Use of triangle singularity in precisely measuring the $X(3872)$ binding energy

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RDP Seventh Autumn PhD School & Workshop, 23–28 Sept. 2019, Tbilisi, Georgia

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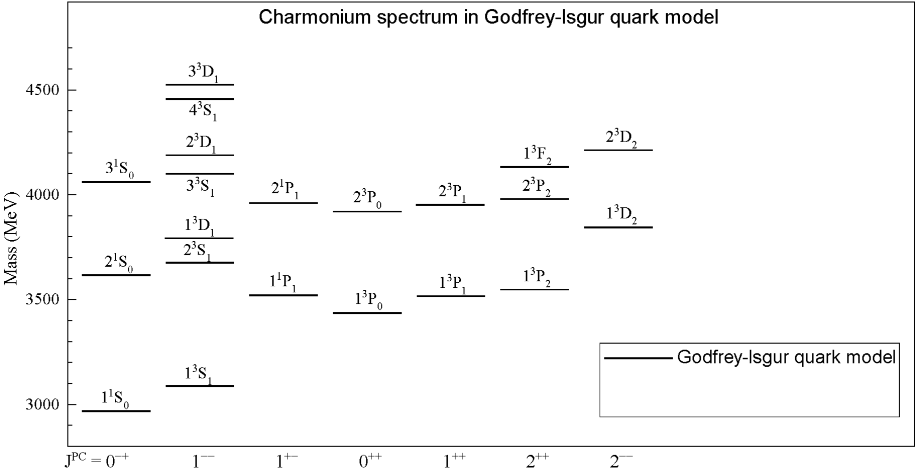
Based on: FKG, Phys. Rev. Lett. 122, 202002 (2019) [arXiv:1902.11221]

Charmonia and XYZ states



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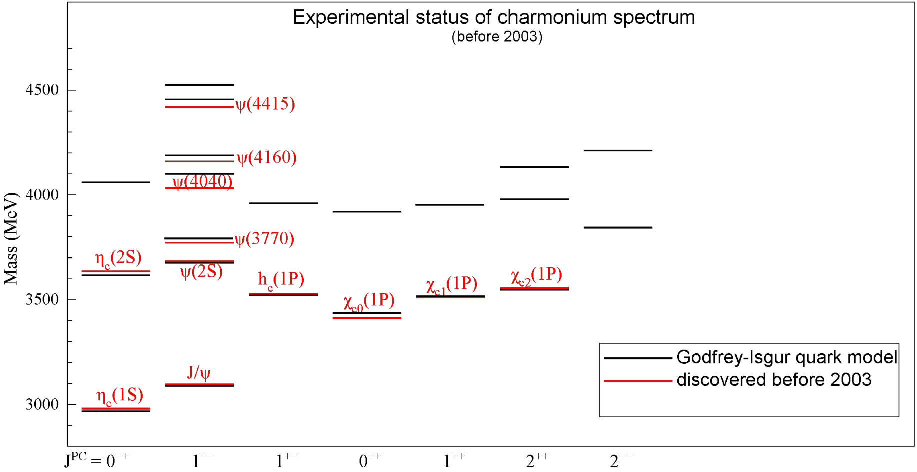

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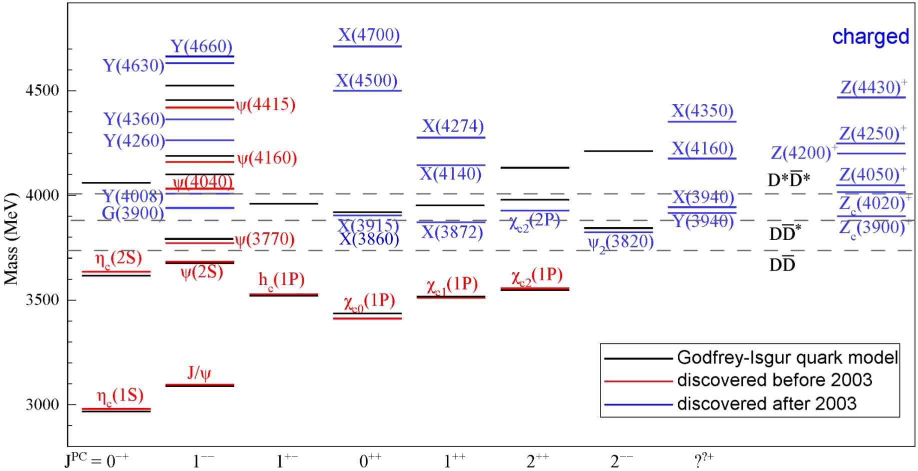


Note:

- $X(3915)$ is probably just the $\chi_{c2}(2P)$ with 2^{++} Z.-Y. Zhou et al., PRL115(2015)022001
- $\psi_3(3^{--})$ recently discovered: $M = 3842.71 \pm 0.20$ MeV LHCb, JHEP1907(2019)035

Naming convention

For states with properties in conflict with naive quark model (normally):

- X: $I=0$, J^{PC} other than 1^{--} or unknown
- Y: $I=0$, $J^{PC}=1^{--}$
- Z: $I=1$

PDG2018 naming scheme:

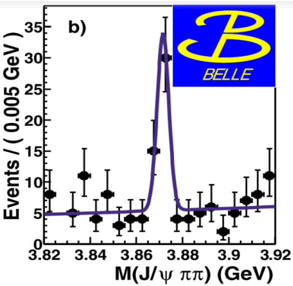
$J^{PC} =$ Minimal quark content	0^{-+}, 2^{-+}, $\vdots$	1^{+-}, 3^{+-}, $\vdots$	1^{--}, 2^{--}, $\vdots$	0^{++}, 1^{++}, $\vdots$
$u\bar{d}, u\bar{u}-d\bar{d}, d\bar{u}$ $(I=1)$	π	b	ρ	a
$d\bar{d}+u\bar{u}$ and/or $s\bar{s}$ $(I=0)$	η, η'	h, h'	ω, ϕ	f, f'
$c\bar{c}$	η_c	h_c	$\psi^\dagger$	χ_c
$b\bar{b}$	η_b	h_b	Υ	χ_b
$I=1$ with $c\bar{c}$	(Π_c)	Z_c	R_c	(W_c)
$I=1$ with $b\bar{b}$	(Π_b)	Z_b	(R_b)	(W_b)

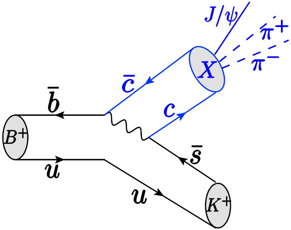
†The J/ψ remains the J/ψ.

So $X(3872)$ is also called $\chi_{c1}(3872)$

$X(3872)$: properties (1)

Belle, PRL91(2003)262001





- The beginning of the XYZ story, discovered in $B^\pm \to K^\pm J/\psi\pi\pi$

  $M_X = (3871.69 \pm 0.17)$ MeV
- $\Gamma < 1.2$ MeV   Belle, PRD84(2011)052004
- Confirmed in many experiments: Belle, BaBar, BESIII, CDF, CMS, D0, LHCb, ...
- 10 years later, $J^{PC} = 1^{++}$

  LHCb, PRL110(2013)222001

  $\Rightarrow$ S-wave coupling to $D\bar{D}^*$

Mysterious properties:

- Mass coincides with the $D^0\bar{D}^{*0}$ threshold:

  $M_{D^0} + M_{D^{*0}} - M_X = (0.00 \pm 0.18)$ MeV

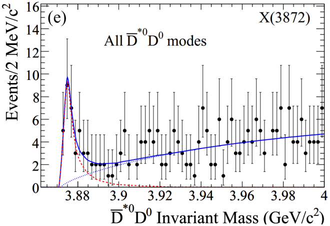
$X(3872)$: properties (2)

Mysterious properties (cont.):

- Large coupling to $D^0\bar{D}^{*0}$:

  $\mathcal{B}(X \to D^0\bar{D}^{*0}) > 30\%$ Belle, PRD81(2010)031103

  $\mathcal{B}(X \to D^0\bar{D}^0\pi^0) > 40\%$ Belle, PRL97(2006)162002



- No isospin partner observed $\Rightarrow I = 0$

  but, large isospin breaking:

  $$\frac{\mathcal{B}(X \to \omega J/\psi)}{\mathcal{B}(X \to \pi^+\pi^- J/\psi)} = 0.8 \pm 0.3$$

  BaBar, PRD77(2008)011102

  $$C(X) = +, C(J/\psi) = - \quad \Rightarrow C(\pi^+\pi^-) = - \quad \Rightarrow I(\pi^+\pi^-) = 1$$

- Radiative decays:

  $$\frac{\mathcal{B}(X \to \gamma\psi')}{\mathcal{B}(X \to \gamma J/\psi)} = 2.6 \pm 0.6$$

  PDG18 average of BaBar(2009) and LHCb(2014)

  Upper limit at Belle: < 2.1 at 90% C.L. Belle, PRL107(2011)091803

  Notice: New BESIII result: < 0.59 at 90% C.L. see C.-Z. Yuan, talk at Lattice2019

$X(3872)$: important observables

$$\delta \equiv M_{D^0} + M_{D^{*0}} - M_X = (0.00 \pm 0.18) \text{ MeV}$$

- Long-distance wave function (below th.) given by $D^0\bar{D}^{*0}$: Braaten, Voloshin, ...

$$\psi_X(k^2) = \frac{\sqrt{8\pi\gamma_X}}{k^2 + \gamma_X^2} \qquad \text{binding momentum : } \gamma_X \equiv \sqrt{2\delta \frac{M_{D^{*0}} M_{D^0}}{M_{D^{*0}} + M_{D^0}}}$$

- Line shapes, mass and width are important to understand the $X(3872)$

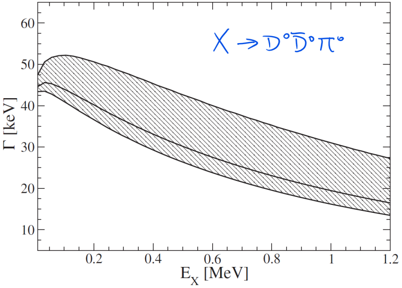
  Hanhart et al., PRD76(2007)034007; Artoisenet, Braaten, Kang, PRD82(2010)014013; ...

- Width: sensitivity of $\lesssim 100$ keV at PANDA

  PANDA, EPJA55(2019)42

- Mass $\Rightarrow$ Coupling of X to $D^0\bar{D}^{*0}$ for the molecular component

  Is the X above or below the $D^0\bar{D}^{*0}$ threshold? High precision!



Prediction in XEFT

Fleming et al., PRD76(2007)034006

Current precision of the binding energy

Precision of the binding energy δ (± 180 keV) is limited by

- Precision of the $X(3872)$ mass

  PDG2018 average from the $J/\psi\pi\pi$ and $J/\psi\pi\pi\pi$ modes

$\chi_{c1}(3872)$ MASS FROM $J/\psi X$ MODE

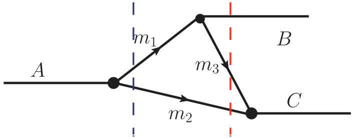
INSPIRE search

VALUE (MeV)	*EVTS*		*DOCUMENT ID*		*TECN*	*COMMENT*
3871.69 ± 0.17	**OUR AVERAGE**					
$3871.9 \pm 0.7 \pm 0.2$	20 ± 5		ABLIKIM	2014	BES3	$e^+ e^- \to J/\psi\pi^+\pi^-\gamma$
$3871.95 \pm 0.48 \pm 0.12$	0.6k		AAIJ	2012H	LHCB	$p\,p \to J/\psi\pi^+\pi^- X$
$3871.85 \pm 0.27 \pm 0.19$	~ 170	1	CHOI	2011	BELL	$B \to K\pi^+\pi^- J/\psi$
$3873\ ^{+1.8}_{-1.6} \pm 1.3$	27 ± 8	2	DEL-AMO-SANCH..	2010B	BABR	$B \to \omega J/\psi K$
$3871.61 \pm 0.16 \pm 0.19$	6k	3, 2	AALTONEN	2009AU	CDF2	$p\,\bar{p} \to J/\psi\pi^+\pi^- X$
$3871.4 \pm 0.6 \pm 0.1$	93.4		AUBERT	2008Y	BABR	$B^+ \to K^+ J/\psi\pi^+\pi^-$
$3868.7 \pm 1.5 \pm 0.4$	9.4		AUBERT	2008Y	BABR	$B^0 \to K^0_S\, J/\psi\pi^+\pi^-$
$3871.8 \pm 3.1 \pm 3.0$	522	4, 2	ABAZOV	2004F	D0	$p\,\bar{p} \to J/\psi\pi^+\pi^- X$

- Precision of D^0, D^{*0} masses (PDG AVERAGE):

$$M_{D^0} = (1864.84 \pm 0.05)\ \text{MeV}, \quad M_{D^{*0}} = (2006.85 \pm 0.05)\ \text{MeV}$$

Triangle singularity



$$\frac{1}{2m_A}\sqrt{\lambda(m_A^2, m_1^2, m_2^2)} \equiv \boxed{p_{2,\text{left}} = p_{2,\text{right}}} \equiv \gamma\,(\beta\, E_2^* - p_2^*)$$

on-shell momentum of m_2 at the left and right cuts in the A rest frame

$\beta = |\vec{p}_{23}|/E_{23}, \gamma = 1/\sqrt{1-\beta^2}$ Bayar et al., PRD94(2016)074039

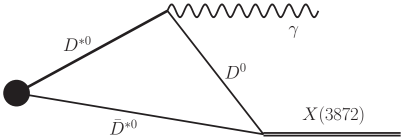
- $p_2 > 0$, $p_3 = \gamma\,(\beta\, E_3^* + p_2^*) > 0 \Rightarrow$ m_2 and m_3 move in the same direction
- velocities in the A rest frame: $v_3 > \beta > v_2$

$$v_2 = \beta\,\frac{E_2^* - p_2^*/\beta}{E_2^* - \beta\, p_2^*} < \beta\,, \qquad v_3 = \beta\,\frac{E_3^* + p_2^*/\beta}{E_3^* + \beta\, p_2^*} > \beta$$

- Conditions (Coleman–Norton theorem): Coleman, Norton (1965); Bronzan (1964)
  - ☞ all three intermediate particles can go on shell simultaneously
  - ☞ $\vec{p}_2 \parallel \vec{p}_3$, particle-3 can catch up with particle-2 (as a classical process)
- needs very special kinematics $\Rightarrow$ process dependent! (contrary to pole position)

New method for measuring the $X(3872)$ mass (1)

FKG, PRL122(2019)202002



Use of triangle singularity:

- Massless photon
- TS for the $X\gamma$ invariant mass ($\delta = M_{D^{*0}} + M_{D^0} - M_X$):

$$E_{X\gamma}^{\text{TS}} = 2M_{D^{*0}} + \frac{1}{4M_{D^0}}\left(M_{D^{*0}} - M_{D^0} - 2\sqrt{-M_{D^0}\delta} + \delta\right)^2 + \mathcal{O}\left(\frac{(..)^3}{M_{D^0}^2}\right)$$

$\Rightarrow$ TS in $E_{X\gamma}$ around the $D^{*0}\bar{D}^{*0}$ thr. (S-wave $D^{*0}\bar{D}^{*0}$ with $J^PC = 1^{+-}$)

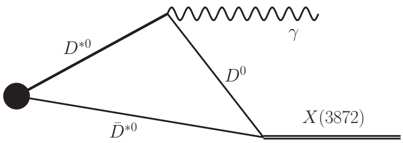
- D^{0*} width is tiny:

$\Gamma(D^{*\pm}) = (83.4 \pm 1.8)$ keV, $\mathcal{B}(D^{*\pm} \to \pi^0 D^\pm) = (67.7 \pm 0.5)\%$,

$\mathcal{B}(D^{*0} \to \pi^0 D^0) = (64.7 \pm 0.9)\%$

$\Rightarrow \Gamma(D^{*0}) = (55.3 \pm 1.4)$ keV

FKG, PRL122(2019)202002



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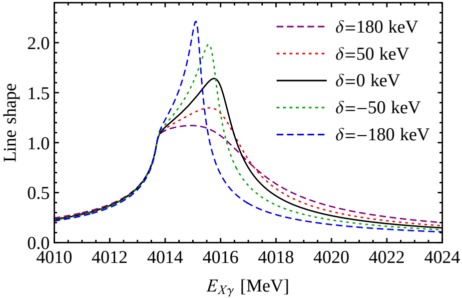
$$E_{X\gamma}^{\text{TS}} \simeq 2M_{D^{*0}} + \frac{1}{2M_{D^0}}\left(M_{D^{*0}} - M_{D^0} - 2\sqrt{-M_{D^0}\delta} + \delta\right)^2$$



- For X blow $D^0\bar{D}^{*0}$ threshold, $\delta > 0$, TS is complex, smooth shape
- For X above $D^0\bar{D}^{*0}$ threshold, $\delta < 0$, TS is real $\Rightarrow$ logarithmic divergent peak if neglecting D^{*0} width
- Sharper peak when $\delta < 0$

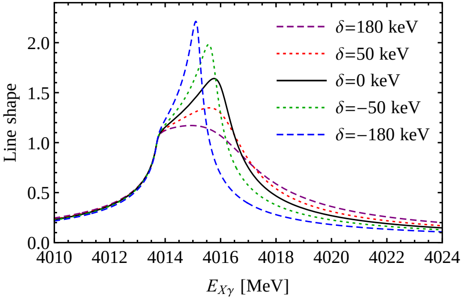
Line shape normalized at the $D^{*0}\bar{D}^{*0}$ threshold:

$$F(E_{X\gamma}) = \frac{|I(E_{X\gamma})|^2}{|I(2m_*)|^2}\frac{E_\gamma^3}{\left[(4m_*^2 - m_X^2)/(4m_*)\right]^3}$$

here $I(E_{X\gamma})$: the triangle loop integral

New method for measuring the $X(3872)$ mass (2)

$$E_{X\gamma}^{\text{TS}} \simeq 2M_{D^{*0}} + \frac{1}{2M_{D^0}}\left(M_{D^{*0}} - M_{D^0} - 2\sqrt{-M_{D^0}\delta} + \delta\right)^2$$



- Cusp fixed at the $D^{*0}\bar{D}^{*0}$ threshold
- Peak fixed at the TS energy:

δ (keV)	$E_{X\gamma}^{\text{TS}}$ (MeV)
-180	$4015.2 - i0.1$
-50	$4015.7 - i0.2$
0	$4016.0 - i0.4$

Line shape normalized at the $D^{*0}\bar{D}^{*0}$ threshold:

$$F(E_{X\gamma}) = \frac{|I(E_{X\gamma})|^2}{|I(2m_*)|^2}\,\frac{E_\gamma^3}{\left[(4m_*^2 - m_X^2)/(4m_*)\right]^3}$$

here $I(E_{X\gamma})$: the triangle loop integral

Sensitivity study (1)

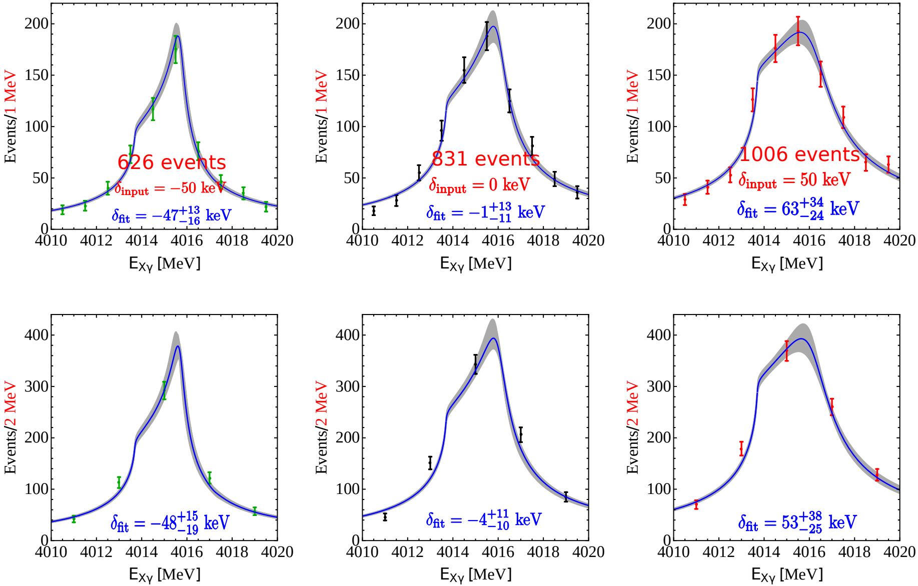
Sensitivity study from a simple Monte Carlo simulation:

(1) Generate synthetic events following the distribution

$$F(E_{X\gamma}) = \frac{|I(E_{X\gamma})|^2}{|I(2m_*)|^2} \frac{E_\gamma^3}{\left[(4m_*^2 - m_X^2)/(4m_*)\right]^3}$$

(2) Fit to the synthetic data treating δ as a free parameter

Sensitivity study (2)



Sensitivity study (3)

A	$\delta_{\text{in}} = -50$ keV (127 events)	$\delta_{\text{in}} = 0$ (164 events)	$\delta_{\text{in}} = 50$ keV (192 events)
10 bins	-24^{+24}_{-28}	11^{+31}_{-20}	22^{+41}_{-23}
5 bins	-17^{+24}_{-27}	30^{+64}_{-29}	40^{+67}_{-31}
B	$\delta_{\text{in}} = -50$ keV (626 events)	$\delta_{\text{in}} = 0$ (831 events)	$\delta_{\text{in}} = 50$ keV (1006 events)
10 bins	-47^{+13}_{-16}	-1^{+13}_{-11}	63^{+34}_{-24}
5 bins	-48^{+15}_{-19}	-4^{+11}_{-10}	53^{+38}_{-25}
C	$\delta_{\text{in}} = -50$ keV (3133 events)	$\delta_{\text{in}} = 0$ (4027 events)	$\delta_{\text{in}} = 50$ keV (5015 events)
10 bins	-53^{+7}_{-8}	-2 ± 5	55^{+13}_{-11}
5 bins	-52^{+7}_{-8}	-2^{+7}_{-6}	61^{+17}_{-14}

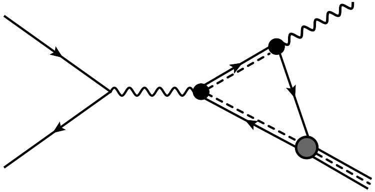
10 bins: 1 MeV/bin

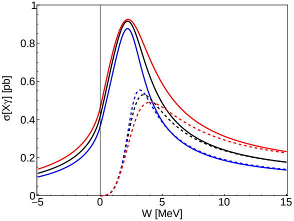
5 bins: 2 MeV/bin

Triangle singularity for $e^+e^- \to \gamma X(3872)$

- There can also be triangle singularity for $e^+e^- \to D^{*0}\bar{D}^{*0} \to \gamma X(3872)$
- $D^{*0}\bar{D}^{*0}$ in P-wave, thus suppressed at threshold, no threshold cusp
- But, still enhanced cross section and a narrow peak at about 2.2 MeV above the $D^{*0}\bar{D}^{*0}$ threshold

E. Braaten, L.-P. He, K. Ingles, PRD100(2019)031501





- For BESIII: to measure cross section for $e^+e^- \to \gamma X(3872)$ between 4009 MeV and 4020 MeV

Summary

- Unprecedented data allow us to study analytic structures of QFT other than the resonance poles
- Triangle singularity as a tool:
  - ☞ TS ⇒ enhanced production of near-threshold states
  - ☞ New method for precisely measuring the $X(3872)$ mass:

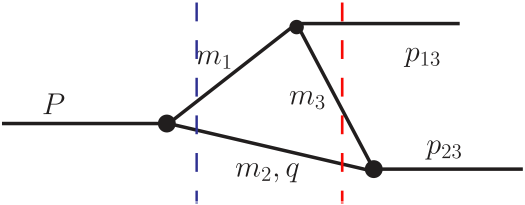
    to measure the $X\gamma$ line shape between 4.01 and 4.02 GeV
- A few possibilities:
  - ☞ BESIII + STCF + Belle-II (ISR):

$$e^+e^- \to \pi^0 D^{*0}\bar{D}^{*0} \to \pi^0\gamma X(3872) \ @ \ \sqrt{s}_{e^+e^-} \sim 4.4 \text{ GeV}$$

  - ☞ B factories: $B \to K D^{*0}\bar{D}^{*0} \to K\gamma X(3872)$
  - ☞ PANDA: $p\bar{p} \to D^{*0}\bar{D}^{*0} \to \gamma X(3872)$

THANK YOU FOR YOUR ATTENTION!

Backup slides



Consider the scalar three-point loop integral

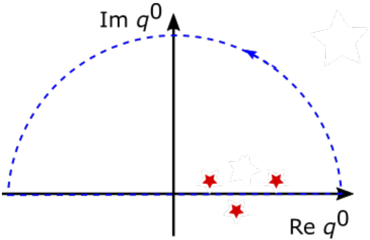
$$I = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{\left[(P-q)^2 - m_1^2 + i\epsilon\right] \left(q^2 - m_2^2 + i\epsilon\right) \left[(p_{23}-q)^2 - m_3^2 + i\epsilon\right]}$$

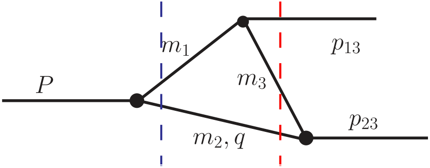
Rewriting a propagator into two poles:

$$\frac{1}{q^2 - m_2^2 + i\epsilon} = \frac{1}{\left(q^0 - \omega_2 + i\epsilon\right)\left(q^0 + \omega_2 - i\epsilon\right)} \quad \text{with} \quad \omega_2 = \sqrt{m_2^2 + \vec{q}^{\,2}}$$

focus on the positive-energy poles

$$I \simeq \frac{i}{8 m_1 m_2 m_3} \int \frac{dq^0 d^3\vec{q}}{(2\pi)^4} \frac{1}{\left(P^0 - q^0 - \omega_1 + i\epsilon\right)\left(q^0 - \omega_2 + i\epsilon\right)\left(p_{23}^0 - q^0 - \omega_3 + i\epsilon\right)}$$





Contour integral over $q^0 \Rightarrow$

$$I \propto \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{[P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon][p_{23}^0 - \omega_2(q) - \omega_3(\vec{p}_{23} - \vec{q}\,) + i\,\epsilon]}$$

$$\propto \int_0^\infty dq\, \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon} f(q)$$

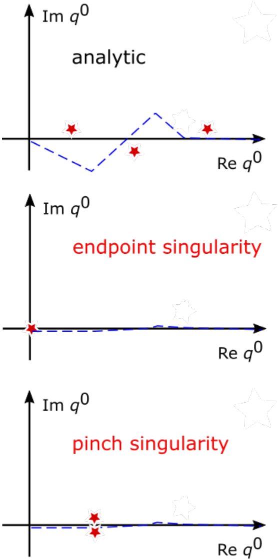
The second cut:

$$f(q) = \int_{-1}^{1} dz\, \frac{1}{p_{23}^0 - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 - 2p_{23}qz} + i\,\epsilon}$$

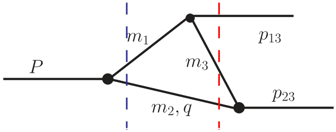
TS: some details (3)

Relation between singularities of integrand and integral

- singularity of integrand does not necessarily give a singularity of integral:
  integral contour may be deformed to avoid the singularity
- Two cases that a singularity cannot be avoided:
  - ☞ endpoint singularity
  - ☞ pinch singularity



TS: some details (4)



$$I \propto \int_0^\infty dq\, \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon} f(q)$$

$$f(q) = \int_{-1}^{1} dz\, \frac{1}{A(q,z)} \equiv \int_{-1}^{1} dz\, \frac{1}{p_{23}^0 - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 - 2p_{23}qz} + i\,\epsilon}$$

Singularities of the **integrand of I** in the rest frame of initial particle ($P^0 = M$):

- 1st cut: $M - \omega_1(l) - \omega_2(l) + i\,\epsilon = 0 \Rightarrow$

$$q_{\text{on}\pm} \equiv \pm\left(\frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_2^2)} + i\,\epsilon\right)$$

- 2nd cut: $A(q, \pm 1) = 0 \Rightarrow$ endpoint singularities of $f(q)$

$$z = +1: \quad q_{a+} = \gamma\,(\beta\, E_2^* + p_2^*) + i\,\epsilon\,, \qquad q_{a-} = \gamma\,(\beta\, E_2^* - p_2^*) - i\,\epsilon\,,$$

$$z = -1: \quad q_{b+} = \gamma\,(-\beta\, E_2^* + p_2^*) + i\,\epsilon\,, \quad q_{b-} = -\gamma\,(\beta\, E_2^* + p_2^*) - i\,\epsilon$$

$$\beta = |\vec{p}_{23}|/E_{23}, \qquad \gamma = 1/\sqrt{1-\beta^2} = E_{23}/m_{23}$$

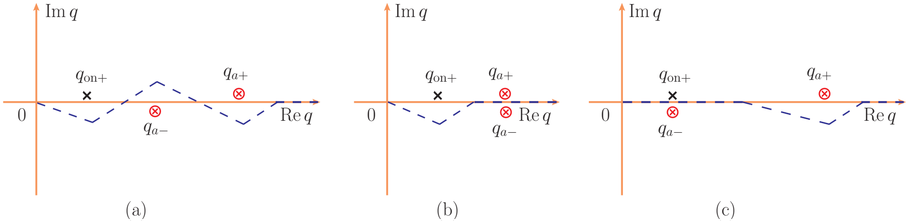
$E_2^*(p_2^*)$: energy (momentum) of particle-2 in the cmf of the (2,3) system

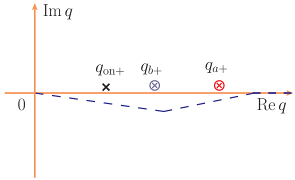
TS: some details (5)

All singularities of the integrand of I:

$$q_{\text{on}+}, \qquad q_{a+} = \gamma\,(\beta\, E_2^* + p_2^*) + i\,\epsilon, \qquad q_{a-} = \gamma\,(\beta\, E_2^* - p_2^*) - i\,\epsilon,$$

$$q_{\text{on}-} < 0, \quad q_{b-} = -q_{a+} < 0 \text{ (for } \epsilon = 0), \qquad q_{b+} = -q_{a-},$$





2-body threshold singularity at

$m_{23} = m_2 + m_3$

triangle singularity at

$p_{2,\text{left}} = p_{2,\text{right}}$

here $p_{2,\text{left}} = q_{\text{on}+}$, $p_{2,\text{right}} = q_{a-}$