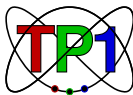


Lecture 2

Effective Theory Tools

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What is the Problem?

- Weak interaction: Transitions between quarks
- Observations: Transitions between Hadrons
- We have to deal with nonperturbative QCD
- Extraction of fundamental parameters (CKM Elements, CP Phases) requires precise predictions, including error estimates
- → Simple models are out ...
- Effective Field Theory methods
- QCD Sum Rules
- Lattice QCD

Contents

- 1 Effective Field Theories
 - EFT in a nutshell
 - Effective Weak Hamiltonian
 - Introduction to Renormalization Group
- 2 Heavy Mass Limit
 - Heavy Quark Effective Theory
 - Heavy Quark Expansion
 - Soft Collinear Effective Theory
- 3 QCD Sum Rules

Effective Field Theories

- Weak decays:
Very different mass scales are involved:
 - $\Lambda_{\text{QCD}} \sim 200 \text{ MeV}$: Scale of strong interactions
 - $m_c \sim 1.5 \text{ GeV}$: Charm Quark Mass
 - $m_b \sim 4.5 \text{ GeV}$: Bottom Quark Mass
 - $m_t \sim 175 \text{ GeV}$ and $M_W \sim 81 \text{ GeV}$:
Top Quark Mass and Weak Boson Mass
 - Λ_{NP} Scale of “new physics”
- At low scales the high mass particles / high energy degrees of freedom are irrelevant.
- Construct an “effective field theory” where the massive / energetic degrees of freedom are removed (“integrated out”)

Integrating out heavy degrees of freedom

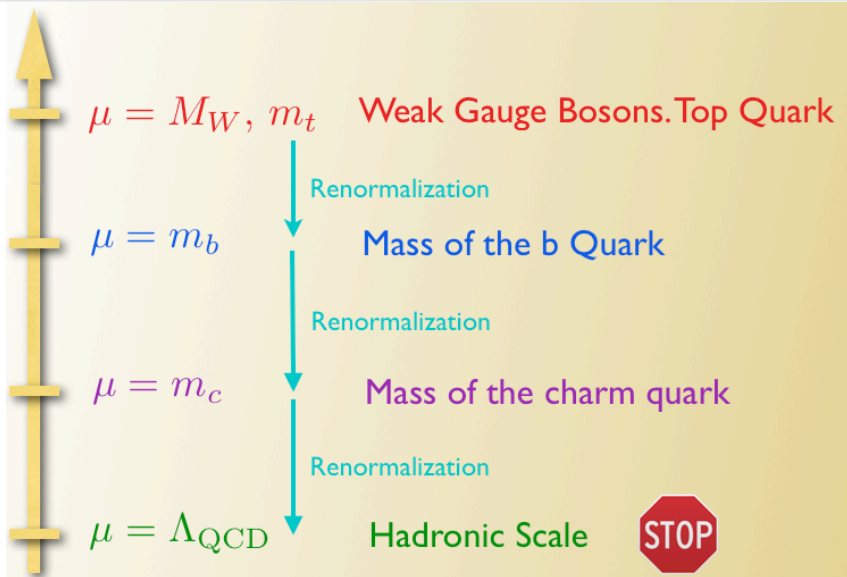
- ϕ : light fields, Φ : heavy fields with mass Λ
- Generating functional as a functional integral
Integration over the heavy degrees of freedom

$$\begin{aligned} Z[j] &= \int [d\phi][d\Phi] \exp \left(\int d^4x [\mathcal{L}(\phi, \Phi) + j\phi] \right) \\ &= \int [d\phi] \exp \left(\int d^4x [\mathcal{L}_{\text{eff}}(\phi) + j\phi] \right) \text{ with} \\ &\quad \exp \left(\int d^4x \mathcal{L}_{\text{eff}}(\phi) \right) = \int [d\Phi] \exp \left(\int d^4x \mathcal{L}(\phi, \Phi) \right) \end{aligned}$$

- For length scales $x \gg 1/\Lambda$: local effective Lagrangian
- Technically: (Operator Product) Expansion in inverse powers of Λ

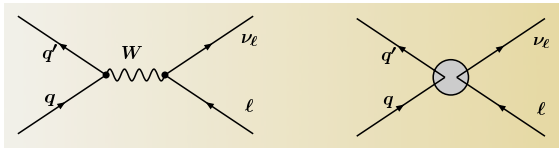
$$\mathcal{L}_{\text{eff}}(\phi) = \mathcal{L}_{\text{eff}}^{(4)}(\phi) + \frac{1}{\Lambda} \mathcal{L}_{\text{eff}}^{(5)}(\phi) + \frac{1}{\Lambda^2} \mathcal{L}_{\text{eff}}^{(6)}(\phi) + \dots$$

- \mathcal{L}_{eff} is in general non-renormalizable, but ...
- $\mathcal{L}_{\text{eff}}^{(4)}$ is the renormalizable piece
- For a fixed order in $1/\Lambda$: Only a finite number of insertions of $\mathcal{L}_{\text{eff}}^{(4)}$ is needed!
- \rightarrow can be renormalized
- Renormalizability is not an issue here



Effective Weak Hamiltonian

- Start out from the Standard Model
- W^\pm, Z^0 , top: much heavier than any hadron mass
- “integrate out” these particles at the scale $\mu \sim M_{\text{Hadron}}$



- W has zero range in this limit:

$$\langle 0 | T[W_\mu^*(x) W_\nu(y)] | 0 \rangle \rightarrow g_{\mu\nu} \frac{1}{M_W^2} \delta^4(x - y)$$

- Effective Interaction (Fermi Coupling)

$$H_{\text{eff}} = \frac{g^2}{\sqrt{2}M_W^2} V_{q'q} [\bar{q}' \gamma_\mu (1 - \gamma_5) q] [\bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \ell] = \frac{4G_F}{\sqrt{2}} V_{q'q} j_{\mu, \text{had}} j_{\mu, \text{lep}}^\mu$$

Decays of Hadrons

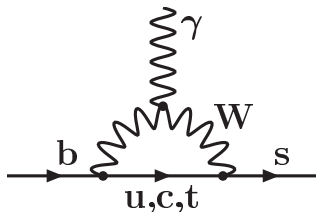
- Leptonic and semi-leptonic decays

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{q'q} [\bar{q}' \gamma_\mu \frac{(1 - \gamma_5)}{2} q] [\bar{\nu}_\ell \gamma_\mu \frac{(1 - \gamma_5)}{2} \ell]$$

- Hadronic decays

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{q'q} V_{QQ'}^* [\bar{q}' \gamma_\mu \frac{(1 - \gamma_5)}{2} q] [\bar{Q} \gamma_\mu \frac{(1 - \gamma_5)}{2} Q']$$

- Rare (FCNC) Decays: **Loop Corrections**
(QCD and electroweak)



- Example: $b \rightarrow s\gamma$

$$\mathcal{A}(b \rightarrow s\gamma) = V_{ub} V_{us}^* f(m_u) + V_{cb} V_{cs}^* f(m_c) + V_{tb} V_{ts}^* f(m_t)$$

- In case of degenerate masses up-type masses:

$$\mathcal{A}(b \rightarrow s\gamma) = f(m) [V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^*] = 0$$

Renormalization Group Running

- H_{eff} is defined at the scale Λ , where we integrated out the particles with mass Λ : **General Structure**

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \lambda_{\text{CKM}} \sum_k \hat{C}_k(\Lambda) \mathcal{O}_k(\Lambda)$$

- $\mathcal{O}_k(\Lambda)$: The matrix elements of \mathcal{O}_k have to be evaluated (“normalized”) at the scale Λ .
- $\hat{C}_k(\Lambda)$: Short distance contribution, contains the information about scales $\mu > \Lambda$
- Matrixelements of $\mathcal{O}_k(\Lambda)$: Long Distance Contribution, contains the information about scales $\mu < \Lambda$

- We could as well imagine a situation with a different definition of “long” and “short” distances, defined by a scale μ , in which case

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \lambda_{\text{CKM}} \sum_k C_k(\Lambda/\mu) \mathcal{O}_k(\mu)$$

- Key Observation: The matrix elements of H_{eff} are physical Quantities, thus cannot depend on the arbitrary choice of μ

$$0 = \mu \frac{d}{d\mu} H_{\text{eff}}$$

- compute this

$$0 = \sum_i \left(\mu \frac{d}{d\mu} C_i(\Lambda/\mu) \right) \mathcal{O}_i(\mu) + C_i(\Lambda/\mu) \left(\mu \frac{d}{d\mu} \mathcal{O}_i(\mu) \right)$$

- Operator Mixing: **Change in scale can turn the operator \mathcal{O}_i into a linear combination of operators (of the same dimension)**

$$\mu \frac{d}{d\mu} \mathcal{O}_i(\mu) = \sum_j \gamma_{ij}(\mu) \mathcal{O}_j(\mu)$$

and so

$$\sum_i \sum_j \left(\left[\delta_{ij} \mu \frac{d}{d\mu} + \gamma_{ij}(\mu) \right] C_i(\Lambda/\mu) \right) \mathcal{O}_j(\mu) = 0$$

- Assume: The operators \mathcal{O}_j from a basis, then

$$\sum_i \left[\delta_{ij} \mu \frac{d}{d\mu} + \gamma_{ij}^T(\mu) \right] C_j(\Lambda/\mu) = 0$$

- QCD: Coupling constant α_s depends on μ : β -function

$$\mu \frac{d}{d\mu} \alpha_s(\mu) = \beta(\alpha_s(\mu))$$

- C_j depend also on α_s

$$\mu \frac{d}{d\mu} = \left(\mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right)$$

- In an appropriate scheme γ_{ij} depend on μ only through α_s :
 $\gamma_{ij}(\mu) = \gamma_{ij}(\alpha_s(\mu))$

- Renormalization Group Equation (RGE) for the coefficients

$$\sum_i \left[\delta_{ij} \left(\mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) + \gamma_{ij}^T(\alpha_s) \right] C_j(\Lambda/\mu, \alpha_s) = 0$$

- This is a system of linear differential equations:
→ Once the initial conditions are known, the solution is in general unique
- RGE Running: Use the RGE to relate the coefficients at different scales

- The coefficients are at $\mu = \Lambda$ (at the “matching scale”)

$$C_i(\Lambda/\mu = 1, \alpha_s) = \sum_n a_i^{(n)} \left(\frac{\alpha_s}{4\pi} \right)^n \quad \text{perturbative calculation}$$

- Perturbative calculation of the RG functions β and γ_{ij}

$$\beta(\alpha_s) = \alpha_s \sum_{n=0} \beta^{(n)} \left(\frac{\alpha_s}{4\pi} \right)^{n+1} \quad \gamma_{ij}(\alpha_s) = \sum_{n=0} \gamma_{ij}^{(n)} \left(\frac{\alpha_s}{4\pi} \right)^{n+1}$$

- RG functions can be calculated from loop diagrams:

$$\beta^{(0)} = -\frac{2}{3}(33 - 2n_f) \quad \gamma_{ij} \text{ depends on the set of } \mathcal{O}_i$$

- Structure of the perturbative expansion of the coefficient at some other scale

$$\begin{aligned}
 c_i(\Lambda/\mu, \alpha_s) = & \\
 & b_i^{00} \\
 + & b_i^{11} \left(\frac{\alpha_s}{4\pi} \right) \ln \frac{\Lambda}{\mu} + b_i^{10} \left(\frac{\alpha_s}{4\pi} \right) \\
 + & b_i^{22} \left(\frac{\alpha_s}{4\pi} \right)^2 \ln^2 \frac{\Lambda}{\mu} + b_i^{21} \left(\frac{\alpha_s}{4\pi} \right)^2 \ln \frac{\Lambda}{\mu} + b_i^{20} \left(\frac{\alpha_s}{4\pi} \right)^2 \\
 + & b_i^{33} \left(\frac{\alpha_s}{4\pi} \right)^3 \ln^3 \frac{\Lambda}{\mu} + b_i^{32} \left(\frac{\alpha_s}{4\pi} \right)^3 \ln^2 \frac{\Lambda}{\mu} + b_i^{31} \left(\frac{\alpha_s}{4\pi} \right)^3 \ln \frac{\Lambda}{\mu} + \dots,
 \end{aligned}$$

- LLA (Leading Log Approximation):
Resummation of the b_i^{nn} terms

$$C_i(\Lambda/\mu, \alpha_s) = \sum_{n=0}^{\infty} b_i^{nn} \left(\frac{\alpha_s}{4\pi} \right)^n \ln^n \frac{\Lambda}{\mu}$$

→ leading terms in the expansion of the RG functions

- NLLA (Next-to-leading log approximation):

$$C_i(\Lambda/\mu, \alpha_s) = \sum_{n=0}^{\infty} \left[b_i^{nn} + b_i^{n+1,n} \left(\frac{\alpha_s}{4\pi} \right) \right] \left(\frac{\alpha_s}{4\pi} \right)^n \ln^n \frac{\Lambda}{\mu}$$

→ next-to-leading terms of the RG functions

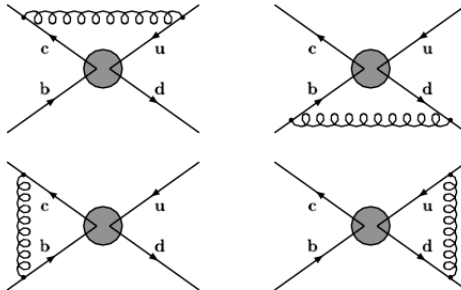
- Typical Procedure:
 - “Matching” at the scale $\mu = M_W$
 - “Running” to a scale of the order $\mu = m_b$
 - \rightarrow includes operator mixing
- Resummation of the large logs $\ln(M_W^2/m_b^2)$
 - “Matching” at the scale $\mu = m_b$
 - “Running” to the scale m_c
- Resummation of the “large” logs $\ln(m_b^2/m_c^2)$
- ...
- Untli $\alpha_s(\mu)$ becomes too large ...

H_{eff} for b decays at low scales

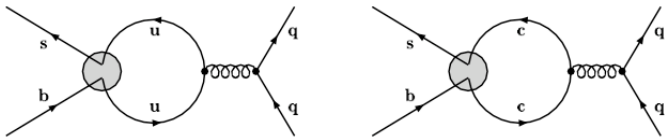
- Effective interaction: $H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \lambda_{\text{CKM}} \sum_k \hat{C}_k(\Lambda) \mathcal{O}_k(\Lambda)$
- "Tree" Operators"

$$\mathcal{O}_1 = (\bar{c}_{L,i} \gamma_\mu s_{L,j}) (\bar{d}_{L,j} \gamma_\mu u_{L,i}) ,$$

$$\mathcal{O}_2 = (\bar{c}_{L,i} \gamma_\mu s_{L,i}) (\bar{d}_{L,j} \gamma_\mu u_{L,j}) .$$



• If two flavours are equal: **QCD Penguin Operators**



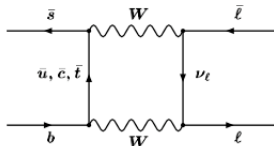
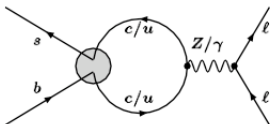
$$\mathcal{O}_3 = (\bar{s}_{L,i} \gamma_\mu b_{L,i}) \sum_{q=u,d,s,c,b} (\bar{q}_{L,j} \gamma^\mu q_{L,j}) ,$$

$$\mathcal{O}_4 = (\bar{s}_{L,i} \gamma_\mu b_{L,i}) \sum_{q=u,d,s,c,b} (\bar{q}_{L,j} \gamma^\mu q_{L,i}) ,$$

$$\mathcal{O}_5 = (\bar{s}_{L,i} \gamma_\mu b_{L,i}) \sum_{q=u,d,s,c,b} (\bar{q}_{R,j} \gamma^\mu q_{R,j}) ,$$

$$\mathcal{O}_6 = (\bar{s}_{L,i} \gamma_\mu b_{L,i}) \sum_{q=u,d,s,c,b} (\bar{q}_{R,j} \gamma^\mu q_{R,i}) .$$

- Electroweak Penguins:
Replace the Gluon by a Z_0 or Photon: $\mathcal{P}_7 \cdots \mathcal{P}_{10}$
- Rare (FCNC) Processes:



$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}_{L,\alpha} \sigma_{\mu\nu} b_{R,\alpha}) F^{\mu\nu}$$

$$\mathcal{O}_8 = \frac{g}{16\pi^2} m_b (\bar{s}_{L,\alpha} T_{\alpha\beta}^a \sigma_{\mu\nu} b_{R,\alpha}) G^{a\mu\nu}$$

$$\mathcal{O}_9 = \frac{1}{2} (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \ell)$$

$$\mathcal{O}_{10} = \frac{1}{2} (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

• Coefficients of the Operators (One Loop)

$C_i(\mu)$	$\mu = 10.0 \text{ GeV}$	$\mu = 5.0 \text{ GeV}$	$\mu = 2.5 \text{ GeV}$
C_1	0.182	0.275	0.40
C_2	-1.074	-1.121	-1.193
C_3	-0.008	-0.013	-0.019
C_4	0.019	0.028	0.040
C_5	-0.006	-0.008	-0.011
C_6	0.022	0.035	0.055

$C_i(\mu)$	$\mu = 2.5 \text{ GeV}$	$\mu = 5 \text{ GeV}$	$\mu = 10 \text{ GeV}$
C_7^{eff}	-0.334	-0.299	-0.268
C_8^{eff}	-0.157	-0.143	-0.131
$\frac{2\pi}{\alpha} C_9$	1.933	1.788	1.494

Heavy Quark Limit

Isgur, Wise, Voloshin, Shifman, Georgi, Grinstein, ...

- $1/m_Q$ Expansion: Substantial Theoretical Progress!
- Static Limit: $m_b, m_c \rightarrow \infty$ with fixed (four)velocity

$$v_Q = \frac{p_Q}{m_Q}, \quad Q = b, c$$

- In this limit we have

$$\left. \begin{aligned} m_{Hadron} &= m_Q \\ p_{Hadron} &= p_Q \end{aligned} \right\} v_{Hadron} = v_Q$$

- For $m_Q \rightarrow \infty$ the heavy quark does not feel any recoil from the light quarks and gluons (Cannon Ball)
- This is like the H-atom in Quantum Mechanics !!

Heavy Quark Symmetries

- The interaction of gluons is **identical for all quarks**
- Flavour enters QCD only through the mass terms
 - $m \rightarrow 0$: (Chiral) Flavour Symmetry (Isospin)
 - $m \rightarrow \infty$ **Heavy Flavour Symmetry**
 - Consider b and c heavy: Heavy Flavour SU(2)
- **Coupling of the heavy quark spin to gluons:**

$$H_{int} = \frac{g}{2m_Q} \bar{Q}(\vec{\sigma} \cdot \vec{B})Q \xrightarrow{m_Q \rightarrow \infty} 0$$

- Spin Rotations become a symmetry
 - Heavy Quark Spin Symmetry: SU(2) Rotations
- **Spin Flavour Symmetry Multiplets**

Mesonic Ground States

Bottom:

$$|(b\bar{u})_{J=0}\rangle = |B^-\rangle$$

$$|(b\bar{d})_{J=0}\rangle = |\bar{B}^0\rangle$$

$$|(b\bar{s})_{J=0}\rangle = |\bar{B}_s\rangle$$

$$|(b\bar{u})_{J=1}\rangle = |B^{*-}\rangle$$

$$|(b\bar{d})_{J=1}\rangle = |\bar{B}^{*0}\rangle$$

$$|(b\bar{s})_{J=1}\rangle = |\bar{B}_s^*\rangle$$

Charm:

$$|(c\bar{u})_{J=0}\rangle = |D^0\rangle$$

$$|(c\bar{d})_{J=0}\rangle = |D^+\rangle$$

$$|(c\bar{s})_{J=0}\rangle = |D_s\rangle$$

$$|(c\bar{u})_{J=1}\rangle = |D^{*0}\rangle$$

$$|(c\bar{d})_{J=1}\rangle = |D^{*+}\rangle$$

$$|(c\bar{s})_{J=1}\rangle = |D_s^*\rangle$$

Baryonic Ground States

$$|[(ud)_0 \textcolor{red}{Q}]_{1/2}\rangle = |\Lambda_Q\rangle$$

$$|[(uu)_1 \textcolor{red}{Q}]_{1/2}\rangle, |[(ud)_1 \textcolor{red}{Q}]_{1/2}\rangle, |[(dd)_1 \textcolor{red}{Q}]_{1/2}\rangle = |\Sigma_Q\rangle$$

$$|[(uu)_1 \textcolor{red}{Q}]_{3/2}\rangle, |[(ud)_1 \textcolor{red}{Q}]_{3/2}\rangle, |[(dd)_1 \textcolor{red}{Q}]_{3/2}\rangle = |\Sigma_Q^*\rangle$$

$$|[(us)_0 \textcolor{red}{Q}]_{1/2}\rangle, |[(ds)_0 \textcolor{red}{Q}]_{1/2}\rangle = |\Xi_Q\rangle$$

$$|[(us)_1 \textcolor{red}{Q}]_{1/2}\rangle, |[(ds)_1 \textcolor{red}{Q}]_{1/2}\rangle = |\Xi_Q'\rangle$$

$$|[(us)_1 \textcolor{red}{Q}]_{3/2}\rangle, |[(ds)_1 \textcolor{red}{Q}]_{3/2}\rangle = |\Xi_Q^*\rangle$$

$$|[(ss)_1 \textcolor{red}{Q}]_{1/2}\rangle = |\Omega_Q\rangle \quad |[(ss)_1 \textcolor{red}{Q}]_{3/2}\rangle = |\Omega_Q^*\rangle$$

Wigner Eckart Theorem for HQS

- HQS imply a “Wigner Eckart Theorem”

$$\langle H^{(*)}(\nu) | Q_\nu \Gamma Q_{\nu'} | H^{(*)}(\nu') \rangle = C_\Gamma(\nu, \nu') \xi(\nu \cdot \nu')$$

with $H^{(*)}(\nu) = D^{(*)}(\nu)$ or $B^{(*)}(\nu)$

- $C_\Gamma(\nu, \nu')$: Computable Clebsh Gordan Coefficient
- $\xi(\nu \cdot \nu')$: Reduced Matrix Element
- $\xi(\nu \cdot \nu')$: universal non-perturbative Form Faktor:
Isgur Wise Funktion
- Normalization of ξ at $\nu = \nu'$:

$$\xi(\nu \cdot \nu' = 1) = 1$$

Heavy Quark Effective Theory

- The heavy mass limit can be formulated as an effective field theory
- Expansion in inverse powers of m_Q
- Define the static field h_v for the velocity v

$$h_v(x) = e^{im_Q v \cdot x} \frac{1}{2} (1 + \not{v}) b(x) \quad p_Q = m_Q v + k$$

- HQET Lagrangian

$$\mathcal{L} = \bar{h}_v (i v \cdot D) h_v + \frac{1}{2m_Q} \bar{h}_v (i \not{D})^2 h_v + \dots$$

- Dim-4 Term: Feynman rules, loops, renormalization...

Application: Determination of V_{cb} from $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$

- Kinematic variable for a heavy quark: Four Velocity v
- Differential Rates

$$\frac{d\Gamma}{d\omega}(B \rightarrow D^* \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m_{D^*}^3 (\omega^2 - 1)^{1/2} P(\omega) (\mathcal{F}(\omega))^2$$

$$\frac{d\Gamma}{d\omega}(B \rightarrow D \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} (\mathcal{G}(\omega))^2$$

- with $\omega = vv'$ and
- $P(\omega)$: Calculable Phase space factor
- \mathcal{F} and \mathcal{G} : Form Factors

Heavy Quark Symmetries

- Normalization of the Form Factors is known at $v v' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

$$\mathcal{F}(\omega) = \eta_{\text{QED}} \eta_A \left[1 + \delta_{1/\mu^2} + \dots \right] (\omega - 1) \rho^2 + \mathcal{O}((\omega - 1)^2)$$

$$\mathcal{G}(1) = \eta_{\text{QED}} \eta_V \left[1 + \mathcal{O} \left(\frac{m_B - m_D}{m_B + m_D} \right) \right]$$

- Parameter of HQS breaking: $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$,
 $\delta_{1/\mu^2} = -(8 \pm 4)\%$, $\eta_{\text{QED}} = 1.007$

$B \rightarrow D^{(*)}$ Form Factors

- Lattice Calculations of the deviation from unity

$$\mathcal{F}(1) = 0.903 \pm 0.13$$

$$\mathcal{G}(1) = 1.033 \pm 0.018 \pm 0.0095$$

A. Kronfeld et al.

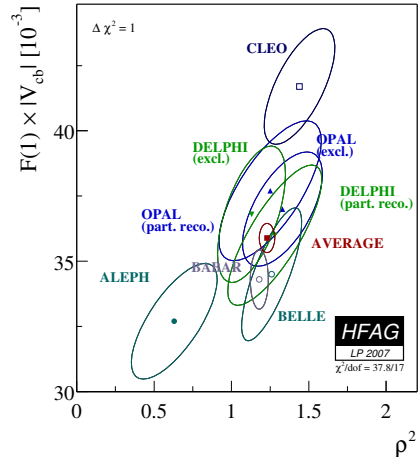
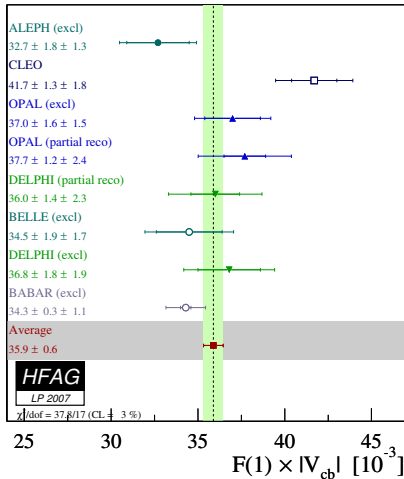
- Zero Recoil Sum Rules

$$\mathcal{F}(1) = 0.86 \pm 0.04$$

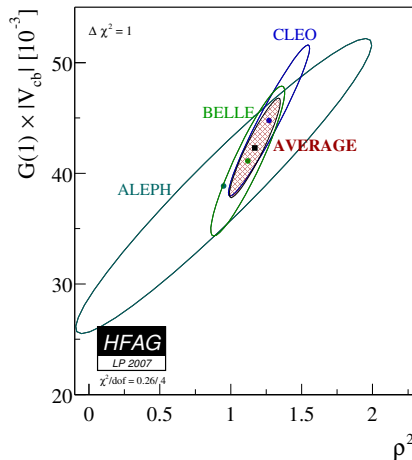
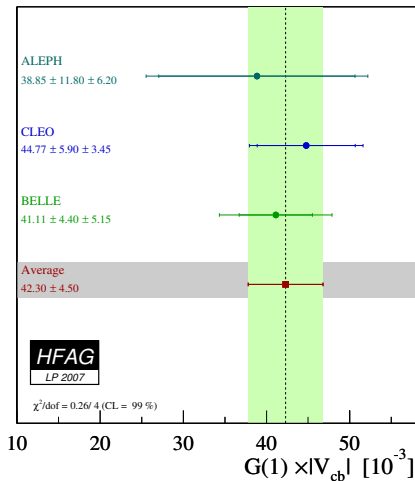
$$\mathcal{G}(1) = 1.04 \pm 0.02$$

P. Gambino et al.

$$B \rightarrow D^* \ell \bar{\nu}_\ell$$



$B \rightarrow D \ell \bar{\nu}_\ell$



Inclusive Decays: Heavy Quark Expansion

Operator Product Expansion = Heavy Quark Expansion

(Chay, Georgi, Bigi, Shifman, Uraltsev, Vainshtein, Manohar, Wise, Neubert, M,...)

$$\begin{aligned}
 \Gamma &\propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{\text{eff}} | B(v) \rangle|^2 \\
 &= \int d^4x \langle B(v) | \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}^\dagger(0) | B(v) \rangle \\
 &= 2 \operatorname{Im} \int d^4x \langle B(v) | T \{ \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}^\dagger(0) \} | B(v) \rangle \\
 &= 2 \operatorname{Im} \int d^4x e^{-im_b v \cdot x} \langle B(v) | T \{ \tilde{\mathcal{H}}_{\text{eff}}(x) \tilde{\mathcal{H}}_{\text{eff}}^\dagger(0) \} | B(v) \rangle
 \end{aligned}$$

- Last step: $p_b = m_b v + k$,
Expansion in the residual momentum k

- Perform an OPE: m_b is much larger than any scale appearing in the matrix element

$$\begin{aligned} \int d^4x e^{-im_b vx} T\{\tilde{\mathcal{H}}_{\text{eff}}(x) \tilde{\mathcal{H}}_{\text{eff}}^\dagger(0)\} \\ = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu) \mathcal{O}_{n+3} \end{aligned}$$

→ The rate for $B \rightarrow X_c \ell \bar{\nu}_\ell$ can be written as

$$\Gamma = \Gamma_0 + \frac{1}{m_Q} \Gamma_1 + \frac{1}{m_Q^2} \Gamma_2 + \frac{1}{m_Q^3} \Gamma_3 + \dots$$

- The Γ_i are power series in $\alpha_s(m_Q)$:
→ Perturbation theory!

- Γ_0 is the decay of a free quark (“Parton Model”)
- Γ_1 vanishes due to Heavy Quark Symmetries
- Γ_2 is expressed in terms of two parameters

$$2M_H\mu_\pi^2 = -\langle H(v) | \bar{Q}_v (iD)^2 Q_v | H(v) \rangle$$

$$2M_H\mu_G^2 = \langle H(v) | \bar{Q}_v \sigma_{\mu\nu} (iD^\mu)(iD^\nu) Q_v | H(v) \rangle$$

μ_π : Kinetic energy and μ_G : Chromomagnetic moment

- Γ_3 two more parameters

$$2M_H\rho_D^3 = -\langle H(v) | \bar{Q}_v (iD_\mu)(ivD)(iD^\mu) Q_v | H(v) \rangle$$

$$2M_H\rho_{LS}^3 = \langle H(v) | \bar{Q}_v \sigma_{\mu\nu} (iD^\mu)(ivD)(iD^\nu) Q_v | H(v) \rangle$$

ρ_D : Darwin Term and ρ_{LS} : Chromomagnetic moment

New: $1/m_b^4$ Contribution Γ_4 (Dassinger, Turczyk, M.)

- Five new parameters:

$\langle \vec{E}^2 \rangle$: Chromoelectric Field squared

$\langle \vec{B}^2 \rangle$: Chromomagnetic Field squared

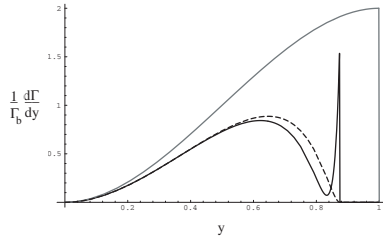
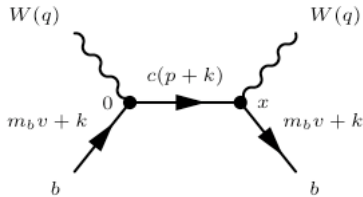
$\langle (\vec{p}^2)^2 \rangle$: Fourth power of the residual b quark momentum

$\langle (\vec{p}^2)(\vec{\sigma} \cdot \vec{B}) \rangle$: Mixed Chromomag. Mom. and res. Mom. sq.

$\langle (\vec{p} \cdot \vec{B})(\vec{\sigma} \cdot \vec{p}) \rangle$: Mixed Chromomag. field and res. helicity

- Some of these can be estimated in naive factorization

Spectra of Inclusive Decays



- Endpoint region: $\rho = m_c^2/m_b^2$, $y = 2E_\ell/m_b$

$$\frac{d\Gamma}{dy} \sim \Theta(1-y-\rho) \left[2 + \frac{\lambda_1}{(m_b(1-y))^2} \left(\frac{\rho}{1-y} \right)^2 \left\{ 3 - 4 \frac{\rho}{1-y} \right\} \right]$$

- Reliable calculation in HQE possible for the moments of the spectrum

Application: V_{cb} from $b \rightarrow c \ell \bar{\nu}$ inclusive

- Tree level terms up to and including $1/m_b^5$ known
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate known
- $\mathcal{O}(\alpha_s)$ for the μ_π^2/m_b^2 is known
- QCD inspired modelling for the HQE matrix elements
- **New:** Complete α_s/m_b^2 , including the μ_G terms
Alberti, Gambino, Nandi (arXiv:1311.7381)
ThM, Pivovarov, Rosenthal (arXiv:1405.5072,
arXiv:1506.08167)
- **This was the remaining parametrically largest uncertainty**

- Alberti et al.: **Phys.Rev.Lett. 114 (2015) 6, 061802**
and **JHEP 1401 (2014) 147**
 - Calculation of the differential rate including the charm mass
 - partially numerical calculation
- ThM, Pivovarov, Rosenthal:
Phys.Lett. B741 (2015) 290-294
 - Fully analytic calculation
 - limit $m_c \rightarrow 0$
 - Possibility to include m_c in a Taylor series
- Results do agree,
surprisingly steep m_c dependence

Result for V_{cb}

- Inclusive Decay (HQE / OPE)

$$V_{cb} = 42.21 \pm 0.78 \quad (\text{Gambino et al. 2015})$$

- Exclusive decay (Lattice FF)

$$V_{cb} = 39.36 \pm 0.78 \quad (\text{Fermilab/Milc 2015})$$

- Exclusive decay (Zero Recoil Sum rule)

$$V_{cb} = 41.4 \pm 0.9 \quad (\text{Gambino et al. 2015})$$

S_{oft} C_{ollinear} E_{ffective} T_{heory}

- Problem: How to deal with “energetic” light degrees of freedom = Endpoint regions of the spectra ?
- **More than two scales involved!**
- Inclusive Rates in the Endpoint become (Korchinski, Sterman)

$$d\Gamma = H * J * S$$

with $*$ = Convolution

- H : Hard Coefficient Function, Scales $\mathcal{O}(m_b)$
- J : Jet Function, Scales $\mathcal{O}(\sqrt{m_b \Lambda_{\text{QCD}}})$
- S : Shape function, Scales $\mathcal{O}(\Lambda_{\text{QCD}})$

Basics of Soft Collinear Effective Theory

- Heavy-to-light decays:

Kinematic Situations with energetic light quarks

hadronizing into jets or energetic light mesons

p_{fin} : Momentum of a light final state meson

$$p_{fin}^2 \sim \mathcal{O}(\Lambda_{\text{QCD}} m_b) \quad v \cdot p_{fin} \sim \mathcal{O}(m_b)$$

- Use light-cone vectors $n^2 = \bar{n}^2 = 0, n \cdot \bar{n} = 2$:

$$p_{fin} = \frac{1}{2}(n \cdot p_{fin})\bar{n} \quad \text{and} \quad v = \frac{1}{2}(n + \bar{n})$$

- Momentum of a light quark in such a meson:

$$p_{light} = \frac{1}{2}[(n \cdot p_{light})\bar{n} + (\bar{n} \cdot p_{light})n] + p_{light}^\perp$$

SCET Power Counting

- Define the parameter $\lambda = \sqrt{\Lambda_{\text{QCD}}/m_b}$
- The light quark invariant mass (or virtuality) is assumed to be

$$p_{\text{light}}^2 = (n \cdot p_{\text{light}})(\bar{n} \cdot p_{\text{light}}) + (p_{\text{light}}^\perp)^2 \sim \lambda^2 m_b^2$$

- The components of the quark momentum have to scale as

$$(n \cdot p_{\text{light}}) \sim m_b \quad (\bar{n} \cdot p_{\text{light}}) \sim \lambda^2 m_b \quad p_{\text{light}}^\perp \sim \lambda m_b$$

A brief look at SCET

(Bauer, Stewart, Pirjol, Beneke, Feldmann ...)

- QCD quark field q is split into a collinear component ξ and a soft one with $\xi = \frac{1}{4}\not{n}_-\not{n}_+ q$
- The Lagrangian $\mathcal{L}_{\text{QCD}} = \bar{q}(i\not{D})q$ is rewritten in terms of the collinear field

$$\mathcal{L} = \frac{1}{2}\bar{\xi}\not{n}_+(in_-D)\xi - \bar{\xi}i\not{D}_\perp \frac{1}{in_+D + i\epsilon} \frac{\not{n}_+}{2} i\not{D}_\perp \xi$$

- Expansion according to the above power counting:

$$in_+D = in_+\partial + gn_+A_c + gn_+A_{\text{us}} = in_+D_c + gn_+A_{\text{us}}$$

- Leading \mathcal{L} becomes **non-local**: Wilson lines

Practical Consequences of SCET

- Similar to HQS: **Relations between form factors at large momentum transfer**

$$\langle B(v) | \bar{b} \Gamma q | \pi(p) \rangle \propto \zeta(vp), \zeta_{//}(vp), \zeta_{\perp}(vp)$$

For energetic pion only three independent form factors (Charles et al.)

- Correction can be calculated as in HQET

Basic Idea (Shifman, Vainshtein, Zakharov, 1978)

- Start from a suitably chosen correlation function, e.g.

$$T(q^2) = \int d^4x e^{-iqx} \langle 0 | T[j(x)j^\dagger(0)] | 0 \rangle$$

- This can be calculated perturbatively as $q^2 \rightarrow -\infty$.
- On the other hand, it has a dispersion relation

$$T(q^2) = \int \frac{ds}{2\pi} \frac{\rho(s)}{s - q^2 + i\epsilon} + \text{possible subtractions}$$

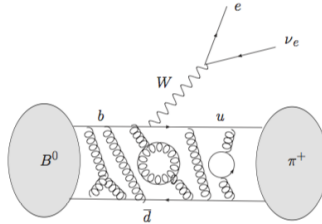
$$\text{with } \rho(s) \sim \langle 0 | j(x) j^\dagger(0) | 0 \rangle = \sum_n \langle 0 | j(x) | n \rangle \langle n | j^\dagger(0) | 0 \rangle$$

- Estimates for $\langle 0 | j(x) | n \rangle$ from e.g. positivity statements

Application: Determination of V_{ub} from exclusive $b \rightarrow u\ell\bar{\nu}$

□ $B \rightarrow \pi\ell\nu_\ell$, determination of $|V_{ub}|$

- decay amplitude parametrized by
hadronic form factors



$$\langle \pi^+(p) | \bar{u} \gamma_\mu b | \bar{B}^0(p+q) \rangle = f_{B\pi}^+(q^2) [\dots]_\mu + f_{B\pi}^0(q^2) [\dots]_\mu$$

- $|V_{ub}|$ determination [BaBar,Belle]

$$\left(\frac{1}{\tau_B} \right) \frac{dBR(\bar{B}^0 \rightarrow \pi^+ l^- \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} p_\pi^3 |f_{B\pi}^+(q^2)|^2 + O(m_l^2)$$

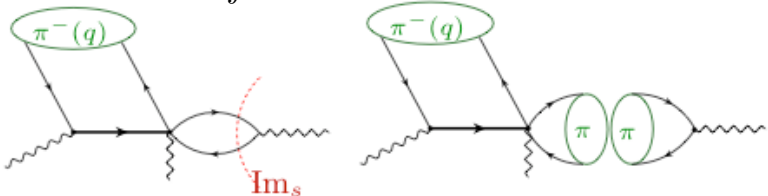
$$0 < q^2 < (m_B - m_\pi)^2 \sim 26 \text{ GeV}^2,$$

- form factors accessible in lattice QCD at $q^2 \gtrsim 16 \text{ GeV}^2$

$f_+(q^2)$ from QCD Sum Rules (Ball, Zwicky, Khodjamirian, ...)

- Dispersion Relation and Light Cone Expansion
- Study a Correlation Function

$$F_\lambda(p, q) = i \int d^4x \, e^{ipx} \langle \pi^+(q) | T \{ \bar{u} \gamma_\lambda b(x) \, m_b \bar{b} i \gamma_5 d(0) \} | 0 \rangle$$



- Yields an estimate for $f_B f_+(q^2)$
- Limited to small q^2

Results from LCSR

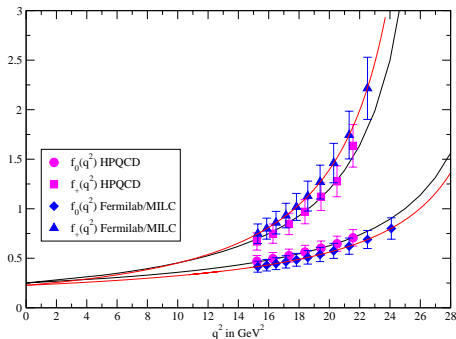
- Uncertainties from
 - Higher Twists (≥ 4)
 - b quark mass and renormalization scale
 - Values of the condensates
 - Threshold and Borel parameters
 - Pion Distribution amplitude

$$f_+(0) = 0.27 \times \left[1 \pm (5\%)_{tw>4} \pm (3\%)_{m_{b,\mu}} \pm (3\%)_{\langle \bar{q}q \rangle} \pm (3\%)_{s_0^B, M} \pm (8\%)_{a_{2,4}^\pi} \right]$$

- Extrapolation to $q^2 \neq 0$ by a pole model

Lattice QCD for Heavy to Light Form Factors

- Results reliable for large q^2
- **Unquenched results are available**
- Extrapolation to small q^2 by a pole model Becirevic, Kaidalov



Rate for $q^2 \geq 16 \text{ GeV}^2$

$$|V_{ub}|^2 \times (1.31 \pm 0.33) \text{ ps}^{-1}$$

$$|V_{ub}|^2 \times (1.80 \pm 0.48) \text{ ps}^{-1}$$

(HPQCD / Fermilab MILC)

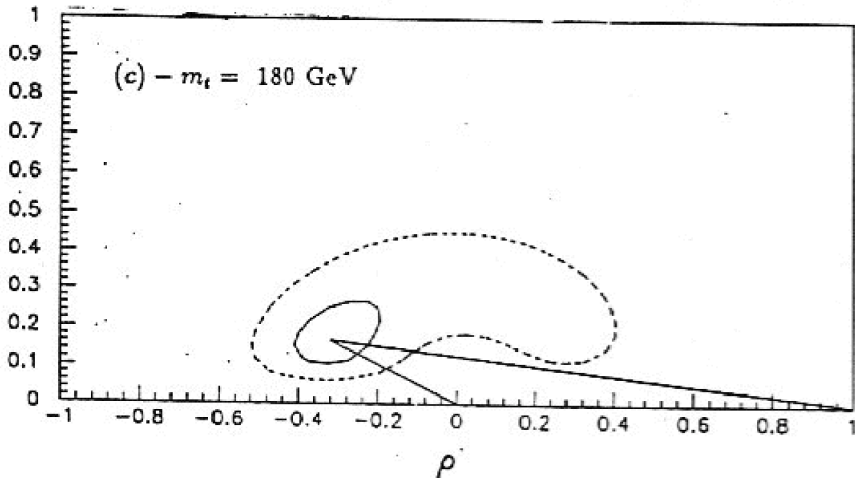
Status of V_{ub}

- From QCD LCSR: $V_{ub} = (3.32 \pm 0.26) \times 10^{-3}$
- PDG 2104:
 - Inclusive (LC-OPE): $V_{ub} = (4.41 \pm 0.25) \times 10^{-3}$
 - Exclusive (Combined): $V_{ub} = (3.28 \pm 0.29) \times 10^{-3}$
- This is the famous tension between the V_{ub} s

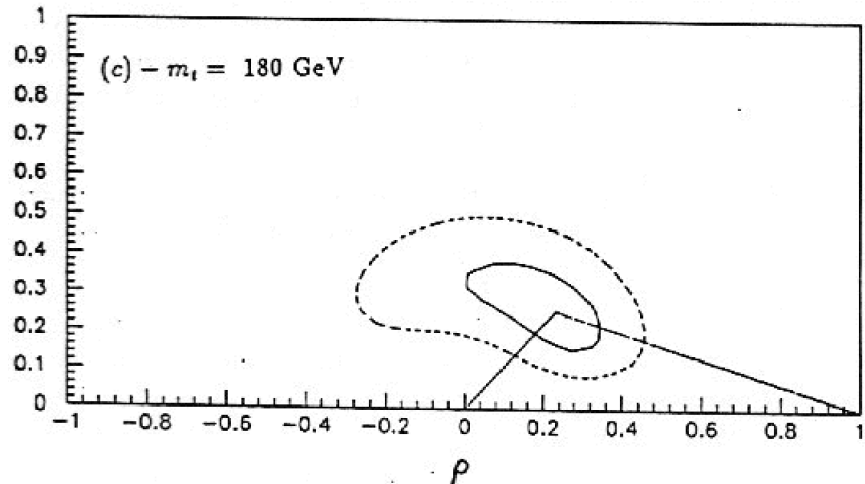
The history of the UT since ~ 1993

- Situation in 1993:
 - HQET was still young (~ 3 years)
 - Hadronic Matrix elements for $\Delta m_d \sim f_B^2$ were quite uncertain
 - V_{ub}/V_{cb} was known at the level of $\sim 20\%$
 - The top quark mass was still $m_t \sim (140 \pm 40) \text{ GeV}$
 - No CP violation has been observed except ϵ_K
- The UT still could have been “flat”

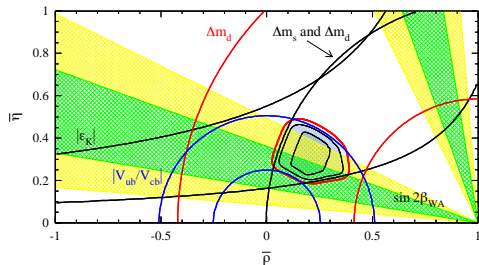
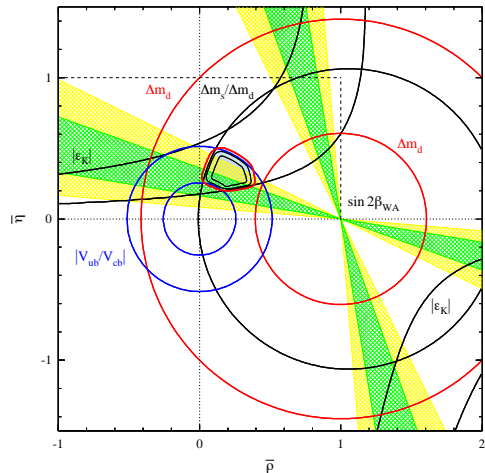
Unitarity triangle 1993: $f_B = 135 \pm 25$ MeV



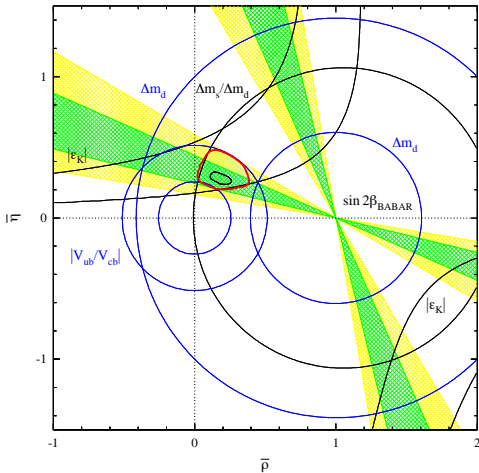
Unitarity triangle 1993: $f_B = 200 \pm 30$ MeV



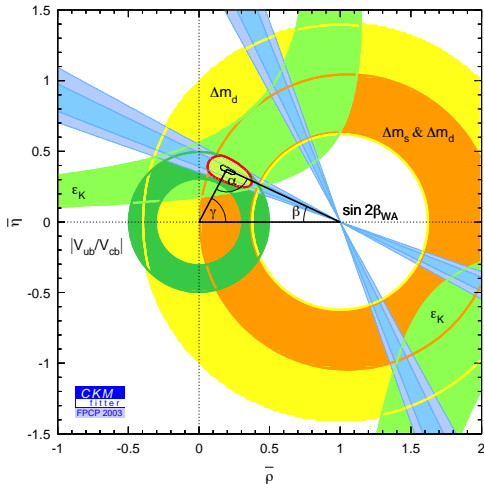
2001: First observation of “Non-Kaon CPV”



Unitarity Triangle 2001

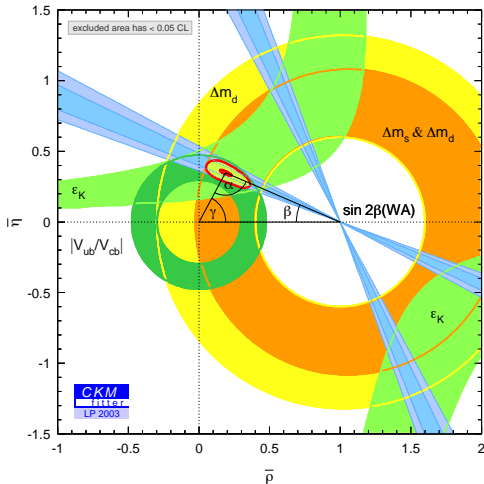


Unitarity Triangle 2002



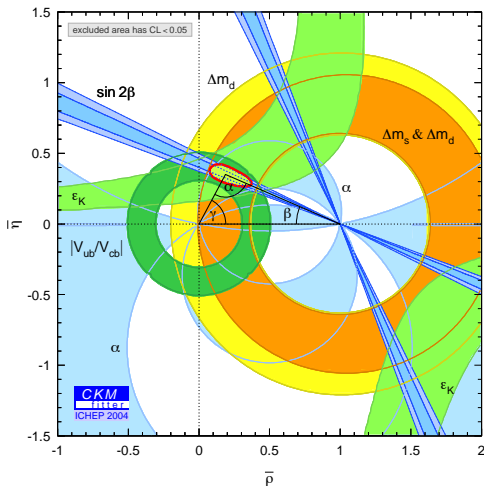
- Some improvement of V_{ub}/V_{cb} through the Heavy Quark Expansion
- More data on $\mathcal{A}_{CP}(B \rightarrow J/\psi K_s)$

Unitarity Triangle 2003



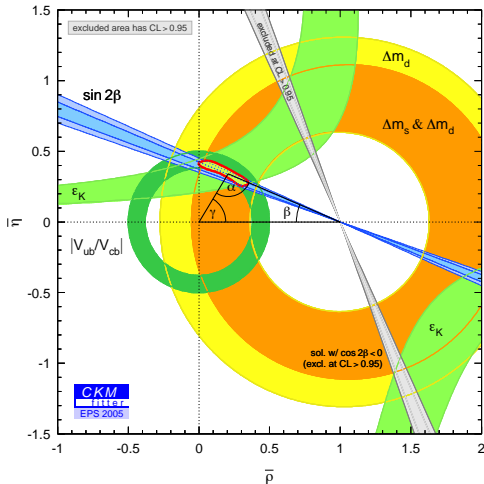
- Slight improvement of $f_B^2 B_B$ from lattice calculations
- Still more data on $\mathcal{A}_{\text{CP}}(B \rightarrow J/\psi K_s)$
- Central value of V_{ub}/V_{cb} slightly moved

Unitarity Triangle 2004



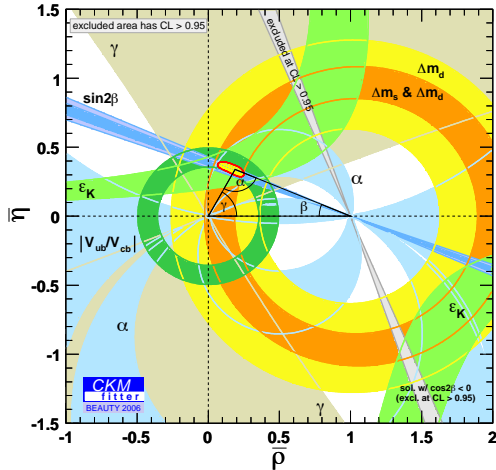
- More improvement of $f_B^2 B_B$ from lattice calculations
- Still more data on $\mathcal{A}_{CP}(B \rightarrow J/\psi K_s)$
- **First constraints on the angle α from $B \rightarrow \rho\rho$**

Unitarity Triangle 2005



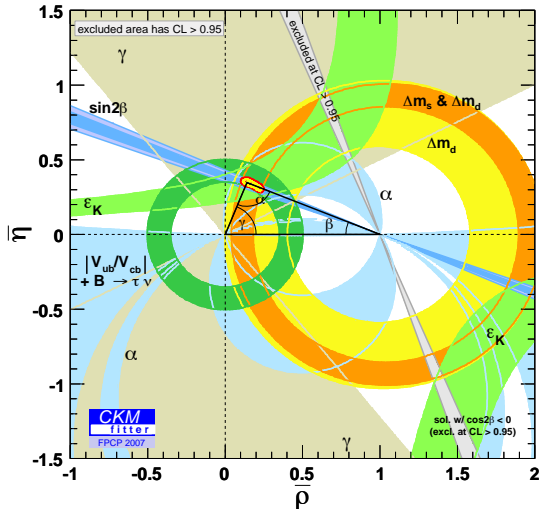
- Still more data on $\mathcal{A}_{CP}(B \rightarrow J/\psi K_s)$
- Exclusion of the “wrong branch” of β
- **Dramatic Improvement of V_{ub} from the HQE**

Unitarity Triangle 2006

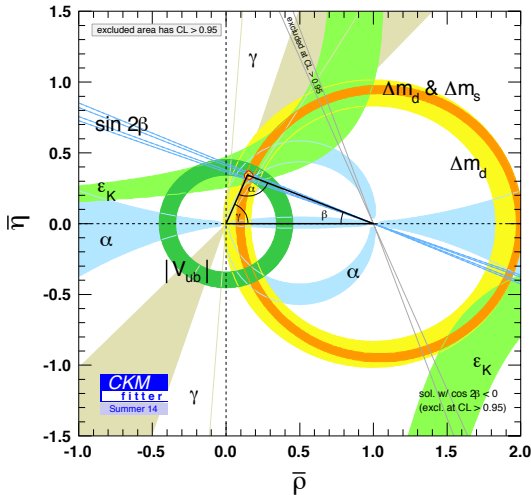


- **TEVATRON measurement of Δm_s**
- Tighter constraints on α
- First constraints on γ from CPV in $B \rightarrow K\pi$

Unitarity triangle 2007



Unitarity Triangle Now



Conclusions on Tools and Phenomenology

- There are more phenomenological methods which have not been mentioned
 - QCD factorization for non-leptonic decays
 - HQE calculations of neutral meson mixings
 - Multibody decays
 - **CP Violation measurements and phenomenology**
- We are in the era of precision flavor physics
- Sensitivity to NEW PHYSICS?