RDP Seventh Autumn PhD School

Frontiers of QCD

Tbilisi, September 23-25, 2019

Hadrons in the muon anomalous magnetic moment

Arkady Vainshtein

William Fine Theoretical Physics Institute
University of Minnesota
and
Kavli Institute for Theoretical Physics, UCSB

Lepton magnetic moments

$$H = -\boldsymbol{\mu}\,\boldsymbol{B} \qquad\qquad \boldsymbol{\mu} = g\,\frac{e\hbar}{2mc}\,\boldsymbol{s}$$

The present experimental values

Electron: Hanneke, Fogwell, and Gabrielse ’08

g/2=1.001 159 652 180 73 (28)

0.28×10^{-12} $[0.28\ \mathrm{ppt}]$

Muon: BNL E821 ’06

g/2=1.001 165 920 89 (63) [630 ppt]

Tau: Delphi at LEP2 ’04

g/2=0.985(32)

Anatomy of muon g-2

$$a_\mu = \frac{g_\mu - 2}{2} \qquad a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm had} + a_\mu^{\rm EW}$$

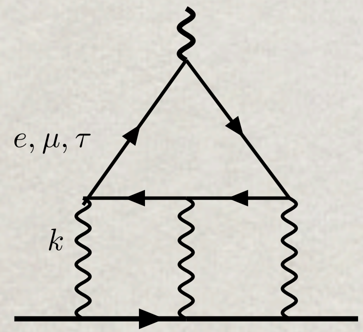
$$a_\mu^{\rm QED} = 116\,584\,718.85(0.36) \times 10^{-11}$$ Kinoshita et al

4th and 5th loop including.

Two enhancement parameters

$$\ln \frac{m_\mu}{m_e} \sim 5, \quad \pi^2 \sim 9$$

E.g., three loops are dominated by light-by-light with the electron in the loop



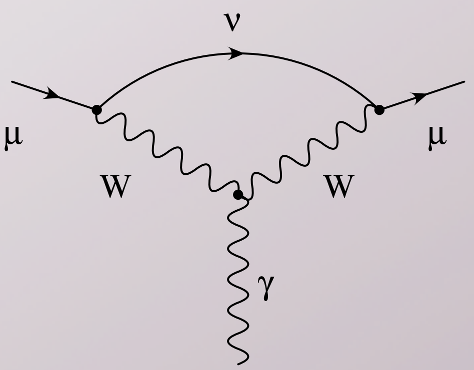
$$\sim \left(\frac{\alpha}{\pi}\right)^3 \frac{2\pi^2}{3} \ln \frac{m_\mu}{m_e}$$

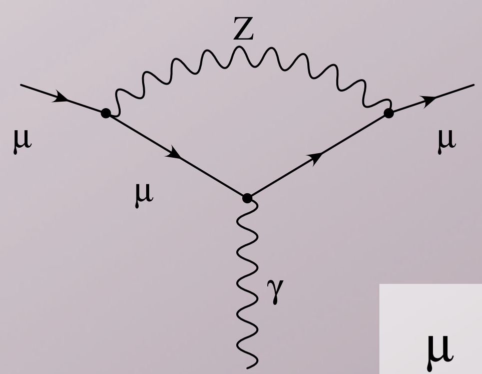
Electroweak part

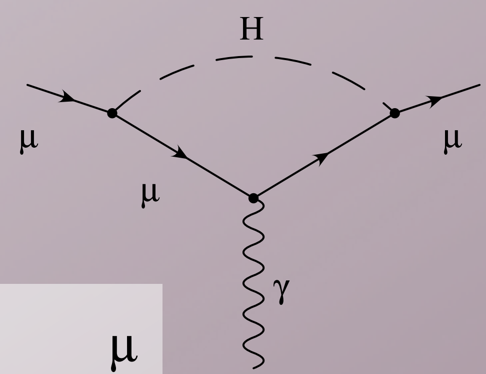
$$a_\mu^{\mathrm{EW}} = 154(1)(2) \times 10^{-11}$$

Czarnecki, Marciano, AV '02

$$a_\mu^{\mathrm{EW}}(1\text{-loop}) = \frac{5\, G_\mu\, m_\mu^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}(1 - 4\sin^2\theta_W)^2 + \mathcal{O}\left(\frac{m_\mu^2}{m_{W,H}^2}\right)\right] = 194.8 \times 10^{-11}$$



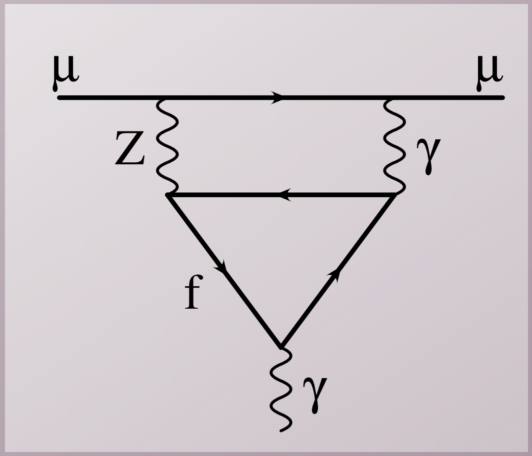




Hadrons at the second loop

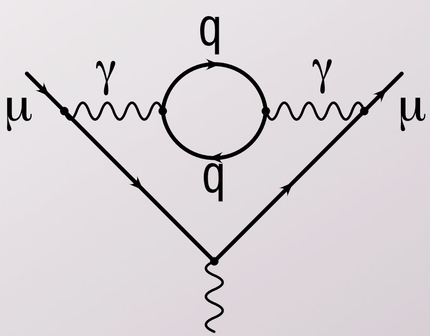
Nonrenormalization of triangle

Longitudinal as wel as transversal.

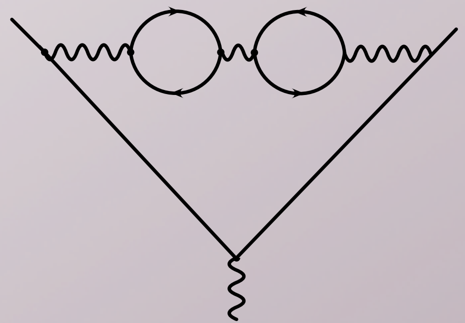


Hadronic contributions

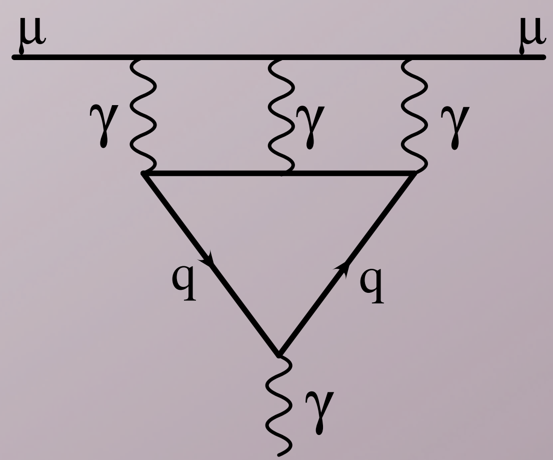
$$a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,HO} + a_\mu^{\rm LBL}$$



Lowest order hadronic contribution represented by a quark loop



An example of higher order hadronic contribution



Light-by-light scattering contribution

In theory

$$a_\mu^{\rm had,LO} = \left(\frac{\alpha\, m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{{\rm d}s}{s^2} K(s) R(s)$$

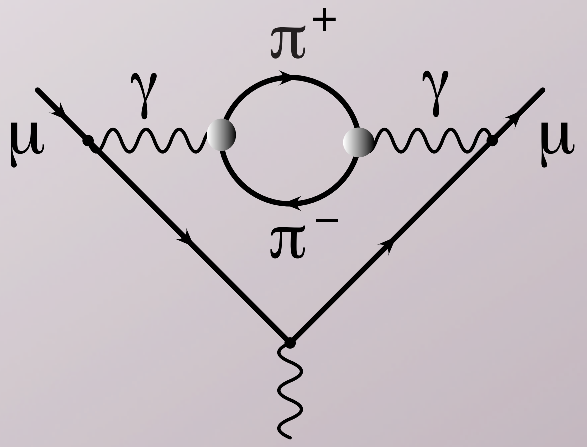
$K(s)$ is the known function, $K(s) \to 1, \quad s \gg m_\mu^2$

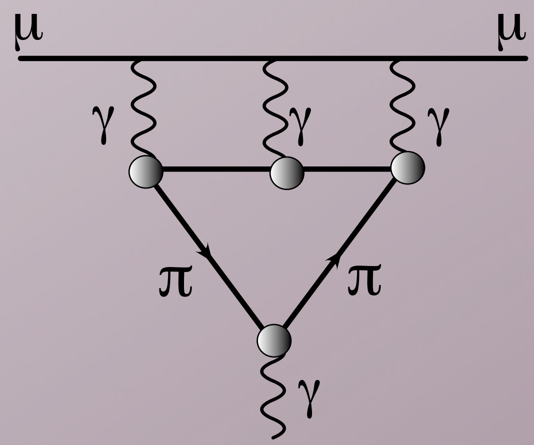
$R(s)$ is the cross section of e^+e^- annihilation into hadrons in units of $\sigma(e^+e^- \to \mu^+\mu^-)$.

In difference with $a_\mu^{\mathrm{had,LO}}$ there is no experimental input for the light-by-light contribution.What are possible theoretical parameters to exploit?

Smallness of chiral symmetry breaking, $m_\rho^2/m_\pi^2 \gg 1$

$$a_\mu^{(n)} \sim c_1 \left(\frac{\alpha}{\pi}\right)^n \frac{m_\mu^2}{m_\pi^2}, \qquad \mathrm{LO}: n=2, \quad \mathrm{LbL}: n=3$$

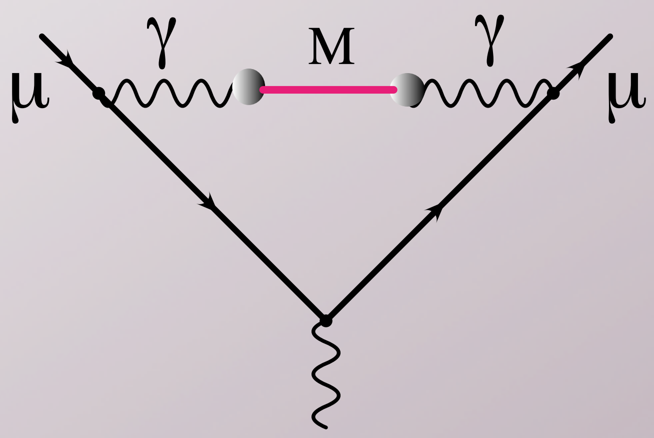


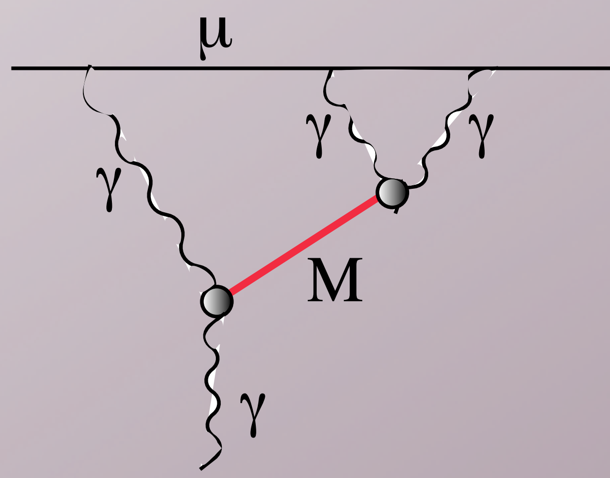


The Goldstone nature of pion implies $m_\pi^2 \propto m_q$ much less than typical $M_{\mathrm{had}}^2 \sim m_\rho^2$. Thus, the threshold range in pion loops produces the $1/m_\pi^2$ enhancement.

Large number of colors, N_c

Quark loops clearly give $a_\mu \propto N_c$. Dual not to pion loops but to meson exchanges.





No continuum in the large N_c limit.
$M = \rho^0, \omega, \phi, \rho', \ldots$ for the polarization operator
$M = \pi^0, \eta, \eta', a_0, a_1, \ldots$ (and any C-even meson) for the light-by-light

$$a_\mu^{(n)} \sim c_2 \left(\frac{\alpha}{\pi}\right)^n N_c \, \frac{m_\mu^2}{m_\rho^2}$$

We can check for $a_\mu^{\mathrm{had,LO}}$

Two regions. The threshold region $s \sim 4m_\pi^2$ where

$$R(s) \approx \frac{1}{4}\left(1 - \frac{4m_\pi^2}{s}\right)^{3/2}$$

and the resonance region $s \sim m_\rho^2$ where by quark-hadron duality on average

$$R(s) \approx N_c \sum Q_q^2$$

The chirally enhanced threshold region gives numerically

$$a_\mu^{\mathrm{had,LO}}(4m_\pi^2 \leq s \leq m_\rho^2/2) \approx 400 \times 10^{-11}$$

Compare with the N_c enhanced ρ peak,

$$a_\mu^{\mathrm{had,LO}}(\rho) = \frac{m_\mu^2\, \Gamma(\rho \to e^+ e^-)}{\pi\, m_\rho^3} \approx 5000 \times 10^{-11}$$

This contribution is enhanced by N_c,

$$a_\mu(\rho) \sim c_2 \left(\frac{\alpha}{\pi}\right)^2 N_c \frac{m_\mu^2}{m_\rho^2}$$

What is a lesson from this exercise? We see that the large N_c enhancement prevails over chiral one.

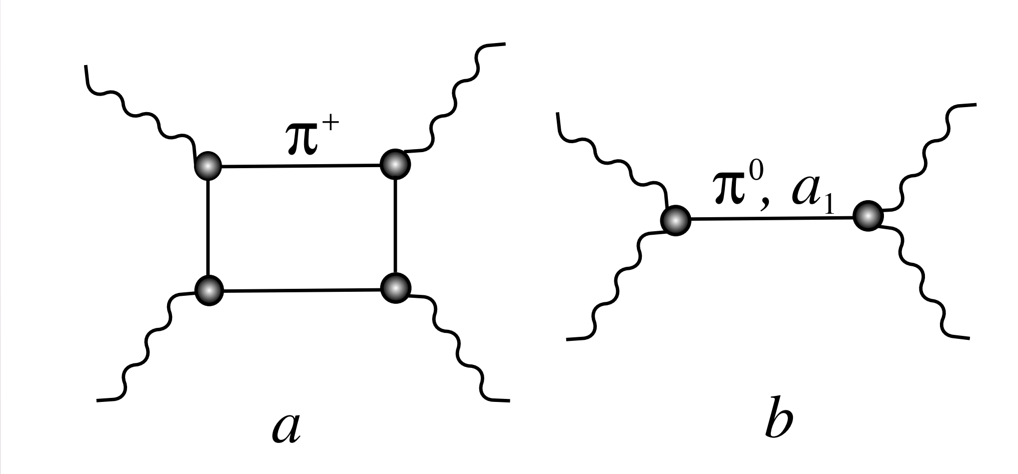
In the chiral perturbation theory

$$a_\mu^{2\pi} = \frac{1}{40}\left(\frac{\alpha}{3\pi}\right)^2 \frac{m_\mu^2}{m_\pi^2}\left[1+40\, m_\pi^2 F'_{\pi\pi\gamma *}(0)\ln\frac{m_\rho}{2m_\pi}\right]$$

$$= \frac{1}{40}\left(\frac{\alpha}{3\pi}\right)^2 \frac{m_\mu^2}{m_\pi^2}\left[1+40\,\frac{m_\pi^2}{m_\rho^2}\ln\frac{m_\rho}{2m_\pi}\right]$$

Chiral perturbation theory does not work. The leading term is suppressed by p-wave nature.

In light-by-light



The chirally enhanced pion box contribution does not result in large number, it is actually rather small,

$$a_\mu^{\mathrm{LbL}}(\text{pion box}) \approx -4 \times 10^{-11}$$ Hayakawa, Kinoshita, Sanda; Melnikov

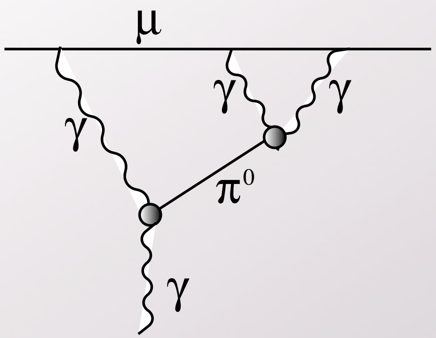
similarly to the hadronic polarization case above.
A larger value (-19) for the pion box was obtained by Bijnens, Pallante, Prades

Instability of the number is due to relatively large pion momenta in the loop, of order of $4m_\pi$ as we estimated. Then details of the model becomes important and theoretical control is lost. In HSL model few first terms of m_π^2/m_ρ^2 expansion are

$$a_\mu(\text{charged pion loop})\times 10^{11} = -46.37+35.46+10.98-4.7+\ldots = -4.9$$

If momenta were small compared with m_ρ the result would be close to the leading term – free pion loop.

In case of polarization operator the suppression of the leading term in the chiral expansion (larger momenta) can be related to the p-wave p^3 suppression. There is a suppression for s-wave in two-pion intermediate state near threshold in the case of LbL.



Hayakawa, Kinoshita, Sanda
Bijnens, Pallante, Prades
Barbieri, Remiddi
Pivovarov
Bartos, Dubničkova,Dubnička, Kuraev, Zemlyanaya
Knecht, Nyffeler
Knecht, Nyffeler, Perrotttet, de Rafael
Ramsey-Musolf, Wise
Blokland, Czarnecki, Melnikov
Melnikov, A.V.

Different models: constituent quark loop, extended Nambu–Jano-Lasinio model (ENJL), hidden local symmetry (HLS) model . . .

The π^0 pole part of LbL contains besides N_c the chiral enhancement in the logarithmic form, leading to the model-independent analytical expression

$$a_\mu^{\rm LbL}(\pi^0) = \left(\frac{\alpha}{\pi}\right)^3 N_c \frac{m_\mu^2 N_c}{48\pi^2 F_\pi^2} \ln^2 \frac{m_\rho}{m_\pi} + \ldots$$

However next, model dependent, terms are comparable with the the leading or

Numerically

$$a_\mu^{\rm LbL}(\pi^0) = 58(10)\times 10^{-11}$$

Knecht, Nyffeler

Massive quark loop (Laporta, Remiddi '91)

$$a^{\mathrm{HLbL}}(\text{quark loop}) = \left(\frac{\alpha}{\pi}\right)^3 N_c Q_q^4 \left\{ \underbrace{\left[\frac{3}{2}\zeta(3) - \frac{19}{16}\right]}_{0.62} \frac{m_\mu^2}{m_q^2} + \mathcal{O}\left[\frac{m_\mu^4}{m_q^4}\log^2\frac{m_\mu^2}{m_q^2}\right] \right\}$$

For c-quark with $m_c \approx 1.5\ \mathrm{GeV}$,

$$a^{\mathrm{HLbL}}(\mathrm{c}) = 2.3 \times 10^{-11}$$

Light quark estimate for the constituent mass 300 MeV

$$a^{\mathrm{HLbL}}(u,d,s) = 64 \times 10^{-11}$$

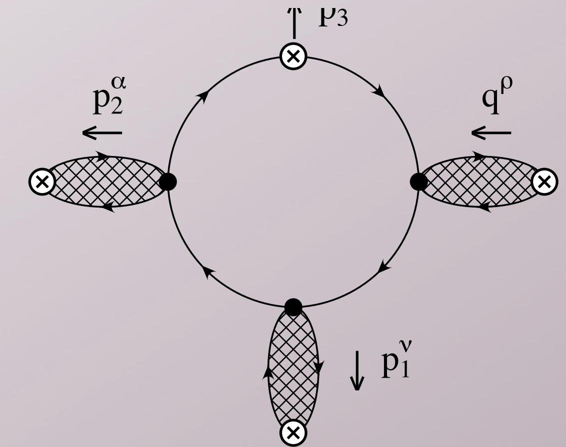
Together with the neutral pion exchange it gives

$$a^{\mathrm{HLbL}} \approx 120 \times 10^{-11}$$

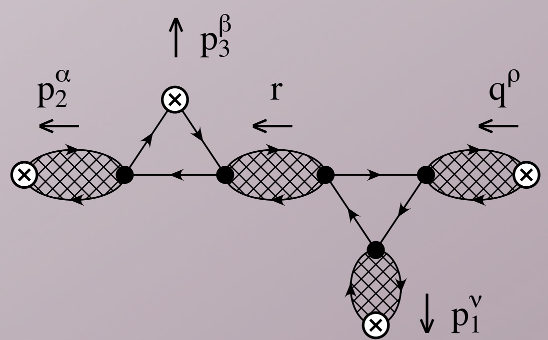
Models

HLS model is a modification the Vector Meson Dominance model.

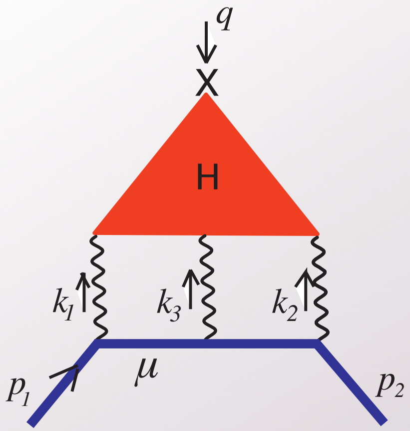
ENJL model is represented by the following graphs



(a)



(b)



OPE constraints and hadronic model

$$\epsilon_i^\mu(q_i)\,, \quad i=1,2,3,4, \quad \sum q_i = 0$$

ϵ_4 represents the external magnetic field $f^{\gamma\delta} = q_4^\gamma \epsilon_4^\delta - q_4^\delta \epsilon_4^\gamma, \quad q_4 \to 0.$

The LbL amplitude

$$\begin{aligned}\mathcal{M} &= \alpha^2 N_c \,\mathrm{Tr}\,[\hat{Q}^4]\,\mathcal{A} = \alpha^2 N_c\,\mathrm{Tr}\,[\hat{Q}^4]\,\mathcal{A}_{\mu_1\mu_2\mu_3\gamma\delta}\epsilon_1^{\mu_1}\epsilon_2^{\mu_2}\epsilon_3^{\mu_3} f^{\gamma\delta} \\ &= -e^3 \int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{e}^{-iq_1x-iq_2y}\,\epsilon_1^{\mu_1}\epsilon_2^{\mu_2}\epsilon_3^{\mu_3}\langle 0|T\left\{j_{\mu_1}(x)\,j_{\mu_2}(y)\,j_{\mu_3}(0)\right\}|\gamma\rangle\end{aligned}$$

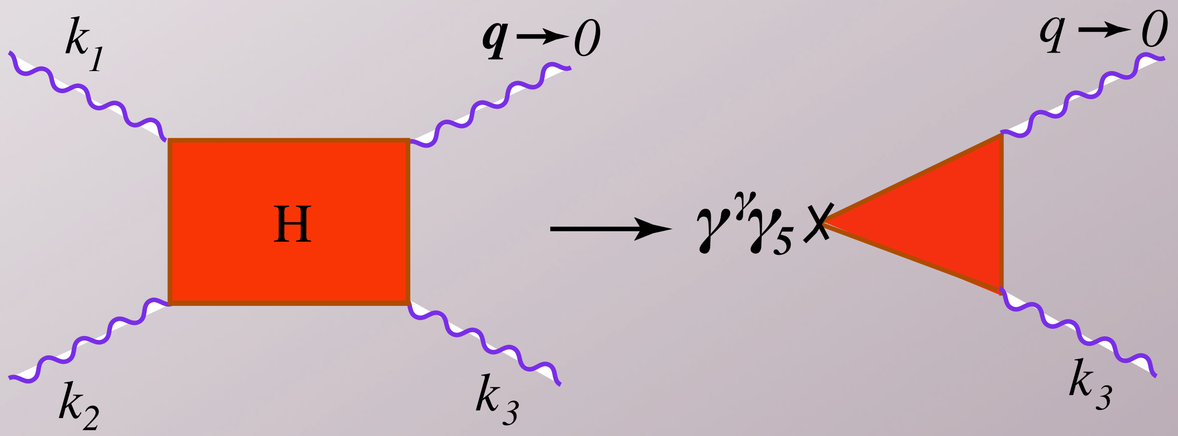
The electromagnetic current $j_\mu = \bar{q}\,\hat{Q}\gamma_\mu q, \quad q=\{u,d,s\}$

Three Lorentz invariants: q_1^2, q_2^2, q_3^2

Consider the Euclidian range $q_1^2 \approx q_2^2 \gg q_3^2 \gg \Lambda_{\mathrm{QCD}}^2$

Short distance QCD constraints

Operator Product Expansion leads to constraints



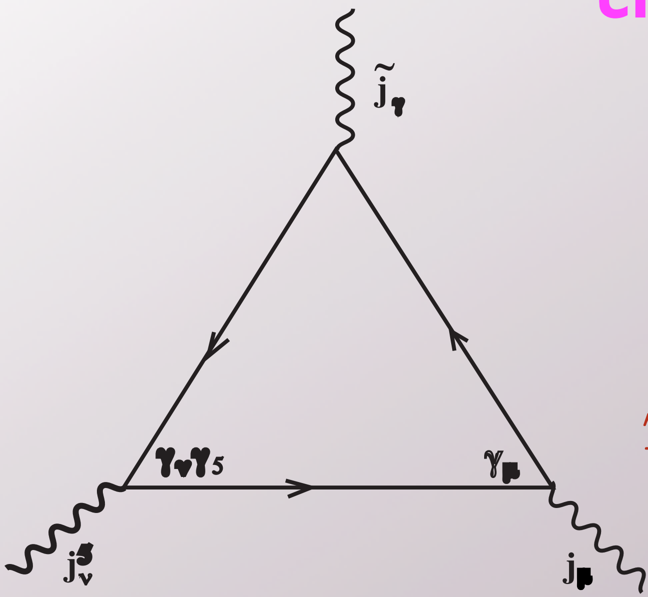
In the range where $k_1^2 \approx k_2^2 \gg k_3^2$ and $k_3^2 \gg \Lambda^2_{\text{QCD}}$

$$\int d^4x_1 \int d^4x_2 \, \mathrm{e}^{-ik_1\cdot x_1 - ik_2 \cdot x_2} \, T\{j_\nu(x_1), j_\rho(x_2)\} =$$

$$\frac{2}{\hat{k}^2} \, \epsilon_{\nu\rho\delta\gamma} \hat{k}^\delta \int d^4z \, \mathrm{e}^{-ik_3\cdot z} \, j_5^\gamma(z) + \mathcal{O}\left(\frac{1}{\hat{k}^3}\right)$$

$$\hat{k} = (k_1 - k_2)/2 \approx k_1 \approx -k_2$$

Perturbative and nonperturbative triangle amplitudes



$$j_\mu = \bar{q}\, V \gamma_\mu q, \qquad j^5_\nu = \bar{q}\, A \gamma_\nu \gamma_5 q$$

$$T_{\mu\gamma\nu} = -\int \mathrm{d}^4x \mathrm{d}^4y\, \mathrm{e}^{iqx - iky}\, \langle 0|\, T\{j_\mu(x)\, \tilde{j}_\gamma(y)\, j^5_\nu(0)\}|0\rangle$$

$$T_{\mu\nu} = T_{\mu\gamma\nu} e^\gamma(k) = i \int \mathrm{d}^4x\, \mathrm{e}^{iqx}\, \langle 0|\, T\{j_\mu(x)\, j^5_\nu(0)\}|\, \gamma(k)\rangle$$

$$T_{\mu\nu} = -\frac{i}{4\pi^2}\left[w_T(q^2)\left(-q^2 \tilde{f}_{\mu\nu} + q_\mu q^\sigma \tilde{f}_{\sigma\nu} - q_\nu q^\sigma \tilde{f}_{\sigma\mu}\right) + w_L(q^2)\, q_\nu q^\sigma \tilde{f}_{\sigma\mu}\right]$$

$$\tilde{f}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\gamma\delta} f^{\gamma\delta}, \qquad f_{\mu\nu} = k_\mu e_\nu - k_\nu e_\mu\,.$$

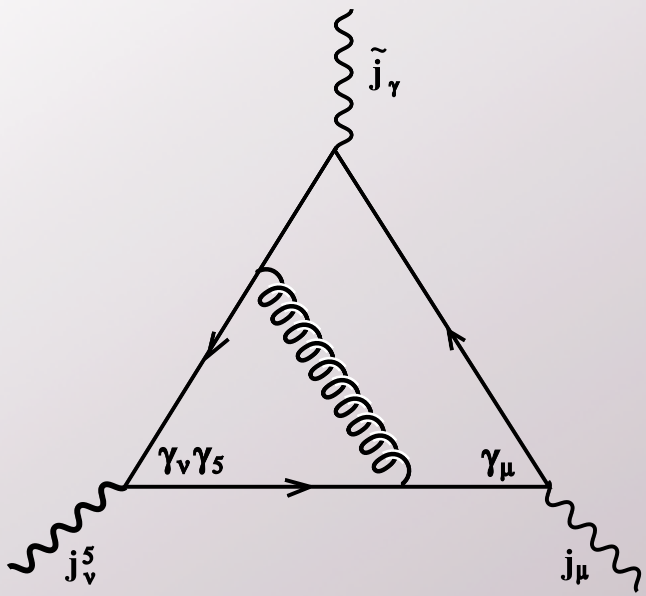
$$w_L^{1-\mathrm{loop}}[m=0] = 2\, w_T^{1-\mathrm{loop}}[m=0] = \frac{2N_c\, \mathrm{Tr}\,(A\, V\, \widetilde{V})}{Q^2}$$

Nonrenormalization theorem

AV '02

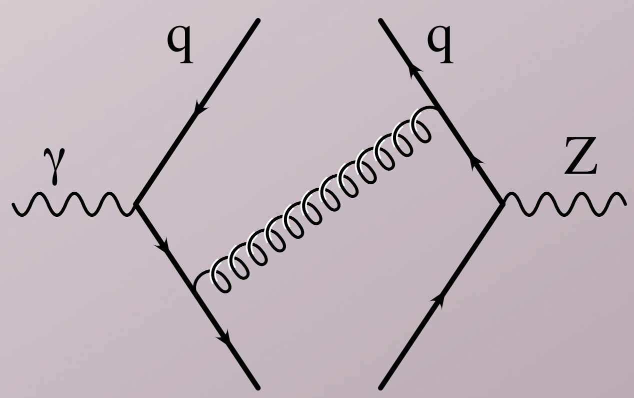
Czarnecki, Marciano, AV '02

Knecht, Peris, Perrottet, de Rafael '03



No pertubative corrections both in longitudinal and transversal parts in the chiral limit. Pole in the longitudinal part corresponds to massless pion.

But it should be no massless pole in the transversal part.
A shift from zero is provided by nonperturbative effects.
Four-fermion operators in OPE.

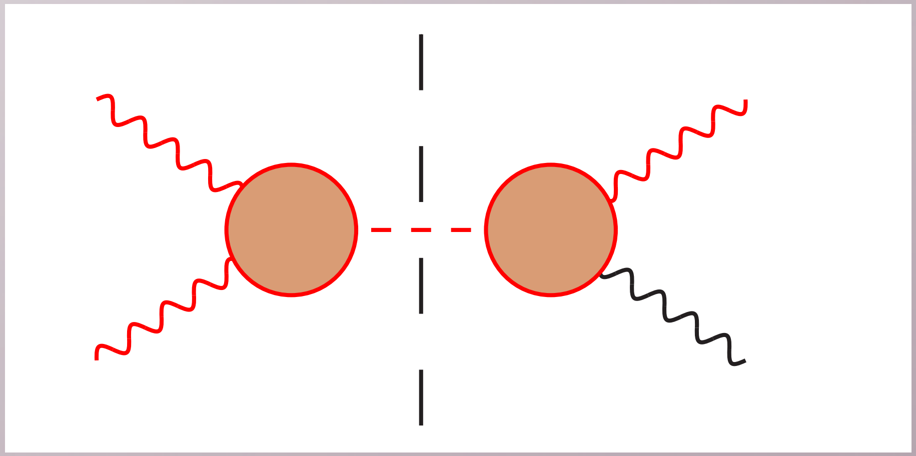


$$w_T[u,d] = \frac{1}{m_{a_1}^2 - m_\rho^2}\left[\frac{m_{a_1}^2 - m_\pi^2}{Q^2 + m_\rho^2} - \frac{m_\rho^2 - m_\pi^2}{Q^2 + m_{a_1}^2}\right]$$

Dispersive approach and pion exchange

Let me comment on the procedure of dispersion reconstruction suggested by Colangelo, Hoferichter, Procura and Stoffer.

The one-pion exchange is represented by the diagram



$$F_{\gamma^*\gamma^*\pi}(q_1^2, q_2^2)\,\frac{1}{(q_3+q_4)^2 - m_\pi^2}\,F_{\gamma^*\gamma\pi}(q_3^2, 0)$$

In application to g-2 we take the limit $q_4 \to 0$, so come to

$$F_{\gamma^*\gamma^*\pi}(q_1^2,q_2^2)\,\frac{1}{q_3^2-m_\pi^2}\,F_{\gamma^*\gamma\pi}(q_3^2,0)$$

On the other hand, if we put $q_4=0$ at the beginning and use dispersion relations in the variable q_3^2 , we come to

$$F_{\gamma^*\gamma^*\pi}(q_1^2,q_2^2)\,\frac{1}{q_3^2-m_\pi^2}\,F_{\gamma^*\gamma\pi}(m_\pi^2,0)$$

where suppression due to the transitional form factor is absent. The difference

$$\frac{F_{\gamma^*\gamma\pi}(q_3^2,0)-F_{\gamma^*\gamma\pi}(m_\pi^2,0)}{q_3^2-m_\pi^2}$$

is the smooth function near the pion pole but should be explained. I'll return to an explanation a bit later.

We can use the short distance constraints from OPE to verify the absence of the transition form factor in the vertex with the soft photon.

We discuss the isovector part of the axial current j_5^γ relevant to the pion exchange. We also implying the chiral limit $m_\pi^2 = 0$.

Then the longitudinal part of the axial current associated with pion exchange is fixed by the ABJ anomaly,

$$\partial_\gamma j_5^\gamma = C F_{\mu\nu} \tilde{F}^{\mu\nu}$$

It means that the longitudinal part of the axial current matrix element $\langle 0 | j_5^\gamma | \gamma(q_3), \gamma(q_4) \rangle$ is completely fixed at any q_3, q_4. No transition form factor present! It is just nonrenormalization of the axial anomaly.

Note, that another form factor $F_{\gamma^*\gamma^*\pi}(q_1^2, q_2^2)$ is represented by $\frac{2}{\hat{q}^2}$ factor in the OPE which is just the asymptotic of this form factor.

Now back to explanation of

$$\frac{F_{\gamma^*\gamma\pi}(q_3^2,0) - F_{\gamma^*\gamma\pi}(m_\pi^2,0)}{q_3^2 - m_\pi^2}$$

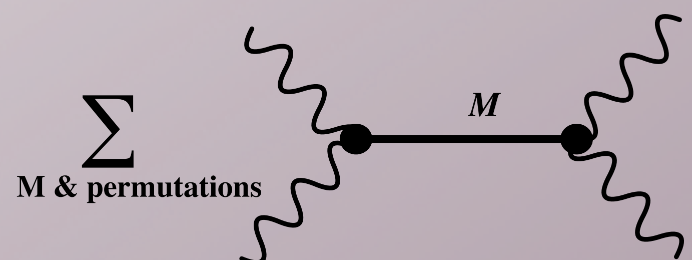
difference between different limits. The difference should be attributed to the transversal part of the axial current, i.e., it is a part of axial vector particles exchange. This, actually, was discussed long ago in our with Prades and de Rafael Glasgow white paper of 2008. In difference with the longitudinal part corrections to the transversal part of the AVV triangle do exist.

Summary for LbL

In our '08 mini-review with Prades, de Rafael we combined different calculations with some educated guesses about possible errors to come to:

$$a^{\mathrm{HLbL}} = (105 \pm 26) \times 10^{-11}$$

However the error estimates are quite subjective and further study of different exchanges is certainly needed. Experimental data on two-photon production and radiative decays can be a help.



pseudoscalar mesons $\pi^0,\ \eta,\ \eta'$; scalars $f_0,\ a_0$; vectors π_1^0; pseudovectors $a_1^0,\ f_1,\ f_1^*$

spin 2 $f_2,\ a_2,\ \eta_2,\ \pi_2$

Do we see NP in the muon g-2?

QED	$116\,584\,718.85(0.36)\times 10^{-11}$
Electroweak	$154(2)\times 10^{-11}$
Hadronic LO	$6\,901(35)(21)\times 10^{-11}$
Hadronic HO	$-\,99(1)\times 10^{-11}$
Hadronic LbL	$105(26)\times 10^{-11}$

Jegerllehner & Benayoun

Total SM	$116\,591\,779\,(52)\times 10^{-11}$	
Experimental a	$116\,592\,080\,(63)\times 10^{-11}$	
Δa	$300\,(82)\times 10^{-11}$	$3.6\,\sigma$

Both experimental and theoretical uncertainty should be reduced to be sure of NP.

Conclusions

Having in mind the new g-2 experiment in Fermilab more theoretical efforts are going on to improve accuracy for the hadronic light-by-light contribution.

It should also involve new measurements of hadronic two-photon production which provide a good test of theoretical models for HLbL.