GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

The CKM Unitarity Problem: A Trace of New Physics at the TeV Scale?

Benedetta Belfatto

B. B., R. Beradze and Z. Berezhiani, arXiv:1906.02714 [hep-ph]

B. B. and Z. Berezhiani, Eur. Phys. J. C 79, no. 3, 202 (2019)

STANDARD MODEL

- Three families of fermions, left-handed particles $q_{Li} = (u_L, d_L)_i$, $l_{Li} = (\nu_L, e_L)_i$ doublets of $SU(2) \times U(1)$, right-handed components singlets of $SU(2)$.
- Weak eigenstates are not mass eigenstates;
- fermion mass matrices $m_{ij}^{(f)} = Y_{ij}^{f} v_{\rm EW}$ can be diagonalized $V_L^{(f)\dagger} m^{(f)} V_R^{(f)} = m_{\rm diag}^{(f)}$ ($v_{\rm EW} = 174$ GeV);
- The Lagrangian for the charged current interaction is:

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \left(\begin{array}{ccc} \overline{u_L} & \overline{c_L} & \overline{t_L} \end{array} \right) \gamma^\mu \mathbf{V}_{\rm CKM} \left(\begin{array}{c} d \\ s \\ b \end{array} \right)_L W_\mu^+ + h.c.$$

- $V_{CKM} = V_L^{(u)\dagger} V_L^{(d)} = \left(\begin{array}{ccc} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{array} \right)$ is unitary;
- unitarity implies $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$.

CKM UNITARITY PROBLEM

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

CKM UNITARITY PROBLEM

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Determination of V_{us} and the ratio V_{us}/V_{ud} from:

$$\text{A}: \quad f_+(0)|V_{us}| = 0.21654(41) \qquad \text{B}: \quad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.27599(38)$$

- $|V_{us}| = 0.2238(8)$, $\left|\frac{V_{us}}{V_{ud}}\right| = 0.2315(10)$ (PDG 2018);
- New determinations of the ratio for kaon and pion decay constant $f_{K^\pm}/f_{\pi^\pm}$ (FLAG 2019) and of the form factor relevant for semileptonic decay $f_+(0)$ (Fermilab Lattice and MILC);
- $\mathbf{|V_{us}| = 0.22333(60)}$, $\mathbf{\left|\frac{V_{us}}{V_{ud}}\right| = 0.23130(50)}$

CKM UNITARITY PROBLEM

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

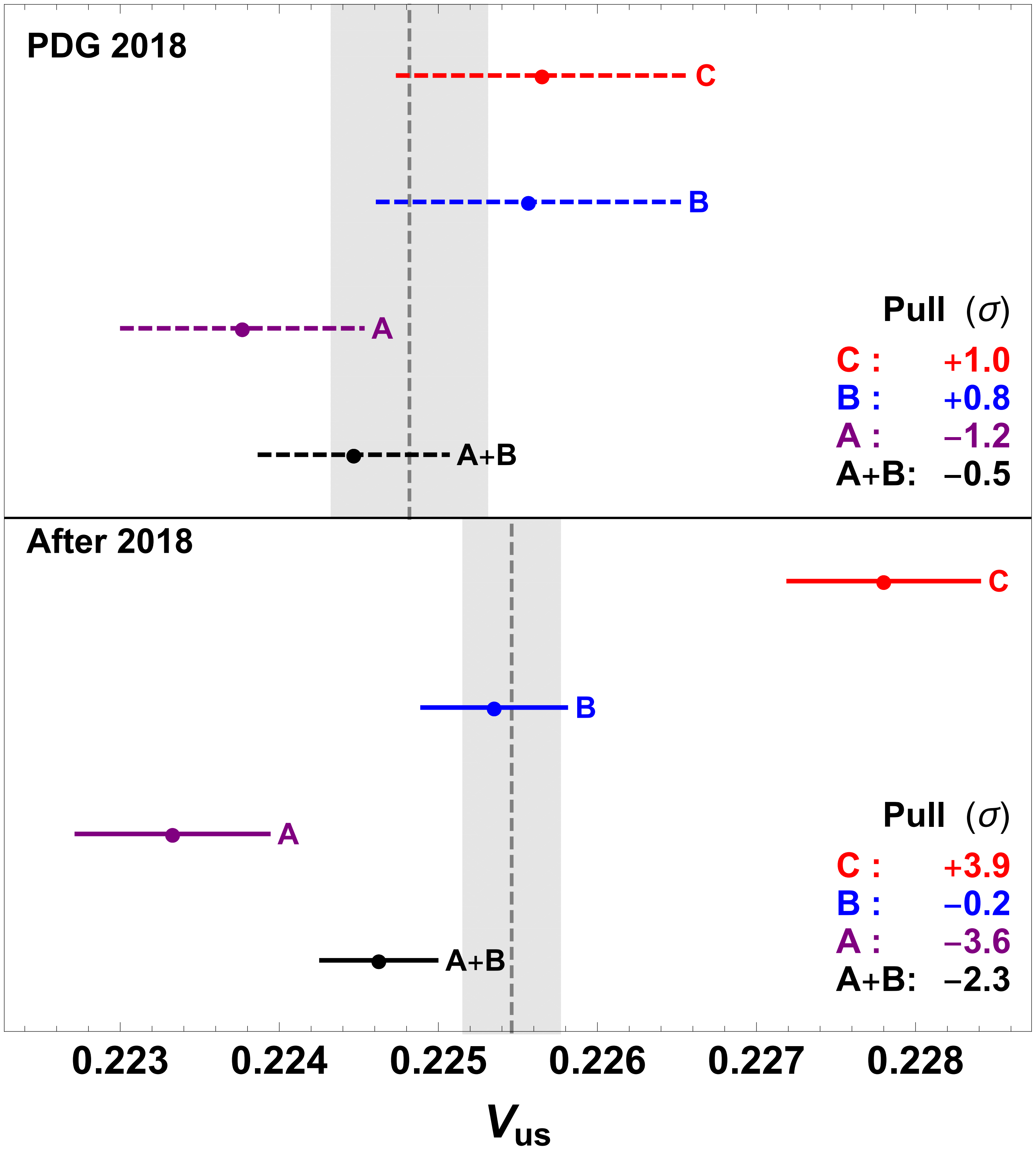
- Superallowed nuclear beta-decays ($0^+ \to 0^+$ Fermi transitions) determine:

$$\mathrm{C}: \quad G_V = G_F |V_{ud}|$$

- within the SM $\mathbf{G_F = G_\mu}$;
- $V_{ud} = 0.97420 \pm 0.00021$ (PDG2018);
- New computation of radiative corrections with reduced hadronic uncertainty (Seng et al. PRL 2018):

$$|\mathbf{V_{ud}}| = \frac{\mathbf{G_V^{exp}}}{\mathbf{G_F(= G_\mu)}} = \mathbf{0.97366(15)}$$

CKM UNITARITY PROBLEM

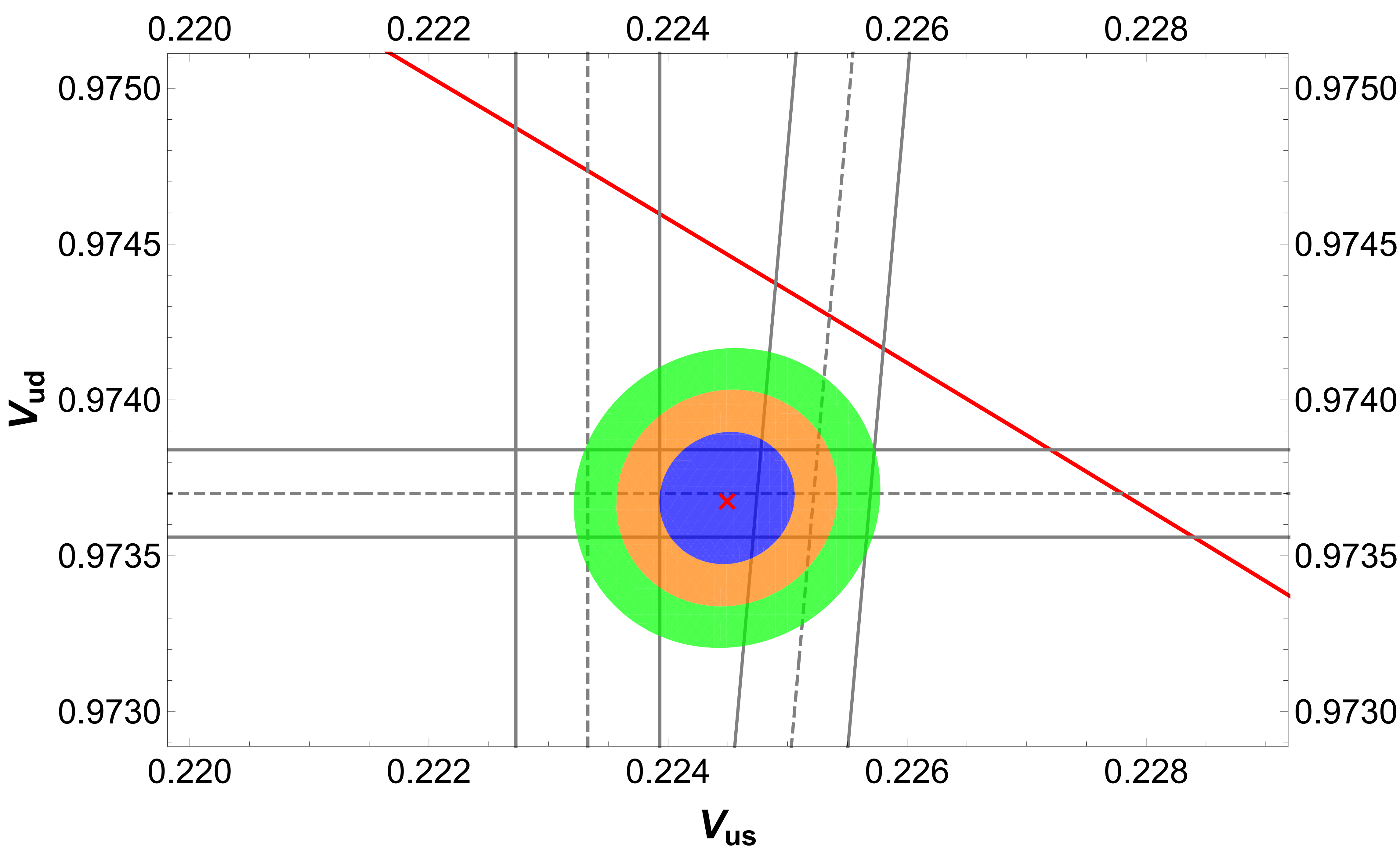


$$\chi^2_{dof} = 1.7$$

$$\rightarrow \quad \chi^2_{dof} = 13.9$$

Discrepancy between average $\overline{A+B}$ and C is 4.5σ.

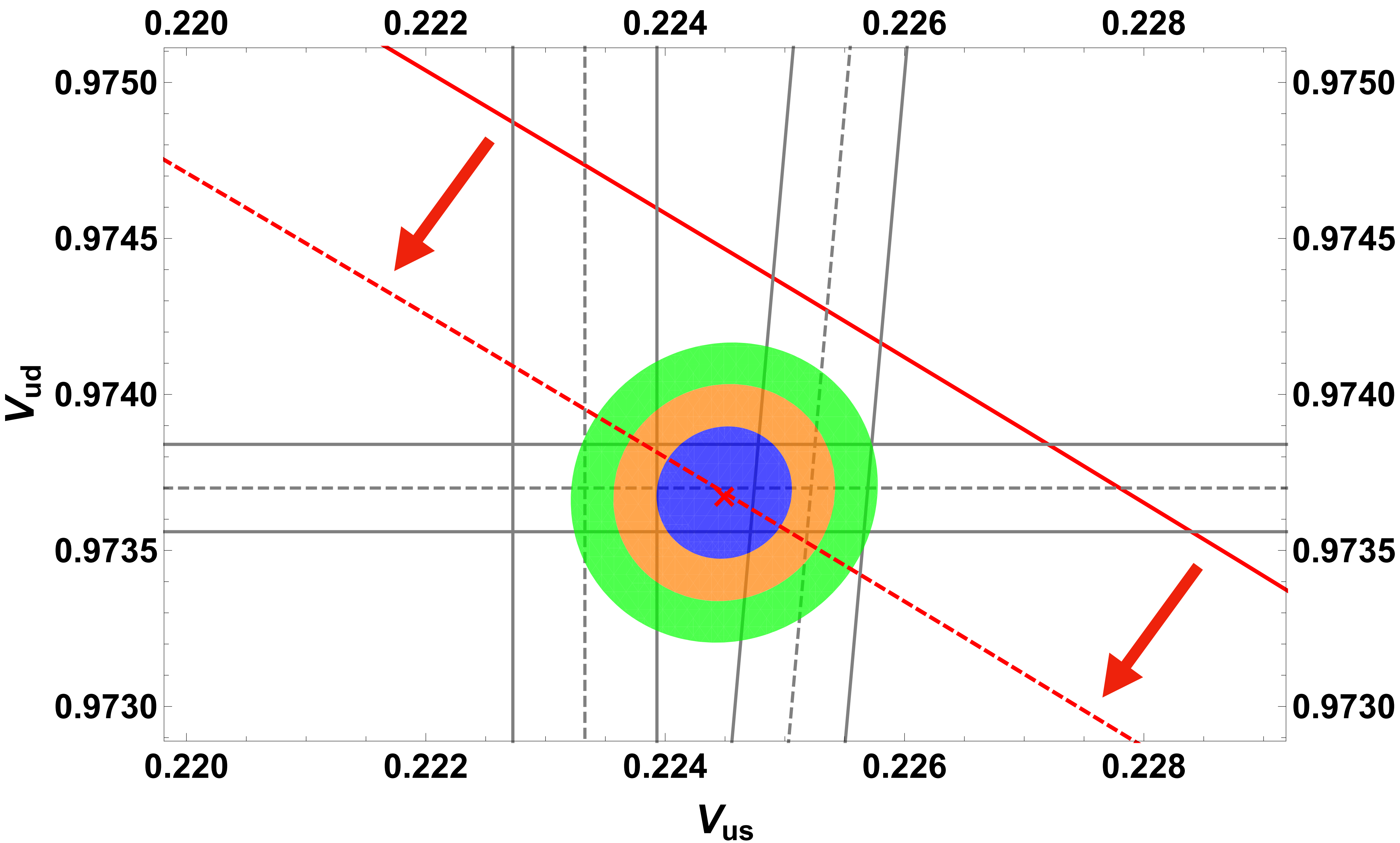
CKM UNITARITY PROBLEM



$4.3\,\sigma$ far from unitarity curve (excluded at 99.998% C.L.)

SOLUTION #1

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{ub'}|^2\,, \qquad |V_{ub'}| = 0.04$$



SOLUTION #1

- Forth vector-like quark $d_{4L,R}$ whose left and right components are both $SU(2)$ singlets involved in quark mixing:

$$... + h_i \phi \overline{q_{Li}} d_{4R} + M \overline{d_{4L}} b_{4R} + h.c.$$

- $\overline{d_{Li}} \mathbf{m}^{(d)}_{ij} d_{Rj} = (\overline{d_{1L}}, \overline{d_{2L}}, \overline{d_{3L}}, \overline{d_{4L}}) \left(\begin{array}{ccc|c} & & & h_d v \\ & \mathbf{m}^{(d)}_{3\times 3} & & h_s v \\ & & & h_b v \\ \hline 0 & 0 & 0 & M \end{array} \right) \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix}_R$

- $\tilde{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \end{pmatrix} = V_L^{(u)\dagger} \tilde{V}_L^{(d)}$;

- $\tilde{V}_L^{(d)}$ is the 3×4 submatrix of $V_L^{(d)}$, $V_L^{(d)\dagger} \mathbf{m}^{(d)} V_R^{(d)} = \mathbf{m}^{(d)}_{\text{diag}}$.

- Since $V_{ub'} \simeq h_d v_w / M$, assuming $|V_{ub'}| > 0.03$ (95% C.L.) and $h_d < 1$, then $M < 6$ TeV.

SOLUTION #1

- The forth quark has tree level flavor-changing couplings with the Higgs boson and with Z-boson. So for down quarks:

$$\mathcal{L}_{\rm nc} = -\frac{1}{2}\frac{g}{\cos\theta_W} Z_\mu \begin{pmatrix} \overline{d_L} & \overline{s_L} & \overline{b_L} & \overline{b'_L} \end{pmatrix} \gamma^\mu \tilde{V}_L^{(d)\dagger} \tilde{V}_L^{(d)} \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix} + \text{diagonal}$$

Elements	Constraint	Process	$\lvert V_{ub'}\rvert = 0.04$
$\lvert V_{ub'}V^*_{cb'}\rvert$	$< 5\cdot 10^{-5}$	$K^+ \to \pi^+\nu\bar{\nu}$	$\lvert V_{cb'}\rvert < 0.0013$
$\lvert {\rm Im}V_{ub'}V^*_{cb'}\rvert$	$< 8\cdot 10^{-6}$	$K_S \to \mu^+\mu^-$	
$\lvert {\rm Re}V_{ub'}V^*_{cb'}\rvert$	$< 1.5\cdot 10^{-5}$	$K_L \to \mu^+\mu^-$	
$\lvert V_{ub'}V^*_{tb'}\rvert$	$< 4\cdot 10^{-4}$	$B^+ \to \pi^+\ell^+\ell^-$	$\lvert V_{tb'}\rvert < 0.01$
$\lvert {\rm Re}V_{ub'}V^*_{tb'}\rvert$	< 0.0001	$B \to \mu^+\mu^-$	
$\lvert V_{cb'}V^*_{tb'}\rvert$	< 0.002	$B^0 \to X_s\mu^+\mu^-$	
$\lvert {\rm Re}V_{cb'}V^*_{tb'}\rvert$	< 0.0006	$B^0_s \to \mu^+\mu^-$	

SOLUTION #1

- Forth vector-like up-type quark $u_{4L,R}$ whose left and right components are both $SU(2)$ singlets involved in quark mixing:

$$\ldots + h_i \tilde{\phi} \overline{q_{Li}} u_{4R} + M_u \overline{u_{4L}} u_{4R} + h.c.$$

- $$\tilde{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{t'd} & V_{t's} & V_{t'b} \end{pmatrix} = \tilde{V}_L^{(u)\dagger} V_L^{(d)} \;;$$

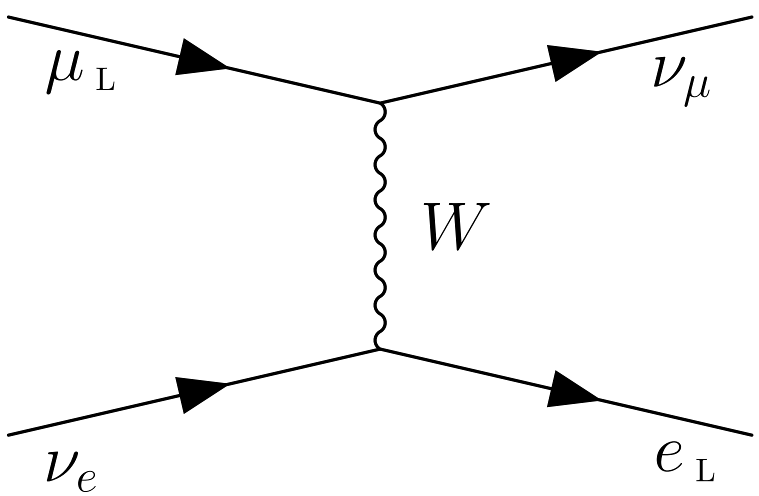
- $\tilde{V}_L^{(u)}$ is the 3×4 submatrix of $V_L^{(u)}$.

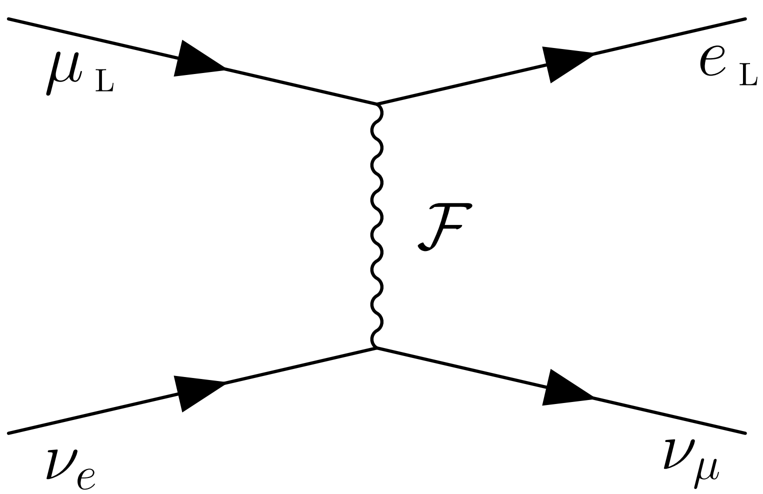
Elements	Constraint	Process	$\lvert V_{t'd}\rvert = 0.04$
$\lvert V^*_{t'd} V_{t's}\rvert$	0.00012	D^0 mixing	$\lvert V_{t's}\rvert < 0.003$
$\lvert \mathrm{Re} V^*_{t'd} V_{t's}\rvert$	0.003	$D^0 \to \mu^+\mu^-$	
$\lvert V^*_{t'b} V_{t'd}\rvert$	0.002	B^0 mixing	$\lvert V_{t'b}\rvert < 0.05$
$\lvert V^*_{t'b} V_{t's}\rvert$	0.01	B^0_s mixing	

SOLUTION #2

- Suppose the existence of **flavor changing bosons**.

$$-\frac{4G_{\rm F}}{\sqrt{2}}(\overline{\nu_\mu}\gamma^\alpha\mu_L)(\overline{e_L}\gamma_\alpha\nu_e) \qquad -\frac{4G_{\rm H}}{\sqrt{2}}(\overline{e_L}\gamma^\alpha\mu_L)(\overline{\nu_\mu}\gamma_\alpha\nu_e)$$





- Horizontal interactions have positive interference with SM;
- After Fierz transformation, the sum of the diagrams gives the operator:

$$-\frac{4G_\mu}{\sqrt{2}}(\overline{\nu_\mu}\gamma^\alpha\mu_L)(\overline{e_L}\gamma_\alpha\nu_e)$$

- $G_\mu = G_F + G_H = G_F(1+\delta_\mu)$ $\boxed{G_\mu \neq G_F}$

SOLUTION #2

- Different $\mathbf{G}_\mu = \mathbf{G_F} + \mathbf{G_H} = \mathbf{G_F}(\mathbf{1} + \delta_\mu) = \mathbf{1} + \mathbf{v_w^2}/\mathbf{v_\ell^2}$
- The values of $V_{\rm us}$, $V_{\rm ud}$ (and corresponding errorbars) should be rescaled:

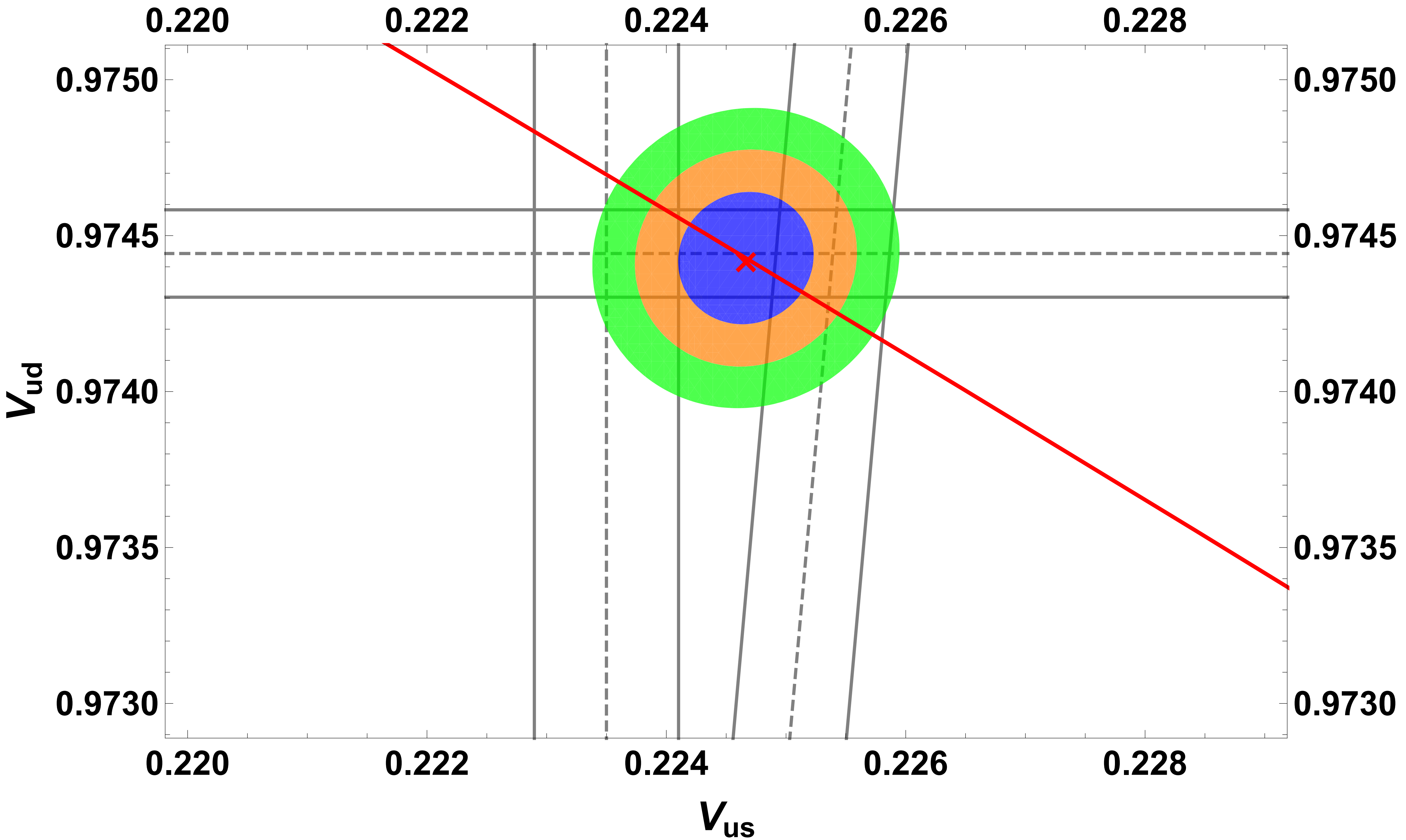
$$|V_{\rm us}| = 0.22333(60) \times (1+\delta_\mu)\,, \qquad |V_{\rm ud}| = 0.97370(14) \times (1+\delta_\mu)$$

  while the ratio is not affected.
- Unitarity recovered: $\left(\frac{G_F}{G_\mu}\right)^2 |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \frac{2G_{\rm H}}{G_{\rm F}}$
- After performing the fit of $V_{\rm us}$, $V_{\rm ud}$ and their ratio with two parameters $V_{\rm us}$ and δ_μ by imposing unitarity, the minimum $\chi^2_{\mathbf{dof}} = \mathbf{3.0}$ is found for $\delta_\mu = \mathbf{0.00076}$, or

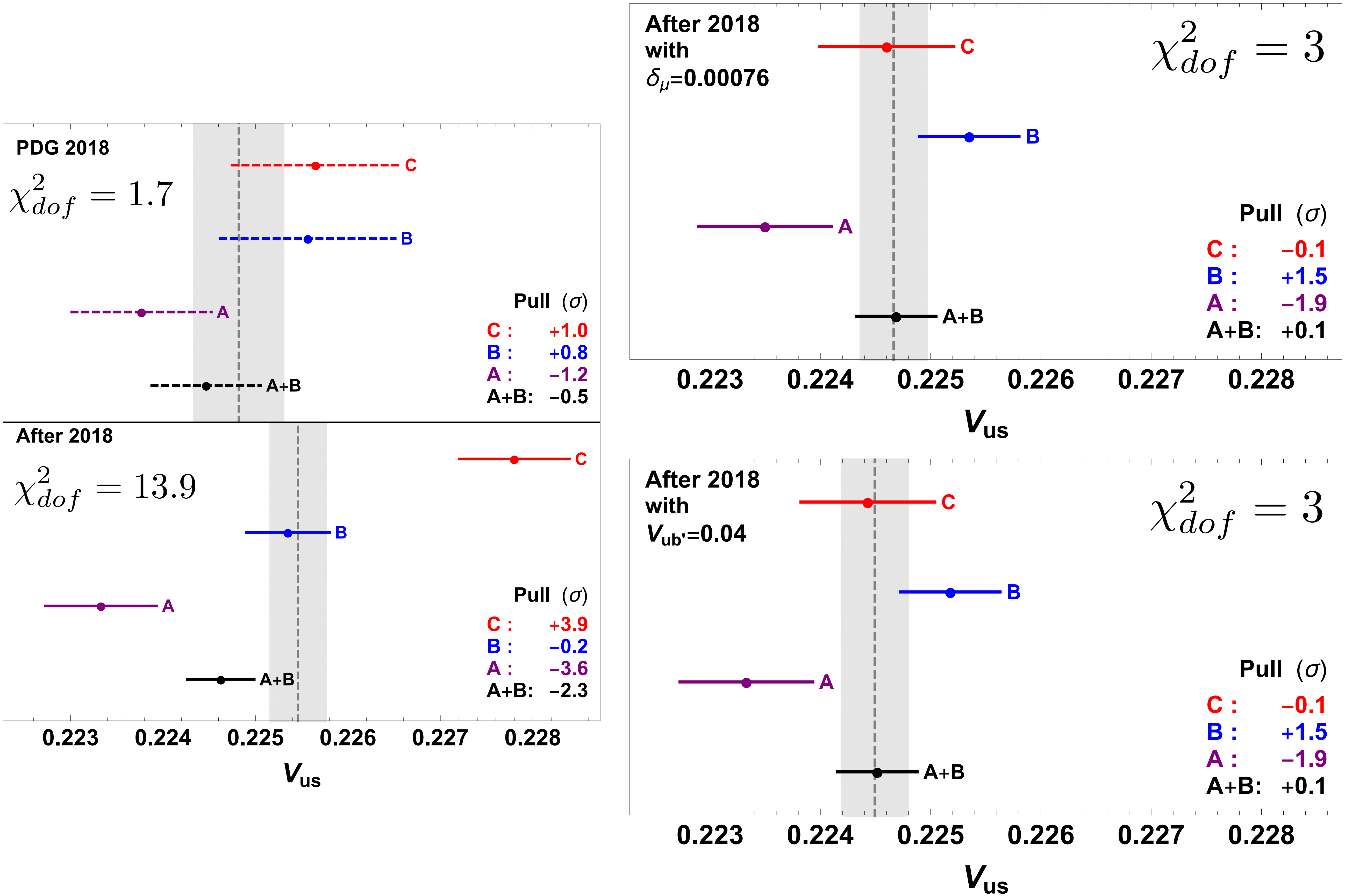
$$\boxed{v_\ell = 6.3\ \text{TeV}}$$

SOLUTION #2

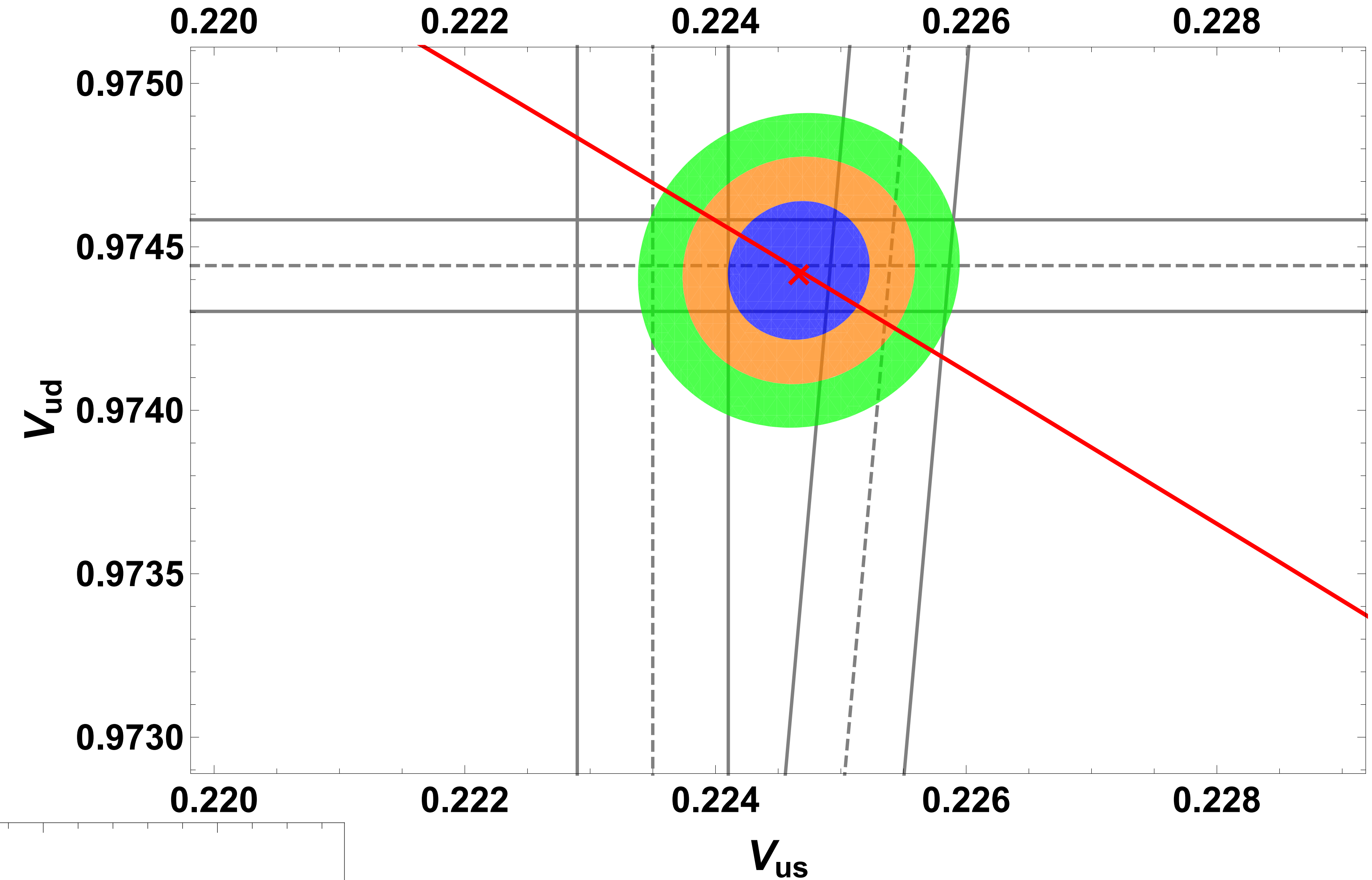
$$\delta_\mu = 0.000\,76 \quad \rightarrow \quad v_\ell = 6.3\ \text{TeV}$$



CKM UNITARITY

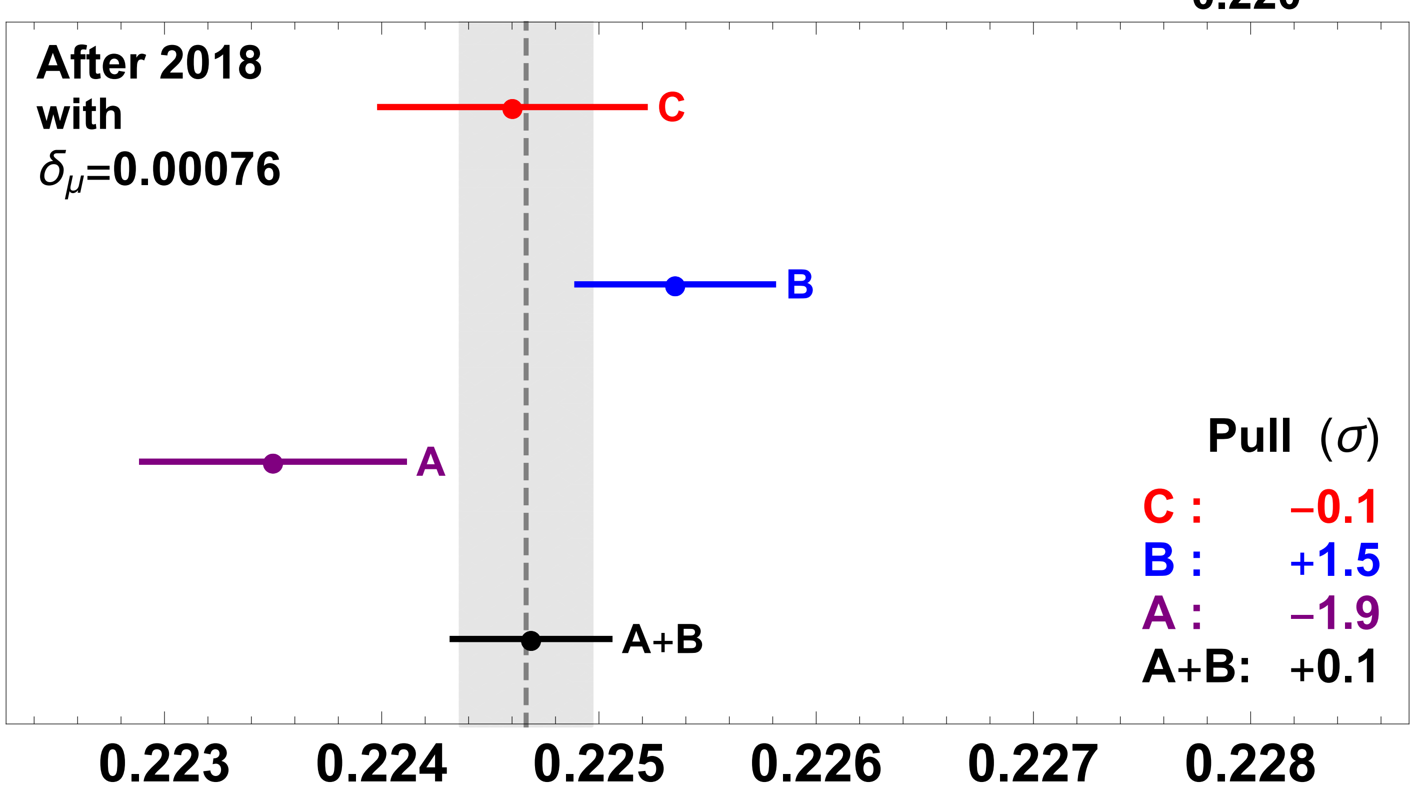


MORE ABOUT #2



$$v_\ell = 6.3\ \mathrm{TeV}$$

$$\chi^2_{dof} = 3$$



STANDARD MODEL

- Weak eigenstates are not mass eigenstates;
- fermion mass matrices

$$m_{ij}^{(f)} = Y_{ij}^{f} v_{\rm EW}$$

  $v_{\rm EW} = 174$ GeV, can be diagonalized $V_L^{(f)\dagger} m^{(f)} V_R^{(f)} = m_{\rm diag}^{(f)}$;
- all masses proportional to Higgs VEV;
- fermion mixing in charged currents is

$$V_{\rm CKM} = V_L^{(u)\dagger} V_L^{(d)} \qquad U_{\rm PMNS} = V_L^{(\nu)\dagger} V_L^{(e)};$$

- Yukawa couplings, and photon/Z couplings ($V^\dagger V = 1$), are diagonal in mass basis: **no flavour changing neutral currents at tree level**;
- all flavour changing and CP-violation is originated from loop diagrams;
- no mixing in the right particles sector (unless right W bosons exist).

MORE ABOUT #2

Something not explained in the SM:

- Replication of fermion **families**;
- inter-family **mass hierarchy** (**Yukawa hierarchy**);
- weak **mixing pattern**: small angles for quarks, large angles for neutrinos;
- neutrino masses: very small (seesaw?), mass hierarchy yet unknown.

Hierarchy between quarks and CKM angles parametrized by $\epsilon \sim 1/20$:

$$m_d : m_s : m_b \sim \epsilon^2 : \epsilon : 1 \qquad\qquad m_u : m_c : m_t \sim \epsilon^4 : \epsilon^2 : 1$$

$$\sin\theta^q_{12} \sim \sqrt{\epsilon} \sim 4\epsilon; \quad \sin\theta^q_{23} \sim \epsilon; \quad \sin\theta^q_{13} \sim \epsilon^2$$

Hierarchy between charged leptons parametrized by same $\epsilon \sim 1/20$:

$$m_e : m_\mu : m_\tau \sim k^{-1}\epsilon^2 : k\epsilon : k$$

$k \simeq 3$ (factor $O(1)$).

Technically natural: SM tolerates Yukawa hierarchy but cannot explain it.

STANDARD MODEL $\otimes$ FAMILY SYMMETRY

- A gauge **family symmetry** can be introduced and the **mass hierarchy** between families can be related to the **hierarchy of the symmetry breaking**.

- Family symmetry should not allow fermions to have mass until this symmetry is spontaneously broken. Left-handed and right-handed particles transform in different representations.

In grand unification SU(5):

$$U(3)^2 = U(3)_l \times U(3)_e$$

$$\bar{5}_L = (\ell, d^c)_L \sim (3_\ell, 1), \qquad 10_L = (e^c, u^c, q)_L \sim (1, 3_e)$$

Maximal family symmetry in the SM:

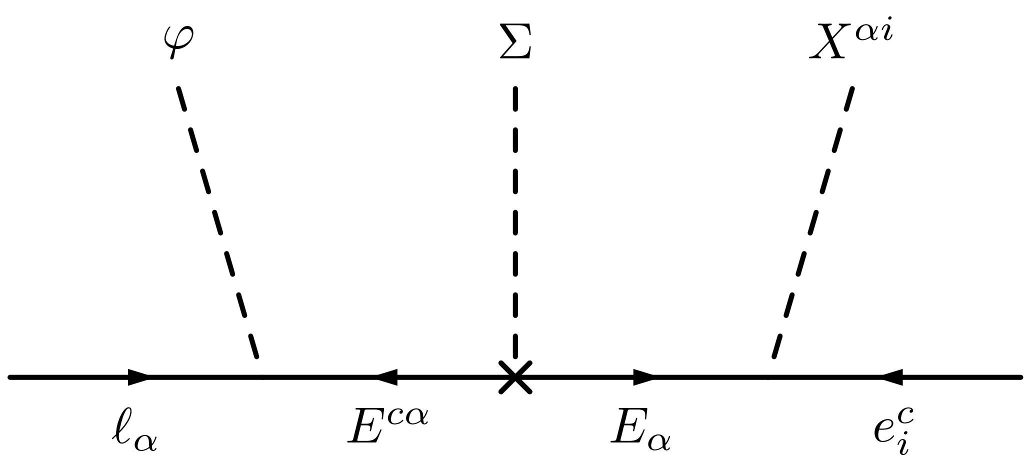
$$U(3)_q \times U(3)_u \times U(3)_d \times U(3)_l \times U(3)_e$$

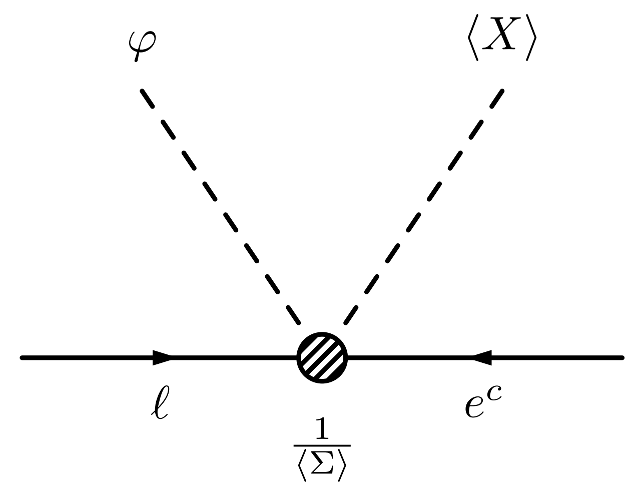
$$q_{Li} \sim 3_q \qquad u_R^j \sim \bar{3}_u \qquad d_R^\alpha \sim \bar{3}_d \qquad l_{L\beta} \sim 3_l \qquad e_R^k \sim \bar{3}_e$$

$SU(3)_f$ gauged, U(1) global

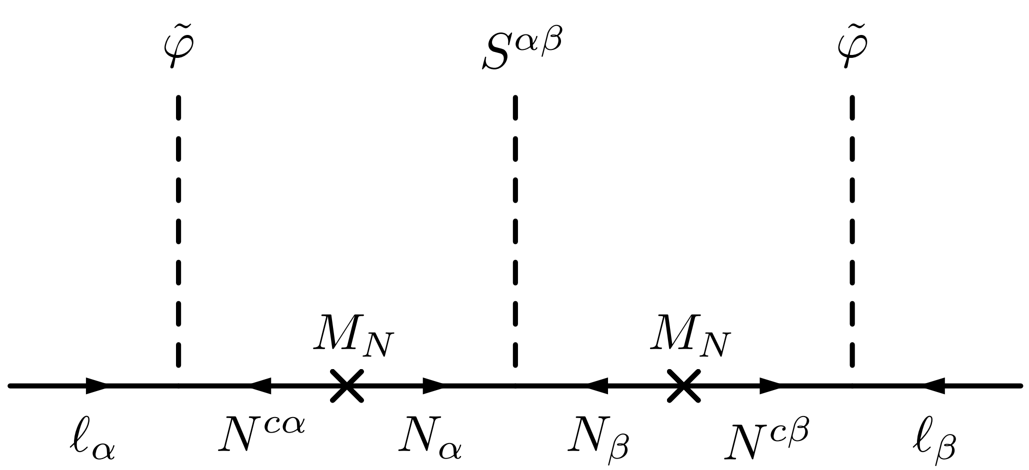
EFFECTIVE OPERATORS FOR FERMION MASSES

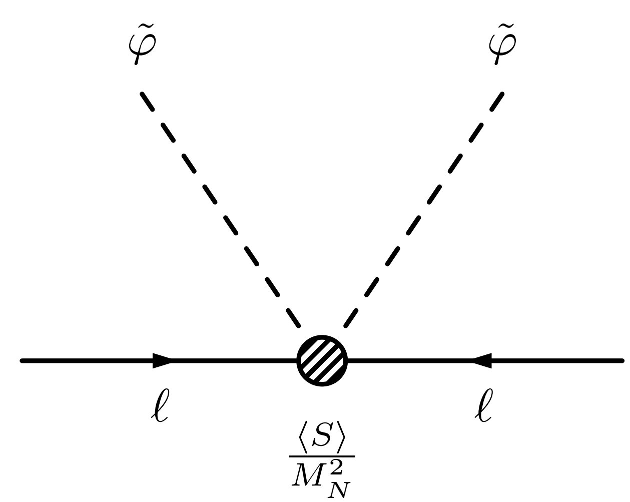
$$X_e^{\beta k} \sim (\bar{3}_l, \bar{3}_e) \qquad X_d^{i\alpha} \sim (\bar{3}_q, \bar{3}_d) \qquad X_u^{ij} \sim (\bar{3}_q, \bar{3}_u)$$





$$S_{\alpha\beta} \sim 6_l$$





SSB & FERMION MASSES

Effective Operators for Fermion Masses

$$\frac{\xi^i \xi^j}{M^2}\bar{\phi}\bar{u}_j q_i + \frac{\xi^i \eta^\alpha}{M^2}\phi \bar{d}_\alpha q_i + \frac{\xi^k \eta^\beta}{M^2}\phi \bar{e}_k l_\beta + \frac{\eta^\alpha \eta^\beta}{M^3}\bar{\phi}\bar{\phi} l_\alpha l_\beta + h.c.$$

In order to have all fermions massive:

$\eta^\alpha_{(n)} \sim \bar{3}_l$ $\quad SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2} \text{nothing}$ $\quad u_3 : u_2 : u_1 = \epsilon_L^2 : \epsilon_L : 1$ $\quad \epsilon_L \simeq \frac{1}{3}$

$n = 1, 2, 3$

$\xi^i_{(n)} \sim \bar{3}_e$ $\quad SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing}$ $\quad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1$ $\quad \epsilon = \frac{1}{20}$

$n = 1, 2, 3$

Then mass ratios, $V_{\rm CKM}$, U_{PMNS} are obtained.

SSB & FERMION MASSES

Effective Operators for Fermion Masses

$$\frac{\xi^i \xi^j}{M^2}\bar{\phi}\bar{u}_j q_i + \frac{\xi^i \eta^\alpha}{M^2}\phi \bar{d}_\alpha q_i + \frac{\xi^k \eta^\beta}{M^2}\phi \bar{e}_k l_\beta + \frac{\eta^\alpha \eta^\beta}{M^3}\bar{\phi}\bar{\phi} l_\alpha l_\beta + h.c.$$

$$SU(3)_l \times SU(3)_e$$

In order to have all fermions massive:

$$\eta^\alpha_{(n)} \sim \bar{3}_l \qquad SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2} \text{nothing} \qquad u_3 : u_2 : u_1 = \epsilon_L^2 : \epsilon_L : 1 \qquad \epsilon_L \simeq \frac{1}{3}$$

$n = 1, 2, 3$

$$\xi^i_{(n)} \sim \bar{3}_e \qquad SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

$n = 1, 2, 3$

$$m^{(e)} \sim \begin{pmatrix} O(\epsilon^2 \epsilon_{\mathrm{L}}^2) & O(\epsilon \epsilon_{\mathrm{L}}^2) & O(\epsilon_{\mathrm{L}}^2) \\ O(\epsilon^2 \epsilon_{\mathrm{L}}) & O(\epsilon \epsilon_{\mathrm{L}}) & O(\epsilon_{\mathrm{L}}) \\ O(\epsilon^2) & O(\epsilon) & O(1) \end{pmatrix} \cdot \frac{v_3 u_3}{M^2} v_{EW}$$

Then mass ratios, V_{CKM}, U_{PMNS} are obtained.

SSB & FERMION MASSES

Effective Operators for Fermion Masses

$$\frac{\xi^i\xi^j}{M^2}\bar{\phi}\bar{u}_j q_i + \frac{\xi^i\eta^\alpha}{M^2}\phi\bar{d}_\alpha q_i + \frac{\xi^k\eta^\beta}{M^2}\phi\bar{e}_k l_\beta + \frac{\eta^\alpha\eta^\beta}{M^3}\bar{\phi}\bar{\phi}l_\alpha l_\beta + h.c.$$

$$SU(3)_l \times SU(3)_e$$

In order to have all fermions massive:

$$\epsilon_L \simeq \frac{1}{3}$$

$$\eta^\alpha_{(n)} \sim \bar{3}_l \qquad SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2} \text{nothing} \qquad u_3 : u_2 : u_1 = \epsilon_L^2 : \epsilon_L : 1$$

$n = 1, 2, 3$

$$\epsilon = \frac{1}{20}$$

$$\xi^i_{(n)} \sim \bar{3}_e \qquad SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1$$

$n = 1, 2, 3$

$$m^{(e)} \sim \begin{pmatrix} O(\epsilon^2\epsilon_{\mathrm{L}}^2) & O(\epsilon\epsilon_{\mathrm{L}}^2) & O(\epsilon_{\mathrm{L}}^2) \\ O(\epsilon^2\epsilon_{\mathrm{L}}) & O(\epsilon\epsilon_{\mathrm{L}}) & O(\epsilon_{\mathrm{L}}) \\ O(\epsilon^2) & O(\epsilon) & O(1) \end{pmatrix} \cdot \frac{v_3 u_3}{M^2} v_{EW}$$

Then mass ratios, V_{CKM}, U_{PMNS} are obtained.

GAUGE BOSON MASSES

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

$$\frac{1}{2}\begin{pmatrix} \theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_1 - i\theta_2 & \theta_4 - i\theta_5 \\ \theta_1 + i\theta_2 & -\theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_6 - i\theta_7 \\ \theta_4 + i\theta_5 & \theta_6 + i\theta_7 & -\frac{2}{\sqrt{3}}\theta_8 \end{pmatrix}^{(e)}$$

GAUGE BOSON MASSES

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

$$\frac{1}{2}\begin{pmatrix} \theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_1 - i\theta_2 & \theta_4 - i\theta_5 \\ \theta_1 + i\theta_2 & -\theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_6 - i\theta_7 \\ \theta_4 + i\theta_5 & \theta_6 + i\theta_7 & -\frac{2}{\sqrt{3}}\theta_8 \end{pmatrix}^{(e)}$$

1) $$M^2_{\theta_{4,5,6,7}} = \frac{g^2 v_3^2}{2}$$

GAUGE BOSON MASSES

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

$$\frac{1}{2}\begin{pmatrix} \theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_1 - i\theta_2 & \theta_4 - i\theta_5 \\ \theta_1 + i\theta_2 & -\theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_6 - i\theta_7 \\ \theta_4 + i\theta_5 & \theta_6 + i\theta_7 & -\frac{2}{\sqrt{3}}\theta_8 \end{pmatrix}^{(e)}$$

1) $$M^2_{\theta_{4,5,6,7}} = \frac{g^2 v_3^2}{2}$$

2) $$m^2_{\theta_{1,2}} = \frac{g^2 v_2^2}{2}$$

GAUGE BOSON MASSES

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

$$\frac{1}{2}\begin{pmatrix} \theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_1 - i\theta_2 & \theta_4 - i\theta_5 \\ \theta_1 + i\theta_2 & -\theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_6 - i\theta_7 \\ \theta_4 + i\theta_5 & \theta_6 + i\theta_7 & -\frac{2}{\sqrt{3}}\theta_8 \end{pmatrix}^{(e)}$$

1) $$M^2_{\theta_{4,5,6,7}} = \frac{g^2 v_3^2}{2}$$

2) $$m^2_{\theta_{1,2}} = \frac{g^2 v_2^2}{2}$$

$$M^2_{\theta_{3,8}} = \frac{g^2}{2}(\theta_8, \theta_3)\begin{pmatrix} \frac{4}{3}v_3^2 & -\frac{1}{\sqrt{3}}v_2^2 \\ -\frac{1}{\sqrt{3}}v_2^2 & v_2^2 \end{pmatrix}\begin{pmatrix} \theta_8 \\ \theta_3 \end{pmatrix}$$

GAUGE BOSON MASSES

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

$$\frac{1}{2}\begin{pmatrix} \theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_1 - i\theta_2 & \theta_4 - i\theta_5 \\ \theta_1 + i\theta_2 & -\theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_6 - i\theta_7 \\ \theta_4 + i\theta_5 & \theta_6 + i\theta_7 & -\frac{2}{\sqrt{3}}\theta_8 \end{pmatrix}^{(e)}$$

1) $$M^2_{\theta_{4,5,6,7}} = \frac{g^2 v_3^2}{2}$$

2) $$m^2_{\theta_{1,2}} = \frac{g^2 v_2^2}{2}$$

$$M^2_{\theta_{3,8}} = \frac{g^2}{2}(\theta_8, \theta_3)\begin{pmatrix} \frac{4}{3}v_3^2 & -\frac{1}{\sqrt{3}}v_2^2 \\ -\frac{1}{\sqrt{3}}v_2^2 & v_2^2 \end{pmatrix}\begin{pmatrix} \theta_8 \\ \theta_3 \end{pmatrix}$$

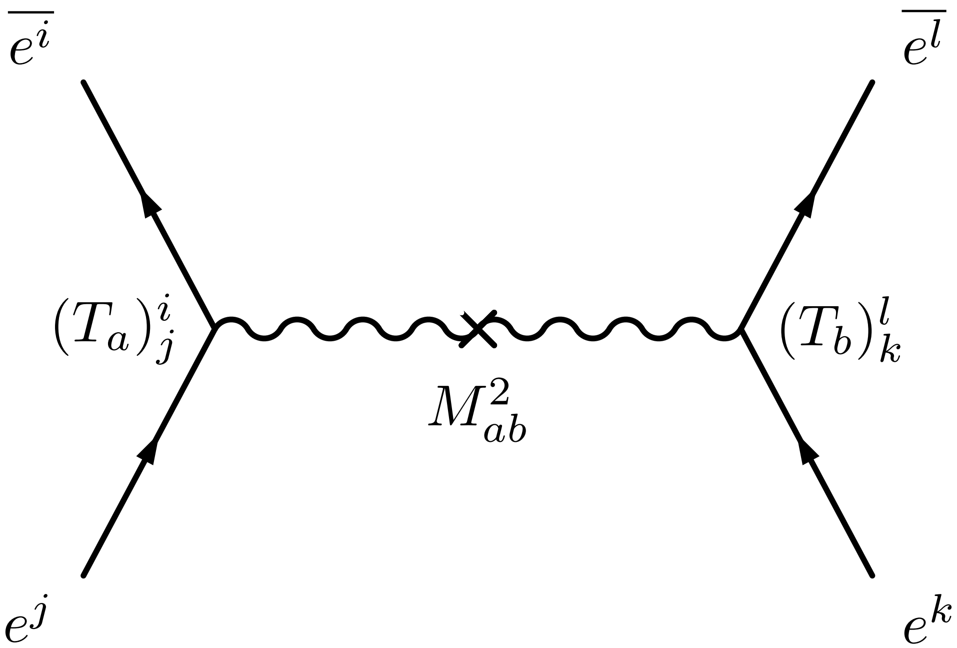
$$v_2 \gtrsim \,?$$

FLAVOR CHANGING NEUTRAL CURRENTS

- Yukawa couplings, and photon/Z couplings ($V^\dagger V = 1$), are diagonal in mass basis: **no flavour changing neutral currents at tree level in the SM**.

Here in principle:

$$\mathcal{L}_{eff} = -\frac{g_e^2}{2} J_a^{(e)\dagger} (\mathbf{M}_{\theta_e}^{-2})_{ab} J_b^{(e)}$$



$$J_a^{(e)\mu} = \frac{1}{2} \begin{pmatrix} \bar{e} & \bar{\mu} & \bar{\tau} \end{pmatrix}_R \gamma^\mu V_R^{(e)\dagger} \lambda_a^* V_R^{(e)} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R$$

BUT FLAVOR CHANGING CAN ALWAYS BE CONTROLLED!

FLAVOR CHANGING NEUTRAL CURRENTS

In $\mathbf{SU(2)_e}$ gauge symmetry limit ($v_3 \gg v_2$):

- $SU(2)_e$ gauge bosons have equal masses;
- there are no FCNC thanks to **CUSTODIAL SYMMETRY**, no matter if two families are mixed:

$$\mathcal{L}_{eff} = -\frac{1}{4v_2^2}(\overline{\mathbf{e}_{\rm R}}\tau^{a*}\gamma^\mu \mathbf{e}_{\rm R})(\overline{\mathbf{e}_{\rm R}}\tau^{a*}\gamma_\mu \mathbf{e}_{\rm R})$$

$$= -\frac{1}{4v_2^2}(\overline{e_{\rm R}}_1\gamma_\mu e^1 + \overline{e_{\rm R}}_2\gamma_\mu e^2_{\rm R})^2 =$$

$$= -\frac{1}{4v_2^2}((\begin{array}{ccc}\bar{e} & \bar{\mu} & \bar{\tau}\end{array})\gamma^\mu V^{(e)\dagger}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}V^{(e)}\begin{pmatrix}e\\ \mu\\ \tau\end{pmatrix})^2_R$$

NO MIXING WITH 3rd FAMILY $\longrightarrow$ NO FCNC.

- Then constraints on masses are proportional to violation of custodial symmetry (corrections of order $\epsilon = v_2/v_3$):

$$\mathcal{L}_{eff} = -\frac{1}{4v_2^2}(J_{(2)})^2 - \frac{1}{4v_3^2}(J_3 + \sqrt{3}J_8)^2 - \frac{1}{v_3^2}\sum_{a=4}^{7}(J_a)^2$$

FCNC & COMPOSITENESS LIMITS

Compositeness limits:

$$\mathcal{L}_C = \pm \frac{g^2}{(1+\delta_{ef})\Lambda_{RR}^2} \bar{e}_R \gamma_\mu e_R \bar{f}_R \gamma^\mu f_R$$

$$\frac{g^2}{4\pi} = 1$$

$$\Lambda_{RR}^-(eeee) > 10.2\,\text{TeV}$$

$$\Lambda_{RR}^-(ee\mu\mu) > 9.1\,\text{TeV}$$

$$\Lambda_{RR}^-(ee\tau\tau) > 5.5\,\text{TeV}$$

$$v_2 > 2\,\text{TeV} \quad \rightarrow \quad v_3 > 40\text{TeV}$$

LFV mode	Exp. $\Gamma_i/\Gamma_\mu(\Gamma_\tau)$	Main contribution to $\frac{\Gamma_i}{\Gamma_{\mu/\tau}}$	Predicted value of $\frac{\Gamma_i}{\Gamma_{\mu/\tau}}$
$\mu \to eee$	$< 1.0 \cdot 10^{-12}$	$\frac{1}{8}\left(\frac{v_{\rm EW}}{v_2}\right)^4 \left\lvert V_{3e}^* V_{3\mu} + V_{2e}^* V_{2\mu}\epsilon^2\right\rvert^2$	$\leq 1.1 \cdot 10^{-13} \left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^4 \tilde{\epsilon}_{20}^2$
$\tau^- \to \mu^- e^+ e^-$	$< 1.8 \cdot 10^{-8}$	$\frac{1}{4}\left(\frac{v_{\rm EW}}{v_2}\right)^4 \left\lvert V_{3\mu}^* V_{3\tau}\right\rvert^2 \frac{\Gamma_{\rm w}}{\Gamma_\tau}$	$= 6.2 \cdot 10^{-9} \left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^2$
$\tau \to \mu\mu\mu$	$< 2.1 \cdot 10^{-8}$	$\frac{1}{8}\left(\frac{v_{\rm EW}}{v_2}\right)^4 \left\lvert V_{3\mu}^* V_{3\tau}\right\rvert^2 \frac{\Gamma_{\rm w}}{\Gamma_\tau}$	$= 3.1 \cdot 10^{-9} \left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^2$
$\mu \to e\gamma$	$< 4.2 \cdot 10^{-13}$	$\frac{3\alpha}{2\pi}\left(\frac{v_{\rm EW}}{v_2}\right)^4 \left\lvert V_{3e}^* V_{3\mu}\right\rvert^2$	$= 3.1 \cdot 10^{-15} \left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^4 \tilde{\epsilon}_{20}^2$
$\tau \to \mu\gamma$	$< 4.4 \cdot 10^{-8}$	$\frac{3\alpha}{2\pi}\left(\frac{v_{\rm EW}}{v_2}\right)^4 \left\lvert V_{3\mu}^* V_{3\tau}\right\rvert^2 \frac{\Gamma_{\rm w}}{\Gamma_\tau}$	$= 8.7 \cdot 10^{-11} \left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^2$

LEFT-HANDED LEPTONS

$$SU(3)_l \times SU(3)_e$$

$$\frac{1}{2}\begin{pmatrix} \theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_1 - i\theta_2 & \theta_4 - i\theta_5 \\ \theta_1 + i\theta_2 & -\theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_6 - i\theta_7 \\ \theta_4 + i\theta_5 & \theta_6 + i\theta_7 & -\frac{2}{\sqrt{3}}\theta_8 \end{pmatrix}^{(\ell)}$$

$$M_{4,5}^2 = \frac{g^2}{2}(u_3^2 + u_1^2) \qquad M_{6,7}^2 = \frac{g^2}{2}(u_3^2 + u_2^2) \qquad M_{1,2}^2 = \frac{g^2}{2}(u_2^2 + u_1^2)$$

$$M_{38}^2 = \frac{g^2}{2}\begin{pmatrix} u_2^2 + u_1^2 & \frac{1}{\sqrt{3}}(u_1^2 - u_2^2) \\ \frac{1}{\sqrt{3}}(u_1^2 - u_2^2) & \frac{1}{3}(4u_3^2 + u_1^2 + u_2^2) \end{pmatrix}$$

LEFT-HANDED LEPTONS

$$SU(3)_l \times SU(3)_e$$

$$\frac{1}{2}\begin{pmatrix} \theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_1 - i\theta_2 & \theta_4 - i\theta_5 \\ \theta_1 + i\theta_2 & -\theta_3 + \frac{1}{\sqrt{3}}\theta_8 & \theta_6 - i\theta_7 \\ \theta_4 + i\theta_5 & \theta_6 + i\theta_7 & -\frac{2}{\sqrt{3}}\theta_8 \end{pmatrix}^{(\ell)}$$

$$M^2_{4,5} = \frac{g^2}{2}(u_3^2 + u_1^2) \qquad M^2_{6,7} = \frac{g^2}{2}(u_3^2 + u_2^2) \qquad M^2_{1,2} = \frac{g^2}{2}\boxed{(u_2^2 + u_1^2)}$$

$$M^2_{38} = \frac{g^2}{2}\begin{pmatrix} u_2^2 + u_1^2 & \frac{1}{\sqrt{3}}(u_1^2 - u_2^2) \\ \frac{1}{\sqrt{3}}(u_1^2 - u_2^2) & \frac{1}{3}(4u_3^2 + u_1^2 + u_2^2) \end{pmatrix}$$

Muon decay from : $\mathcal{L}^{e\nu}_{\text{eff}} = -\frac{2G_H}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\,\gamma^\mu \frac{\lambda_a}{x_a}\,\boldsymbol{e}_L\right)\left(\overline{\boldsymbol{\nu}_L}\,\gamma_\mu \frac{\lambda_a}{x_a}\,\boldsymbol{\nu}_L\right)$

$$\boxed{v_\ell^2 = u_1^2 + u_2^2}$$

LEFT-HANDED LEPTONS

- But also:

$$\mathcal{L}_{\text{eff}}^{\nu\nu} = -\frac{G_H}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{\nu}_L}\, \gamma_\mu \frac{\lambda_a}{x_a}\, \boldsymbol{\nu}_L\right)^2 \qquad \mathcal{L}_{\text{eff}}^{ee} = -\frac{G_H}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\, \gamma_\mu \frac{\lambda_a}{x_a}\, \boldsymbol{e}_L\right)^2$$

- Constraint comes from compositeness limits:

$$v_\ell > 3\text{TeV}$$

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}_{\rm L} = V_L^{(e)} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\rm L} = \begin{pmatrix} V_{1e} & V_{1\mu} & V_{1\tau} \\ V_{2e} & V_{2\mu} & V_{2\tau} \\ V_{3e} & V_{3\mu} & V_{3\tau} \end{pmatrix}_{\rm L}^{(e)} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\rm L}$$

FCNC?

LEFT-HANDED LEPTONS

Considering $\mathbf{SU(3)}_\ell$ gauge symmetry, if a symmetry between flavons η holds and

$$u_3 = u_2 = u_1$$

then

- Gauge bosons have equal masses
- They do not mix, $\lambda_a \to V^\dagger \lambda_a V$ is simply a basis redetermination of the Gell-Mann matrices
- From Fierz identities for λ matrices:

$$\mathcal{L}_{eff} = -\frac{1}{4v_\ell^2}(\overline{\mathbf{e}_\mathrm{L}}\,\lambda^a\,\gamma^\mu \mathbf{e}_\mathrm{L})(\overline{\mathbf{e}_\mathrm{L}}\,\lambda^a\,\gamma_\mu \mathbf{e}_\mathrm{L}) = -\frac{1}{3v_2^2}(\overline{\mathbf{e}_\mathrm{L}}\,\mathbb{I}\,\gamma_\mu \mathbf{e}_\mathrm{L})^2$$

That is **no FCNC**, the global $SO(8)_\ell$ symmetry acts as a custodial symmetry.

LEFT-HANDED LEPTONS

- In general case e.g. $\mu \to 3e$ decay:

$$\frac{\Gamma(\mu \to ee\bar{e})}{\Gamma(\mu \to e\nu_\mu \bar{\nu}_e)} \simeq \frac{1}{8}\left(\delta_\mu C(r)|U^*_{3e}U_{3\mu}|\right)^2$$

  $r = 2u_3^2/v_\ell^2$, $|C(r)| < 1$. $|U_{3\mu}|$ and $|U_{3e}|$ can be almost as large as $\sin\theta_C = V_{us}$.

- The experimental limits on other LFV effects as e.g. $\tau \to 3\mu$ are much weaker.

- Also in this case $\mathbf{v}_\ell \simeq$ **6 TeV** fullfill experimental constraints.

CONCLUSIONS

- 4.3σ deviation from CKM unitarity after present theoretical and experimental indipendent determinations of CKM first row.
- Forth vector-like couple of isosinglets is a possible solution, but with sum unnatural features.
- New operator in positive interference with SM can be a solution.
- The new operator can be related to a gauge flavor symmetry, which also leads to understanding of mass hierarchy among families and mixing angles (analogous analysis can be done for quarks).
- Horizontal gauge bosons can be **as light as few TeV**, without contradiction with the experimental limits (flavour changing processes with leptons, compositeness limits, K-mesons system, B-mesons-system, etc.).
- In order to cancel $SU(3)^3$ anomalies an interesting possibility is to introduce the mirror twins. Mirror matter is a candidate for dark matter.
- The flavor gauge bosons are messengers between the two sectors and so not only they are a portal for direct detection, but also they can mediate new phenomena such as muonium disappearance (conversion into mirror muonium), kaon disappearance, pion disappearance.

backup

BACKUP

	CKM 3	CKM 4	CKM+$\mathcal{F}$
C	0.22780(60)	0.22443(61)	0.22460(61)
B	0.22535(45)	0.22518(45)	0.22535(45)
A	0.22333(60)	0.22333(60)	0.22350(60)
$\overline{A+B}$	0.22463(36)	0.0.22452(36)	0.22469(36)
$\overline{A+B+C}$	0.22546(31)	0.22449(31)	0.22467(31)
	$\chi^2 = 27.71$	$\chi^2 = 6.1$	$\chi^2 = 6.1$

TRIANGLE ANOMALIES

$$SU(3)_\ell \times SU(3)_e \times SU(3)_Q \times SU(3)_u \times SU(3)_d$$

$$\ell_L \sim 3_\ell,\ e_R \sim 3_e,\ Q_L \sim 3_Q,\ u_R \sim 3_u,\ d_R \sim 3_d$$

- In order to cancel $SU(3)^3$ anomalies for each triplet another triplet (SM singlet) with opposite chirality is needed.

- An interesting possibility is to introduce the mirror twins with opposite chirality and analogous representation of mirror SM gauge symmetry $SU(3)' \times SU(2)' \times U(1)'$:

$$\ell'_R \sim 3_\ell,\ e'_L \sim 3_e,\ Q'_R \sim 3_Q,\ u'_L \sim 3_u,\ d'_L \sim 3_d$$

- Couplings with flavons:

$$\frac{g_{in}\xi_n^\alpha}{M}(\phi\overline{\ell_{Li}}e_{R\alpha} + \phi'\overline{\ell'_{Ri}}e'_{L\alpha}) + h.c.$$

TRIANGLE ANOMALIES 2

- As an example, for $SU(3)_e$, mixed triangle anomaly $U(1) \times SU(3)_e^2$ must be cancelled. New leptons

$$\mathcal{E}_{L\alpha} \sim (1, -2, 3_e; X), \qquad \mathcal{E}_{Ri} \sim (1, -2, 1; X)$$

  and for mirror parity

$$\mathcal{E}'_{R\alpha} \sim (1, -2', 3_e; X), \qquad \mathcal{E}'_{Li} \sim (1, -2', 1; X)$$

  cancel the mixed triangle

$$U(1) \times SU(3)_e^2,\ U(1)_X \times SU(3)_e^2,\ U(1) \times U(1)_X^2,\ U(1)_X \times U(1)^2$$

- Masses from Yukawa couplings

$$y_{in} \xi_n^\alpha \overline{\mathcal{E}_{Ri}} \mathcal{E}_{L\alpha} + y_{in} \xi_n^\alpha \overline{\mathcal{E}_{Li}}' \mathcal{E}'_{R\alpha} + h.c.$$

- The lightest has mass $O(100)$ GeV. If $U(1)_X$ is unbroken, then it is stable. Current experimental lower limit on charged new leptons is 102.6 GeV.

COSMOLOGICAL IMPLICATIONS

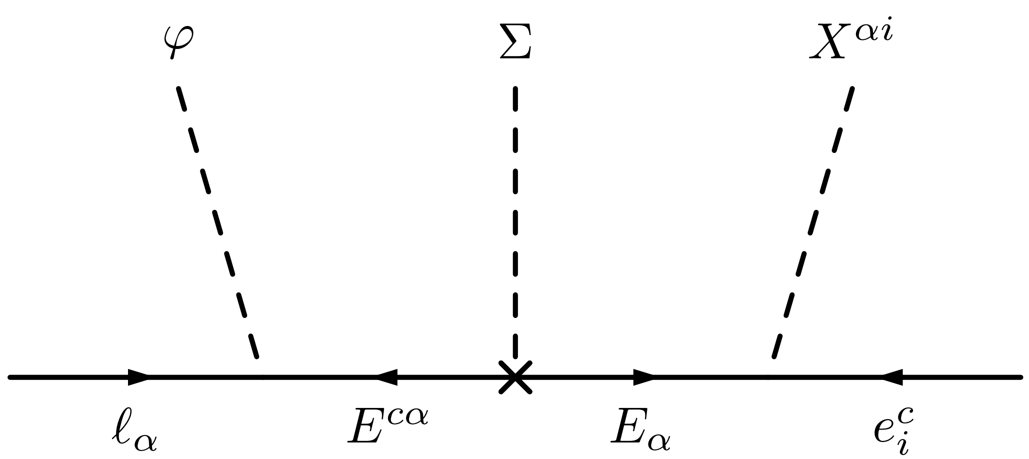
- Mirror matter is a viable candidate for light dark matter dominantly consisting of mirror helium and hydrogen atoms.
- The flavor gauge bosons are messengers between the two sectors and so a portal for direct detection.
- $T'/T < 0.2 \div 0.3$ from CMB and large scale structures.
- Freeze-out temperature of horizontal interactions between the two sectors should not exceed $T_d \simeq (v_2/2)^{\frac{4}{3}} \times 130$ MeV. Or $v'_{\rm EW} \gg v_{\rm EW}$.
- For neutrinos

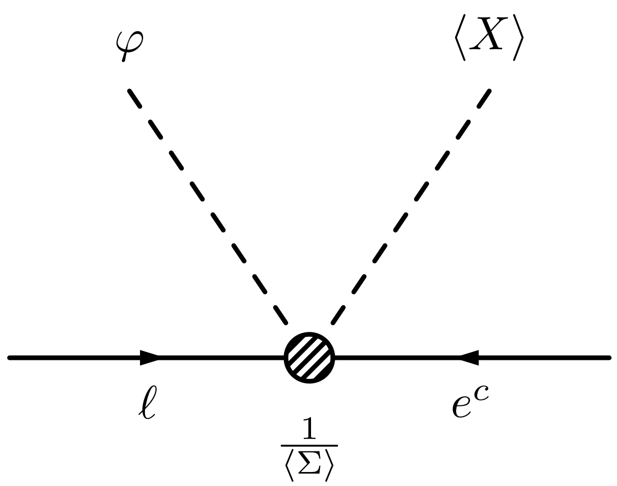
$$\frac{Y_\nu^{ij}}{\mathcal{M}}(\phi\phi l_{Li}^T C l_{Lj} + \phi'\phi' l'^T_{Ri} C l'_{Rj}) + \frac{\tilde{Y}_\nu^{ij}}{\mathcal{M}}\phi\phi' \overline{l_{Li}} l'_{Rj} + h.c.$$

the last operator gives COLEPTOGENESIS.

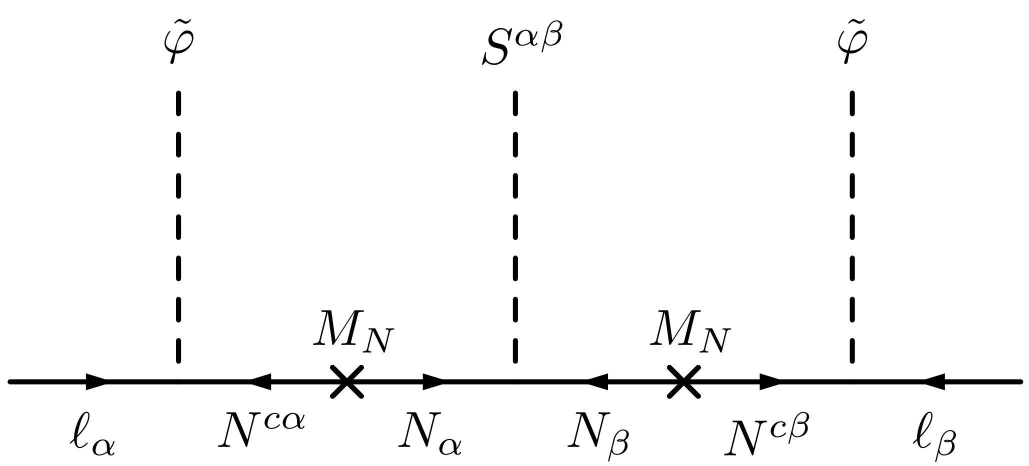
EFFECTIVE OPERATORS FOR FERMION MASSES

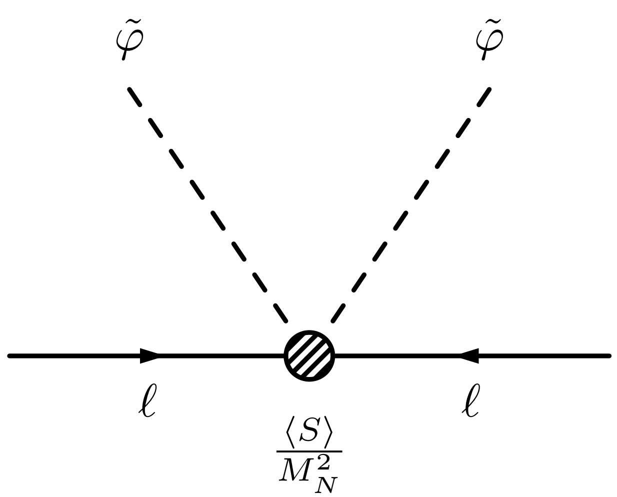
$$X_e^{\beta k} \sim (\bar{3}_l, \bar{3}_e) \qquad X_d^{i\alpha} \sim (\bar{3}_q, \bar{3}_d) \qquad X_u^{ij} \sim (\bar{3}_q, \bar{3}_u)$$





$$S_{\alpha\beta} \sim 6_l$$





$$\mathcal{L} = \frac{\lambda_u}{M_U} X_u^{ij} \bar{\phi} \bar{u}_j q_i + \frac{\lambda_d}{M_D} X_d^{i\alpha} \phi \bar{d}_\alpha q_i + \frac{\lambda_e}{M_E} X_e^{\beta k} \phi \bar{e}_k l_\beta + \lambda_\nu \mathcal{M}^{-1}_{\alpha\beta} \bar{\phi} \bar{\phi} l_\alpha l_\beta + h.c.$$

SSB

$$SU(3)_l \times SU(3)_e$$

$$SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2} \text{nothing} \qquad u_3 : u_2 : u_1 = \epsilon_L^2 : \epsilon_L : 1 \qquad \epsilon_L \simeq \frac{1}{2} - \frac{1}{3}$$

$$\eta^\alpha_{(n)} \sim \bar{3}_l \qquad \langle \eta_{(3)} \rangle = \begin{pmatrix} 0 \\ 0 \\ u_3 \end{pmatrix} \quad \langle \eta_{(2)} \rangle = \begin{pmatrix} 0 \\ u_2 \\ 0 \end{pmatrix} \quad \langle \eta_{(1)} \rangle = \begin{pmatrix} u_1 \\ 0 \\ 0 \end{pmatrix}$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

$$\xi^i_{(n)} \sim \bar{3}_e \qquad \langle \xi_{(3)} \rangle = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix} \quad \langle \xi_{(2)} \rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix} \quad \langle \xi_{(1)} \rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}$$

- inducing VEV to X_e and S via couplings

$$\mu_X^* \xi_{(l)} X^\dagger \eta_{(n)} + \text{h.c.} \qquad \langle X \rangle \sim \langle X^{33} \rangle \cdot \begin{pmatrix} O(\epsilon^2 \epsilon_{\rm L}^2) & O(\epsilon \epsilon_{\rm L}^2) & O(\epsilon_{\rm L}^2) \\ O(\epsilon^2 \epsilon_{\rm L}) & O(\epsilon \epsilon_{\rm L}) & O(\epsilon_{\rm L}) \\ O(\epsilon^2) & O(\epsilon) & O(1) \end{pmatrix}$$

FERMION MASSES

$$SU(3)_l \times SU(3)_e$$

$$SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2} \text{nothing} \qquad u_3 : u_2 : u_1 = \epsilon_L^2 : \epsilon_L : 1 \qquad \epsilon_L \simeq \frac{1}{2} - \frac{1}{3}$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

From Yukawa coupling:

$$\frac{\lambda_e}{M_E} X_e^{\beta k} \phi \bar{e}_k l_\beta \longrightarrow \qquad m^{(e)\dagger} \propto \langle X_e \rangle \simeq \begin{pmatrix} O(\epsilon^2) & 0 & 0 \\ O(\epsilon^2) & O(\epsilon) & 0 \\ O(\epsilon^2) & O(\epsilon) & O(1) \end{pmatrix} \cdot \frac{|\mu_X| v_3 u_3}{M_X^2}$$

$$V_{\mathrm{L}}^{(e)\dagger} m^{(e)} V_{\mathrm{R}}^{(e)} = m_{\mathrm{diag}}^{(e)} \qquad \overline{\mathbf{e}_{\mathrm{R}}} \mathbf{m}^{(\mathbf{e})\dagger} \mathbf{e}_{\mathrm{L}} = (\overline{\mathbf{e}_{\mathrm{R}}} V_{\mathrm{R}}^{(e)})(V_{\mathrm{R}}^{(e)\dagger} \mathbf{m}^{(\mathbf{e})\dagger} V_{\mathrm{L}}^{(e)})(V_{\mathrm{L}}^{(e)\dagger} \mathbf{e}_{\mathrm{L}})$$

$$\begin{pmatrix} e^1 \\ e^2 \\ e^3 \end{pmatrix}_{\mathrm{R}} = V_R^{(e)} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\mathrm{R}} = \begin{pmatrix} V_{1e} & V_{1\mu} & V_{1\tau} \\ V_{2e} & V_{2\mu} & V_{2\tau} \\ V_{3e} & V_{3\mu} & V_{3\tau} \end{pmatrix}_{\mathrm{R}}^{(e)} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\mathrm{R}}$$

FCNC

$$\mathcal{L}_{eff} = -\frac{1}{4v_2^2}\left(\left(\begin{array}{ccc}\bar{e} & \bar{\mu} & \bar{\tau}\end{array}\right)\gamma^\mu V^{(e)\dagger}\left(\begin{array}{ccc}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{array}\right)V^{(e)}\left(\begin{array}{c}e\\ \mu\\ \tau\end{array}\right)\right)_R^2$$

$$-\frac{1}{4v_3^2}(J_3+\sqrt{3}J_8)^2 - \frac{1}{v_3^2}\sum_{a=4}^{7}(J_a)^2$$

LFV mode	Experimental limit	Contribution to $-4\frac{G_{\mu/\tau l l' l''}}{\sqrt{2}}$ at L.O. from: $-\frac{1}{4v_2^2}J_{(2)}^{(e)}J_{(2)}^{(e)}$	Heavy bosons
$\mu \to eee$	$\frac{1}{\sqrt{2}}\frac{\lvert G_{\mu eee}\rvert}{G_F} < 10^{-6}$	$\frac{V_{3e}^* V_{3\mu}}{2v_2^2}$	$-\frac{(V_{1e}^* V_{1\mu} - V_{2e}^* V_{2\mu})\epsilon^2}{4v_2^2}$
$\mu^- e^+ \to \mu^+ e^-$	$\frac{\lvert G_{\mu e\mu e}\rvert}{G_F} < 3 \cdot 10^{-3}$	$\frac{-(V_{3e}^* V_{3\mu})^2}{4v_2^2}$	$-\frac{(V_{1e}^* V_{1\mu} - V_{2e}^* V_{2\mu})^2\epsilon^2}{16v_2^2}$
$\tau^- \to \mu^- e^+ e^-$	$\frac{\lvert G_{\tau\mu ee}\rvert}{G_F} < 1.34 \cdot 10^{-4}$	$\frac{V_{3\mu}^* V_{3\tau}}{2v_2^2}$	$\frac{(V_{2\mu}^* V_{2\tau} - V_{3\mu}^* V_{3\tau})\epsilon^2}{4v_2^2}$
$\tau \to \mu\mu\mu$	$\frac{1}{\sqrt{2}}\frac{\lvert G_{\tau\mu\mu\mu}\rvert}{G_F} < 1.45 \cdot 10^{-4}$	$\frac{V_{3\mu}^* V_{3\tau}}{2v_2^2}$	$-\frac{V_{3\mu}^* V_{3\tau}\epsilon^2}{v_2^2}$
$\tau \to eee$	$\frac{1}{\sqrt{2}}\frac{\lvert G_{\tau eee}\rvert}{G_F} < 1.64 \cdot 10^{-4}$	$\frac{V_{3e}^* V_{3\tau}}{2v_2^2}$	$-\frac{(V_{1e}^* V_{1\tau} - V_{2e}^* V_{2\tau} + V_{3e}^* V_{3\tau})\epsilon^2}{4v_2^2}$
$\tau \to e^- \mu^- \mu^+$	$\frac{\lvert G_{\tau e\mu\mu}\rvert}{G_F} < 1.64 \cdot 10^{-4}$	$\frac{V_{3e}^* V_{3\tau}}{2v_2^2}$	$\frac{(3V_{3e}^* V_{3\tau} + V_{2e}^* V_{2\tau} - V_{1e}^* V_{1\tau})\epsilon^2}{4v_2^2} - \frac{(V_{3\mu}^* V_{1\mu} + V_{3\mu}^* V_{2e}^*)\epsilon^2}{v_2^2}$

FCNC & COMPOSITENESS LIMITS

LFV mode	Operator	Experimental limit	Main contribution	Constraint
$\mu \to eee$	$-\frac{4G_{\mu eee}}{\sqrt{2}}\overline{e_R}\gamma^\mu \mu_R \overline{e_R}\gamma^\mu e_R$	$\Gamma_{\mu eee}/\Gamma_\mu < 1.0 \cdot 10^{-12}$	$-\frac{4G_{\mu eee}}{\sqrt{2}} \simeq -\frac{\epsilon^3}{2v_2^2}$	$v_2 > 1.16$ TeV
$\tau^- \to \mu^- e^+ e^-$	$-\frac{4G_{\tau\mu ee}}{\sqrt{2}}\overline{\mu_R}\gamma^\mu \tau_R \overline{e_R}\gamma^\mu e_R$	$\Gamma_{\tau\mu ee}/\Gamma_\tau < 1.8 \cdot 10^{-8}$	$-\frac{4G_{\tau\mu ee}}{\sqrt{2}} \simeq -\frac{\epsilon}{2v_2^2}$	$v_2 > 1.5$ TeV
$\mu \to e\gamma$	$-\mu_{\mu e}(\overline{e_L}\frac{\sigma^{\delta\rho}}{2}\mu_R + \overline{e_R}\frac{\sigma^{\delta\rho}}{2}\mu_L)F_{\delta\rho}$	$\Gamma_{\mu e\gamma}/\Gamma_\mu < 4.2 \cdot 10^{-13}$	$-\mu_{\mu e} \simeq -\frac{e}{8\pi^2}\frac{\epsilon^3 m_\mu}{v_2^2}$	$v_2 > 1$ TeV
$\tau \to \mu\gamma$	$-\mu_{\tau\mu}(\overline{\mu_L}\frac{\sigma^{\delta\rho}}{2}\tau_R + \overline{\mu_R}\frac{\sigma^{\delta\rho}}{2}\tau_L)F_{\delta\rho}$	$\Gamma_{\tau e\gamma}/\Gamma_\tau < 4.4 \cdot 10^{-8}$	$-\mu_{\tau\mu} \simeq -\frac{e}{8\pi^2}\frac{\epsilon m_\tau}{v_2^2}$	$v_2 > 0.7$ TeV

Compositeness limits:

$$\mathcal{L}_C = \pm \frac{g^2}{(1+\delta_{ef})\Lambda_{RR}^2}\bar{e_R}\gamma_\mu e_R \bar{f_R}\gamma^\mu f_R$$

$$\frac{g^2}{4\pi} = 1$$

$$\Lambda_{RR}^-(eeee) > 10.2\,\text{TeV}$$

$$\Lambda_{RR}^-(ee\mu\mu) > 9.1\,\text{TeV}$$

$$\Lambda_{RR}^-(ee\tau\tau) > 5.5\,\text{TeV}$$

$$v_2 > 2\,\text{TeV} \quad \rightarrow \quad v_3 > 40\text{TeV}$$

BACKUP

$$\Gamma(K^+ \to \pi^+ \mu^- e^+)/\Gamma_{\text{total}} < 5.2 \cdot 10^{-10}$$

$$v_2 > 30\,\text{TeV}$$

$$M_{12} = \frac{G_F^2 m_W^2}{4\pi^2} \tilde{F}^* \langle K^0 | (\overline{d_L} \gamma^\mu s_L)(\overline{d_L} \gamma_\mu s_L) | \bar{K}^0 \rangle =$$
$$= -\frac{G_F^2 m_W^2}{12\pi^2} \tilde{F}^* f_K^2 m_K B_K e^{i(\xi_s - \xi_d - \xi)}$$

$$\mathcal{L}_{K\bar{K}\text{SM}} = \frac{-G_F^2 M_W^2}{4\pi^2} (\overline{s_L} \gamma^\mu d_L)(\overline{s_L} \gamma_\mu d_L) \tilde{F} + h.c. =$$
$$= -\frac{1 - i\alpha_{SM}}{\Lambda_{SM}^2} (\overline{s_L} \gamma^\mu d_L)(\overline{s_L} \gamma_\mu d_L) + h.c.$$
$$\sim -\frac{1 - i10^{-2}}{(1,6 \cdot 10^6 \text{GeV})^2} (\overline{s_L} \gamma^\mu d_L)(\overline{s_L} \gamma_\mu d_L) + h.c.$$

$$\left|\frac{1 - i\alpha_{ds}^{(d)}}{\Lambda_{ds}^{(d)2}}\right| = \left|\frac{(V_{3d}^{(d)} V_{3s}^{(d)*})^2}{2v_2^2}\right| =$$
$$= X_d \left|\frac{(V_{Ltd} V_{Lts}^*)^2}{2v_2^2}\right|$$
$$\approx \frac{(\epsilon^3)^2}{2v_2^2}$$