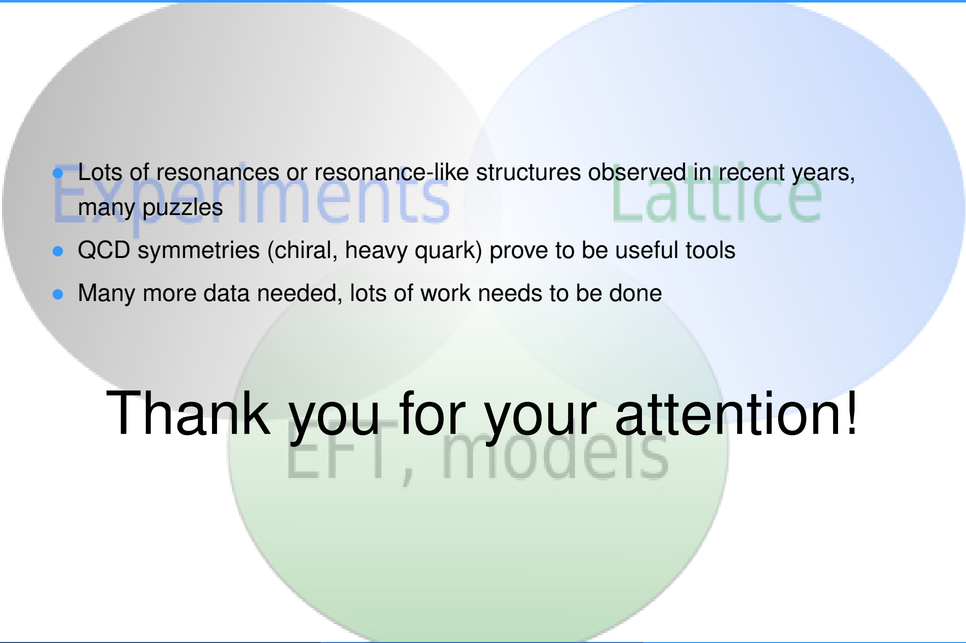
- narrow peak produced by TS, if S-wave between $\chi_{c1}p$, $J^P = \frac{1}{2}^+, \frac{3}{2}^+$





- χ_{c1} very narrow ($\Gamma = (0.84 \pm 0.04)$ MeV), has little effect on the peak
- impossible to produce a narrow peak for other $\chi_{c1}p$ partial waves
- strength unkown
- LHCb 2019: $P_c(4440)$ & $P_c(4457)$ cannot be just such a TS, but there might still be a nontrivial interference (dip at 4.45 GeV)

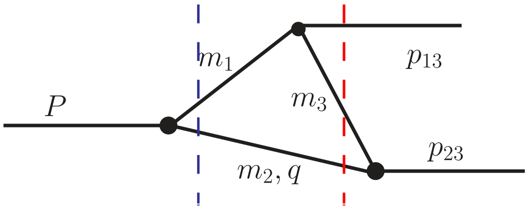




- Lots of resonances or resonance-like structures observed in recent years, many puzzles
- QCD symmetries (chiral, heavy quark) prove to be useful tools
- Many more data needed, lots of work needs to be done

Thank you for your attention!

Backup slides



Consider the scalar three-point loop integral

$$I = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{[(P-q)^2 - m_1^2 + i\epsilon]\,(q^2 - m_2^2 + i\epsilon)\,[(p_{23}-q)^2 - m_3^2 + i\epsilon]}$$

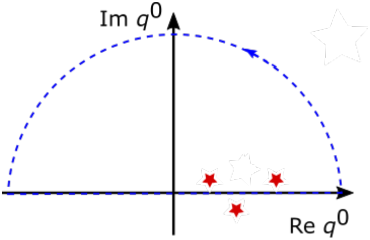
Rewriting a propagator into two poles:

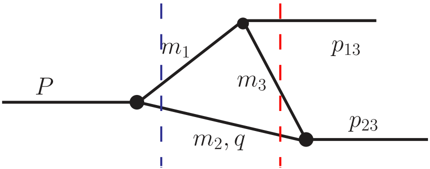
$$\frac{1}{q^2 - m_2^2 + i\epsilon} = \frac{1}{(q^0 - \omega_2 + i\epsilon)\,(q^0 + \omega_2 - i\epsilon)} \quad \text{with} \quad \omega_2 = \sqrt{m_2^2 + \vec{q}^{\,2}}$$

focus on the positive-energy poles

$$I \simeq \frac{i}{8m_1m_2m_3} \int \frac{dq^0 d^3\vec{q}}{(2\pi)^4} \frac{1}{(P^0 - q^0 - \omega_1 + i\epsilon)\,(q^0 - \omega_2 + i\epsilon)\,(p_{23}^0 - q^0 - \omega_3 + i\epsilon)}$$

TS: some details (2)





Contour integral over $q^0 \Rightarrow$ cut-1 cut-2

$$I \propto \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{[P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon][p_{23}^0 - \omega_2(q) - \omega_3(\vec{p}_{23} - \vec{q}\,) + i\,\epsilon]}$$

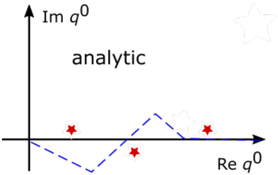
$$\propto \int_0^\infty dq\, \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon} f(q)$$

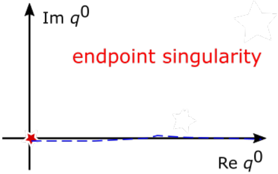
The second cut:

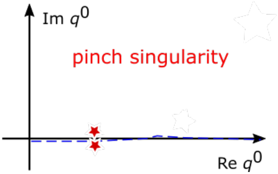
$$f(q) = \int_{-1}^{1} dz\, \frac{1}{p_{23}^0 - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 - 2p_{23}qz} + i\,\epsilon}$$

Relation between singularities of integrand and integral

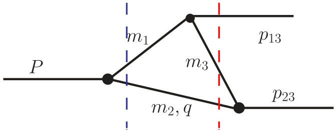
- singularity of integrand does not necessarily give a singularity of integral:
  integral contour may be deformed to avoid the singularity
- Two cases that a singularity cannot be avoided:
  - ☞ endpoint singularity
  - ☞ pinch singularity







TS: some details (4)



$$I \propto \int_0^\infty dq \, \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon} f(q)$$

$$f(q) = \int_{-1}^{1} dz \, \frac{1}{A(q,z)} \equiv \int_{-1}^{1} dz \, \frac{1}{p_{23}^0 - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 - 2p_{23}qz} + i\,\epsilon}$$

Singularities of the **integrand of I** in the rest frame of initial particle ($P^0 = M$):

- 1st cut: $M - \omega_1(l) - \omega_2(l) + i\,\epsilon = 0 \Rightarrow$

$$q_{\rm on\pm} \equiv \pm \left(\frac{1}{2M} \sqrt{\lambda(M^2, m_1^2, m_2^2)} + i\,\epsilon \right)$$

- 2nd cut: $A(q, \pm 1) = 0 \Rightarrow$ endpoint singularities of $f(q)$

$$z = +1: \quad q_{a+} = \gamma\,(\beta\, E_2^* + p_2^*) + i\,\epsilon\,, \qquad q_{a-} = \gamma\,(\beta\, E_2^* - p_2^*) - i\,\epsilon\,,$$

$$z = -1: \quad q_{b+} = \gamma\,(-\beta\, E_2^* + p_2^*) + i\,\epsilon\,, \quad q_{b-} = -\gamma\,(\beta\, E_2^* + p_2^*) - i\,\epsilon$$

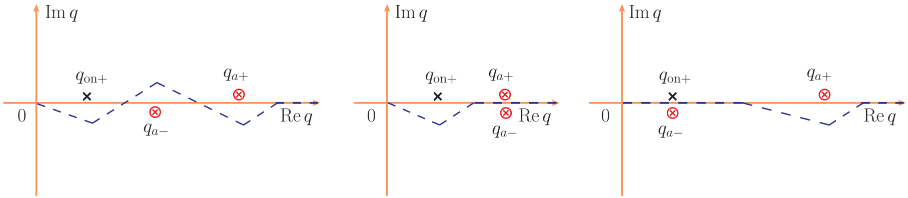
$$\beta = |\vec{p}_{23}|/E_{23}, \qquad \gamma = 1/\sqrt{1-\beta^2} = E_{23}/m_{23}$$

$E_2^*(p_2^*)$: energy (momentum) of particle-2 in the cmf of the (2,3) system

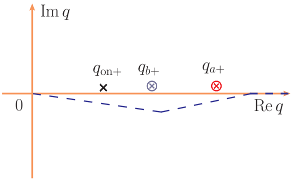
All singularities of the integrand of I:

$$q_{\rm on+}, \qquad q_{a+} = \gamma\,(\beta\, E_2^* + p_2^*) + i\,\epsilon, \qquad q_{a-} = \gamma\,(\beta\, E_2^* - p_2^*) - i\,\epsilon,$$

$$q_{\rm on-} < 0, \quad q_{b-} = -q_{a+} < 0 \text{ (for } \epsilon = 0), \qquad q_{b+} = -q_{a-},$$



(a) (b) (c)



2-body threshold singularity at

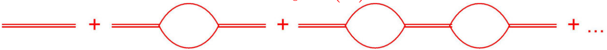
$m_{23} = m_2 + m_3$

triangle singularity at

$p_{2,\text{left}} = p_{2,\text{right}}$

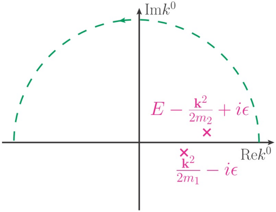
here $p_{2,\text{left}} = q_{\rm on+}$, $p_{2,\text{right}} = q_{a-}$

We may also start from a QFT (for very small E_B, nonrelativistic)

Free field	Interacting field
$\frac{i}{E-E_0}$	$\frac{i}{E-E_0-\Sigma(E)}$
	

Here $E_0 = M_0 - m_1 - m_2$ with M_0 the bare mass, $\Sigma(E)$ is the self-energy (g_0: bare coupling constant)

$$\Sigma(E) = i g_0^2 \int \frac{d^4k}{(2\pi)^4} \left[\left(k^0 - \frac{\boldsymbol{k}^2}{2m_1} + i\epsilon \right) \left(E - k^0 - \frac{\boldsymbol{k}^2}{2m_2} + i\epsilon \right) \right]^{-1}$$



$$= -i2\mu g_0^2 (2\pi i) \int^{\Lambda} \frac{d^3\boldsymbol{k}}{(2\pi)^4} \frac{1}{2\mu E - \boldsymbol{k}^2 + i\epsilon}$$

$$= g_0^2 \frac{\mu}{2\pi} \sqrt{-2\mu E - i\epsilon} + \text{constant}$$

$$= g_0^2 \frac{\mu}{2\pi} \left[\sqrt{-2\mu E}\,\theta(-E) - i\sqrt{2\mu E}\,\theta(E) \right] + \text{constant}$$

Compositeness (6)

The physical mass is $M = m_1 + m_2 + E_B (E_B \geq 0)$ with $-E_B$ the solution of $E - E_0 - \Sigma(E) = 0$, i.e.

$$E_B = -E_0 - \Sigma(-E_B)$$

Expanding the self-energy around the pole, we rewrite the propagator

$$\begin{aligned}
\frac{i}{E - E_0 - \Sigma(E)} &= \frac{i}{E - E_0 - \left[\Sigma(-E_B) + (E + E_B)\Sigma'(-E_B) + \tilde{\Sigma}(E)\right]} \\
&= \frac{i}{E + E_B - (E + E_B)\Sigma'(-E_B) - \tilde{\Sigma}(E)} \\
&= \frac{iZ}{E + E_B - Z\tilde{\Sigma}(E)}
\end{aligned}$$

Z is the wave function renormalization constant

$$\boxed{Z = \frac{1}{1 - \Sigma'(-E_B)} = \left[1 + \frac{g_0^2 \mu^2}{2\pi\sqrt{2\mu E_B}}\right]^{-1}}$$

The physical coupling constant

$$\tilde{g}^2 = Z g_0^2 = \frac{1}{\frac{1}{g_0^2} + \frac{\mu^2}{2\pi\sqrt{2\mu E_B}}} = (1-Z)\frac{2\pi}{\mu^2}\sqrt{2\mu E_B}$$

Taking into account the nonrel. normalization, we get the one with rel. normalization

$$\boxed{g^2 = 8m_1 m_2(m_1+m_2)\tilde{g}^2 = 16\pi(1-Z)(m_1+m_2)^2\sqrt{\frac{2E_B}{\mu}}}$$

If the ERE is dominated by the scattering length (when the pole is extremely close to threshold),

$$T(E) = \frac{2\pi/\mu}{-1/a - \sqrt{-2\mu E - i\epsilon}}$$

At LO, effective coupling strength for bound state

$$|g_{\rm NR}|^2 = \lim_{E\to -E_B}(E+E_B)\,T(E) = -\frac{2\pi}{\mu}\left(\frac{d}{dE}\sqrt{-2\mu E - i\epsilon}\right)^{-1}_{E=-E_B}$$

$$= \frac{2\pi}{\mu^2}\sqrt{2\mu E_B} \quad \Rightarrow \quad Z = 0 \text{ at this leading order approximation}$$