Theory of quarkonium electromagnetic transitions

Antonio Vairo

Technische Universität München

Bibliography

(1) N. Brambilla, Y. Jia and A. Vairo
Model-independent study of magnetic dipole transitions in quarkonium
Phys. Rev. D73 (2006) 054005 `hep-ph/0512369`

(2) N. Brambilla, P. Roig and A. Vairo
Precise determination of the η_c mass and width in the radiative $J/\psi \to \eta_c\gamma$ decay
AIP Conf. Proc. 1343 (2011) 418 `arXiv:1012.0773`

(3) N. Brambilla, P. Pietrulewicz and A. Vairo
Model-independent study of electric dipole transitions in quarkonium
Phys. Rev. D85 (2012) 094005 `arXiv:1203.3020`

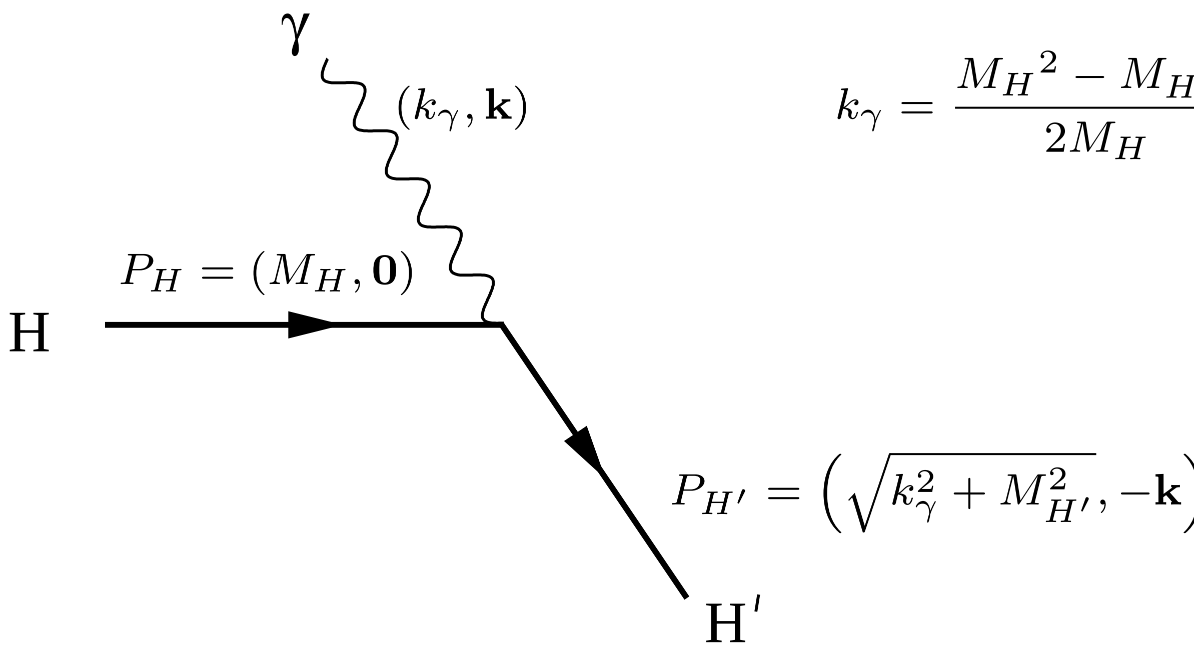
(4) A. Pineda and J. Segovia
Improved determination of heavy quarkonium magnetic dipole transitions in potential NRQCD
Phys. Rev. D87 (2013) 074024 `arXiv:1302.3528`

(5) J. Segovia, S. Steinbeißer and A. Vairo
Electric dipole transitions of $1P$ bottomonia
Phys. Rev. D99 (2019) 074011 `arXiv:1811.07590`

Introduction

Radiative transitions: basics

Two dominant single-photon-transition processes:
(1) magnetic dipole transitions (M1)
(2) electric dipole transitions (E1)



$$k_\gamma = \frac{{M_H}^2 - {M_{H'}}^2}{2M_H}$$

○ QWG coll CERN-2005-005 Yellow Book hep-ph/0412158

M1 transitions in the non-relativistic limit

(1) M1 transitions in the non-relativistic limit:

$$\Gamma^{\rm M1}_{n^3S_1 \to n'^1S_0\,\gamma} = \frac{4}{3}\,\alpha\, e_Q^2\, \frac{k_\gamma^3}{m^2} \left| \int_0^\infty dr\, r^2\, R_{n'0}(r)\, R_{n0}(r)\, j_0\left(\frac{k_\gamma r}{2}\right) \right|^2$$

If $k_\gamma \langle r \rangle \ll 1$ $\quad j_0(k_\gamma r/2) = 1 - (k_\gamma r)^2/24 + \ldots$

- $n = n'$ allowed transitions
- $n \neq n'$ hindered transitions

$\Gamma_{J/\psi \to \eta_c \gamma}$

At leading order in the multipole expansion, M1 (allowed) transition rates are independent from the low-energy dynamics (i.e. the quarkonium wave-function).

As an example consider

$$\Gamma_{J/\psi \to \eta_c \gamma} = \frac{16}{27}\, \alpha\, \frac{k_\gamma^3}{m_c^2} \approx 2.83\,\text{keV}$$

from $M_{J/\psi} \approx 3097$ MeV and $M_{\eta_c} \approx 2984$ MeV ($k_\gamma \approx 111$ MeV).

To be compared with the PDG value $\Gamma_{J/\psi \to \eta_c\, \gamma} = 1.6 \pm 0.4\,\text{keV}$.

E1 transitions in the non-relativistic limit

(2) E1 transitions in the non-relativistic limit:

$$\Gamma^{\rm E1}_{n^{2S+1}L_J \to n'^{2S+1}L'_{J'}\,\gamma} = \frac{4}{3}\,\alpha e_Q^2\, k_\gamma^3\, [\mathcal{E}(nL \to n'L')]^2\, (2J'+1)\, \text{max}_{\{L,L'\}} \begin{Bmatrix} J & 1 & J' \\ L' & S & L \end{Bmatrix}^2$$

where

$$\begin{aligned} \mathcal{E}(nL \to n'L') &= \int_0^\infty dr\, r^2\, R_{n'L'}(r)\, R_{nL}(r) \left[\frac{k_\gamma r}{2} j_0\left(\frac{k_\gamma r}{2}\right) - j_1\left(\frac{k_\gamma r}{2}\right)\right] \\ &\approx I_3(nL \to n'L') \times \left[1 + \mathcal{O}\left((k_\gamma r)^2\right)\right] \quad \text{if } k_\gamma \langle r\rangle \ll 1 \\ I_N(nL \to n'L') &= \int_0^\infty dr\, r^N\, R_{n'L'}(r)\, R_{nL}(r) \end{aligned}$$

Note that, for equal energies and masses, M1 transitions are suppressed by a factor $1/(m\langle r\rangle)^2 \sim v^2$ with respect to E1 transitions, which are much more common.

E.g. $\mathcal{B}(\chi_{b0}(1P) \to \Upsilon(1S)\gamma) = (1.94 \pm 0.27)\,\%$, $\mathcal{B}(\chi_{b1}(1P) \to \Upsilon(1S)\gamma) = (35.0 \pm 2.1)\,\%$, $\mathcal{B}(\chi_{b2}(1P) \to \Upsilon(1S)\gamma) = (18.8 \pm 1.1)\,\%$, and $\mathcal{B}(h_b(1P) \to \eta_b(1S)\gamma) = (52^{+6}_{-5})\%$.

$\Gamma_{\chi_c(1P)\to J/\psi\gamma}/\Gamma_{\chi_b(3P)\to\Upsilon(3S)\gamma}$

Even at leading order in the multipole expansion, E1 transition rates depend on the low-energy dynamics (i.e. on the quarkonium wave-function).

As an example consider

$$\frac{\Gamma_{\chi_c(1P)\to J/\psi\gamma}}{\Gamma_{\chi_b(3P)\to\Upsilon(3S)\gamma}} \approx \frac{e_c^2\; k_\gamma^{(c)\,3}\;\; \langle r^2\rangle^{(c)}}{e_b^2\; k_\gamma^{(b)\,3}\;\; \langle r^2\rangle^{(b)}} \approx 33^{+16}_{-9}$$

from $M_{\chi_c(1P)} \approx h_c(1P) \approx 3525$ MeV, $M_{J/\psi} \approx 3097$ MeV, $M_{\chi_b(3P)} \approx 10530$ MeV, $M_{\Upsilon(3S)} \approx 10355$ MeV [$k_\gamma^{(c)} \approx 402$ MeV and $k_\gamma^{(b)} \approx 174$ MeV], and assuming $\langle r^2\rangle^{(b)} \approx (1.5 \pm 0.5) \times \langle r^2\rangle^{(c)}$.

Relativistic corrections

- Relativistic corrections may be sizeable:
  about 30% for charmonium ($v_c^2 \approx 0.3$) and 10% for bottomonium ($v_b^2 \approx 0.1$).

- For quarkonium radiative transitions, essentially one model-dependent calculation has been used for over twenty years to account for relativistic corrections, based upon:

  relativistic equation with scalar and vector potentials;

  non-relativistic reduction;

  a somewhat imposed relativistic invariance to calculate recoil corrections.

◦ `Grotch Owen Sebastian PR D30 (1984) 1924`
`see also QWG CERN Yellow Book CERN-2005-005, hep-ph/0412158`

Effective Field Theories

Relativistic corrections and EFTs

Nowadays, however, effective field theories (EFTs) for quarkonium allow

- to derive expressions of radiative transitions directly from QCD;
- with a well specified range of applicability;
- to determine a reliable error associated with the theoretical determinations;
- to improve the theoretical determinations in a systematic way.

◦ Brambilla Pineda Soto Vairo RMP 77 (2005) 1423

Energy scales

- $p \sim \frac{1}{r} \sim mv$, $E \sim mv^2$; in a non-relativistic system $v \ll 1$
- $\Lambda_{\rm QCD}$
- k_γ

$mv^2 \gtrsim \Lambda_{\rm QCD}$ for weakly-coupled quarkonia (J/ψ, η_c, $\Upsilon(1S)$, $\eta_b(1S)$, ...);
$mv^2 \ll \Lambda_{\rm QCD} \lesssim mv$ for strongly-coupled quarkonia (excited states);

$k_\gamma \sim mv^2$ for hindered M1 transitions, most E1 transitions;
$k_\gamma \sim mv^4$ for allowed M1 transitions.

$\Rightarrow \quad k_\gamma r \ll 1$

Degrees of freedom

- Degrees of freedom at scales of order mv^2:

  Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\rm QCD}$, mv^2 and momentum $\lesssim mv$
  $\Rightarrow$ *(i)* singlet $\rm S$ *(ii)* octet $\rm O$ [if $mv^2 \gtrsim \Lambda_{\rm QCD}$]

  Gluons with energy and momentum $\sim \Lambda_{\rm QCD}$, mv^2 [if $mv^2 \gtrsim \Lambda_{\rm QCD}$]

  Photons of energy and momentum of order mv^2.

- Power counting:

  $p \sim \frac{1}{r} \sim mv$;
  all gauge fields are multipole expanded: $A(R,r,t) = A(R,t) + \mathbf{r}\cdot\boldsymbol{\nabla} A(R,t) + \dots$
  and scale like $(\Lambda_{\rm QCD}$ or $mv^2)^{\rm dimension}$.

EFT Lagrangian

$$\mathcal{L}_{\rm pNRQCD} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} - \frac{1}{4}F^{\rm em}_{\mu\nu}F^{\mu\nu\,\rm em} + \sum_{\ell=1}^{n_\ell}\bar{q}_\ell\,(i\gamma_\mu D^\mu - m_\ell)\,q_\ell$$

$$+\int d^3r\,{\rm Tr}\left\{{\rm S}^\dagger\left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s\right){\rm S}\right.$$

$$\left. + {\rm O}^\dagger\left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o\right){\rm O}\right\}$$

LO in r

$$+{\rm Tr}\left\{{\rm O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,{\rm S} + {\rm S}^\dagger\mathbf{r}\cdot g\mathbf{E}\,{\rm O}\right\}$$

$$+\frac{1}{2}{\rm Tr}\left\{{\rm O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,{\rm O} + {\rm O}^\dagger{\rm O}\mathbf{r}\cdot g\mathbf{E}\right\}$$ [if $mv^2 \gtrsim \Lambda_{\rm QCD}$]

$$+\cdots$$

NLO in r

$$+\mathcal{L}_\gamma$$

$\mathcal{L}_\gamma$

$$\mathcal{L}_\gamma = \mathcal{L}_\gamma^{\mathrm{M1}} + \mathcal{L}_\gamma^{\mathrm{E1}} + \ldots$$

$$\begin{aligned}
\mathcal{L}_\gamma^{\mathrm{M1}} = \mathrm{Tr}\Bigg\{ & \frac{1}{2m} V_1^{\mathrm{M1}} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma} \cdot ee_Q \mathbf{B}^{\mathrm{em}} \right\} \mathrm{S} \\
& + \frac{1}{2m} V_1^{\mathrm{M1}} \left\{ \mathrm{O}^\dagger, \boldsymbol{\sigma} \cdot ee_Q \mathbf{B}^{\mathrm{em}} \right\} \mathrm{O} \qquad [\text{if } mv^2 \gtrsim \Lambda_{\mathrm{QCD}}] \\
& + \frac{1}{4m^2} \frac{V_2^{\mathrm{M1}}}{r} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma} \cdot \left[\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times ee_Q \mathbf{B}^{\mathrm{em}} \right) \right] \right\} \mathrm{S} \\
& + \frac{1}{4m^2} \frac{V_3^{\mathrm{M1}}}{r} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma} \cdot ee_Q \mathbf{B}^{\mathrm{em}} \right\} \mathrm{S} \\
& + \frac{1}{4m^3} V_4^{\mathrm{M1}} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma} \cdot ee_Q \mathbf{B}^{\mathrm{em}} \right\} \boldsymbol{\nabla}_r^2 \mathrm{S} + \cdots \Bigg\}
\end{aligned}$$

- Brambilla Jia Vairo PR D73 (2006) 054005

$\mathcal{L}_\gamma$

$$
\begin{aligned}
\mathcal{L}_\gamma^{\mathrm{E1}} \quad = \quad & \mathrm{Tr}\Big\{ V_1^{\mathrm{E1}}\, \mathrm{S}^\dagger \mathbf{r}\cdot ee_Q \mathbf{E}^{\mathrm{em}} \mathrm{S} \\
& + V_1^{\mathrm{E1}}\, \mathrm{O}^\dagger \mathbf{r}\cdot ee_Q \mathbf{E}^{\mathrm{em}} \mathrm{O} \qquad [\text{if } mv^2 \gtrsim \Lambda_{\mathrm{QCD}}] \\
& + \frac{1}{24}\, V_2^{\mathrm{E1}}\, \mathrm{S}^\dagger \mathbf{r}\cdot [(\mathbf{r}\cdot \boldsymbol{\nabla})^2 ee_Q \mathbf{E}^{\mathrm{em}}]\mathrm{S} \\
& + \frac{i}{4m}\, V_3^{\mathrm{E1}}\, \mathrm{S}^\dagger \{\boldsymbol{\nabla}\cdot, \mathbf{r}\times ee_Q \mathbf{B}^{\mathrm{em}}\}\mathrm{S} \\
& + \frac{i}{12m}\, V_4^{\mathrm{E1}}\, \mathrm{S}^\dagger \{\boldsymbol{\nabla}_r\cdot, \mathbf{r}\times [(\mathbf{r}\cdot\boldsymbol{\nabla}) ee_Q \mathbf{B}^{\mathrm{em}}]\}\mathrm{S} \\
& + \frac{1}{4m}\, V_5^{\mathrm{E1}}\, [\mathrm{S}^\dagger, \boldsymbol{\sigma}]\cdot [(\mathbf{r}\cdot\boldsymbol{\nabla}) ee_Q \mathbf{B}^{\mathrm{em}}]\mathrm{S} \\
& - \frac{i}{4m^2}\, V_6^{\mathrm{E1}}\, [\mathrm{S}^\dagger, \boldsymbol{\sigma}]\cdot (ee_Q \mathbf{E}^{\mathrm{em}} \times \boldsymbol{\nabla}_r)\mathrm{S} + \cdots \Big\}
\end{aligned}
$$

- Brambilla Pietrulewicz Vairo PR D85 (2012) 094005

Matching

The matching consists in the calculation of the coefficients V.
They get contributions from

- hard modes ($\sim m$):

$$\bar{\psi}(i\not{D} - m)\psi \to \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{c_F^{\rm em}}{2m}\boldsymbol{\sigma}\cdot ee_Q\mathbf{B}^{\rm em} + \cdots \right)\psi$$

  From HQET:

$$c_F^{\rm em} \equiv 1 + \kappa^{\rm em} = 1 + \frac{2}{3}\frac{\alpha_{\rm s}}{\pi} + \ldots$$

  is the quark magnetic moment.

  ○ Grozin Marquard Piclum Steinhauser NP B789 (2008) 277 (3 loops)

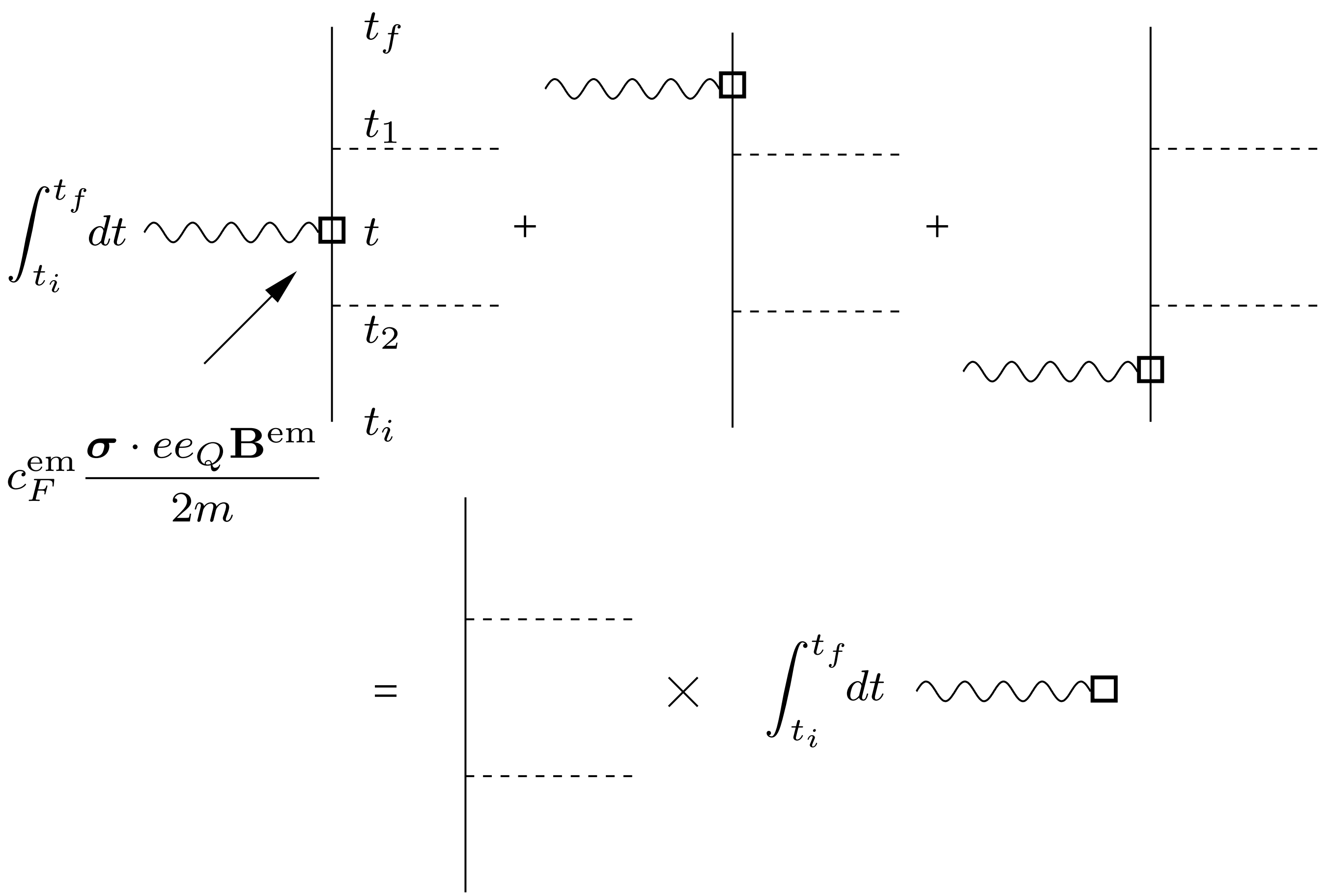
- soft modes ($\sim mv$).

M1 operator at $\mathcal{O}(1)$

$$V_1^{\mathrm{M1}} \left\{ S^\dagger, \frac{\boldsymbol{\sigma} \cdot e\mathbf{B}^{\mathrm{em}}}{2m} \right\} S$$

$$V_1^{\mathrm{M1}} = \left(\text{hard}\right) \times \left(\text{soft}\right)$$

- $\left(\text{hard}\right) = c_F^{\mathrm{em}} = 1 + \dfrac{2\alpha_{\mathrm{s}}(m)}{3\pi} + \cdots$
- Since $\boldsymbol{\sigma} \cdot e\mathbf{B}^{\mathrm{em}}(\mathbf{R})$ behaves like the identity operator to all orders V_1^{M1} does not get soft contributions.



Diagrammatic factorization of the magnetic dipole coupling in the SU(3)$_f$ limit.

The argument is similar to the factorization of the QCD corrections in $b \to u\, e^- \bar{\nu}_e$*, which leads to*

$\mathcal{L}_{\rm eff} = -4G_F/\sqrt{2}\; V_{ub}\; \bar{e}_L\gamma_\mu\nu_L\; \bar{u}_L\gamma^\mu b_L$ *to all orders in* $\alpha_{\rm s}$.

M1 operator at $\mathcal{O}(1)$

$$V_1^{\mathrm{M1}} \left\{ S^\dagger, \frac{\boldsymbol{\sigma} \cdot e\mathbf{B}^{\mathrm{em}}}{2m} \right\} S$$

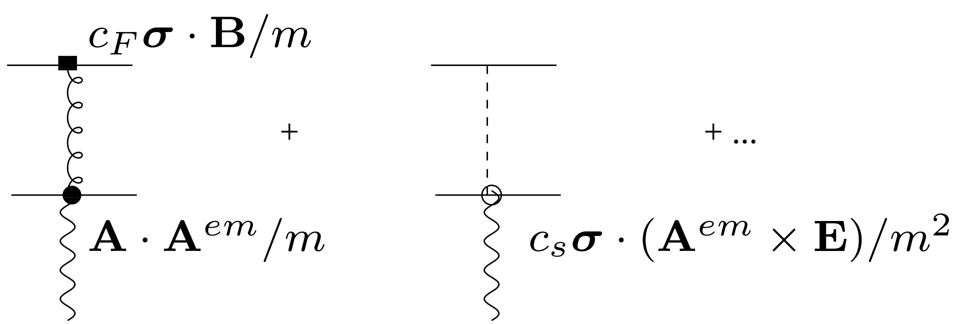
$$V_1^{\mathrm{M1}} = 1 + \frac{2\alpha_{\mathrm{s}}(m)}{3\pi} + \cdots$$

No large quarkonium anomalous magnetic moment!

- Dudek Edwards Richards PR D73 (2006) 074507 (lattice)

M1 operators at $\mathcal{O}(v^2)$

$$\frac{1}{4m^2}\,\frac{V_2^{\rm M1}}{r}\,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times ee_Q\mathbf{B}^{\rm em}\right)\right]\right\}\mathrm{S} \quad \text{and} \quad \frac{1}{4m^2}\,\frac{V_3^{\rm M1}}{r}\,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q\mathbf{B}^{\rm em}\right\}\mathrm{S}$$



$$= \left(\text{hard}\right)\times\left(\text{soft}\right)$$

- to all orders $\left(\text{hard}\right) = 2c_F - c_s = 1$; $\left(\text{soft}\right) = r^2 V_s'/2$
  - Brambilla Gromes Vairo PL B576 (2003) 314 (Poincaré invariance)
    Luke Manohar PL B286 (1992) 348 (reparameterization invariance)

$$V_2^{\rm M1} = r^2 V_s'/2 \quad \text{and} \quad V_3^{\rm M1} = 0$$

No (effective) scalar interaction!

M1 operators at $\mathcal{O}(v^2)$

$$V_4^{\rm M1} \left\{ S^\dagger, \frac{\boldsymbol{\sigma}\cdot e\mathbf{B}^{em}}{4m^3} \right\} \boldsymbol{\nabla}_r^2 S$$

$$V_4^{\rm M1} = \left(\text{hard}\right) \times \left(\text{soft}\right)$$

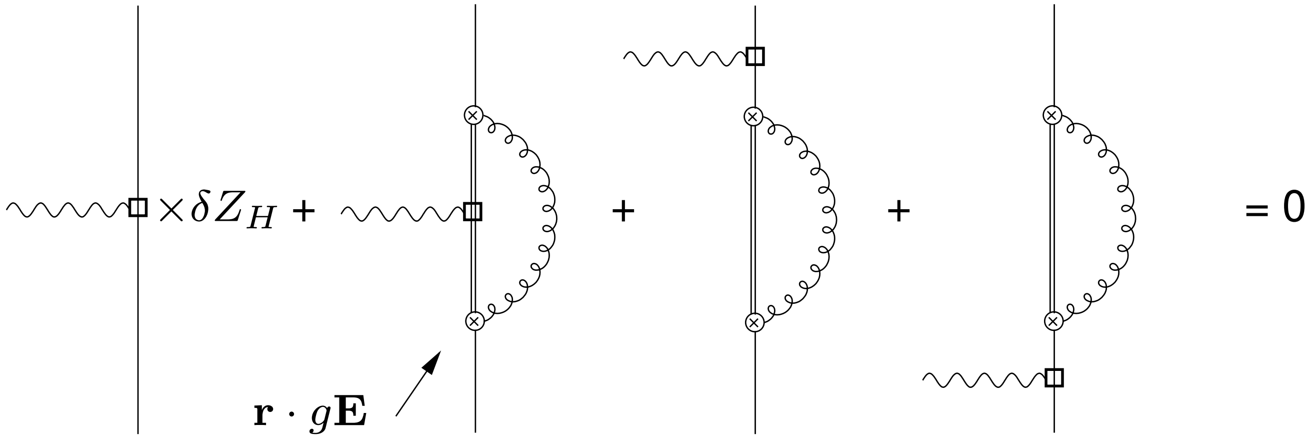
- $\left(\text{hard}\right) = 1$
  - Manohar PR D56 (1997) 230 (reparameterization invariance)

- $\left(\text{soft}\right) = 1$ to all orders
  - Brambilla Pietrulewicz Vairo PR D85 (2012) 094005

$$V_4^{\rm M1} = 1$$

$\mathcal{O}(v^2)$ wave-function corrections to weakly-coupled quarkonia

Coupling of photons with octets: $V_1^{\rm M1}\left\{O^\dagger, \frac{\boldsymbol{\sigma}\cdot e\mathbf{B}^{em}}{2m}\right\}O$ [if $mv^2 \gtrsim \Lambda_{\rm QCD}$]



- If $mv^2 \sim \Lambda_{\rm QCD}$ the above graphs are potentially of order $\Lambda_{\rm QCD}^2/(mv)^2 \sim v^2$.
- The contribution vanishes, for $\boldsymbol{\sigma}\cdot e\mathbf{B}^{em}(\mathbf{R})$ behaves like the identity operator.
- There are no non-perturbative contributions at $\mathcal{O}(v^2)$!

- This is not the case for strongly-coupled quarkonia:

  non-perturbative corrections affect the operator $\frac{1}{m^3}\,\frac{V_5^{\rm M1}}{r^2}\,\left\{{\rm S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q\mathbf{B}^{\rm em}\right\}{\rm S}$.

M1 hindered transitions

- One new operator contributes:

$$-\frac{1}{16m^2}\, c_S^{\rm em}\, \left[{\rm S}^\dagger, \boldsymbol{\sigma}\cdot\left[-i\boldsymbol{\nabla}_r\times, {\bf r}^i(\boldsymbol{\nabla}^i e e_Q {\bf E}^{\rm em})\right]\right]{\rm S}$$

- Two new wave-function corrections contribute:

(1) induced by the spin-spin potential $V^{\rm ss}$;

(2) recoil correction induced by the spin-orbit potential.

Due to the recoil, the final state develops a nonzero P-wave component suppressed by a factor $v\, k_\gamma/m$ (through the spin-orbit operator $-\frac{1}{4\,m^2}\frac{V_S^{(0)\,\prime}}{2}\,{\rm Tr}\left\{\{{\rm S}^\dagger, \boldsymbol{\sigma}\}\cdot[\hat{\bf r}\times(-i\boldsymbol{\nabla})]\,{\rm S}\right\}$), which, in a $n^3S_1 \to n'\,^1S_0\,\gamma$ transition, can be reached from the initial $^3S_1$ state through a $1/v$ enhanced E1 transition.

M1 transitions

$$\Gamma_{n^3S_1 \to n^1S_0\,\gamma} = \frac{4}{3}\alpha e_Q^2 \frac{k_\gamma^3}{m^2}\left[1 + \frac{4\alpha_s(m)}{3\pi} - \frac{5}{3}\langle nS|\frac{\mathbf{p}^2}{m^2}|nS\rangle\right]$$

$$\Gamma_{n^3S_1 \to n'^1S_0\,\gamma} = \frac{4}{3}\alpha e_Q^2 \frac{k_\gamma^3}{m^2}\left[\langle n'S|\left(-\frac{k_\gamma^2\mathbf{r}^2}{24} - \frac{5}{6}\frac{\mathbf{p}^2}{m^2}\right)|nS\rangle + \frac{1}{m^2}\frac{\langle n'S|V^{\rm ss}(\mathbf{r})|nS\rangle}{E_n^{(0)} - E_{n'}^{(0)}}\right]^2 \quad \text{for} \quad n \neq n'$$

$$\Gamma_{n^3P_J \to n^1P_1\,\gamma} = \frac{4}{3}\alpha e_Q^2 \frac{k_\gamma^3}{m^2}\left[1 + \frac{4\alpha_s(m)}{3\pi} - d_J\,\langle nP|\frac{\mathbf{p}^2}{m^2}|nP\rangle\right]$$

$$\Gamma_{n^1P_1 \to n^3P_J\,\gamma} = (2J+1)\frac{4}{9}\alpha e_Q^2 \frac{k_\gamma^3}{m^2}\left[1 + \frac{4\alpha_s(m)}{3\pi} - d_J\,\langle nP|\frac{\mathbf{p}^2}{m^2}|nP\rangle\right]$$

where $d_0 = 1$, $d_1 = 2$ and $d_2 = 8/5$.

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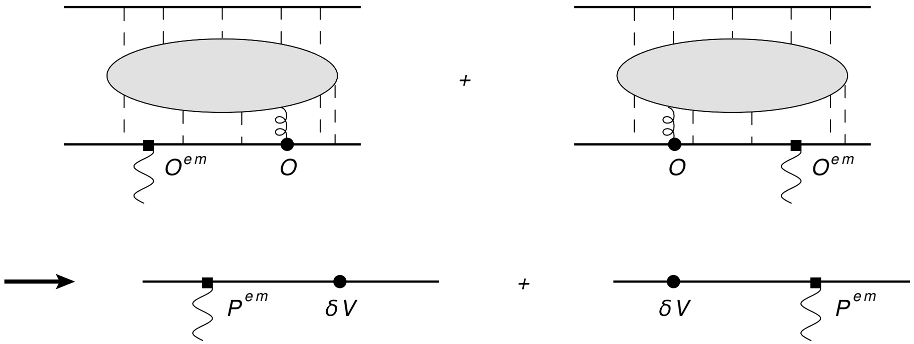
E1 transitions

E1 transitions always involve excited states.

- Operators contributing at relative order v^2 to E1 transitions are not affected by (perturbative or non-perturbative) soft corrections.



$$V_1^{\mathrm{E1}} = V_2^{\mathrm{E1}} = V_3^{\mathrm{E1}} = V_4^{\mathrm{E1}} = 1$$

$$V_5^{\mathrm{E1}} = c_F^{\mathrm{em}} = 1 + \frac{2\alpha_{\mathrm{s}}(m)}{3\pi} + \cdots, \qquad V_6^{\mathrm{E1}} = 2c_F^{\mathrm{em}} - 1 = 1 + \frac{4\alpha_{\mathrm{s}}(m)}{3\pi} + \cdots$$

○ Brambilla Pietrulewicz Vairo PR D85 (2012) 094005

E1 transitions

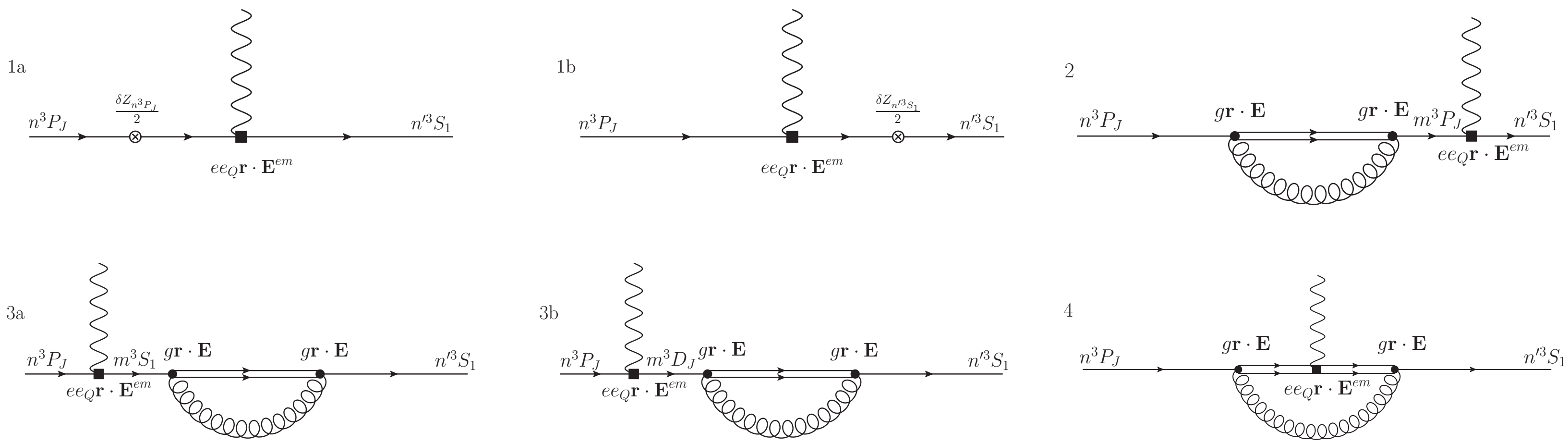
E1 transitions always involve excited states.

- However, non-perturbative corrections affect the quarkonium wave-functions: at large distances the quarkonium potentials are non-perturbative.
- For weakly-coupled quarkonia, non-perturbative corrections to the quarkonium wave-functions also involve octet fields and are of

  - relative order v^2 (if $\Lambda_{\rm QCD} \sim mv^2$)
  - or suppressed by $\alpha_{\rm s} v^2$ (if $\Lambda_{\rm QCD} \ll mv^2$).

  Unlike M1 dipoles, E1 dipoles do not commute with the octet Hamiltonian.

E1 transitions: color octet contributions



where

$$
\begin{aligned}
\delta Z_{H(\lambda)} &= -\frac{\partial \delta E_{H(\lambda)}}{\partial E_H^{(0)}} \\
&= -\frac{1}{6}\int_0^\infty dt\, t\, \langle 0|g\mathbf{E}^a(\mathbf{R},t)\phi(t,0)^{\text{adj}}_{ab}g\mathbf{E}^b(\mathbf{R},0)|0\rangle \, \langle H(\lambda)|\mathbf{r}e^{-i(\mathcal{H}_O^{(0)}-E_H^{(0)})t}\mathbf{r}|H(\lambda)\rangle
\end{aligned}
$$

and similarly for the other contributions.

E1 transitions

$$\Gamma_{n^3P_J\to n'^3S_1\,\gamma} = \Gamma^{\rm E1}_{n^3P_J\to n'^3S_1\,\gamma}\left[1+R^{S=1}_{nn'}(J)-\frac{k_\gamma^2}{60}\frac{I_5(n1\to n'0)}{I_3(n1\to n'0)} -\frac{k_\gamma}{6m}+\kappa^{\rm em}\frac{k_\gamma}{2m}\left(\frac{J(J+1)}{2}-2\right)\right]$$

$$\Gamma_{n^1P_1\to n'^1S_0\,\gamma} = \Gamma^{\rm E1}_{n^1P_1\to n'^1S_0\,\gamma}\left[1+R^{S=0}_{nn'}-\frac{k_\gamma}{6m}-\frac{k_\gamma^2}{60}\frac{I_5(n1\to n'0)}{I_3(n1\to n'0)}\right]$$

$$\Gamma_{n^3S_1\to n'^3P_J\,\gamma} = \frac{2J+1}{3}\Gamma^{\rm E1}_{n^3S_1\to n'^3P_J\,\gamma}\left[1+R^{S=1}_{nn'}(J)-\frac{k_\gamma^2}{60}\frac{I_5(n'1\to n0)}{I_3(n'1\to n0)} +\frac{k_\gamma}{6m}-\kappa^{\rm em}\frac{k_\gamma}{2m}\left(\frac{J(J+1)}{2}-2\right)\right]$$

where $R^{S=1}_{nn'}(J)$ and $R^{S=0}_{nn'}$ are the (non-perturbative) initial and final state corrections.

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$$J/\psi \rightarrow \eta_c \gamma$$

$J/\psi \to \eta_c \gamma$

$$\Gamma_{J/\psi \to \eta_c \gamma} = \int \frac{d^3k}{(2\pi)^3} \, (2\pi)\delta(E_p^{J/\psi} - k - E_k^{\eta_c}) \, |\langle \gamma(k) \eta_c | \mathcal{L}_\gamma | J/\psi \rangle|^2$$

We assume the J/ψ and η_c to be Coulombic and weakly coupled: $mv^2 \gtrsim \Lambda_{\rm QCD}$.

$J/\psi \to \eta_c\gamma$

Up to order v^2 the transition $J/\psi \to \eta_c\gamma$ is completely accessible by perturbation theory.

$$\Gamma_{J/\psi\to\eta_c\gamma} = \frac{16}{3}\alpha e_c^2 \frac{k_\gamma^3}{M_{J/\psi}^2}\left[1 + 4\frac{\alpha_s(M_{J/\psi}/2)}{3\,\pi} - \frac{32}{27}\alpha_s(p_{J/\psi})^2\right]$$

The normalization scale for the α_s inherited from κ^{em} is the charm mass ($\alpha_s(M_{J/\psi}/2) \approx 0.35 \sim v^2$), and for the α_s, which comes from the Coulomb potential, is the typical momentum transfer $p_{J/\psi} \approx 2m\alpha_s(p_{J/\psi})/3 \approx 0.8$ GeV $\sim mv$.

$$\Gamma_{J/\psi\to\eta_c\gamma} = (1.5 \pm 1.0)\ \text{keV}$$

to be compared with the non-relativistic result ≈ 2.83 keV.

∘ Brambilla Jia Vairo PR D73 (2006) 054005

Improved determination of M1 transitions

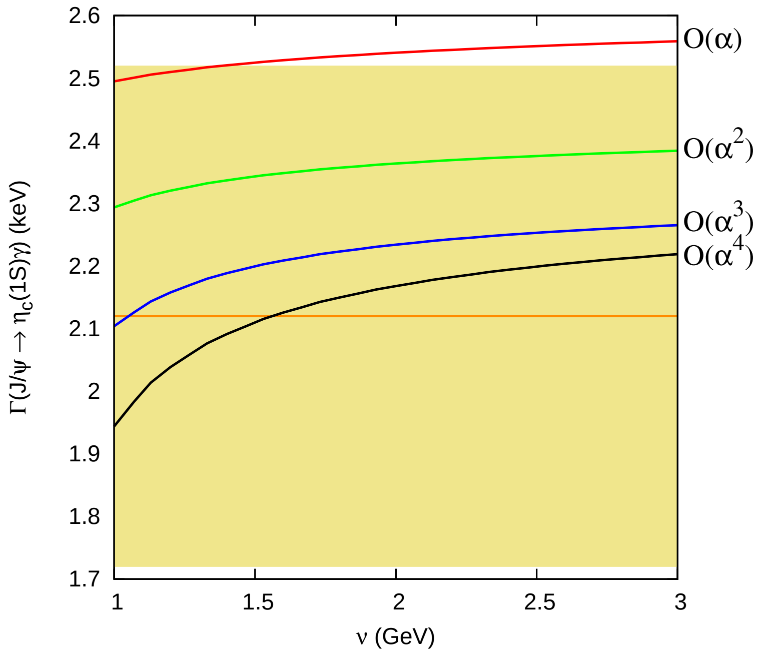
- Exact incorporation of the static potential.
- Renormalon cancellation.

$$V_{s,\mathrm{RS}'}(\nu,\nu_f,\nu_r,r) = \begin{cases} V_s + 2\delta m_{\mathrm{RS}'}\big|_{\nu=1/r} \equiv \sum_{k=0}^{3} V_{s,\mathrm{RS}'}^{(k)} \alpha_{\mathrm{s}}^{k+1}(1/r) & \text{if } r < \nu_r^{-1}\,, \\ V_s + 2\delta m_{\mathrm{RS}'}\big|_{\nu=\nu} \equiv \sum_{k=0}^{3} V_{s,\mathrm{RS}'}^{(k)} \alpha_{\mathrm{s}}^{k+1}(\nu) & \text{if } r > \nu_r^{-1}\,. \end{cases}$$

○ Kiyo Pineda Signer NP B841 (2010) 231

Improved determination of M1 transitions

- Exact incorporation of the static potential.
- Renormalon cancellation.



$$\Gamma_{J/\psi\to\eta_c\gamma} = 2.12 \pm 0.40 \text{ keV}$$

- Pineda Segovia PR D87 (2013) 074024

$\Gamma_{J/\psi\to\eta_c\gamma}$ as a probe of the J/ψ potential

$$\Gamma_{J/\psi\to\eta_c\gamma} = \frac{16}{3}\alpha e_c^2 \frac{k_\gamma^3}{M_{J/\Psi}^2}\left(1 + \frac{4}{3}\frac{\alpha_s(M_{J/\Psi}/2)}{\pi} - \frac{2}{3}\frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2\frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}}\right)$$

- If $V_s = -\frac{4}{3}\frac{\alpha_s(\mu)}{r}$: $-\frac{2}{3}\frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2\frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}} = -\frac{32}{27}\alpha_s(\mu)^2 < 0$
- *If* $V_s = \sigma r$: $-\frac{2}{3}\frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2\frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}} = \frac{4}{3}\frac{\sigma}{M_{J/\Psi}}\langle 1|r|1\rangle > 0$

  A scalar interaction would add a negative contribution: $-2\langle 1|V^{\text{scalar}}|1\rangle/M_{J/\Psi}$.

$J/\psi \to \eta_c\gamma$ (experimental status)

- Only one direct experimental measurement existed for long time:

$$\Gamma_{J/\psi\to\eta_c\gamma} = (1.14 \pm 0.23)\ \text{keV}$$

  - Crystal Ball coll. PR D34 (1986) 711

- The situation changed in the last few years:

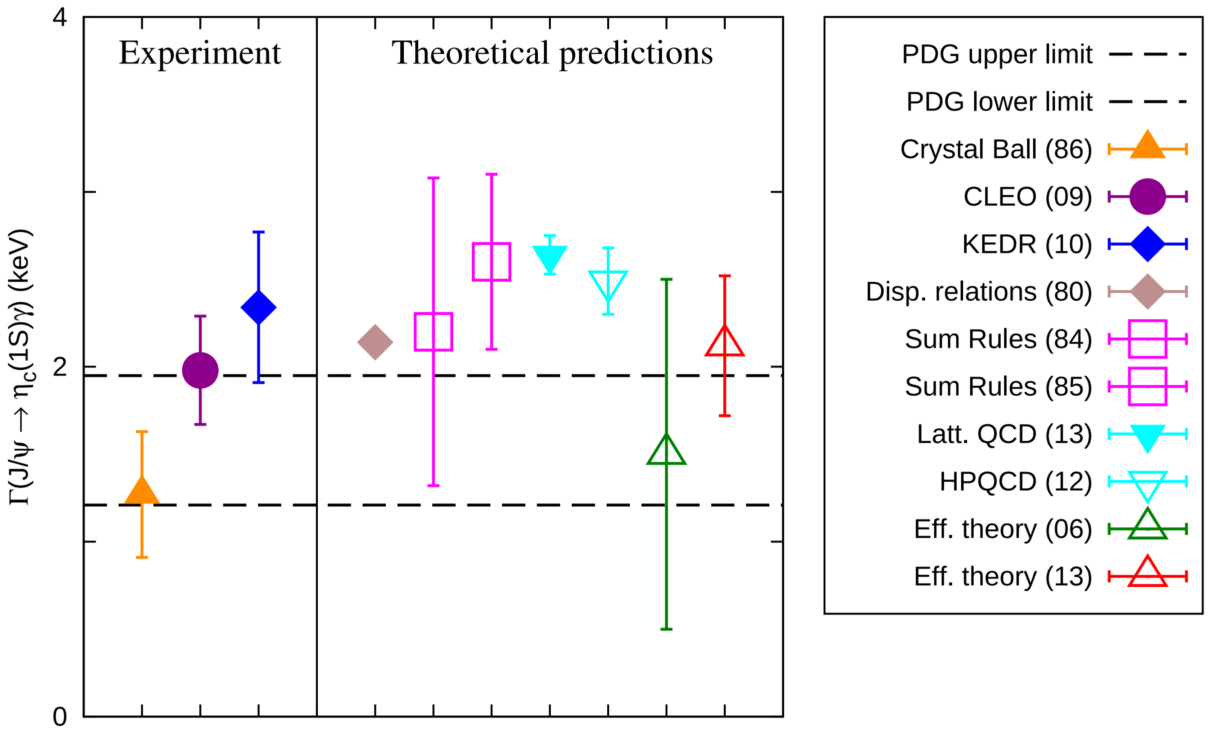
$$\Gamma_{J/\psi\to\eta_c\gamma} = (1.85 \pm 0.08 \pm 0.28)\ \text{keV}$$

  - CLEO coll. PRL 102 (2009) 011801

$$\Gamma_{J/\psi\to\eta_c\gamma} = (2.98 \pm 0.18^{+0.15}_{-0.33})\ \text{keV}$$

  - from KEDR coll. PL B738 (2014) 391
    number not reported by the PDG!

$J/\psi \to \eta_c\gamma$ (experimental & theoretical status)



- Pineda Segovia PR D87 (2013) 074024

$$\Gamma_{\Upsilon(1S)\to\eta_b(1S)\gamma}$$

$\Gamma_{\Upsilon(1S)\to\eta_b(1S)\gamma}$

We assume the $\Upsilon(1S)$ and $\eta_b(1S)$ to be Coulombic and weakly coupled: $mv^2 \gtrsim \Lambda_{\rm QCD}$.

In the improved perturbative framework

$$\Gamma_{\Upsilon(1S)\to\eta_b(1S)\gamma} = (15.18 \pm 0.51)\ {\rm eV}$$

- Pineda Segovia PR D87 (2013) 074024

To be compared with the NLO calculation (without resummation):

$$\Gamma_{\Upsilon(1S)\to\eta_b(1S)\gamma} = (k_\gamma/71\ \text{MeV})^3\,(15.1 \pm 1.5)\ \text{eV}$$

- Brambilla Jia Vairo PR D73 (2006) 054005

M1 hindered transitions

$\Gamma_{\Upsilon(2S)\to\eta_b(1S)\gamma}$, $\Gamma_{h_b(1P)\to\chi_{b0,1}(1P)\gamma}$ and $\Gamma_{\chi_{b2}(1P)\to h_b(1P)\gamma}$

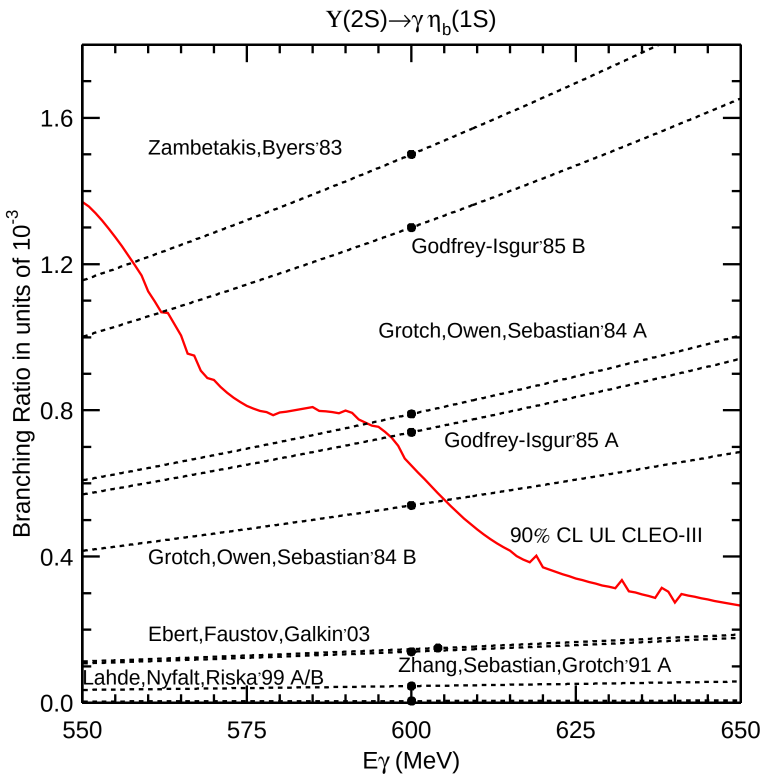
We assume $n = 2$ bottomonia to be Coulombic and weakly coupled: $mv^2 \gtrsim \Lambda_{\rm QCD}$.

$$\Gamma_{h_b(1P)\to\chi_{b0}(1P)\,\gamma} = 0.962 \pm 0.035 \text{ eV}$$
$$\Gamma_{h_b(1P)\to\chi_{b1}(1P)\,\gamma} = 8.99 \pm 0.55 \text{ meV}$$
$$\Gamma_{\chi_{b2}(1P)\to h_b(1P)\,\gamma} = 118 \pm 6 \text{ meV}$$

$$\Gamma_{\Upsilon(2S)\to\eta_b(1S)\gamma} = 6^{+26}_{-6} \text{ eV}$$

- Pineda Segovia PR D87 (2013) 074024

$\Upsilon(2S) \to \eta_b(1S)\gamma$

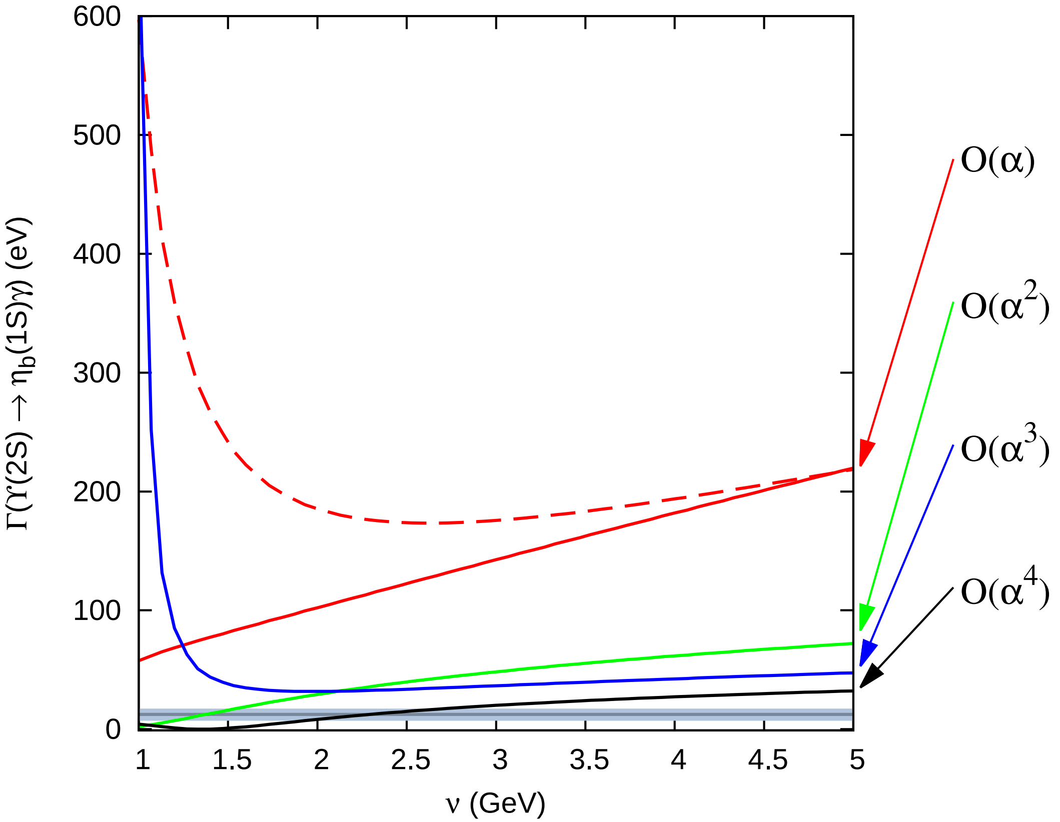


- Already CLEO's upper limit was problematic for many models.
- More recently $\mathrm{BR}_{\Upsilon(2S)\to\eta_b(1S)\gamma} = 3.9 \pm 1.1^{+1.1}_{-0.9} \times 10^{-4}$ [BABAR]
  and $\mathrm{BR}_{\Upsilon(2S)\to\eta_b(1S)\gamma} = 6.1^{+0.6}_{-0.7}{}^{+0.9}_{-0.6} \times 10^{-4}$ [Belle]
  - BABAR PRL 103 (2009) 161801, Belle PRL 121 (2018) 232001

But ok with $\Gamma_{\Upsilon(2S)\to\eta_b(1S)\gamma} = 6^{+26}_{-6}$ eV, $\mathrm{BR}_{\Upsilon(2S)\to\eta_b(1S)\gamma} = 2^{+9}_{-2} \times 10^{-4}$, $k_\gamma = 612$ MeV.

$\Upsilon(2S) \to \eta_b(1S)\gamma$

Resummation of the static potential contributions is crucial for $\Gamma_{\Upsilon(2S)\to\eta_b(1S)\gamma}$.



- Strong cancellation between $-\langle 1S|5\mathbf{p}^2/(6m^2)|2S\rangle$ and $\langle 1S|V^{ss}/[m^2(E_2^{(0)}-E_1^{(0)})]|2S\rangle$.
  - Pineda Segovia PR D87 (2013) 074024

E1 transitions

E1 charmonium transitions

E1 transitions are sensitive to the quarkonium wave-function even at leading order.

A systematic treatment that includes in a consistent way non-perturbative relativistic corrections to the wave functions (mostly known from lattice) is still missing. Partial results include some (but not all) NLO relativistic corrections.

process	$\Gamma^{\rm LO}_{\rm pNRQCD}/{\rm keV}$	$\Gamma^{\rm NLO}_{\rm pNRQCD}/{\rm keV}$	$\Gamma_{\rm mod}/{\rm keV}$	$\Gamma^{\rm PDG}_{\rm exp}/{\rm keV}$
$\chi_{c0}(1P) \to J/\psi\gamma$	199	158 ± 60	162-183	122 ± 11
$\chi_{c1}(1P) \to J/\psi\gamma$	421	302 ± 126	340-363	296 ± 22
$\chi_{c2}(1P) \to J/\psi\gamma$	568	415 ± 170	413-464	386 ± 27
$h_c(1P) \to \eta_c(1S)\gamma$	909	447 ± 272	-	<600
$\psi(2S) \to \chi_{c0}(1P)\gamma$	53.6	21.4 ± 16.1	26.0-40.3	29.4 ± 1.3
$\psi(2S) \to \chi_{c1}(1P)\gamma$	45.2	30.7 ± 13.6	28.3-37.3	28.0 ± 1.5
$\psi(2S) \to \chi_{c2}(1P)\gamma$	31.6	25.6 ± 9.5	17.5-22.7	26.5 ± 1.3
$\eta_c(2S) \to h_c(1P)\gamma$	38.1	31.0 ± 11.4	-	-

◦ Pietrulewicz PoS ConfinementX (2012) 135

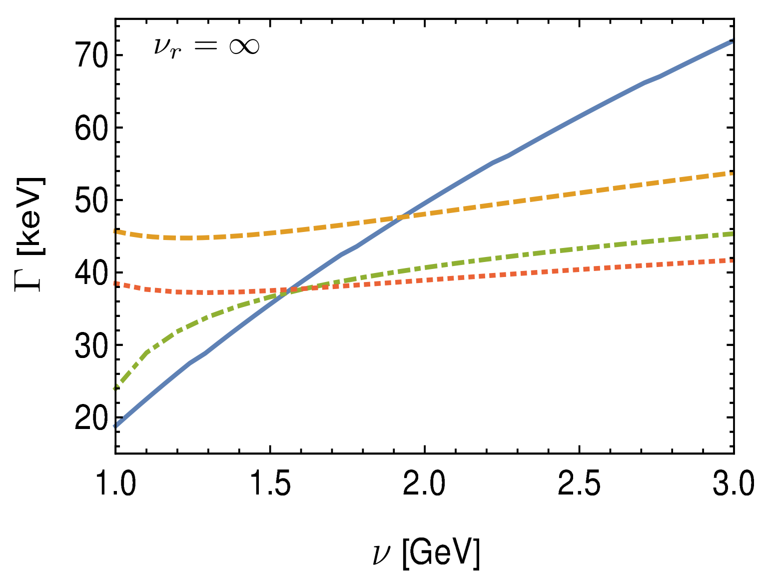
For bottomonium, relying on its Coulombic nature, a more systematic study is available.

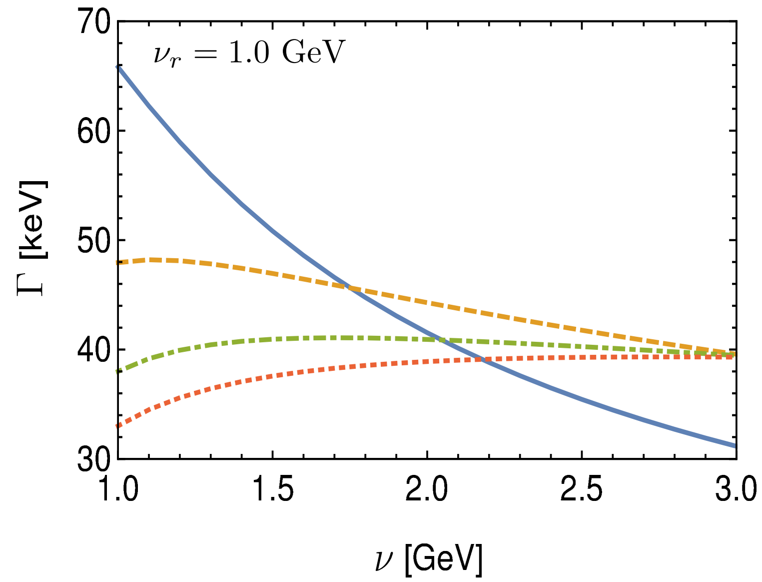
E1 bottomonium transitions: set up

- We consider the E1 transitions $\chi_{bJ}(1P) \to \Upsilon(1S)\gamma$ and $h_b(1P) \to \eta_b(1S)\gamma$.
- We assume $m_b v^2 \gg \Lambda_{\rm QCD}$ and make the calculation accurate up to order v^2. Under this condition:
  - the wave function can be computed perturbatively;
  - non-perturbative corrections to the wave function (coming from the potentials and the octet) may be neglected.
- We take as central value the one corresponding to the renormalization scale $\nu = 1.25$ GeV, which is the scale that solves the self-consistency relation for the $\Upsilon(1S)$ Bohr radius at one loop: $\nu = \dfrac{2m_b\alpha_{\rm s}(\nu)}{3}$
- We estimate the uncertainty as the largest between $1/2$ of the maximum difference between the leading order and the order v^2 result and the variation of the order v^2 result over the range 1 GeV $\leq \nu \leq$ 3 GeV.
- Relaxing the condition $m_b v^2 \gg \Lambda_{\rm QCD}$ to $m_b v^2 \sim \Lambda_{\rm QCD}$ will enlarge the uncertainties of at least a factor 2.

◦ Segovia Steinbeißer Vairo PR D99 (2019) 074011

Improved determinations of E1 transitions

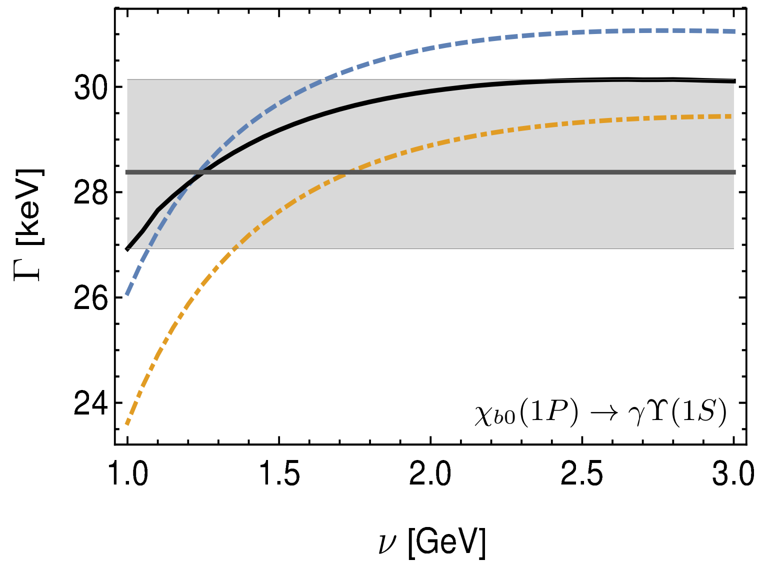




On the example of the $\chi_{b1}(1P) \to \Upsilon(1S)\gamma$ transition:
blue curve: LO; orange curve: NLO; green curve: NNLO; red curve: NNNLO.

○ Segovia Steinbeißer Vairo PR D99 (2019) 074011

$\chi_{b0}(1P) \to \Upsilon(1S)\gamma$

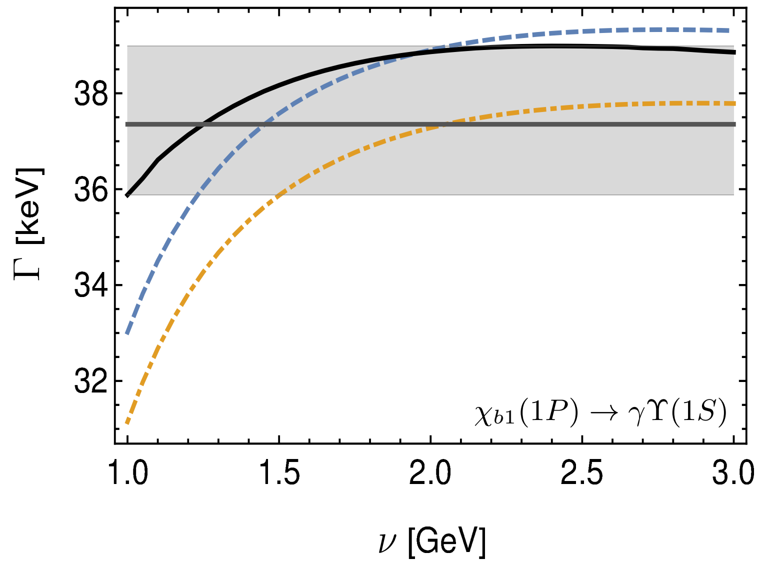


blue curve: LO; orange curve: LO + $\mathcal{L}_\gamma^{\mathrm{E1}}$; black curve: final order v^2 result:

$$\Gamma(\chi_{b0}(1P) \to \Upsilon(1S)\gamma) = 28^{+2}_{-2}\ \text{keV}$$

- Segovia Steinbeißer Vairo PR D99 (2019) 074011

$\chi_{b1}(1P) \to \Upsilon(1S)\gamma$

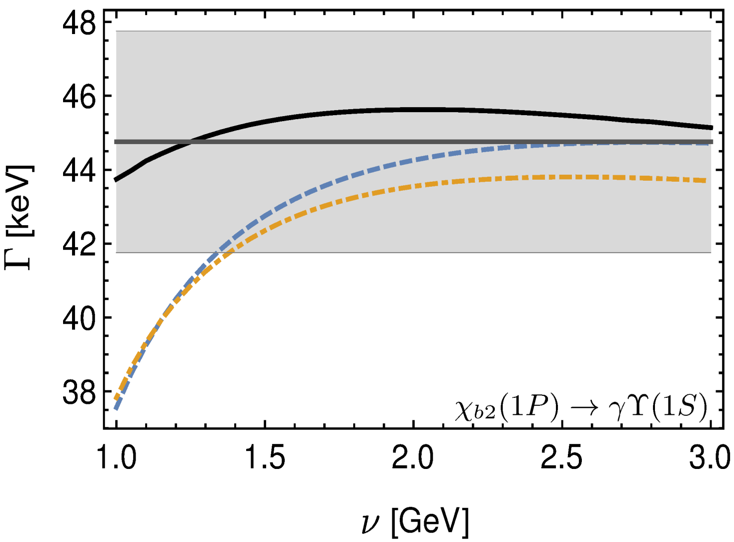


blue curve: LO; orange curve: LO + $\mathcal{L}_\gamma^{\text{E1}}$; black curve: final order v^2 result:

$$\Gamma(\chi_{b1}(1P) \to \Upsilon(1S)\gamma) = 37^{+2}_{-2} \text{ keV}$$

- Segovia Steinbeißer Vairo PR D99 (2019) 074011

$\chi_{b2}(1P) \to \Upsilon(1S)\gamma$

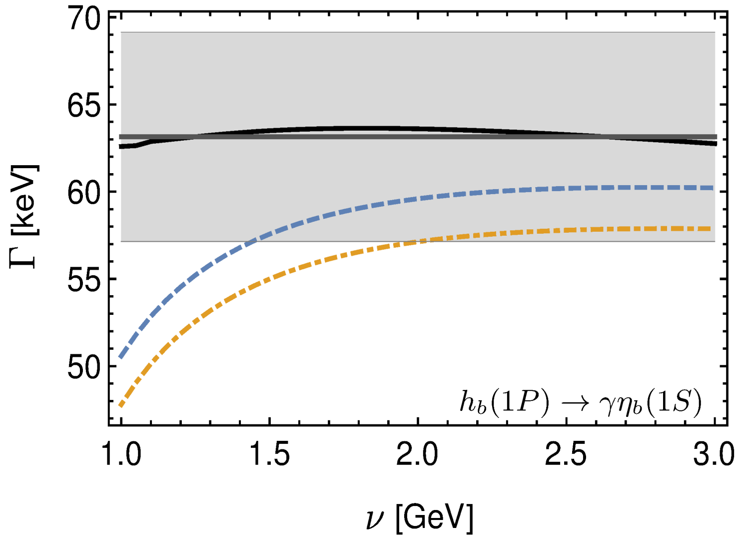


blue curve: LO; orange curve: LO + $\mathcal{L}_\gamma^{\mathrm{E1}}$; black curve: final order v^2 result:

$$\Gamma(\chi_{b2}(1P) \to \Upsilon(1S)\gamma) = 45^{+3}_{-3}\ \text{keV}$$

- Segovia Steinbeißer Vairo PR D99 (2019) 074011

$h_b(1P) \to \eta_b(1S)\gamma$



blue curve: LO; orange curve: LO + $\mathcal{L}_\gamma^{\rm E1}$; black curve: final order v^2 result:

$$\Gamma(h_b(1P) \to \eta_b(1S)\gamma) = 63^{+6}_{-6} \text{ keV}$$

- Segovia Steinbeißer Vairo PR D99 (2019) 074011

Comparison with the literature

Mode	LO	$\mathcal{O}(v^2)$	CQM	R	GI	BT	LFQM	$\text{SNR}_{0/1}$
$\chi_{b0}(1P) \to \Upsilon(1S)\gamma$	28.5	28.4	28.1	29.9	23.8	25.7	-	26.6/24.3
$\chi_{b1}(1P) \to \Upsilon(1S)\gamma$	36.0	37.4	35.7	36.6	29.5	29.8	-	33.6/30.0
$\chi_{b2}(1P) \to \Upsilon(1S)\gamma$	41.0	44.8	39.2	40.2	32.8	33.0	-	38.2/32.6
$h_b(1P) \to \eta_b(1S)\gamma$	55.2	63.2	43.7	52.6	35.7	-	37.5	55.8/36.3

- Segovia Ortega Entem Fernandez PR D93 (2016) 074027 [CQM]
  Ebert Faustov Galkin PR D67 (2003) 014027 [R]
  Godfrey Moats PR D93 (2015) 054034 [GI]
  Grotch Owen Sebastian PR D30 (1984) 1924 [BT]
  Shi EPJ C77 (2017) 253 [LFQM]
  Li Chao CTP 52 (2009) 653 [$\text{SNR}_{0/1}$]

Total widths

Mode	$\mathcal{B}_i = \Gamma_i/\Gamma$ [PDG]	Γ_i	Γ
$\chi_{b0}(1P) \to \Upsilon(1S)\gamma$	$(1.94 \pm 0.27)\%$	28^{+2}_{-2} keV	$1.46^{+0.2}_{-0.2}$ MeV
$\chi_{b1}(1P) \to \Upsilon(1S)\gamma$	$(35.0 \pm 2.1)\%$	37^{+2}_{-2} keV	107^{+9}_{-9} keV
$\chi_{b2}(1P) \to \Upsilon(1S)\gamma$	$(18.8 \pm 1.1)\%$	45^{+3}_{-3} keV	238^{+21}_{-21} keV
$h_b(1P) \to \eta_b(1S)\gamma$	$(52^{+6}_{-5})\%$	63^{+6}_{-6} keV	121^{+18}_{-16} keV

- Segovia Steinbeißer Vairo PR D99 (2019) 074011

Conclusions

EFTs provide a description of quarkonium electromagnetic transitions in terms of systematic expansions in $\alpha_{\rm s}$ and v. This description shows that:

- There is no scalar interaction.
- The quarkonium anomalous magnetic moment is small and positive: $\kappa^{\rm em} = 2\alpha_{\rm s}/(3\pi) + ...$
- M1 transitions involving the lowest quarkonium states may be described at relative order v^2 entirely by perturbation theory.
- Theory expectations for M1 transitions are consistent with data.
- E1 transitions require the calculation of non-perturbative corrections to the quarkonium wave-functions. These can be calculated from the quarkonium potentials evaluated on the lattice, which are mostly known.
- Assuming the $\chi_{bJ}(1P)$ and $h_b(1P)$ to be weakly coupled Coulombic bound states we have predicted the transition widths for $\chi_{bJ}(1P) \to \Upsilon(1S)\gamma$ and $h_b(1P) \to \eta_b(1S)\gamma$ providing eventually also predictions for the total widths.