Pion-nucleon sigma term from lattice QCD

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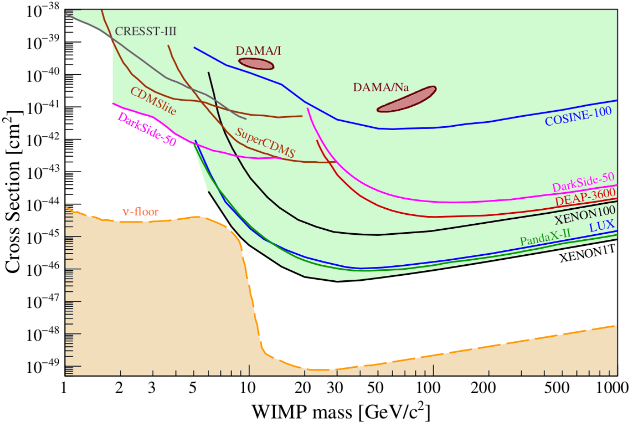
Plan

- Sigma term: motivation and status
- Lattice setup
- Excited-state contamination
- Dependence on the pion mass
- Chiral extrapolation
- Summary and Outlook

Direct detection of dark matter



M. Schumann, 1903.03026 (19)

- Weakly Interacting Massive Particles (WIMP) - a promising candidate
- WIMPs can interact with normal matter by elastic scattering with nuclei

M. W. Goodman, E. Witten, Phys. Rev. **D** 31, 3059 (85)

WIMP-nucleus elastic scattering

- Effective four-fermion interactions (velocity-independent):

$$\mathcal{L} = \underbrace{\sum_i \alpha_{3i}\ \bar{\chi}\chi\bar{q}_i q_i}_{\text{spin-independent}} + \underbrace{\sum_i \alpha_{2i}\ \bar{\chi}\gamma_\mu\gamma_5\chi\bar{q}_i\gamma^\mu\gamma_5 q_i}_{\text{spin-dependent}}$$

  - χ - WIMP dark matter field
  - q_i - quark field with flavour i ($q_1 = u$, $q_2 = d$)

- Spin-independent (SI) cross-section for WIMP-nucleus (A,Z) scattering:

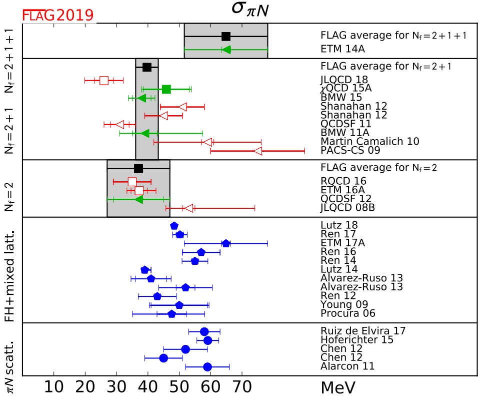
$$\sigma_{SI}^{Z,A} = \frac{4m_r^2}{\pi}\left[Zf_p + (A-Z)f_n\right]^2$$

G. Jungman, M. Kamionkowski, K. Griest, hep-ph/9506380 (96)

  - m_r - the reduced WIMP mass
  - $f_N = \sum_{q=u,d,s} \alpha_{3q}\sigma_q/m_q, \quad \sigma_q \equiv \langle N|m_q\bar{q}q|N\rangle \quad (N = p, n)$

- The SI cross-section is sensitive to the values of sigma terms σ_q

Pion-nucleon sigma term $\sigma_{\pi N}$



- Definition of $\sigma_{\pi N}$:

$$\sigma_{\pi N} = \frac{m_u + m_d}{2} \langle N | \bar{u}u + \bar{d}d | N \rangle$$

- Phenomenological value (Roy-Steiner equations): $\sigma_{\pi N} = 59.1(3.5)$ MeV

  M. Hoferichter et. al., 1506.04142 (15)

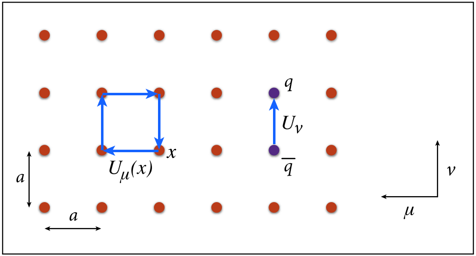
- $N_f = 2 + 1$ lattice QCD (FLAG average): $\sigma_{\pi N} = 39.7(3.6)$ MeV

  S. Aoki et. al., 1902.08191 (19)

- Most recent lattice value: $\sigma_{\pi N} = 41.6(3.8)$ MeV

  C. Alexandrou et. al., 1909.00485 (19)

Lattice-regularised QCD



- Space-time is discretized and finite $\Rightarrow$ natural UV cut-off $1/a$

  K. G. Wilson (74)
- Path integral is written in Euclidean space-time
- Numerical integration is done with Monte Carlo methods
- Physical information is extracted from correlation functions

Correlation functions

- Two- and three-point functions:

$$C_2(\vec{p}, t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}\,\Gamma_{\beta\alpha}\langle \mathcal{N}_\alpha(\vec{x}, t)\bar{\mathcal{N}}_\beta(0)\rangle$$

$$C_{3,\mathcal{O}}(\vec{p}, \vec{p}', t, t_{\rm sep}) = \sum_{\vec{x},\vec{y}} e^{i(\vec{p}'-\vec{p})\cdot\vec{y}} e^{-i\vec{p}'\cdot\vec{x}}\,\Gamma_{\beta\alpha}\langle \mathcal{N}_\alpha(\vec{x}, t_{\rm sep})\mathcal{O}(\vec{y}, t)\bar{\mathcal{N}}_\beta(0)\rangle$$

  - $\mathcal{N}_\alpha$ - nucleon interpolating field
  - $\Gamma = \frac{1}{2}(1+\gamma_0) \times \text{spin projector}$
  - $t_{\rm sep}$ - sink-source time separation
  - $\mathcal{O}(\vec{x}, t)$ - a local operator, e.g., $\mathcal{O} = \bar{q}q$

- Consider the ratio ($\vec{p} = \vec{p}' = 0$):

$$R(t, t_{\rm sep}) = \frac{C_{3,\mathcal{O}}(\vec{0}, \vec{0}, t, t_{\rm sep})}{C_2(\vec{0}, t)} \xrightarrow{t,\, t_{\rm sep}-t\to\infty} \langle N|\mathcal{O}|N\rangle + \underbrace{O\left(e^{-\Delta t}, e^{-\Delta(t_{\rm sep}-t)}\right)}_{\text{excited states}}$$

  - $\Delta \approx 2M_\pi$ - energy gap ($N\pi$ or $N\pi\pi$ state)

Lattice setup

ID	β	T/a	L/a	M_π/MeV	$M_\pi L$	t_{sep}/fm
H102	3.40	96	32	352(4)	4.93	1.0, 1.2, 1.4
H105	3.40	96	32	278(4)	3.90	1.0, 1.2, 1.4
C101	3.40	96	48	223(3)	4.68	1.0, 1.2, 1.4
N401	3.46	128	48	287(4)	5.33	1.1, 1.2, 1.4, 1.5, 1.7
N203	3.55	128	48	347(4)	5.42	1.0, 1.2, 1.3, 1.4, 1.5
N200	3.55	128	48	283(3)	4.42	1.0, 1.2, 1.3, 1.4
D200	3.55	128	64	203(3)	4.23	1.0, 1.2, 1.3, 1.4
N302	3.70	128	48	353(4)	4.28	1.0, 1.1, 1.2, 1.3, 1.4

- Coordinated Lattice Simulations (CLS) initiative
  - $N_f = 2 + 1$ flavors of $O(\alpha)$-improved Wilson fermions (clover term)
  - Lüscher-Weisz gauge action
  - CLS ensembles are along $2m_l + m_s =$const (at fixed β)

Renormalization

- Operator mixing occurs (Wilson fermions $\Rightarrow$ broken chiral symmetry)
- Consider the mass terms in the Lagrangian (continuum):

$$\mathcal{L}_m = \frac{1}{3}(2m_\ell + m_s)\underbrace{(\bar{u}u + \bar{d}d + \bar{s}s)}_{S^{(0)}} + \frac{1}{3}(m_\ell - m_s)\underbrace{(\bar{u}u + \bar{d}d - 2\bar{s}s)}_{S^{(8)}}$$

- $S^{(0)}, S^{(8)}$ - a suitable operator basis in Wilson lattice QCD
- Renormalization and improvement:

$$\sigma_{\pi N} = \frac{2\hat{m}_\ell}{3(2\hat{m}_\ell + \hat{m}_s)}\langle N|\hat{\Sigma}^{(0)}|N\rangle + \frac{\hat{m}_\ell}{3(\hat{m}_\ell - \hat{m}_s)}\langle N|\hat{\Sigma}^{(8)}|N\rangle$$

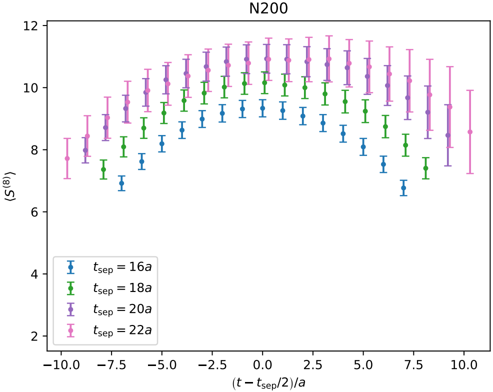
  - $\hat{m}_\ell$, $\hat{m}_s$ - PCAC quark masses (axial Ward identity)
  - The operators $\hat{\Sigma}^{(0)}, \hat{\Sigma}^{(8)}$ involve bare/lattice quantities

$$\hat{\Sigma}^{(0)} = (2m_\ell + m_s)S^{(0)} + O(a), \qquad \hat{\Sigma}^{(8)} = (m_\ell - m_s)S^{(8)} + O(a)$$

- $O(a)$ terms are $\sim 1 - 2\%$ $\rightarrow$ continuum extrapolation in $O(a^2)$

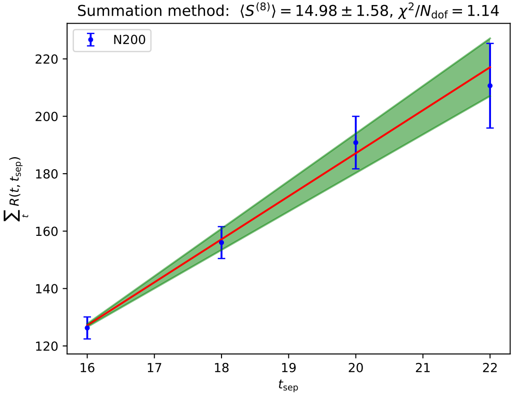
T. Bhattacharya et. al., hep-lat/0511014 (05)

The ratio $R(t, t_{\mathrm{sep}})$



- A typical plot of the ratio $R \to \langle N | S^{(8)} | N \rangle$ (N200 ensemble)
  - Signal-to-noise problem at large sink-source separations t_{sep}
  - Excited-state contamination on all ensembles
- Plateau method is not well suited $\to$ summation method

Summation method

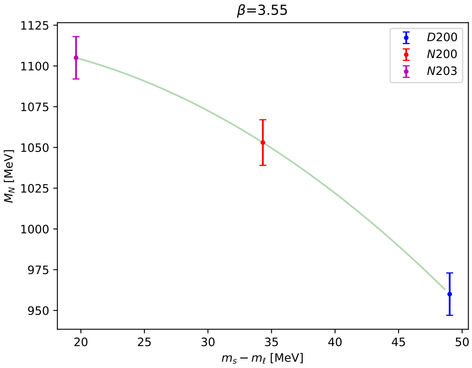


$$\sum_{t=1}^{t_{\rm sep}-1} R(t, t_{\rm sep}) = \text{const} + \langle N|\mathcal{O}|N\rangle\, t_{\rm sep} + O\left(t_{\rm sep} e^{-\Delta t_{\rm sep}}\right)$$

- Additional suppression of excited states

L. Maiani et. al., Nucl. Phys. **B** 293, 420 (87)

Excited-state contamination



- We can estimate the impact of excited states
- Feynman-Hellmann theorem:

$$\langle N|S^8|N\rangle = -3 \left.\frac{\partial M_N}{\partial(m_s - m_\ell)}\right|_{N200}$$

- Fit function: $f(x) = A + Bx + Cx^2$

- Lagrange interpolating polynomial: $f(x) = \sum_{i=1}^{3} C_i P_i(x)$
- Mean value:

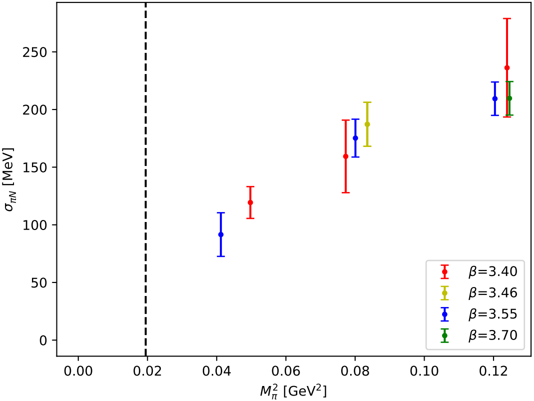
$$\langle S^8 \rangle_{\mathrm{mean}} = -3 \sum_{i=1}^{3} C_i P_i'(x_2), \qquad x_2 \to \mathrm{N200}$$

- Standard deviation σ:

$$\sigma^2 = 9 \sum_{i=1}^{3} \sigma_i^2 [P_i'(x_2)]^2$$

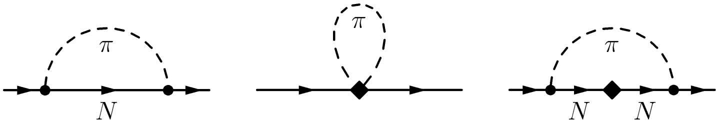
- The value of $\langle S^8 \rangle$:
  - Feynman-Hellmann method: $\langle S^8 \rangle = 14.80 \pm 1.88$
  - Summation method: $\langle S^8 \rangle = 14.98 \pm 1.58$
- Excited-state effects are under control

The values of $\sigma_{\pi N}$ (PRELIMINARY)



- Pion-nucleon sigma term on CLS ensembles ($M_\pi = 203 - 353$ MeV)
- Disconnected contributions are included
- Vertical dashed line $\rightarrow M_\pi^{\rm phys} = 139.57$ MeV

Chiral extrapolation



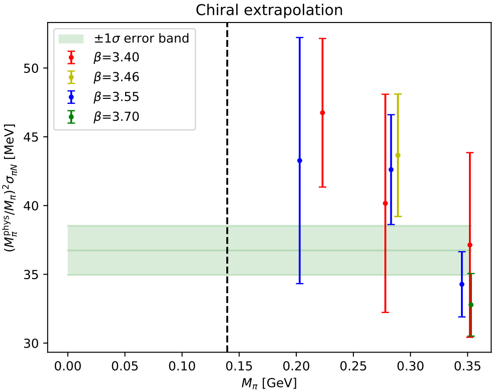
- Chiral expansion of $\sigma_{\pi N}$:

$$\sigma_{\pi N}(M_\pi) = M_\pi^2 \frac{\partial M_N}{\partial M_\pi^2} = -4c_1 M_\pi^2 + \frac{9 g_A^2 M_\pi^3}{64\pi F_\pi^2} + O\left(M_\pi^4 \ln\left(\frac{M_\pi}{\lambda}\right)\right)$$

  - $\lambda \sim m_N$ - renormalization scale
  - $c_1 = -1.11(3)$ GeV^{-1} (phenomenological value)

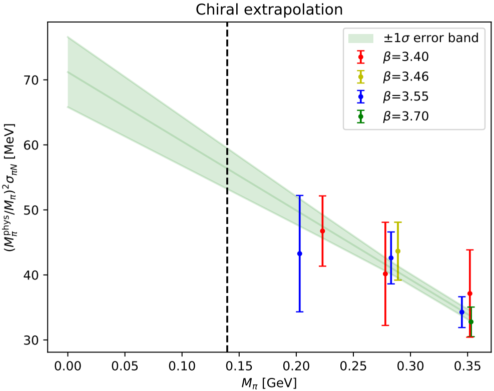
- The prediction of ChPT provides a strong constraint: $\sigma_{\pi N}(0) \equiv 0$

Fit Model I



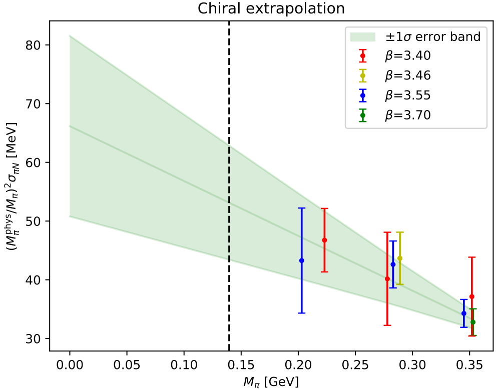
- The plot shows the combination $\frac{(M_\pi^{\rm phys})^2}{M_\pi^2}\sigma_{\pi N}$
- Fit function: $f(M_\pi) = \mathrm{const}$
  - $\sigma_{\pi N} = 36.7(1.8)$ MeV, $\chi^2/N_{\rm dof} = 1.83$

Fit Model II



- Fit function: $f(M_\pi) = A + B\,M_\pi$
  - $\sigma_{\pi N} = 56.3(3.1)$ MeV, $\chi^2/N_{\rm dof} = 0.27$
  - $c_1 = -0.91(10)$ GeV^{-1}

Finite-size effects



- Finite-volume corrections ($\sim e^{-M_\pi L}$, neglected)
- Cut-off effects ($a \neq 0$)
- Fit function (for $\sigma_{\pi N}$): $f(M_\pi) = A\,M_\pi^2 + B\,M_\pi^3 + C\,a^2$
  - $\sigma_{\pi N} = 53.1(9.7)$ MeV, $\chi^2/N_{\text{dof}} = 0.31$, $c_1 = -0.85(20)$ GeV^{-1}

Summary and Outlook

- Direct lattice calculation of $\sigma_{\pi N}$ on $N_f = 2 + 1$ CLS ensembles
- Disconnected contributions were included
- Preliminary analysis gives a range $\sigma_{\pi N} \sim 35 - 60$ MeV

- More statistics on D200 ($M_\pi \approx 200$ MeV)
- A new ensemble E250 at physical M_π
- Alternative test: πN scattering lengths (from phase shifts)