Lectures on

Chiral Perturbation Theory

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STRUCTURE of the LECTURES

I) Short Introduction

II) Chiral Symmetry in QCD

III) Chiral Perturbation Theory: Goldstone Bosons

IV) Chiral Perturbation Theory: Pions and Nucleons

V) Testing Chiral Dynamics in Hadron-Hadron Scattering

- more emphasis on the foundations rather than on specific calculations

Introduction

FORCES in NATURE

type	gauge boson	spin $[\hbar]$	range [m]	strength @ hadronic scale
gravity	graviton	2	∞	10^{-40}
weak int.	W,Z-bosons	1	10^{-17}	10^{-5}
EM int.	photon	1	∞	$1/137$
strong int.	gluons	1	10^{-15}	~ 1

} SU(3)$_C \times$SU(2)$_L \times$U(1)$_Y$

- electro-weak interactions are perturbative at hadronic scales
- strong interactions are really strong $\rightarrow$ non-perturbative

QCD LAGRANGIAN

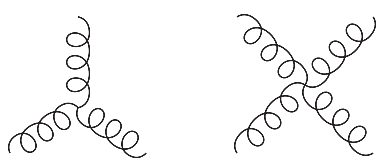
Note: θ-term not discussed here

- $$\mathcal{L}_{\rm QCD} = -\frac{1}{4}\, G^a_{\mu\nu} G^{\mu\nu,a} + \sum_f \bar{q}_f (i\gamma_\mu D^\mu - \mathcal{M}) q_f + \ldots$$

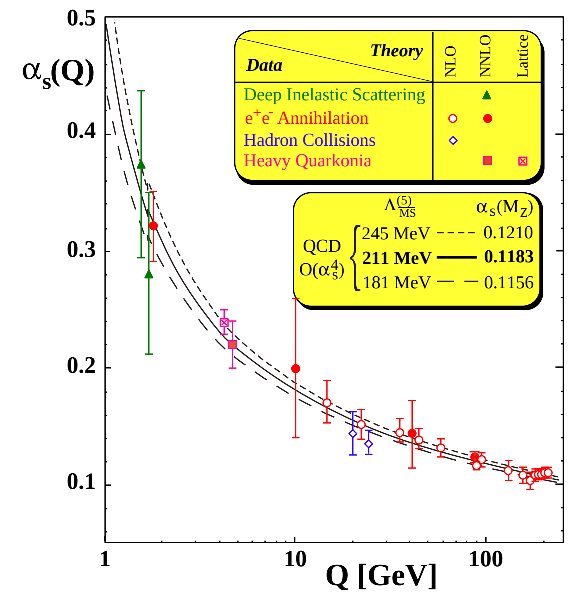
$$\underbrace{G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - ig[A^b_\mu, A^c_\nu],}_{\text{gluon field strength tensor}} \quad \underbrace{D_\mu = \partial_\mu + igA^a_\mu \lambda^a/2}_{\text{covariant derivative}}$$

$$\underbrace{f = (u,d,s,c,b,t)}_{\text{flavors}}\ , \quad \underbrace{\mathcal{M} = {\rm diag}(m_u, m_d, m_s, m_c, m_b, m_t)}_{\text{quark mass matrix}}$$

- local color gauge invariance SU(3)$_C$
- non-linear couplings:



- running of $\alpha_s = g^2/4\pi$
- **light** (u,d,s) and heavy (c,b,t) quark flavors:



$$m_{\rm light} \ll \Lambda_{\rm QCD}, m_{\rm heavy} \gg \Lambda_{\rm QCD}$$

$$\Lambda_{\rm QCD} \simeq 250 \text{ MeV}$$

- quarks and gluons are **confined** within hadrons & nuclei

  $\hookrightarrow$ concentrate on the light quark sector here (pions, nucleons)

QCD chiral dynamics

INTRO: CHIRAL SYMMETRY

- Massless fermions exhibit **chiral symmetry**:

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi$$

- left/right-decomposition:

$$\psi = \tfrac{1}{2}(1-\gamma_5)\psi + \tfrac{1}{2}(1+\gamma_5)\psi = P_L\psi + P_R\psi = \psi_L + \psi_R$$

- projectors:

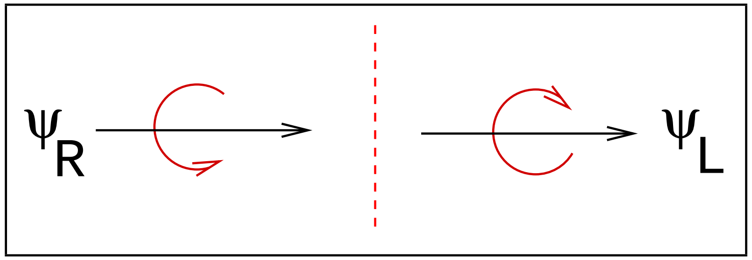
$$P_L^2 = P_L,\ P_R^2 = P_R,\ P_L\cdot P_R = 0,\ P_L + P_R = \mathbb{1}$$

- helicity eigenstates:

$$\tfrac{1}{2}\hat{h}\psi_{L,R} = \pm\tfrac{1}{2}\psi_{L,R} \quad \hat{h} = \vec{\sigma}\cdot\vec{p}/|\vec{p}\,|$$

- L/R fields do **not** interact $\rightarrow$ conserved L/R currents

$$\mathcal{L} = i\bar{\psi}_L\gamma_\mu\partial^\mu\psi_L + i\bar{\psi}_R\gamma_\mu\partial^\mu\psi_R$$



- mass terms break chiral symmetry: $\bar{\psi}\mathcal{M}\psi = \bar{\psi}_R\mathcal{M}\psi_L + \bar{\psi}_L\mathcal{M}\psi_R$

CHIRAL SYMMETRY of QCD

- Three flavor QCD:

$$\boxed{\mathcal{L}_{\rm QCD} = \mathcal{L}^0_{\rm QCD} - \bar{q}\mathcal{M}q}\ , \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} , \quad \mathcal{M} = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

- $\mathcal{L}^0_{\rm QCD}$ is invariant under **chiral** $SU(3)_L \times SU(3)_R$ (split off U(1)'s)

$$\mathcal{L}^0_{\rm QCD}(G_{\mu\nu}, q', D_\mu q') = \mathcal{L}^0_{\rm QCD}(G_{\mu\nu}, q, D_\mu q)$$

$$q' = RP_R q + LP_L q = Rq_R + Lq_L \quad R, L \in SU(3)_{R,L}$$

- conserved L/R-handed [vector/axial-vector] Noether currents:

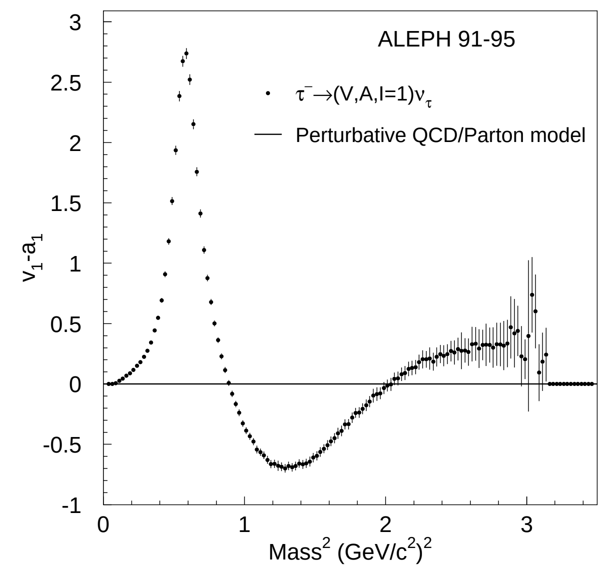
$$J^{\mu,a}_{L,R} = \bar{q}_{L,R}\gamma^\mu \frac{\lambda^a}{2} q_{L,R}\ , \quad a = 1, \ldots, 8$$

$$\partial_\mu J^{\mu,a}_{L,R} = 0 \quad [\text{or}\ \ V^\mu = J^\mu_L + J^\mu_R\ , \quad A^\mu = J^\mu_L - J^\mu_R]$$

- Is this symmety reflected in the vacuum structure/hadron spectrum?

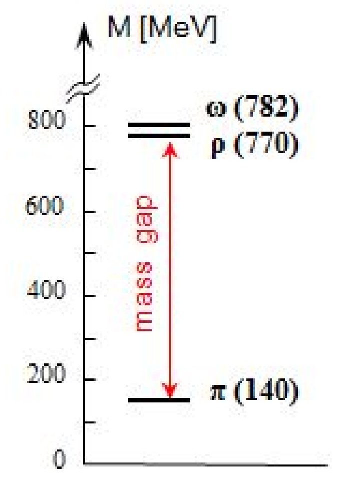
THE FATE of QCD's CHIRAL SYMMETRY

- the chiral symmetry is not "visible" (spontaneously broken)
  - no parity doublets
  - $\langle 0|AA|0\rangle \neq \langle 0|VV|0\rangle$
  - scalar condensate $\bar{q}q$ acquires v.e.v.
  - Vafa-Witten theorem [NPB 234 (1984) 173]
  - (almost) massless pseudoscalar bosons



- the chiral symmetry is realized in the Nambu-Goldstone mode
  - weakly interacting massless pseudoscalar excitations
  - approximate symmetry (small quark masses)
    $\to \pi, K, \eta$ as Pseudo-Goldstone Bosons
  - calls for an effective field theory

$\Rightarrow$ Chiral Perturbation Theory



THE FATE of QCD's CHIRAL SYMMETRY II

- Wigner mode $Q_5^a|0\rangle = Q^a|0\rangle = 0 \; (a = 1,\ldots,8)$?

- parity doublets: $dQ_5^a/dt = 0 \to [H, Q_5^a] = 0$

  single particle state: $H|\psi_p\rangle = E_p|\psi_p\rangle$

  axial rotation: $H(\mathrm{e}^{iQ_5^a}|\psi_p\rangle) = \mathrm{e}^{iQ_5^a} H|\psi_p\rangle = \underbrace{E_p(\mathrm{e}^{iQ_5^a}|\psi_p\rangle)}$

  same mass but opposite parity

- VV and AA spectral functions (without pion pole):

$$\begin{array}{c} \langle 0|VV|0\rangle = \langle 0|(L+R)(L+R)|0\rangle = \langle 0|L^2+R^2+2LR|0\rangle = \langle 0|L^2+R^2|0\rangle \\ \| \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \| \\ \langle 0|AA|0\rangle = \langle 0|(L-R)(L-R)|0\rangle = \langle 0|L^2+R^2-2LR|0\rangle = \langle 0|L^2+R^2|0\rangle \end{array}$$

since L and R are orthogonal

PROPERTIES of GOLDSTONE BOSONS

- GBs are massless [no explicit symmetry breaking]

consider a broken generator $[Q,H]=0$ but $Q|0\rangle \neq 0$

define $|\psi\rangle \equiv Q|0\rangle$

$\rightarrow H|\psi\rangle = HQ|0\rangle = QH|0\rangle = 0$

$\rightarrow$ not only G.S. $|0\rangle$ has $E=0$

> There exist massless excitations, non-interacting as $E,p\rightarrow 0$

[NB: proper argumentation requires more precise use of the infinite volume]

- explicit symmetry breaking, perturbative [small parameter ε]

Goldstone bosons acquire a small mass $M^2_{\rm GB} \sim \varepsilon$

In QCD, this symmetry breaking is given in terms of the light quark masses

$$\Rightarrow M_\pi^2 \sim (m_u+m_d)$$

Chiral Perturbation Theory: Goldstone Bosons

CHIRAL EFT of QCD

Gasser, Leutwyler, Weinberg, van Kolck, Kaplan, Savage, Wise, Bernard, Kaiser, M., . . .

- Starting point: CHIRAL LAGRANGIAN (two flavors)

$$\mathcal{L}_{\rm QCD} \to \mathcal{L}_{\rm EFF} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} \;\; [+\mathcal{L}_{NN} + \ldots]$$

- dofs: quark & gluon fields $\to$ pions, nucleons, external sources
- Spontaneous chiral symmetry breaking of QCD $\to$ pions are Goldstone bosons
- Systematic expansion in powers of q/Λ_χ & M_π/Λ_χ, with $\Lambda_\chi \simeq 1$ GeV
- pion and pion-nucleon sectors are perturbative in q $\to$ chiral perturbation theory
- Parameters in $\mathcal{L}_{\pi\pi}$ and $\mathcal{L}_{\pi N}$ known from CHPT studies $\to$ low-energy constants

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[• $\mathcal{L}_{NN}$ collects short-distance contact terms, to be fitted]

[• NN interaction requires non-perturbative resummation]

[$\to$ chirally expand $V_{\rm NN}$, use in regularized LS equation]

STRUCTURE of the chiral EFT

- Energy expansion [derivative/momentum/. . .]

  dimensional analysis:

  (a) derivatives $\to$ powers of q [small scale]

  (b) be Λ the hard [limiting] scale

  $\to$ any derivative $\partial \sim q/\Lambda$

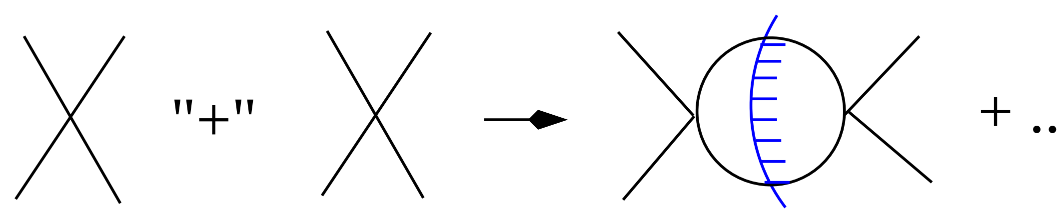
  $\to$ N derivative vertex $\sim q^N/\Lambda^N$

  $\to$ for $E[q] \ll \Lambda$, terms w/ more derivatives are suppressed

- Energy expansion = Loop expansion

  interactions generate loops

  loops generate imaginary parts



$\Rightarrow$ all this is contained in the *power counting*, which assigns a dimension [not the canonical one] to each diagram

POWER COUNTING THEOREM

- Consider $\mathcal{L}_{\rm eff} = \sum_d \mathcal{L}^{(d)}$, d bounded from below
- for interacting Goldstone bosons, $d \geq 2$ and $iD(q) = \dfrac{1}{q^2 - M^2}$
- consider an L-loop diagram with I internal lines and V_d vertices of order d

$$Amp \propto \int (d^4q)^L \, \frac{1}{(q^2)^I} \prod_d (q^d)^{V_d}$$

- let $Amp \sim q^\nu \to \nu = 4L - 2I + \sum_d d V_d$
- topology: $L = I - \sum_d V_d + 1$
- eliminate I: $\to \boxed{\nu = 2 + 2L + \sum_d V_d (d-2)}$ ✓
- Important: construct all possible terms for a given ν

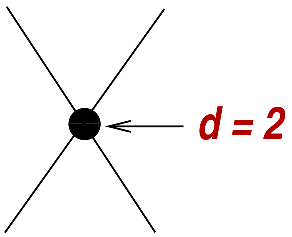
POWER COUNTING for PION-PION SCATTERING

- $\pi(p_1) + \pi(p_2) \to \pi(p_3) + \pi(p_4)$

  leading interaction $\sim \partial\pi\,\partial\pi \Rightarrow d = 2$

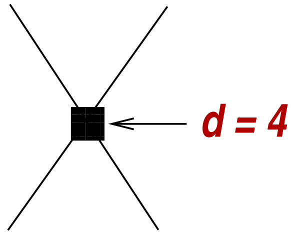
- leading order (LO)

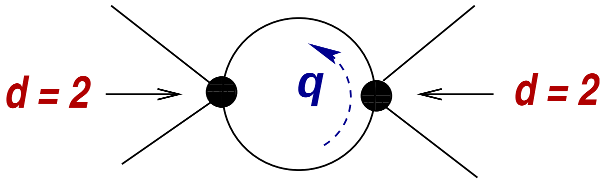
  $d = 2, N_L = 0 \Rightarrow D = 2$



- next-to-leading order (NLO)

  $a)\ d = 4, N_L = 0 \Rightarrow D = 4$ $\qquad$ $b)\ d = 2, N_L = 1 \Rightarrow D = 4$





$$\sim \int d^4q \frac{q_1 \cdot q_2 \;\; q_3 \cdot q_4}{(q^2 - M_\pi^2)(q^2 - M_\pi^2)} \sim \mathcal{O}(q^4)$$

LOW-ENERGY CONSTANTS (LECs)

- consider a covariant & parity-invariant theory of Goldstone bosons parameterized in some matrix-valued field U

$$\mathcal{L}_{\rm eff} = g_2 {\rm Tr}(\partial_\mu U \partial^\mu U^\dagger) + g_4^{(1)} \left[{\rm Tr}(\partial_\mu U \partial^\mu U^\dagger)\right]^2 + g_4^{(2)} {\rm Tr}(\partial_\mu U \partial^\nu U^\dagger) {\rm Tr}(\partial_\nu U \partial^\mu U^\dagger) + \ldots$$

- couplings = **low-energy constants** (LECs)

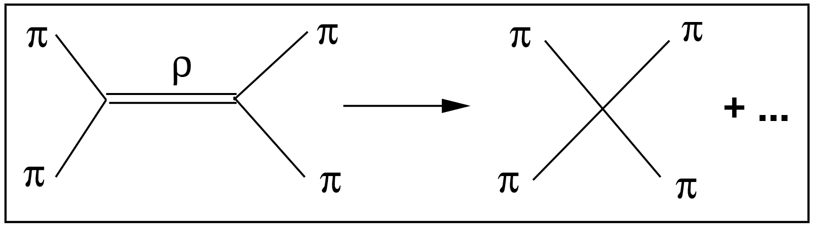
  $g_2 \neq 0$ spontaneous chiral symmetry breaking (cf also the $g^{(i)}_{>2}$)

  $g_4^{(1)}, g_4^{(2)}, \ldots$ must be fixed from data (or calculated from the underlying theory)

- calculations in EFT: fix the LECs from some processes, then make **predictions**

- LECs encode information about the high mass states that are integrated out

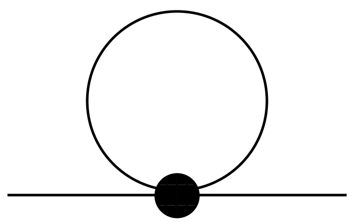
$$\frac{g^2_{\rho\pi\pi}}{M_\rho^2 - q^2} \xrightarrow{q^2 \ll M_\rho^2} \frac{g^2_{\rho\pi\pi}}{M_\rho^2}\left(1 + \frac{q^2}{M_\rho^2} + \ldots\right)$$



LOOPS and DIVERGENCES

- Loop diagrams generate imag. parts, but are mostly **divergent**

$\Rightarrow$ choose a mass-independent & symmetry-preserving regularization scheme
[like dimensional regularization] $\longrightarrow$ consider tadpole as example:



$$= -i\Delta_\pi(0) = \frac{-i}{(2\pi)^d}\int d^dp \frac{1}{M^2 - p^2 - i\varepsilon} \quad \text{[d space-time dim.]}$$

$$= (2\pi)^{-d}\int d^dk \frac{1}{M^2 + k^2} \text{ with } p_0 = ik_0\,, \quad -p^2 = k_0^2 + \vec{k}^{\,2}$$

$$= (2\pi)^{-d}\int d^dk \int_0^\infty d\lambda \exp(-\lambda(M^2 + k^2))$$

$$= (2\pi)^{-d}\int_0^\infty d\lambda \exp(-\lambda M^2) \underbrace{\int d^dk \exp(-\lambda k^2)}_{(\pi/\lambda)^{d/2}}$$

$$= (4\pi)^{-d} M^{d-2}\,\Gamma\left(1 - \frac{d}{2}\right) \quad \text{has a pole at } d = 4$$

$\Rightarrow$ absorb in LECs: $\boxed{g_i \to g_i^{\rm ren} + \beta_i \frac{1}{d-4}}$ always possible!

CHIRAL PERTURBATION THEORY

- Consider first the mesonic chiral effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots$$

$$\mathcal{L}^{(2)} = \frac{F_\pi^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle \qquad [U^\dagger U = UU^\dagger = 1, U \to LUR^\dagger]$$

$$U = \exp(i\Phi/F_\pi)\ , \quad \Phi = \sqrt{2}\begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix}$$

$\chi = 2B\mathcal{M} + \ldots,\ B = |\langle 0|\bar{q}q|0\rangle|/F_\pi^2 \leftarrow$ scalar quark condensate

- Two parameters:

$F_\pi \simeq 92$ MeV = pion decay constant (GB coupling to the vacuum)

$B \simeq 2$ GeV = normalized vacuum condensate

- Goldstone boson masses: $M_{\pi^+}^2 = (m_u + m_d)B\ ,\ \ M_{K^+}^2 = (m_d + m_s)B\ , \ldots$

- has been extended to two loops $\mathcal{O}(q^6)$ in many cases

FROM QUARK to MESON MASSES

- Explicit symmetry breaking Lagrangian:

$$\mathcal{L}_{\rm SB} = \mathcal{M} \times f(U, \not{\partial_\mu} U, \ldots)\ , \quad \mathcal{M} = {\rm diag}(m_u, m_d)$$

- LO invariants: ${\rm Tr}(\mathcal{M}U^\dagger)\ ,\ {\rm Tr}(U\mathcal{M}^\dagger)$

$$\Rightarrow \mathcal{L}_{\rm SB} = \frac{1}{2}F_\pi^2 \left\{ B\,{\rm Tr}(\mathcal{M}U^\dagger + U\mathcal{M}^\dagger)\right\}$$

B is a real constant if CP is conserved

$$= (m_u + m_d)\, B \left[F_\pi^2 - \tfrac{1}{2}\pi^2 + \tfrac{\pi^4}{24F_\pi^2} + \ldots \right]$$

[expand $U = \exp(i\vec{\tau}\cdot\vec{\pi}/F_\pi)$]

First term (vacuum): $\left.\dfrac{\partial \mathcal{L}_{\rm QCD}}{\partial m_q}\right|_{m_q=0} = -\bar{q}q$

$$\Rightarrow \langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = -BF_\pi^2\,(1 + \mathcal{O}(\mathcal{M}))$$

Second term (pion mass): $-\frac{1}{2}M_\pi^2\pi^2 \Rightarrow M_\pi^2 = (m_u + m_d)B$

combined: $\boxed{M_\pi^2 = -(m_u + m_d)\,\langle 0|\bar{u}u|0\rangle / F_\pi^2}$ Gell-Mann–Oakes–Renner rel.

repeat for SU(3) $\Rightarrow 3M_\eta^2 = 4M_K^2 - M_\pi^2$ Gell-Mann–Okubo relation

MESON MASSES $\to$ QUARK MASS RATIOS

- lowest order:

$$M^2_{\pi^+} = (m_u + m_d)B \simeq (0.140\,\text{GeV})^2$$
$$M^2_{K^0} = (m_u + m_s)B \simeq (0.494\,\text{GeV})^2$$
$$M^2_{K^+} = (m_d + m_s)B \simeq (0.497\,\text{GeV})^2$$

$$\xrightarrow{\text{ratios}} \quad \frac{m_u}{m_d} = 0.66\,, \quad \frac{m_s}{m_d} = 20.1\,, \quad \frac{\hat m}{m_s} = \frac{1}{24.2} \quad [\hat m = \tfrac{1}{2}(m_u + m_d)]$$

- corrections: next-to-leading order and beyond

electromagnetism — Weinberg, Gasser, Leutwyler, ...

$$\longrightarrow \quad \boxed{\frac{m_u}{m_d} = 0.553 \pm 0.043\,, \quad \frac{m_s}{m_d} = 18.9 \pm 0.8\,, \quad \frac{\hat m}{m_s} = \frac{1}{24.4 \pm 1.5}}$$

lattice QCD can now get these ratios with better precision [strong]

no large isospin violation since $m_u - m_d$ so small vs hadronic scale

Chiral Perturbation Theory: Pions and Nucleons

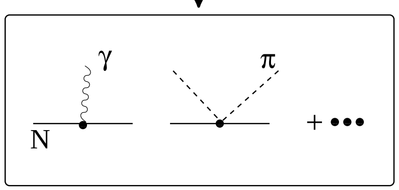
CHIRAL EFECTIVE PION-NUCLEON THEORY

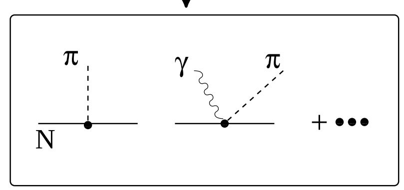
- view nucleon as matter fields [from now on, SU(2) only]
- chiral symmetry *dictates* the couplings to pions & external sources

$\downarrow$ a few steps well documented in the literature

$$\mathcal{L} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} + \ldots$$

$$\mathcal{L}^{(1)}_{\pi N} = \bar{\psi} \left(i \not{D} - m_N + \tfrac{1}{2} g_A \gamma_\mu \gamma_5 u^\mu \right) \psi$$





- tree calculations: tree graphs w/ insertions from $\mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N}$
- one-loop calculations: tree graphs w/ insertions from $\mathcal{L}^{(1)}_{\pi N} + \ldots + \mathcal{L}^{(4)}_{\pi N}$

plus loop graphs w/ (one) insertion(s) from $\mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N}$

EFFECTIVE LAGRANGIAN AT ONE LOOP

- Pion-nucleon Lagrangian: $$\boxed{\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)}}$$

[$^{(n)}$ = chiral dimension]

with

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\,\not{D} - m_N + \frac{1}{2} g_A\,\not{u}\,\gamma_5\right)\Psi$$

[$u_\mu \sim \partial_\mu \phi$]

$$\mathcal{L}_{\pi N}^{(2)} = \sum_{i=1}^{7} c_i \bar{\Psi} O_i^{(2)} \Psi = \bar{\Psi}\left(c_1 \langle \chi_+ \rangle + c_2\left(-\frac{1}{8m_N^2}\langle u_\mu u_\nu \rangle D^{\mu\nu} + \text{h.c.}\right) + c_3 \frac{1}{2}\langle u \cdot u \rangle\right.$$
$$\left. + c_4 \frac{i}{4}[u_\mu, u_\nu]\,\sigma^{\mu\nu} + c_5\,\widetilde{\chi}_+ + c_6 \frac{1}{8m_N} F^+_{\mu\nu}\sigma^{\mu\nu} + c_7 \frac{1}{8m_N}\left\langle F^+_{\mu\nu}\right\rangle \sigma^{\mu\nu}\right)\Psi$$

- dynamical LECs $g_A \sim \partial_\mu \phi$, and $c_2, c_3, c_4 \sim \partial_\mu^2 \phi, \partial_\mu \partial_\nu \phi$
- symmetry breaking LECs $c_1 \sim m_u + m_d$, $c_5 \sim m_u - m_d$
- external probe LECs $c_6, c_7 \sim eQ\mathcal{A}_\mu$

$$\mathcal{L}_{\pi N}^{(3)} = \sum_{i=1}^{23} d_i\,\bar{\Psi}\,O_i^{(3)}\,\Psi\,, \qquad \mathcal{L}_{\pi N}^{(4)} = \sum_{i=1}^{118} e_i\,\bar{\Psi}\,O_i^{(4)}\,\Psi$$

for details, see Fettes et al., Ann. Phys. 283 (2000) 273 [hep-ph/0001308]

POWER-COUNTING in the PION-NUCLEON THEORY

- nucleon mass $m_N \sim 1\,$GeV $\rightarrow$ only three-momenta can be soft

  $\rightarrow$ complicates the power counting (see fig.) Gasser, Sainio, Svarc, Nucl. Phys. B **307** (1988) 779

- solutions:

  (1) Heavy-baryon approach

  Jenkins, Manohar; Bernard, Kaiser, M., . . .

  $1/m_N$ expansion a la Foldy-Wouthuysen of the Lagrangian

  m_N only appears in vertices, no longer in the propagator

  (2) Infrared Regularization [or variants thereof like EOMS]

  Becher, Leutwyler; Kubis, M.; Gegelia, Scherer, . . .

  extraction of the soft parts from the loop integrals

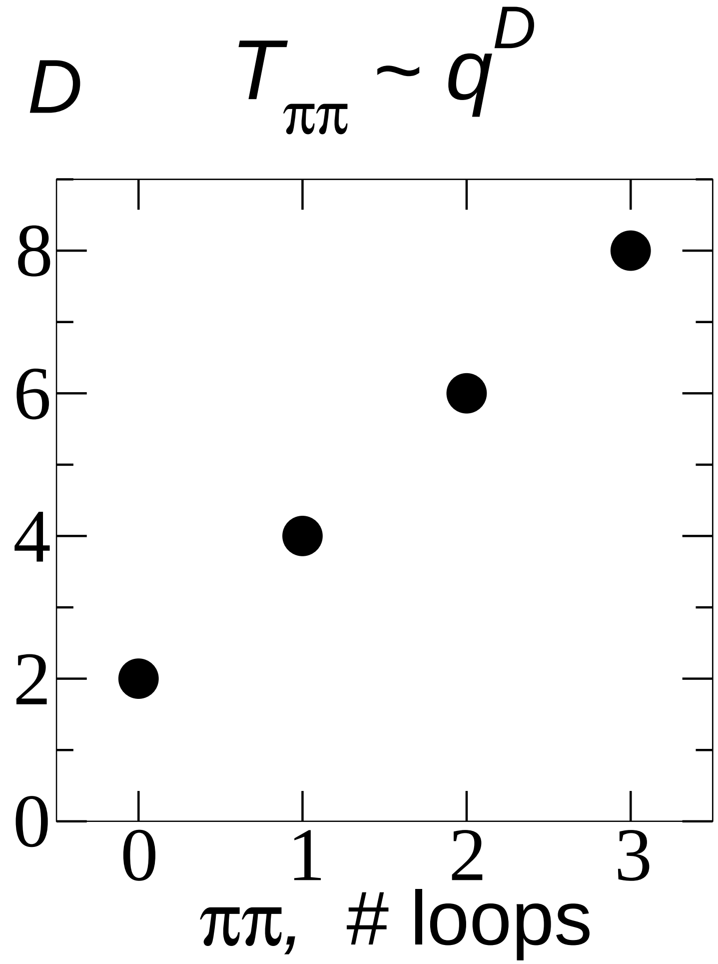
  easier to retain proper analytic structure

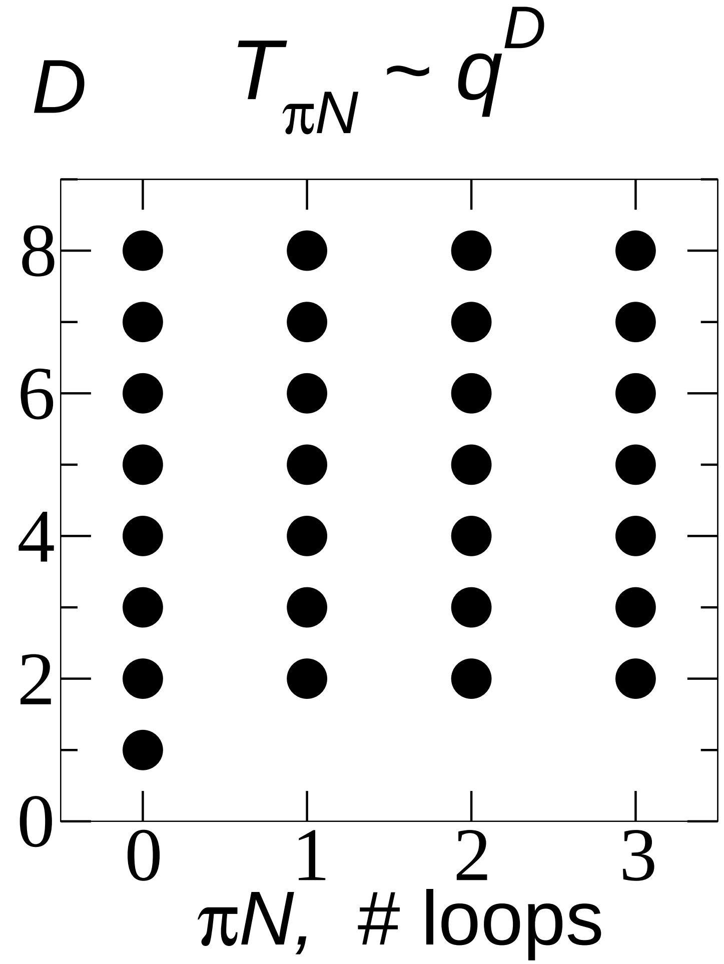
- most calculations at one loop, only two at two loop accuracy (g_A, m_N)

  Bernard, M.; Schindler, Scherer, Gegelia

FAILURE of the POWER-COUNTING

- naive extension of loop graphs from the pion to the pion-nucleon sector





HEAVY BARYON APPROACH I

- consider the nucleon as a static, heavy source $\to$ four-velocity v_μ:

Jenkins, Manohar 1991

$$\boxed{p_\mu = m_N v_\mu + \ell_\mu}\,, \quad v^2 = 1, \quad p^2 = m_N^2, \quad v\cdot\ell \ll m_N$$

- velocity-projection: $\Psi(x) = \exp(-im_N v\cdot x)\,[H(x) + h(x)]$

with $\quad \not{v}H = H,\ \not{v}h = -h \quad$ [“large/small” components]

- H- and h-components decouple, separated by large mas gap $2m_N$:

$$\mathcal{L}^{(1)}_{\pi N} = \bar\Psi\left(i\not{D} - m_N + \tfrac{1}{2}g_A\not{u}\gamma_5\right)\Psi$$

$$u_\mu = iu^\dagger\nabla_\mu U u^\dagger,\ U = u^2$$
$$\nabla_\mu U = \partial_\mu U - ieA_\mu[Q,U],\ Q = \mathrm{diag}(1,0)$$
$$D_\mu\Psi = \partial_\mu\Psi + \tfrac{1}{2}\left(u^\dagger(\partial_\mu - ieA_\mu Q)u + u(\partial_\mu - ieA_\mu Q)u^\dagger\right)\Psi$$

$$\to \boxed{\mathcal{L}^{(1)}_{\pi N} = \bar{H}\,(iv\cdot D + g_A S\cdot u)\,H + \mathcal{O}\left(\frac{1}{m_N}\right)}$$

HEAVY BARYON APPROACH II

- covariant spin-vector à la Pauli-Lubanski:

$$S_\mu = \frac{i}{2}\gamma_5\sigma_{\mu\nu}v^\nu, \;\; S\cdot v = 0, \;\; \{S_\mu, S_\nu\} = \frac{1}{2}(v_\mu v_\nu - g_{\mu\nu}), \;\; S^2 = \frac{1-d}{4}$$

- the Dirac algebra simplifies considerably (only v_μ and S_μ):

$$\bar{H}\gamma_\mu H = v_\mu \bar{H}H, \; \bar{H}\gamma_5 H = \mathcal{O}(\frac{1}{m_N}), \; \bar{H}\gamma_\mu\gamma_5 H = 2\bar{H}S_\mu H, \ldots$$

- propagator:

$$S(\omega) = \frac{i}{\omega + i\eta}, \;\; \omega = v\cdot\ell, \;\; \eta \to 0^+$$

  - mass scale moved from the propagator to $1/m_N$ suppressed vertices
  $\to$ power counting
  - can be systematically extended to arbitrary orders in $1/m_N$

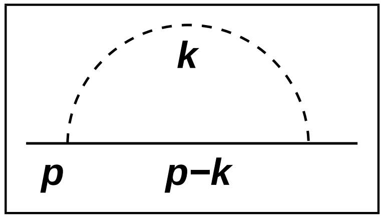
Bernard, Kaiser, Kambor, M., 1992

INFRARED REGULARIZATION I

- relativistic calculation of the nucleon self-energy:

Gasser, Sainio, Švarč, 1988, Becher, Leutwyler 1999

$$H(p^2) = \frac{1}{i}\int \frac{d^dk}{(2\pi)^d}\frac{1}{M_\pi^2 - k^2}\frac{1}{m_N - (p-k)^2}$$



$$\rightarrow H(s_0) = c(d)\frac{M_\pi^{d-3} + m_N^{d-3}}{M_\pi + m_N} = I + R\,, \quad s_0 = (M_\pi + m_N)^2$$

infrared singular piece I: generated by momenta of the order M_π
contains the chiral physics like chiral logs etc.

infrared regular piece R: generated by momenta of the order m_N
leads to the violation of the power counting
polynomial in external momenta and quark masses
$\rightarrow$ can be absorbed in the LECs of the eff. Lagr.

INFRARED REGULARIZATION II

Becher, Leutwyler 1999

- this symmetry-preserving splitting can be *uniquely* defined for any one-loop graph

- method to separate the infrared singular and regular parts (end-point singularity at $z = 1$):

$$H = \int \frac{d^dk}{(2\pi)^d} \frac{1}{AB} = \int_0^1 dz \int \frac{d^dk}{(2\pi)^d} \frac{1}{[(1-z)A + zB]^2}$$

$$= \left\{ \int_0^\infty - \int_1^\infty \right\} dz \int \frac{d^dk}{(2\pi)^d} \frac{1}{[(1-z)A + zB]^2} = I + R$$

$$A = M_\pi^2 - k^2 - i\eta\ , \quad B = m^2 - (p-k)^2 - i\eta\ , \quad \eta \to 0^+$$

- preserves the low-energy analytic structure of any one-loop graph

- extension to higher loop graphs difficult but doable

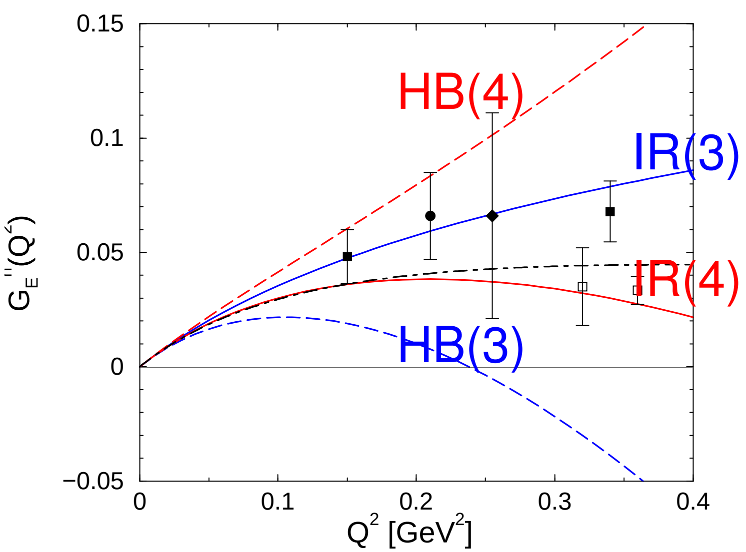
Lehmann, Prezeau, 2002

HEAVY BARYON vs INFRARED REGULARIZATION

- Heavy baryon (HB) is algebraically much simpler than infrared regularization (IR)
- HB can be extended to higher loop orders (IR requires modifications)
- Strict HB approach sometimes at odds with the analytic structure, IR not,
  e.g. anomalous threshold in triangle diagram (isovector em form factors)

$$t_c = 4M_\pi^2 - M_\pi^4/m_N^2 \overset{HB}{=} 4M_\pi^2 + \mathcal{O}(1/m_N^2)$$

- IR resums kinetic energy insertions
  $\longrightarrow$ sometimes improves convergence
  e.g. neutron electric ff $G_E^n(Q^2)$
  Kubis, M., 2001
- for a detailed discussion, see the review
  Bernard, Prog. Nucl. Part. Phys. **60** (2008) 82



EOMS REGULARIZATION I

Fuchs, Gegelia, Japaridze, Scherer 2003

- Extended-on-mass-shell scheme (EOMS), consider the chiral limit $M = 0$:

$$H(p^2, m_N^2, 0; d) = \frac{1}{i}\int \frac{d^dk}{(2\pi)^d} \frac{1}{[k^2 + i\varepsilon]} \frac{1}{[(p-k)^2 - m_N^2 + i\varepsilon]}$$

$\longrightarrow$ modify the integrand by subtracting suitable counterterms:

$$\sum_{\ell=0}^{\infty} \frac{p^2 - m_N^2}{\ell!} \left[\left(\frac{1}{p^2} p_\mu \frac{\partial}{\partial p_\mu} \right)^\ell \frac{1}{[k^2 + i\varepsilon]} \frac{1}{[((p^2 - m_N^2) + k^2 - 2k\cdot p + i\varepsilon]} \right]_{p^2 = m_N^2}$$

$$= \frac{1}{(k^2 + i\varepsilon)(k^2 - 2k\cdot p + i\varepsilon)} \bigg|_{p^2 = m_N^2}$$

$$+(p^2 - m_N^2) \left[\frac{1}{2m_N^2} \frac{1}{(k^2 - 2k\cdot p + i\varepsilon)^2} - \frac{1}{2m_N^2} \frac{1}{(k^2 + i\epsilon)(k^2 - 2k\cdot p + i\varepsilon)} - \frac{1}{(k^2 + i\epsilon)(k^2 - 2k\cdot p + i\varepsilon)^2} \right] + (p^2 - m_N^2)^2 \times \ldots$$

EOMS REGULARIZATION II

Fuchs, Gegelia, Japaridze, Scherer 2003

- Formal definition of the EOMS scheme:

$\to$ subtract from the integrand of those terms of the series which violate the p. c.

$\hookrightarrow$ These terms are always analytic in the small parameter

$\hookrightarrow$ They do not not contain infrared singularities

$\hookrightarrow$ EOMS acts on the integrand, not on the integration boundaries

- Nucleon self-energy: only subtract the first term on the r.h.s.

e.g. the second term (last summand) is IR singular as k^3/k^4

- Can be formulated more elegantly using the generating functional and utilizing heat kernel regularization

Du, Guo, UGM, 2016

POWER COUNTING in the PION-NUCLEON SYSTEM II

- consider the nucleon mass being eliminated, e.g. in the heavy baryon scheme $S(q) \sim 1/(v \cdot q)$ and vertices with $d \geq 1$

- Goldstone bosons as before, $d \geq 2$ and $D(q) \sim 1/(q^2 - M^2)$

- consider an L-loop diagram with I_B internal baryon lines, I_M internal meson lines, V_d^M mesonic vertices and V_d^{MB} meson-nucleon vertices of order d

$$Amp \propto \int (d^4q)^L \frac{1}{(q^2)^{I_M}} \frac{1}{(q)^{I_B}} \prod_d (q^d)^{(V_d^M + V_d^{MB})}$$

- let $Amp \sim q^\nu \to \nu = 4L - 2I_M + I_B + \sum_d d(V_d^M + V_d^{MB})$

- topology: $L = I_M + I_B - \sum_d (V_d^m + V_d^{MB}) + 1$
  and **one** baryon line through the diagram: $\sum_d V_d^{MB} = I_B + 1$

- eliminate I_M: $\boxed{\nu = 1 + 2L + \sum_d V_d^m (d-2) + \sum_d (d-1) V_d^{MB}}$ $\boxed{\to \nu \geq 1}$

STRUCTURE of the PION-NUCLEON INTERACTION

- Pion-nucleon scattering in chiral pertubation theory

Leading order (LO) ($\nu = 1$):

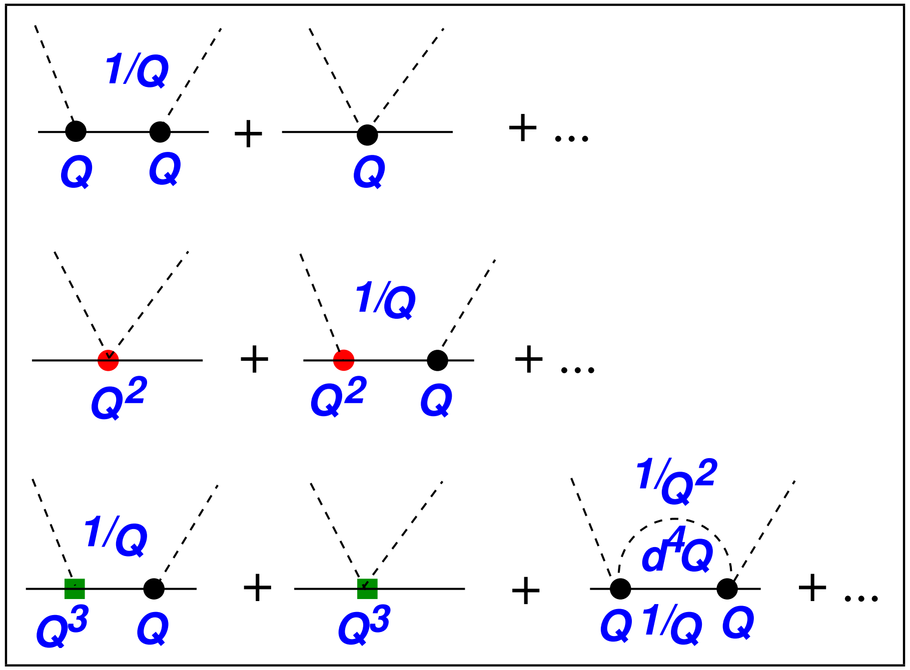
tree graphs w/ insertions with $d = 1$

Next-to-leading order (NLO) ($\nu = 2$):

tree graphs w/ insertions with $d = 1, 2$

Next-to-next-to-leading order (NNLO) ($\nu = 3$):

tree graphs w/ insertions with $d = 1, 2, 3$
and one-loop graphs w/insertion with $d = 1$



- calculations have been performed up to $\nu = 4$ (NNNLO = complete one-loop):

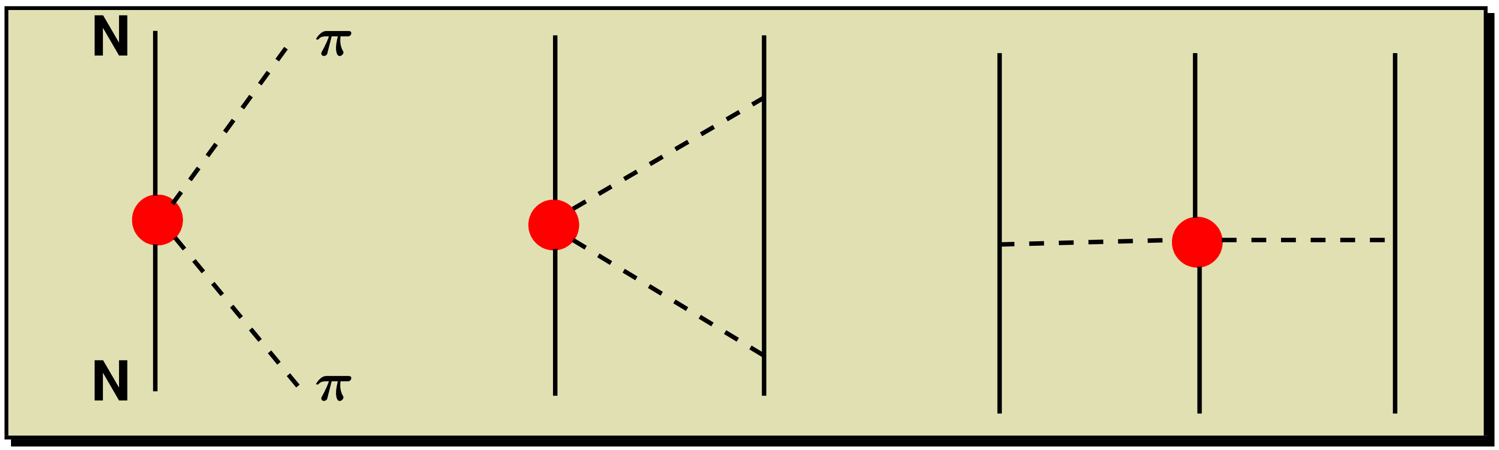
  heavy-baryon scheme Fettes, M., Nucl. Phys. A **676** (2000) 311, Krebs et al., Phys. Rev. C**85** (2012) 054006

  infrared-regularization scheme Becher, Leutwyler, JHEP **06** (2001) 017

  covariant EOMS scheme Alarcon et al., Phys. Rev. C**83** (2011) 055205; Siemens et al., Phys.Rev. C**94** (2016) 014620

APPLICATION: DIMENSION-TWO LECS

- Low-energy constants (LECs) relate ***many*** processes
- e.g. the dimension-two LECs c_i in πN, NN, NNN, ...



= operator from $\mathcal{L}_{\pi N}^{(2)} \propto c_i \quad (i = 1, 2, 3, 4)$

Here:
- determine the c_i from the purest process $\pi N \to \pi N$
- later use in the calculation of nuclear forces

DETERMINATION OF THE LECs

- πN scattering data can be explored in different ways (CHPT or disp. rel.):

- πN scattering inside the Mandelstam triangle:

$\rightarrow$ best convergence, relies on dispersive analysis

$\rightarrow$ not sensitive to all LECs, esp. c_2

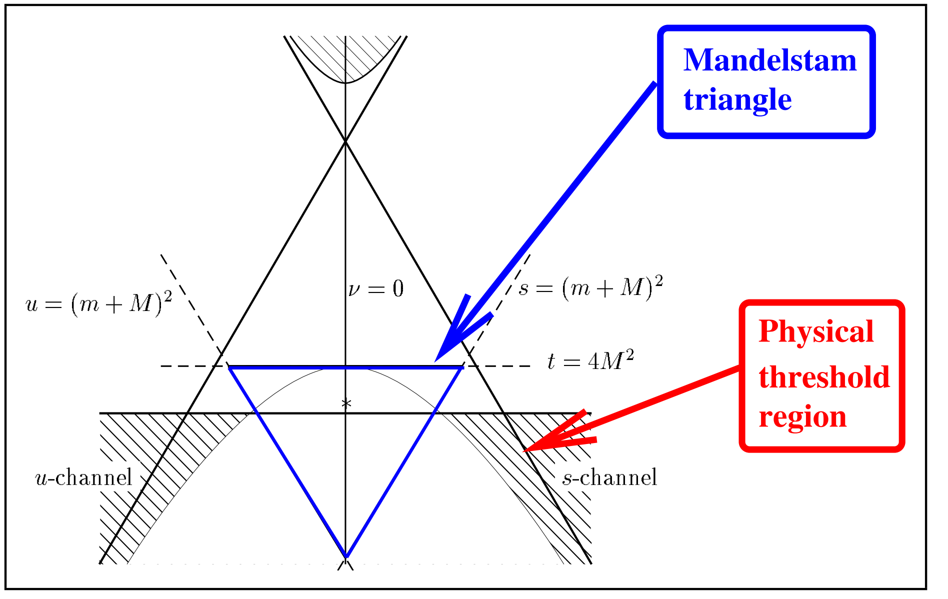
Büttiker, M., Nucl. Phys. A 668 (2000) 97 [hep-ph/9908247]

- πN scattering in the threshold region:

$\rightarrow$ large data basis, not all consistent

$\rightarrow$ use threshold parameters and global fits

$\rightarrow$ sizeable uncertainties remain in some LECs



Fettes, M., Steininger, Nucl. Phys. A 640 (1998) 119 [hep-ph/9803266]

Fettes, M., Nucl. Phys. A 676 (2000) 311 [hep-ph/0002182]

Becher, Leutwyler, JHEP 0106 (2001) 017 [arXiv:hep-ph/0103263]

- πN scattering from Roy-Steiner equations:

$\rightarrow$ hyperbolic partial-wave dispersion relations (unitarity & analyticity & crossing symmetry)

$\rightarrow$ most accurate representation of the πN amplitudes

Hoferichter, Ruiz de Elvira, Kubis, UGM, Phys. Rev. Lett. **115** (2015) 092301; Phys. Rev. Lett. **115** (2015) 192301; Phys. Rept. **625** (2016) 1

RESULTS for the LECs

- Chiral expansion expected to work best at the subthreshold point (polynomial, maximal distance to singularities)

- Express subthreshold parameters in terms of LECs $\to$ invert system

- LECs c_i of the dimension two chiral effective πN Lagrangian:

LEC	RS	KGE 2012	UGM 2005
c_1 [GeV^{-1}]	-1.11 ± 0.03	$-1.13 \ldots -0.75$	$-0.9^{+0.2}_{-0.5}$
c_2 [GeV^{-1}]	3.13 ± 0.03	$3.49 \ldots 3.69$	3.3 ± 0.2
c_3 [GeV^{-1}]	-5.61 ± 0.06	$-5.51 \ldots -4.77$	$-4.7^{+1.2}_{-1.0}$
c_4 [GeV^{-1}]	4.26 ± 0.04	$3.34 \ldots 3.71$	$-3.5^{+0.5}_{-0.2}$

Krebs, Gasparyan, Epelbaum, Phys. Rev. C85 (2012) 054006
UGM, PoS LAT2005 (2006) 009

- also results for pertinent dimension three and four LECs

INTERMEDIATE SUMMARY

- QCD has a chiral symmetry in the light quark sector (neglecting quark masses)
- Chiral symmetry is *spontaneously* and *explicitely* broken

  appearance of almost massless Goldstone bosons (π, K, η)

  Goldstone boson interactions vanish as $E, p \to 0$
- Chiral perturbation theory is the EFT of QCD that explores chiral symmetry
- Meson sector: only even powers in small momenta, many successes
- Single nucleon sector: odd & even powers, quite a few successes
- Low-energy constants relate many processes (in particular the c_i)
- Isospin-breaking can be systematically incorporated $\to$ spares
- NREFT can be set up for hadronic atoms $\to$ extraction of scattering lengths

Testing chiral dynamics in hadron-hadron scattering

WHY HADRON-HADRON SCATTERING?

- Weinberg's 1966 paper "Pion scattering lengths"

Weinberg, Phys. Rev. Lett. **17** (1966) 616

– pion scattering on a target with mass m_t and isospin T_t:

$$a_T = -\frac{L}{1 + M_\pi/m_t}\,[T(T+1) - T_t(T_t+1) - 2]$$

– pion scattering on a pion ["the more complicated case"]:

$$a_0 = \frac{7}{4}L\,, \quad a_2 = -\frac{1}{2}L$$

$$L = \frac{g_V^2 M_\pi}{8\pi F_\pi^2} \simeq 0.1\, M_\pi^{-1}$$

[$F_\pi = 92.1$ MeV]

- amazing predictions - witness to the power of chiral symmetry
- what have we learned since then?

Example 1

ELASTIC PION-PION SCATTERING

- Purest process in two-flavor chiral dynamics (really light quarks)

- scattering amplitude at threshold: two numbers (a_0, a_2)

- History of the prediction for a_0:

LO (tree):	$a_0 = 0.16$	Weinberg 1966
NLO (1-loop):	$a_0 = 0.20 \pm 0.01$	Gasser, Leutwyler 1983
NNLO (2-loop):	$a_0 = 0.217 \pm 0.009$	Bijnens et al. 1996

- even better: match 2-loop representation to Roy equation solution

Roy + 2-loop:	$a_0 = 0.220 \pm 0.005$	Colangelo et al. 2000

$\Rightarrow$ this is an *amazing* prediction!

- same precision for a_2, but corrections very small ...

HOW ABOUT EXPERIMENT?

- Kaon decays (K_{e4} and $K^0 \to 3\pi^0$): most precise

- Lifetime of pionium: experimentally more difficult

Kaon decays:

$$a_0^0 = 0.2210 \pm 0.0047_{\rm stat} \pm 0.0040_{\rm sys}$$

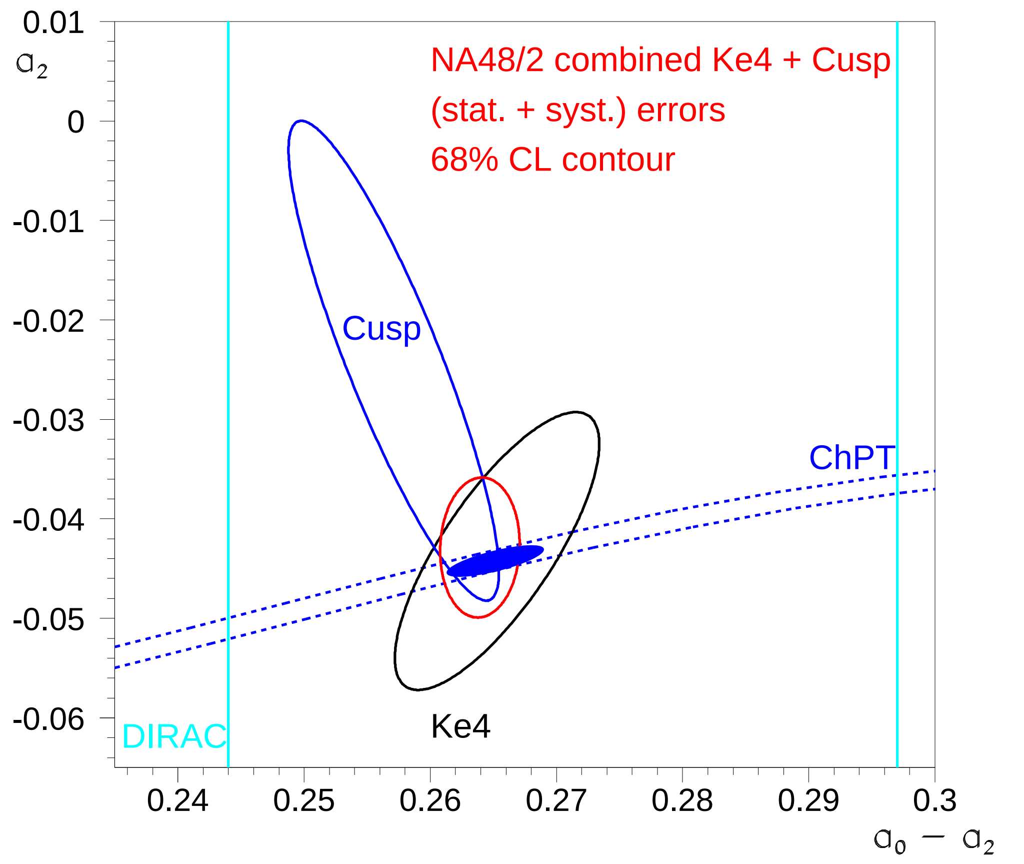
$$a_0^2 = -0.0429 \pm 0.0044_{\rm stat} \pm 0.0028_{\rm sys}$$

J. R. Batley et al. [NA48/2 Coll.] EPJ C 79 (2010) 635

Pionium lifetime:

$$|a_0^0 - a_0^2| = 0.264^{+0.033}_{-0.020}$$

B. Adeva et al. [DIRAC Coll.] PL B 619 (2005) 50

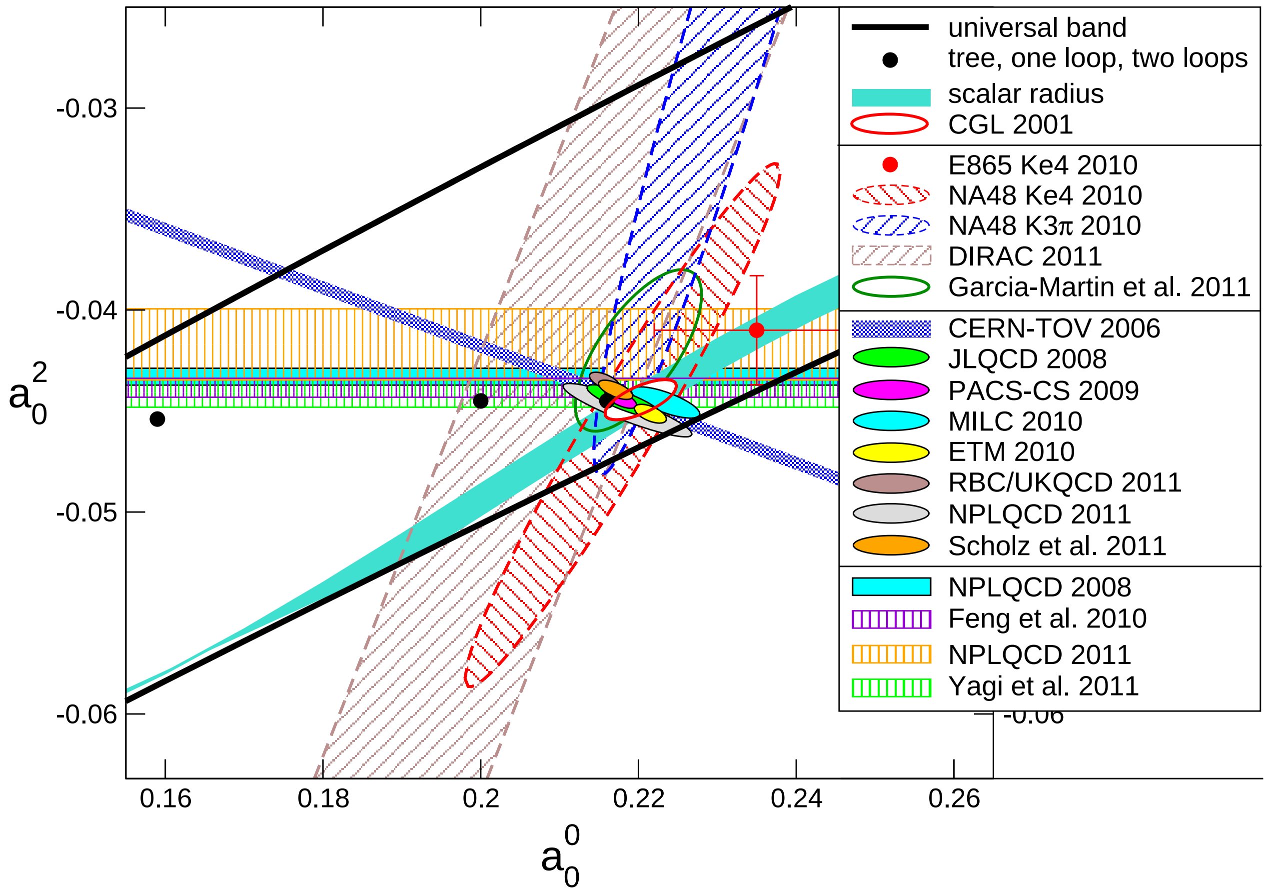


- and how about the lattice?

$\Rightarrow$ direct and indirect determinations of the scattering lengths

THE GRAND PICTURE

Fig. courtesy Heiri Leutwyler 2012



- one of the finest tests of the Standard Model

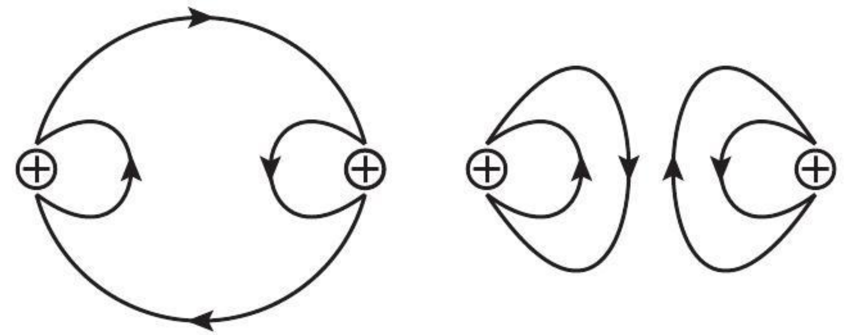
(what about lattice a_0 calcs?)

ELASTIC PION-PION SCATTERING – LATTICE a_0

- Only a few lattice determinations of a_0

  $\hookrightarrow$ disconnected diagrams difficult

  $\hookrightarrow$ quantum numbers of the vacuum



- only a few results:

Author(s)	a_0	Fermions	Pion mass range
Fu	0.214(4)(7)	asqtad staggered	240 - 430 MeV
ETMC	0.198(9)(6)	twisted mass	250 - 320 MeV
GWU	0.213(1)	LW + nHYP	224 - 315 MeV

Fu, PRD87 (2013) 074501; Liu et al. [ETMC], PRD96 (2017) 054516; Mai et la. (GWU), arXiv:1909.05749

$\rightarrow$ use EFT of PQQCD to investigate these contributions

Acharya, Guo, UGM, Seng, Nucl.Phys. B922 (2017) 480, JHEP 1904 (2019) 165

$\rightarrow$ more LQCD calculations needed!

Example 2

STRANGE QUARK MYSTERIES

- Is the strange quark really light? $\boxed{m_s \sim \Lambda_{\rm QCD}}$

$\rightarrow$ expansion parameter: $\xi_s = \frac{M_K^2}{(4\pi F_\pi)^2} \simeq 0.18$ [SU(2): $\xi = \frac{M_\pi^2}{(4\pi F_\pi)^2} \simeq 0.014$]

- many predictions of SU(3) CHPT work quite well, but:

$\hookrightarrow$ indications of bad convergence in some lattice calculations:

$\star$ masses and decay constants — Allton et al. 2008

$\star$ $K_{\ell 3}$-decays — Boyle et al. 2008

$\hookrightarrow$ suppression of the three-flavor condensate?

$\star$ sum rule: $\Sigma(3) = \Sigma(2)[1 - 0.54 \pm 0.27]$ — Moussallam 2000

$\star$ lattice: $\Sigma(3) = \Sigma(2)[1 - 0.23 \pm 0.39]$ — Fukuya et al. 2011

ELASTIC PION-KAON SCATTERING

- Purest process in three-flavor chiral dynamics
- scattering amplitude at threshold: two numbers $(a_0^{1/2}, a_0^{3/2})$
- History of the chiral predictions:

	CA [1]	1-loop [2]	2-loop [3]
$a_0^{1/2}$	0.14	0.18 ± 0.03	0.220 [0.17 ... 0.225]
$a_0^{3/2}$	-0.07	-0.05 ± 0.02	$-0.047[-0.075 \ldots -0.04]$

[1] Weinberg 1966, Griffith 1969 [2] Bernard, Kaiser, UGM 1990 [3] Bijnens, Dhonte, Talavera 2004

- match 1-loop representation to Roy-Steiner equation solution

$$a_0^{1/2} = 0.224 \pm 0.022\ ,\quad a_0^{3/2} = -0.0448 \pm 0.0077$$

Büttiker et al. 2003

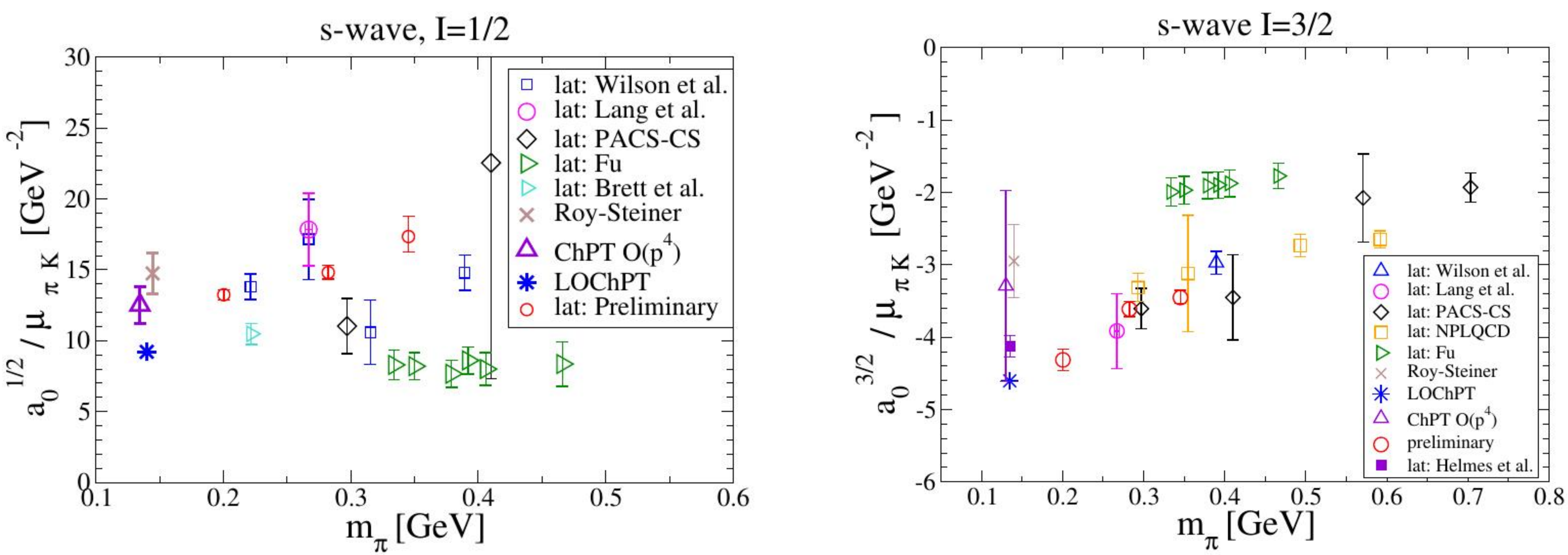
- constrained forward dispersion relations:

$$a_0^{1/2} = 0.22 \pm 0.01\ ,\quad a_0^{3/2} = -0.054^{+0.010}_{-0.014}$$

Pelaez, Rodas 2016

THE GRAND PICTURE

Fig. courtesy Daniel Mohler



- tension between lattice results and/or Roy-Steiner
- need improved lattice results (more direct calculations) ⇒ work required
- see also Pion-Kaon Interactions Workshop at JLab website

https://www.jlab.org/conferences/pki2018/program.html [arXiv:1804.06528]

Example 3

PION-NUCLEON SCATTERING

- simplest scattering process involving nucleons
- intriguing LO prediction for isoscalar/isovector scattering length:

$$a^+_{\rm CA} = 0, \quad a^-_{\rm CA} = \frac{1}{1+M_\pi/m_p}\frac{M_\pi^2}{8\pi F_\pi^2} = 79.5\cdot 10^{-3}/M_\pi,$$

- chiral corrections:
  - chiral expansion for a^- converges fast — Bernard, Kaiser, UGM 1995
  - large cancellations in a^+, even sign not known from scattering data

	$\mathcal{O}(q)$	$\mathcal{O}(q^2)$	$\mathcal{O}(q^3)$	$\mathcal{O}(q^4)$
fit to KA85	0.0	0.46	-1.00	-0.96
fit to EM98	0.0	0.24	0.49	0.45
fit to SP98	0.0	1.01	0.14	0.27

Fettes, UGM 2000

A WONDERFUL ALTERNATIVE: HADRONIC ATOMS

- Hadronic atoms: bound by the static Coulomb force (QED)

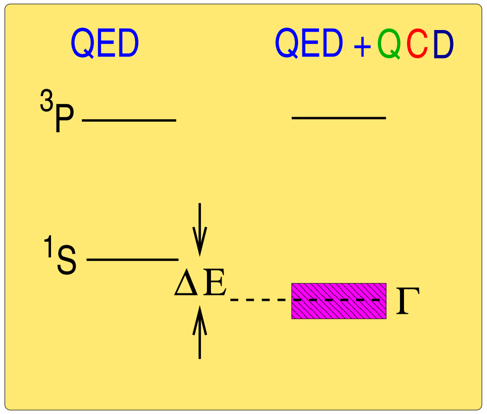
- Many species: $\pi^+\pi^-$, $\pi^\pm K^\mp$, $\pi^- p$, $\pi^- d$, $K^- p$, $K^- d$, ...

- Observable effects of QCD: strong interactions as **small** perturbations

  ⋆ energy shift ΔE

  ⋆ deacy width Γ

  ⇒ access to scattering at zero energy!
  = S-wave scattering lengths



- can be analyzed in suitable NREFTs

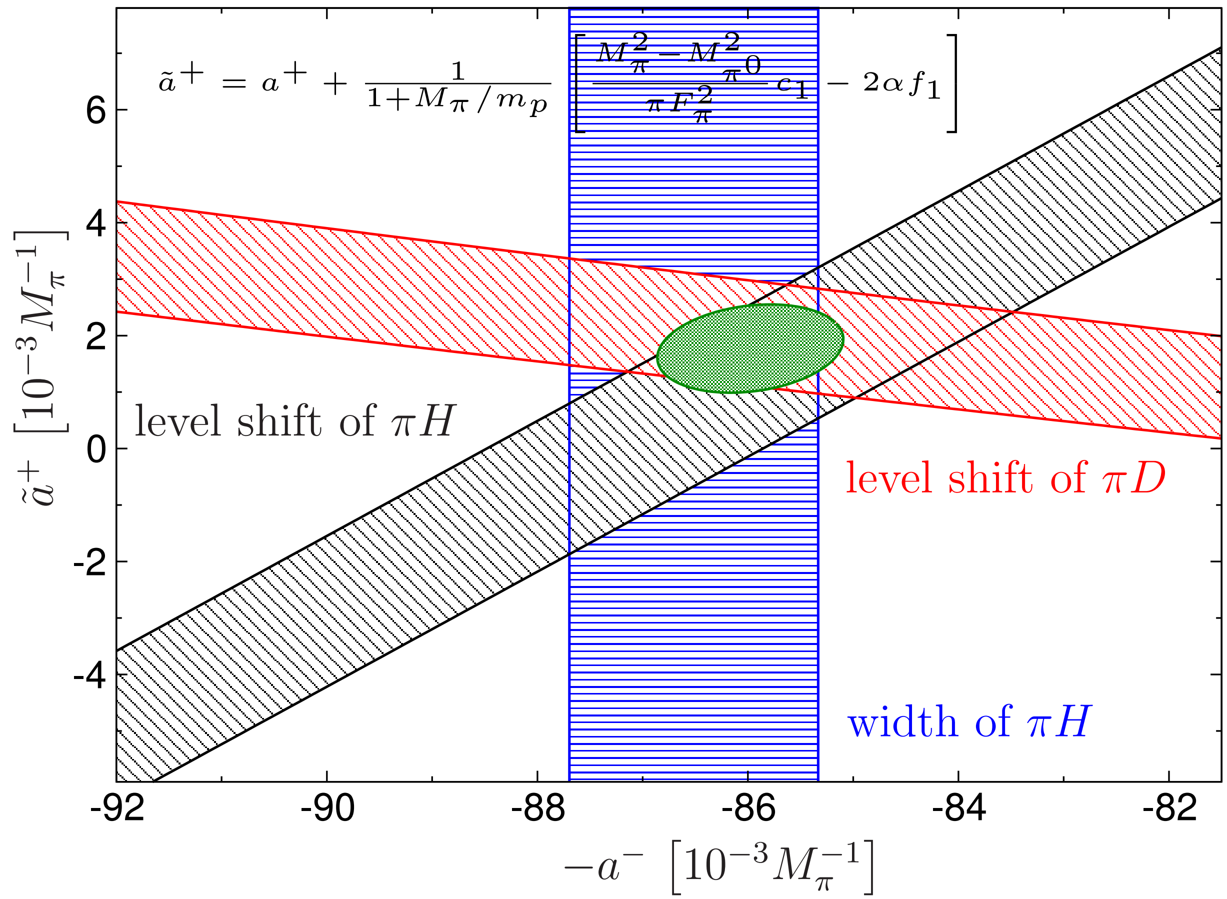
  Pionic hydrogen — Gasser, Rusetsky, . . . 2002

  Pionic deuterium — Baru, Hoferichter, Kubis . . . 2011

PION–NUCLEON SCATTERING LENGTHS

- Superb experiments performed at PSI — Gotta et al.
- Hadronic atom theory (Bern, Bonn, Jülich) — Gasser et al., Baru et al.

Baru, Hoferichter, Hanhart, Kubis, Nogga, Phillips, Nucl. Phys. A **872** (2011) 69



- πH level shift $\Rightarrow \pi^- p \to \pi^- p$
- πD level shift
  $\Rightarrow$ isoscalar $\pi^- N \to \pi^- N$
- πH width $\Rightarrow \pi^- p \to \pi^0 n$

$\downarrow$

$$a^+ = (7.6 \pm 3.1) \cdot 10^{-3}/M_\pi$$

$$a^- = (86.1 \pm 0.9) \cdot 10^{-3}/M_\pi$$

$\Rightarrow$ very precise value for a^- & first time definite sign for a^+

ROLE of the PION-NUCLEON σ-TERM

- Scalar couplings of the nucleon:

$$\langle N|m_q\bar{q}q|N\rangle = f_q^N\, m_N \quad (N = p,n)$$
$$(q = u,d,s)$$

$\hookrightarrow$ Dark Matter detection

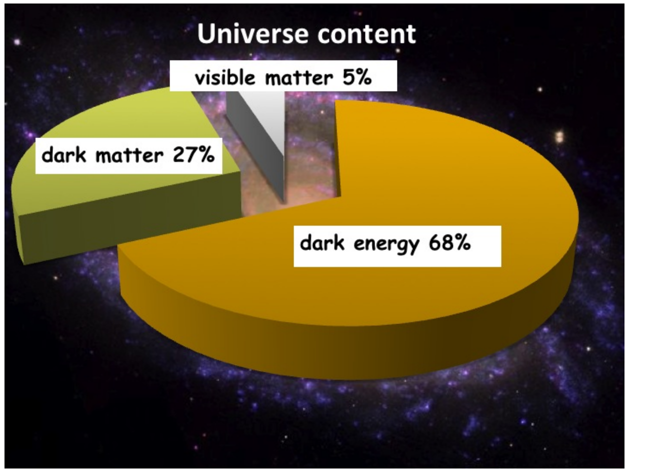
$\hookrightarrow$ $\mu \to e$ conversion in nuclei

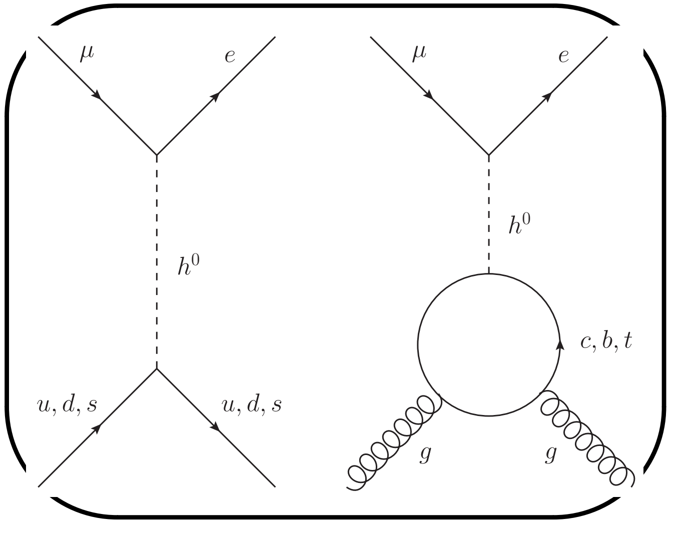
- Condensates in nuclear matter

$$\frac{\langle\bar{q}q\rangle(\rho)}{\langle 0|\bar{q}q|0\rangle} = 1 - \frac{\rho\,\sigma_{\pi N}}{F_\pi^2 M_\pi^2} + \ldots$$

- CP-violating πN couplings

$\hookrightarrow$ hadronic EDMs (nucleon, nuclei)





Crivellin, Hoferichter, Procura

RESULTS for the SIGMA-TERM

- Basic formula: $$\sigma_{\pi N} = F_\pi^2(d_{00}^+ + 2M_\pi^2 d_{01}^+) + \Delta_D - \Delta_\sigma - \Delta_R$$

- Subthreshold parameters output of the RS equations:

$$d_{00}^+ = -1.36(3) M_\pi^{-1} \qquad \text{[KH: } -1.46(10) M_\pi^{-1}\text{]}$$
$$d_{01}^+ = 1.16(3) M_\pi^{-3} \qquad \text{[KH: } 1.14(2)\ M_\pi^{-3}\text{]}$$

- $\Delta_D - \Delta_\sigma = (1.8 \pm 0.2)$ MeV — Hoferichter, Ditsche, Kubis, UGM (2012)
- $\Delta_R \lesssim 2$ MeV — Bernard, Kaiser, UGM (1996)
- Isospin breaking in the CD theorem shifts $\sigma_{\pi N}$ by $+3.0$ MeV

⇒ Final result:

$$\sigma_{\pi N} = (59.1 \pm 1.9_{\rm RS} \pm 3.0_{\rm LET})\ \text{MeV} = (59.1 \pm 3.5)\ \text{MeV}$$

- consistent with scattering data analysis: $\sigma_{\pi N} = 58 \pm 5$ MeV
  Ruiz de Elvira, Hoferichter, Kubis, UGM (2018)
- recover $\sigma_{\pi N} = 45$ MeV if KH80 scattering lengths are used

RESULTS for the SIGMA-TERM

- Recent results from various LQCD collaborations:

collaboration	$\sigma_{\pi N}$ [MeV]	reference	tension to RS
BMW	$38(3)(3)$	Dürr et al. (2015)	3.8σ
χQCD	$45.9(7.4)(2.8)$	Yang et al. (2015)	1.5σ
ETMC	$41.6(3.8)$	Alexandrou et al. (2019)	5.0σ
CRC 55	$35(6)$	Bali et al. (2016)	4.0σ

- We seem to have a problem - do we? [we = RS folks]

- Robust prediction of the RS analysis:

$$\sigma_{\pi N} = (59.1 \pm 3.1)\,\text{MeV} + \sum_{I_s} c_{I_s}\left(a^{I_s} - \bar{a}^{I_s}\right) \quad \left(I_s = \tfrac{1}{2}, \tfrac{3}{2}\right)$$

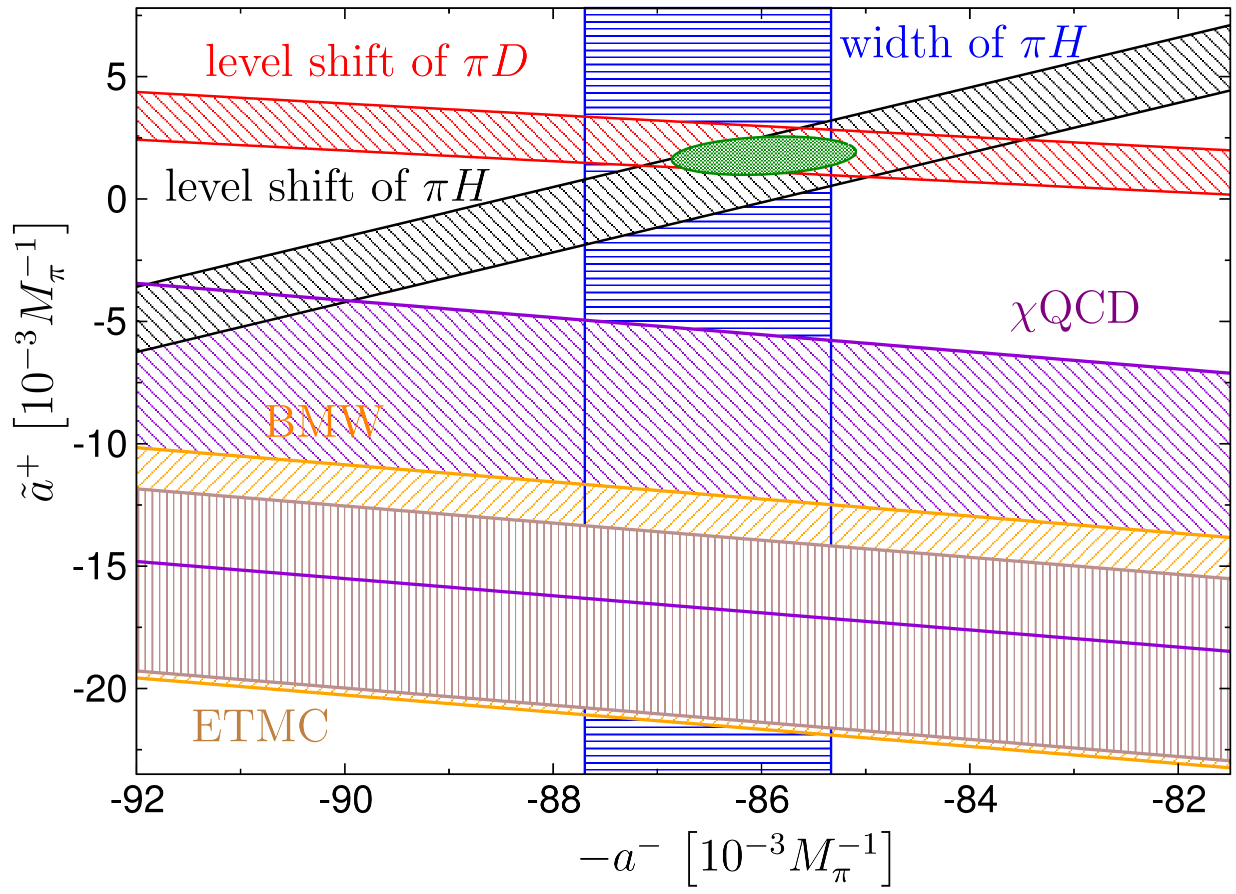
$$c_{1/2} = 0.242\,\text{MeV} \times 10^3 M_\pi, \quad c_{3/2} = 0.874\,\text{MeV} \times 10^3 M_\pi$$

$$\bar{a}^{1/2} = (169.8 \pm 2.0) \times 10^{-3} M_\pi^{-1}, \quad \bar{a}^{3/2} = (-86.3 \pm 1.8) \times 10^{-3} M_\pi^{-1}$$

$\to$ expansion around the reference values from πH and πD

RESULTS for the SIGMA-TERM

- Apply this linear expansion to the lattice data:



$\Rightarrow$ Lattice results clearly at odds with empirical information on the scattering lengths!

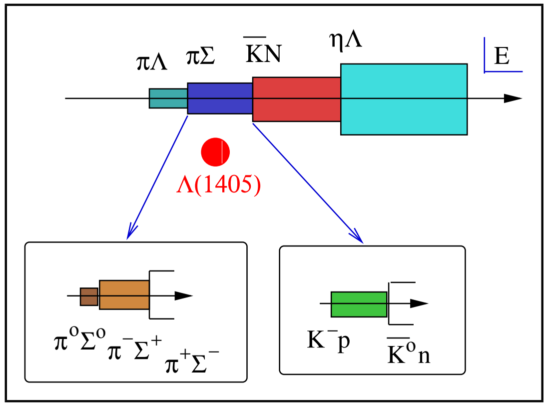
$\Rightarrow$ scattering lengths to $[\mathbf{5 \ldots 10}]\%$ $\rightarrow$ $\boldsymbol{\delta\sigma_{\pi N} = [5.0 \ldots 8.5]}$ MeV

Example 4

ANTIKAON-NUCLEON SCATTERING

- $K^- p \to K^- p$: fundamental scattering process with strange quarks

- coupled channel dynamics



- dynamic generation of the $\Lambda(1405)$

  Dalitz, Tuan 1960

- major playground of **unitarized CHPT**

- chiral Lagrangian + unitarization leads to generation of certain resonances

  like e.g. the $\Lambda(1405)$, $S_{11}(1535)$, $S_{11}(1650)$, ...

  Kaiser, Siegel, Weise, Oset, Ramos, Oller, UGM, Lutz, ...

- two-pole scenario of the $\Lambda(1405)$ emerges Oller, UGM 2001

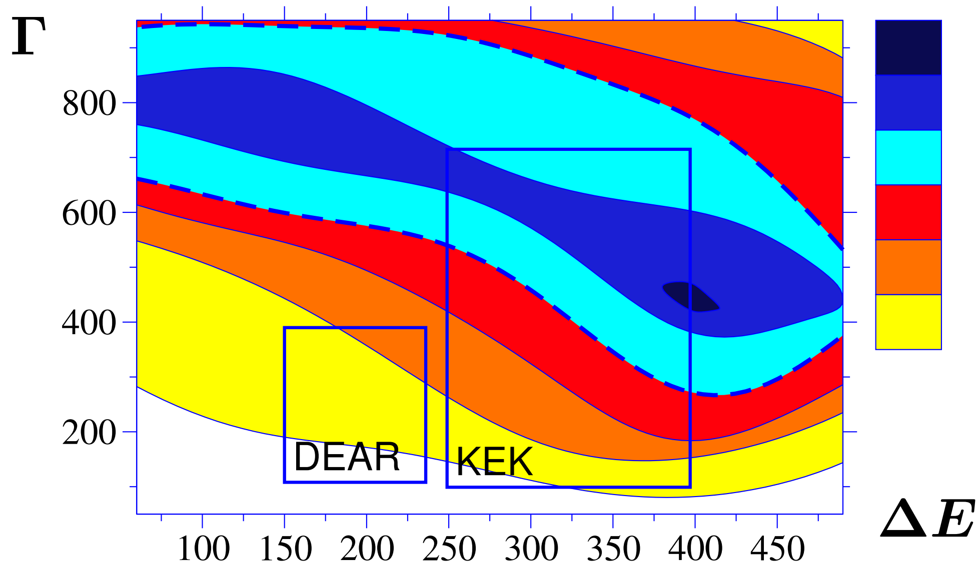
- loopholes: convergence a posteriori, crossing symmetry, on-shell approximation, unphysical poles, ...

A PUZZLE RESOLVED

- DEAR data inconsistent with scattering data

  UGM, Raha, Rusetsky 2004

$\Rightarrow$ vaste number of papers . . .



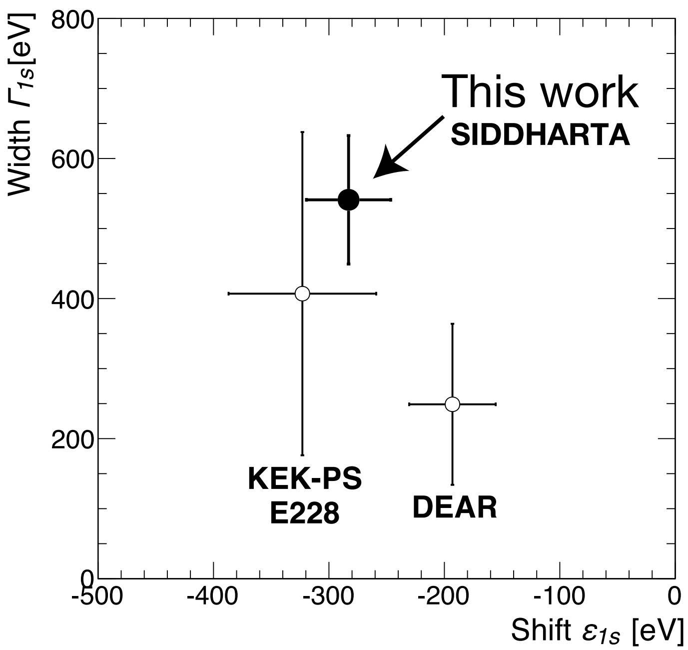
- SIDDHARTA to the rescue

  Bazzi et al. 2011

$\Rightarrow$ more precise, consistent with KpX

$$\epsilon_{1s} = -283 \pm 36(\text{stat}) \pm 6(\text{syst}) \text{ eV}$$

$$\Gamma_{1s} = 541 \pm 89(\text{stat}) \pm 22(\text{syst}) \text{ eV}$$



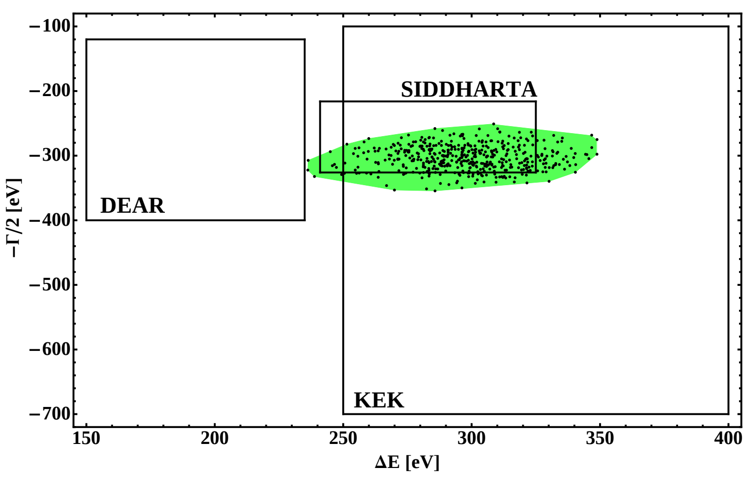
CONSISTENT ANALYSIS

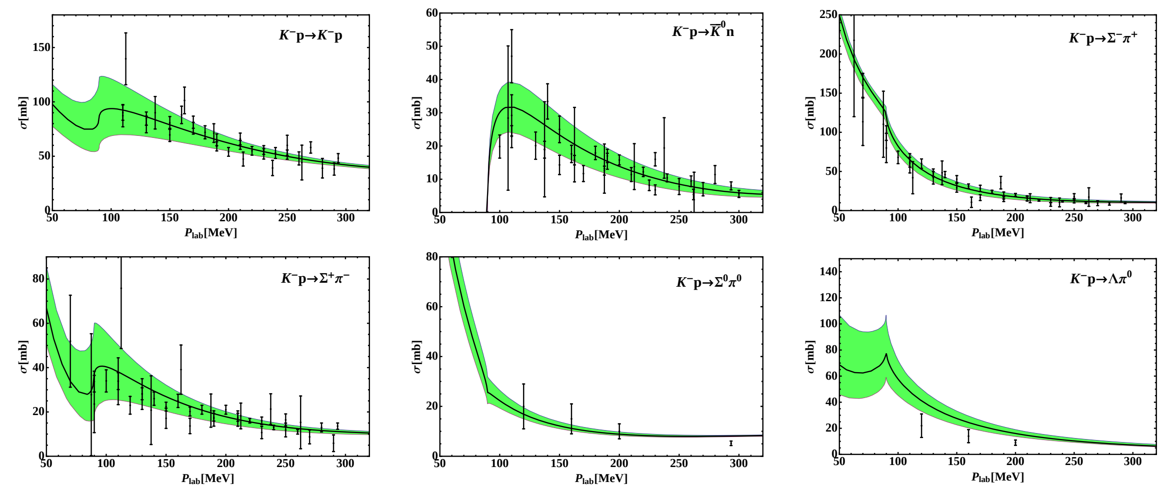
- kaonic hydrogen + scattering data can now be analyzed consistently
- use the chiral Lagrangian at NLO, three groups (different schemes)

Ikeda, Hyodo, Weise 2011; UGM, Mai 2012, Guo, Oller 2012

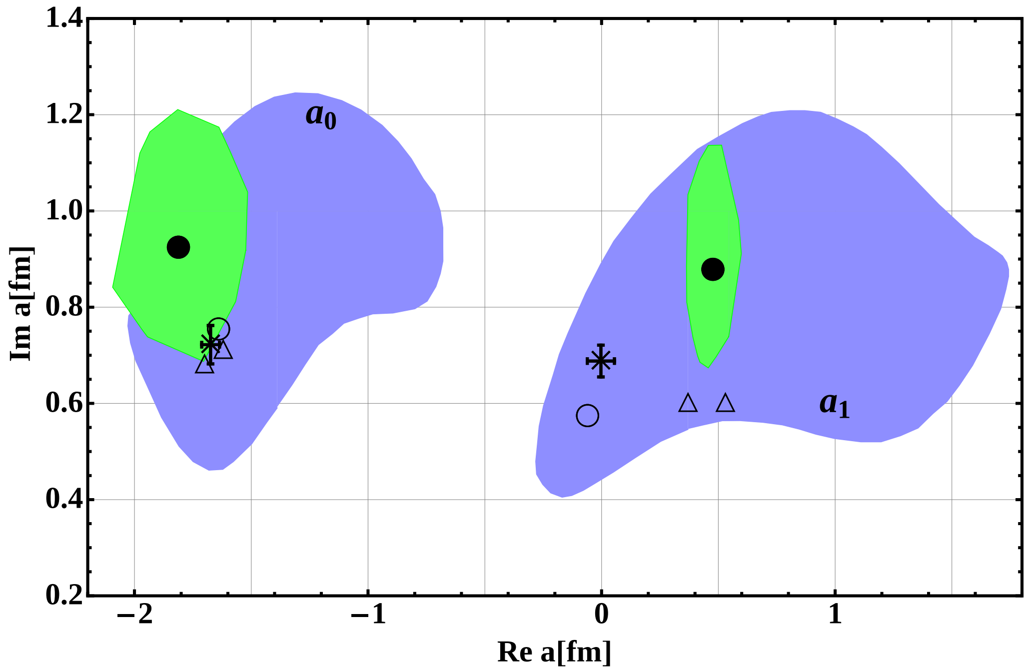
- 14 LECs and 3 subtraction constants to fit

$\Rightarrow$ simultaneous description of the SIDDHARTA and the scattering data





KAON–NUCLEON SCATTERING LENGTHS



$$a_0 = -1.81^{+0.30}_{-0.28} + i\, 0.92^{+0.29}_{-0.23}\ \text{fm}$$

$$a_1 = +0.48^{+0.12}_{-0.11} + i\, 0.87^{+0.26}_{-0.20}\ \text{fm}$$

$$a_{K^-p} = -0.68^{+0.18}_{-0.17} + i\, 0.90^{+0.13}_{-0.13}\ \text{fm}$$

SIDDHARTA only:

$$a_{K^-p} = -0.65^{+0.15}_{-0.15} + i\, 0.81^{+0.18}_{-0.18}\ \text{fm}$$

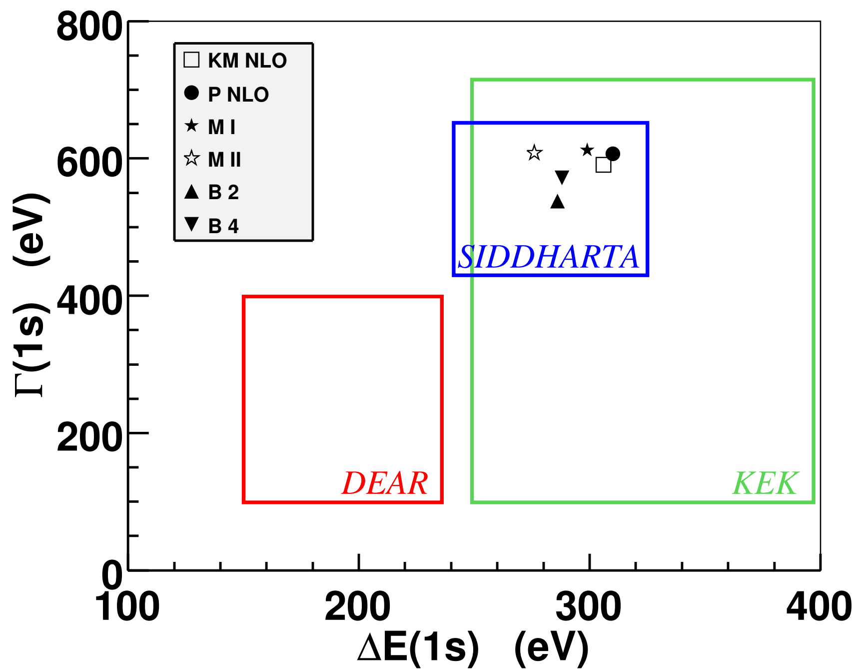
- clear improvement compared to scattering data only

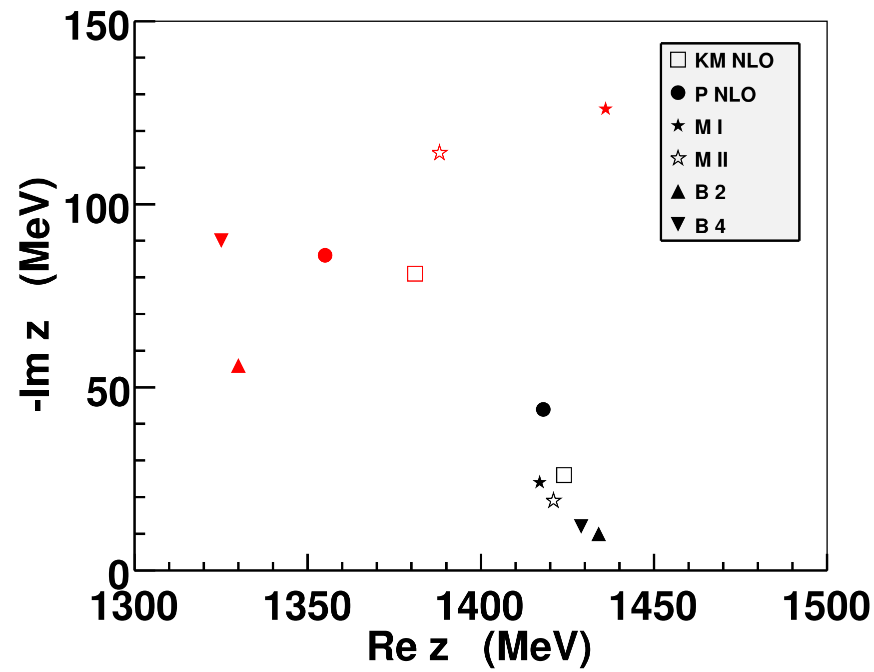
⇒ fundamental parameters to within about 15% accuracy

COMPARISON of VAROIUS APPROACHES

- systematic study of the two-pole scenario of the $\Lambda(1405)$ using various approaches

Cieply, Mai, UGM, Smejkal, Nucl. Phys. A954 (2016) 17





$\hookrightarrow$ higher/lower pole well/not well determined

$\hookrightarrow$ some solutions also include precise data on $\gamma p \to \Sigma K \pi$ from JLab

$\hookrightarrow$ need more data on the $\pi\Sigma$ mass distribution from various reactions

INTERMEDIATE SUMMARY

- Hadron-hadron scattering: important role of chiral symmetry (CHPT)
  $\rightarrow$ combine with dispersion relations, unitarization, lattice
- Pion-pion scattering
  $\rightarrow$ a fine test of the Standard Model
- Pion-kaon scattering
  $\rightarrow$ tension between lattice and Roy-Steiner solution
- Pion-nucleon scattering
  $\rightarrow$ superbe accuracy from EFTs for pionic hydrogen/deuterium
  $\rightarrow$ $\sigma_{\pi N}$: tension between lattice and Roy-Steiner
  $\rightarrow$ lattice determination of scattering lengths required!
- Antikaon-nucleon scattering
  $\rightarrow$ consistent determination of the scattering lengths possible

SPARES

Isospin symmetry and isospin violation

ISOSPIN SYMMETRY

- For $m_u = m_d$, QCD is invariant under *SU(2) isospin* transformations:

$$q \to q' = Uq\,, \quad q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad U = \begin{pmatrix} a^* & b^* \\ -b & a \end{pmatrix}, \quad |a|^2 + |b|^2 = 1$$

  — NB: Charge symmetry = 180° rotation in iso-space

- Rewriting of the QCD quark mass term:

$$\mathcal{H}_{\rm QCD}^{\rm SB} = m_u\,\bar u u + m_d\,\bar d d = \underbrace{\frac{1}{2}(m_u + m_d)(\bar u u + \bar d d)}_{I=0} + \underbrace{\frac{1}{2}(m_u - m_d)(\bar u u - \bar d d)}_{I=1}$$

Strong isospin violation (IV)

- Competing effect: QED $\to$ can be treated in CHPT
- Standard situation: small IV on top of large iso-symmetric background, requires precise machinery to perform accurate calculations

Gasser, Urech, Steininger, Fettes, M., Knecht, Kubis, . . .

ISOSPIN VIOLATION - PIONS & KAONS

Weinberg, Gasser, Leutwyler, Urech, Neufeld, Knecht, M., Müller, Steininger, . . .

- SU(2) effective Lagrangian w/ virtual photons to leading order:

Q = quark charge matrix

$$\mathcal{L}^{(2)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}(\partial_\mu A^\mu)^2 + \frac{F_\pi^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle + C\langle QUQU^\dagger\rangle$$

⋆ pion mass difference of em origin, $M_{\pi^+}^2 - M_{\pi^0}^2 = 2Ce^2/F_\pi^2$

⋆ no strong isospin breaking at LO, absence of D-symbol

⋆ strong and em corrections at NLO worked out

M., Müller, Steininger, Phys. Lett. B 406 (1997) 154

- Three-flavor chiral perturbation theory:

⋆ for $m_u = m_d \Rightarrow M_{K^+}^2 - M_{K^0}^2 = M_{\pi^+}^2 - M_{\pi^0}^2 = \frac{2Ce^2}{F_\pi^2}$ – Dashen's theorem

⋆ for $m_u \neq m_d \Rightarrow$ leading order strong kaon mass difference:

$$(M_{K^0}^2 - M_{K^+}^2)^{\text{strong}} = (m_u - m_d)B_0 + \mathcal{O}(m_q^2)$$

$B_0 = |\langle 0|\bar{q}q|0\rangle|/F_\pi^2$

⋆ strong and em corrections at NLO incl. leptons worked out

Urech, Nucl. Phys. B 433 (1995) 234
Knecht, Neufeld, Rupertsberger, Talavera, Eur. Phys. J. C 12 (2000) 469

ISOSPIN VIOLATION - NUCLEONS

Weinberg, . . ., Fettes, M., Müller, Steininger

- Effective Lagrangian for isospin violation (to leading order):

$$\mathcal{L}_{\pi N}^{(2,\mathrm{IV})} = \bar{N}\Big\{c_5 \underbrace{\left(\chi_+ - \tfrac{1}{2}\langle\chi_+\rangle\right)}_{\sim m_u - m_d} + f_1 \underbrace{\langle \hat{Q}_+^2 - Q_-^2\rangle}_{\sim q_u - q_d} + f_2 \underbrace{\hat{Q}_+\langle Q_+\rangle}_{\sim q_u - q_d}\Big\} N + \mathcal{O}(q^3)$$

  – Three LECs parameterize the leading strong (c_5) & em (f_1, f_2) IV effects
  – These LECs link various observables/processes:

$$m_n - m_p = 4\, c_5\, B_0 (m_u - m_d) + 2\, e^2\, f_2\, F_\pi^2 + \ldots$$ fairly well known

Gasser, Leutwyler, . . .

$$a(\pi^0 p) - a(\pi^0 n) = \mathrm{const}\; (-4\, c_5\, B_0\, (m_u - m_d)) + \ldots$$

extremely hard to measure — Weinberg, M., Steininger

  – IV in πN scattering analyzed in CHPT $\to$ intriguing results

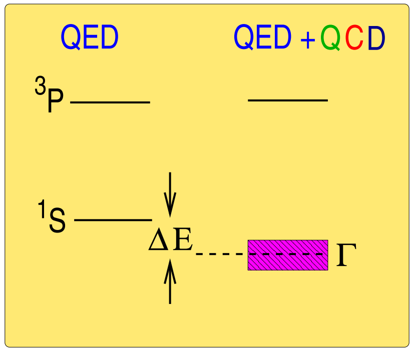
Fettes & M., Nucl. Phys. A 693 (2001) 693; Hoferichter, Kubis, M., Phys. Lett. B 678 (2009) 65

- also access to IV in $np \to d\pi^0$ and $dd \to \alpha\pi^0$ (spin-isospin filter)

$\to$ need to develop a high-precision EFT for few-nucleon systems

BOUND STATE EFT: HADRONIC ATOMS

- Hadronic atoms are bound by the static Coulomb force (QED)
- Many species: $\pi^+\pi^-$, $\pi^\pm K^\mp$, $\pi^- p$, $\boxed{\pi^- d,\, K^- p,\, K^- d,}$ $\ldots$
- Bohr radii $\gg$ typical scale of strong interactions
- Small average momenta $\Rightarrow$ non-relativistic approach
- Observable effects of QCD
  - $\star$ energy shift $\boldsymbol{\Delta E}$ from the Coulomb value
  - $\star$ decay width $\boldsymbol{\Gamma}$



$\Rightarrow$ access to scattering at zero energy! = S-wave scattering lengths

- These scattering lengths are very sensitive to the chiral & isospin symmetry breaking in QCD

Weinberg, Gasser, Leutwyler, . . .

- can be analyzed systematically & consistently in the framework of low-energy Effective Field Theory (including virtual photons)

EFFECTIVE FIELD THEORY for HADRONIC ATOMS

- Three step procedure utilizing *nested* effective field theories

- Step 1:

  Construct non-relativistic effective Lagrangian (complex couplings)
  & solve Coulomb problem exactly, corrections in perturbation theory

- Step 2: *matching*

  relate complex couplings of $\mathcal{L}_{\rm eff}$ to QCD parameters, e.g. scattering lengths
  & express complex energy shift in terms of QCD parameters

- Step 3:

  extract scattering length(s) from the measured complex energy shift

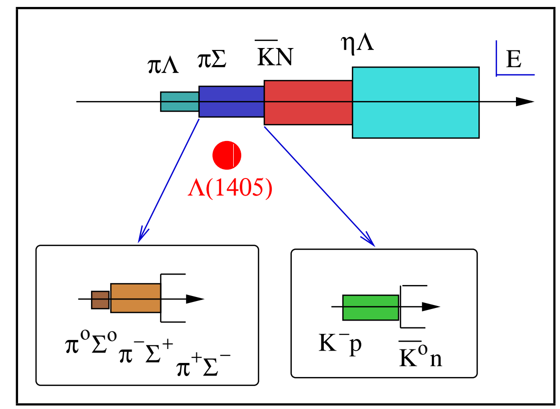
$\Rightarrow$ most precise way of determining hadron-hadron scattering lengths

$\rightarrow$ study kaonic hydrogen as one example

FEATURES OF KAONIC HYDROGEN

- Strong ($K^- p \to \pi^0 \Lambda, \pi^\pm \Sigma^\mp, \ldots$) and weaker electromagnetic ($K^- p \to \gamma \Lambda, \gamma \Sigma^0, \ldots$) decays

  $\to$ complicated (interesting) analytical structure



- Average momentum $\langle p^2 \rangle = \alpha\,\mu \simeq 2$ MeV

  $\to$ highly non-relativistic

- Bohr radius $r_B = 1/(\alpha\,\mu) \simeq 1o0$ fm

- Binding energy $E_{1s} = \frac{1}{2}\,\alpha^2\,\mu + \ldots \simeq 8$ keV

- Width $\Gamma_{1s} \simeq 250$ eV $\ll E_{1s}$

- $\mathcal{M} = m_n + M_{K^0} - m_p + M_{K^-} > 0 \Rightarrow$ unitary cusp

- Isospin breaking, small parameter $\delta \sim \alpha \sim (m_d - m_u)$

$$\Delta E = \underbrace{\delta^3}_{\textbf{LO}} + \underbrace{\delta^4}_{\textbf{NLO}} + \ldots$$

NON-RELATIVISTIC EFFECTIVE LAGRANGIAN

$$
\begin{aligned}
\mathcal{L} \;=\;& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \psi^\dagger \Big\{ i\mathcal{D}_t - m_p + \frac{\mathcal{D}^2}{2m_p} + \frac{\mathcal{D}^4}{8m_p^3} + \cdots \\
&- \mathbf{c_p^F}\,\frac{e\sigma\mathbf{B}}{2m_p} - \mathbf{c_p^D}\,\frac{e(\mathcal{D}\mathbf{E} - \mathbf{E}\mathcal{D})}{8m_p^2} - \mathbf{c_p^S}\,\frac{ie\sigma(\mathcal{D}\times\mathbf{E} - \mathbf{E}\times\mathcal{D})}{8m_p^2} + \cdots \Big\}\psi \\
&+ \chi^\dagger \Big\{ i\partial_t - m_n + \frac{\nabla^2}{2m_n} + \frac{\nabla^4}{8m_n^3} + \cdots \Big\}\chi \\
&+ \sum_\pm (K^\pm)^\dagger \Big\{ iD_t - M_{K^+} + \frac{\mathbf{D}^2}{2M_{K^+}} + \frac{\mathbf{D}^4}{8M_{K^+}^3} + \cdots \mp \mathbf{c_K^R}\,\frac{e(\mathbf{DE} - \mathbf{ED})}{6M_{K^+}^2} + \cdots \Big\} K^\pm \\
&+ (\bar{K}^0)^\dagger \Big\{ i\partial_t - M_{\bar{K}^0} + \frac{\nabla^2}{2M_{\bar{K}^0}} + \frac{\nabla^4}{8M_{\bar{K}^0}^3} + \cdots \Big\} \bar{K}^0 \\
&+ \tilde{\mathbf{d}}_\mathbf{1}\,\psi^\dagger\psi\,(K^-)^\dagger K^- + \tilde{\mathbf{d}}_\mathbf{2}(\psi^\dagger\chi\,(K^-)^\dagger\bar{K}^0 + h.c.) + \tilde{\mathbf{d}}_\mathbf{3}\chi^\dagger\chi\,(\bar{K}^0)^\dagger\bar{K}^0 + \cdots\,.
\end{aligned}
$$

proton

neutron

kaons

→ calculate electromagnetic levels and the strong shift (note: $\tilde{d}_i$ complex!)

ENERGY SHIFT in KAONIC HYDROGEN

a) recoil corrections

b) transverse photon exchange

c) finite size corrections

d) vacuum polarisation

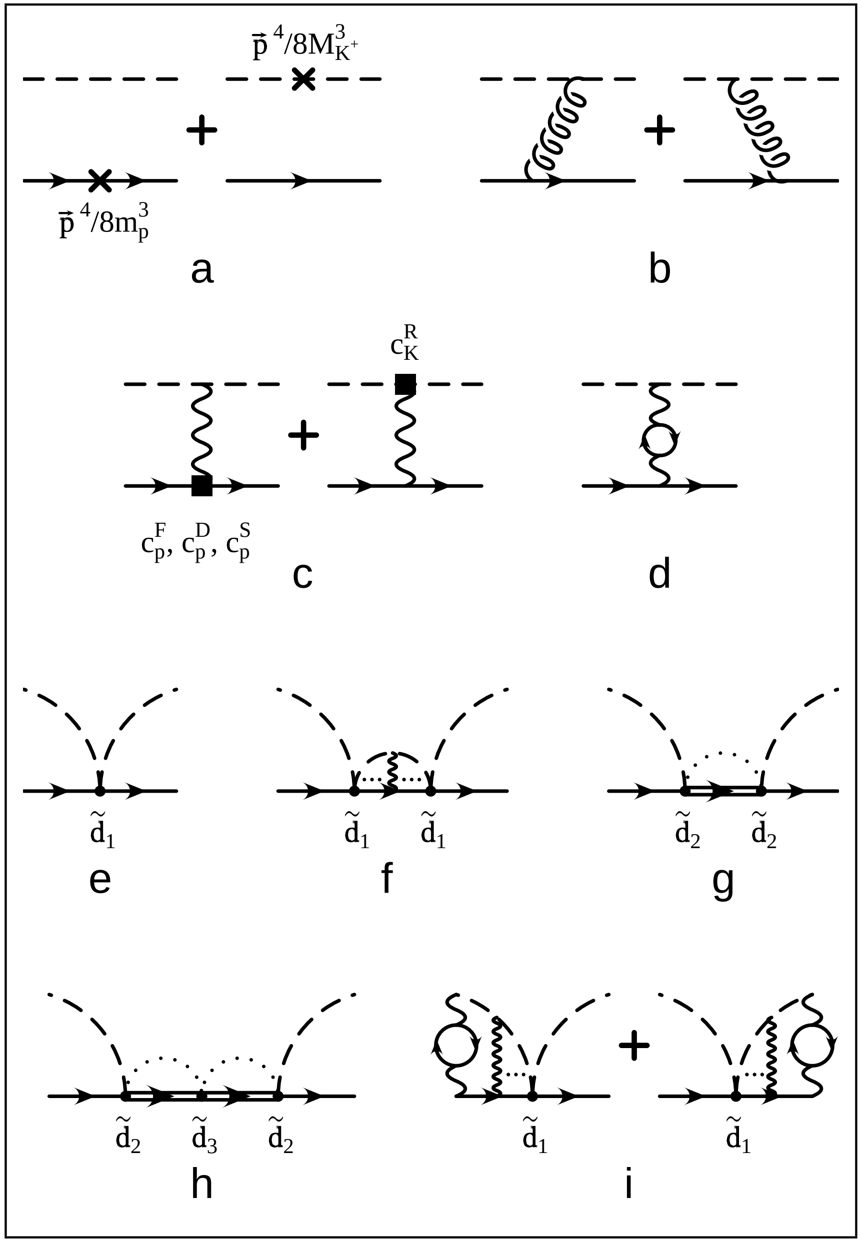
e) leading K^-p interaction

f) K^-p interaction w/ Coulomb ladders

g) leading $\bar{K}^0n$ intermediate state

h) iterated $\bar{K}^0n$ intermediate state

i) Coulomb ladders in the K^-p interaction

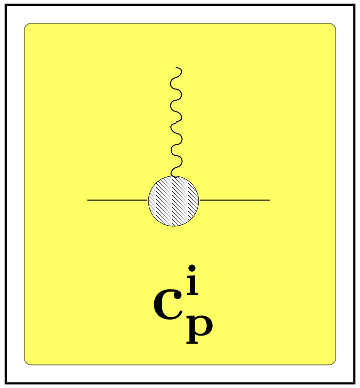


MATCHING CONDITIONS

• Electromagnetic form factors

$$c_p^F = 1 + \mu_p,\; c_p^D = 1 + 2\mu_p + \tfrac{4}{3}\, m_p^2 \langle r_p^2 \rangle,\; c_p^S = 1 + 2\mu_p$$

$$c_K^R = M_{K^+}^2 \langle r_K^2 \rangle$$



• Kaon–nucleon scattering amplitude

matching allows to express the complex strong energy shift in terms of the threshold amplitude (kaon-nucleon scattering lengths a_0 and a_1)

$$\Delta E_n^s - \frac{i}{2}\,\Gamma_n = -\frac{\alpha^3\mu_c^3}{2\pi M_{K^+} n^3}\,\mathcal{T}_{KN}\left\{1 - \frac{\alpha\mu_c^2}{4\pi M_{K^+}}\,\mathcal{T}_{KN}(s_n(\alpha) + 2\pi i) + \delta_n^{\rm vac}\right\}$$

with $\mathcal{T}_{KN} = 4\pi\left(1 + \frac{M_{K^+}}{m_p}\right)\frac{1}{2}\,(a_0 + a_1) + O(\sqrt{\delta})$

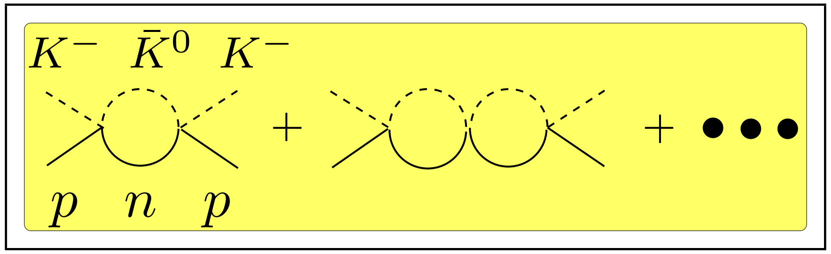
$$s_n(\alpha) = 2(\psi(n) - \psi(1) - \tfrac{1}{n} + \ln\alpha - \ln n)$$

$\Rightarrow$ correct, but not sufficiently accurate

UNITARY CUSP

- Corrections at $\mathcal{O}(\sqrt{\delta})$ can be expressed entirely in terms of a_0 and a_1

$\to$ resum the fundamental bubble
to account for the unitary cusp



$$\mathcal{T}_{KN}^{(0)} = 4\pi\Big(1 + \frac{M_{K^+}}{m_p}\Big)\frac{\frac{1}{2}(a_0+a_1)+q_0 a_0 a_1}{1+\frac{q_0}{2}(a_0+a_1)}, \qquad q_0 = \sqrt{2\mu_0 \Delta\mathcal{M}}$$

⋆ agrees with R.H. Dalitz and S.F.Tuan, Ann. Phys. **3** (1960) 307

⋆ all corrections at $\mathcal{O}(\sqrt{\delta})$ included

$$\mathcal{T}_{KN} = \mathcal{T}_{KN}^{(0)} + \frac{i\alpha\mu_c^2}{2M_{K^+}}(\mathcal{T}_{KN}^{(0)})^2 + \underbrace{\delta\mathcal{T}_{KN}}_{\mathcal{O}(\delta)} + o(\delta)$$

$\Rightarrow$ These further $\mathcal{O}(\delta)$ corrections are expected to be small

FINAL FORMULA to ANALYZE the DATA

$$\Delta E_n^s - \frac{i}{2}\Gamma_n = -\frac{\alpha^3\mu_c^3}{2\pi M_{K^+} n^3}\left(\mathcal{T}_{KN}^{(0)} + \delta\mathcal{T}_{KN}\right)\left\{1 - \frac{\alpha\mu_c^2 s_n(\alpha)}{4\pi M_{K^+}}\mathcal{T}_{KN}^{(0)} + \delta_n^{\rm vac}\right\}$$

- $\mathcal{O}(\sqrt{\delta})$ and $\mathcal{O}(\delta\ln\delta)$ terms:
  - ⋆ Parameter-free, expressed in terms of a_0 and a_1
  - ⋆ Numerically by far dominant
- Estimate of $\delta\mathcal{T}_{KN}$ in CHPT
  - ⋆ $\delta\mathcal{T}_{KN}/\mathcal{T}_{KN} = (-0.5 \pm 0.4)\cdot 10^{-2}$ at $O(p^2)$
  - ⋆ should be improved (loops, unitarization, influence of $\Lambda(1405)$, etc.)
- vacuum polarization calculation: $\delta_n^{\rm vac} \simeq 1\%$

D. Eiras and J. Soto, Phys. Lett. **B 491** (2000) 101 [hep-ph/0005066]