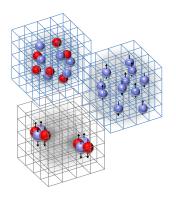
Lattice simulations for nuclear structure and reactions

Serdar Elhatisari

Karamanoğlu Mehmetbey University Nuclear Lattice EFT Collaboration

> School and Workshop: "Frontiers of QCD" Tbilisi, Georgia September 23-28, 2019



▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

Nuclear Lattice Effective Field Theory collaboration

Bundesministerium für Bildung und Forschung



Deutsche Forschungsgemeinschaft DFG



HELMHOLTZ





KARAMAN

Serdar Elhatisari

BOCHUM

Evgeny Epelbaum Herman Krebs Lukas Bovermann

BONN

Ulf-G. Meißner Gianluca Stellin NLEFT Collaboration MSSTATEU

Gautam Rupak

◆□▶ ◆□▶ ▲□▶ ▲□▶ □ のQ@

JÜLICH

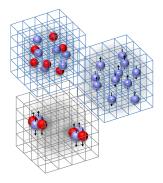
Timo Lähde Thomas Luu Dillon Frame Ulf-G. Meißner MSU

Dean Lee Ning Li Bing-Nan Lu

Outline

Introduction

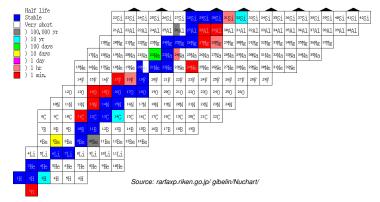
- Chiral effective field theory (χ EFT)
- Lattice effective field theory
- Nuclear structure and reactions
- Degree of locality of nuclear forces
- Recent results
- Summary



◆□▶ ◆□▶ ▲□▶ ▲□▶ □ のQ@

Ab initio nuclear theory

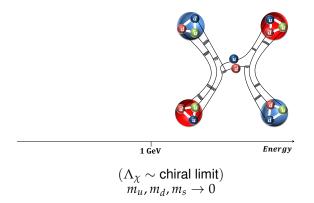
The aim is to predict the properties of atomic nuclei from microscopic nuclear forces



synthesis of the elements in the Universe (nucleosynthesis)
 mechanisms in stars and stellar explosions (nuclear processes)

Nuclear forces from QCD

Quantum chromodynamics (QCD) describes the strong forces by confining quarks (and gluons) into baryons and mesons.

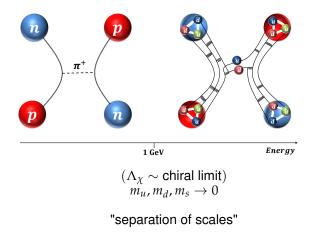


S. Weinberg, Phys. Lett. B 251 (1990) 288, Nucl. Phys. B363 (1991) 3, Phys. Lett. B 295 (1992) 114.

◆□▶ ◆□▶ ▲□▶ ▲□▶ □ のQ@

Nuclear forces from QCD

Quantum chromodynamics (QCD) describes the strong forces by confining quarks (and gluons) into baryons and mesons.

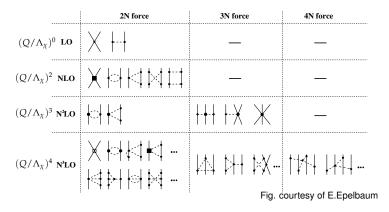


S. Weinberg, Phys. Lett. B 251 (1990) 288, Nucl. Phys. B363 (1991) 3, Phys. Lett. B 295 (1992) 114.

◆□▶ ◆□▶ ▲□▶ ▲□▶ □ のQ@

Chiral EFT for nucleons: nuclear forces

Chiral effective field theory organizes the nuclear interactions as an expansion in powers of momenta and other low energy scales such as the pion mass (Q/Λ_{χ})



Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98,'03,'05,'15; Kaiser '99-'01; Higa et al. '03; ...

◆□▶ ◆□▶ ◆□▶ ◆□▶ ● ● ● ●

Effective field theory

A classical example: Multipole expansion

$$\Phi(r) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r})dV}{|\vec{R} - \vec{r}|}$$

$$\frac{1}{|\vec{R} - \vec{r}|} = \frac{1}{\sqrt{R^2 + r^2 - 2Rr\cos\theta}} = \frac{1}{R} + \frac{r}{R^2}\cos\theta + \frac{r^2}{R^3}\frac{3\cos^2\theta - 1}{2} + \dots$$

$$\Phi(r) = \frac{1}{4\pi\epsilon_0} \left\{ \frac{1}{R} \int_V \rho(\vec{r})dV + \frac{1}{R^2} \int_V \vec{r}\rho(\vec{r})dV + \frac{1}{R^3} \int_V (3r_ir_j - r^2\delta_{ij})\rho(\vec{r})dV \right\} + \mathcal{O}\left(\frac{1}{R^4}\right)$$

□ We do not need to know the dynamics at the short distance to understand the dynamics at the long distance

- \Box the sum converges for $S \ll R$
- □ long distances probes are determined by the bulk properties

Lattice effective field theory

Lattice effective field theory is a powerful numerical method formulated in the framework of chiral effective field theory

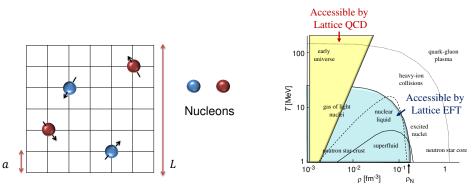


Fig. courtesy of D. Lee

▲□▶▲□▶▲□▶▲□▶ □ のQ@

Lattice formulation of χEFT

a new lattice formulation of χ EFT interactions:

- □ a simpler decomposition into spin channels
- □ the process of fitting to the empirical scattering phase shifts is simplified, and the resulting lattice phase shifts are more accurate.

$$V_{L,L'}^{S,I,J}(\mathbf{n}) = \sum_{I_z,J_z} \sum_{S_z,L_z} \sum_{S'_z,L'_z} \left(\langle SS_z, LL_z | JJ_z \rangle \left[a(\mathbf{n}) \ \nabla^{2M} \ R^*_{L,L_z}(\nabla) \ a(\mathbf{n}) \right]_{S,S_z,I,I_z}^{s_{NL}} \right)^{\dagger} \\ \langle SS'_z, L'L'_z | JJ_z \rangle \left[a(\mathbf{n}) \ \nabla^{2M} \ R^*_{L',L'_z}(\nabla) \ a(\mathbf{n}) \right]_{S,S'_z,I,I_z}^{s_{NL}}$$

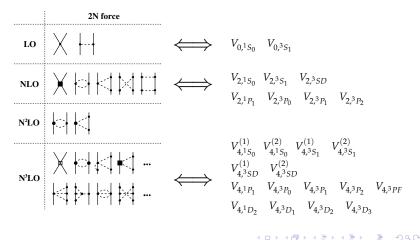
$$[a(\mathbf{n}) a(\mathbf{n}')]_{S,S_z,I,I_z}^{s_{NL}} = \sum_{i,j,i',j'} a_{i,j}^{s_{NL}}(\mathbf{n}) M_{ii'}(S,S_z) M_{jj'}(I,I_z) a_{i,j}^{s_{NL}}(\mathbf{n}')$$

Li, SE, Epelbaum, Lee, Lu, Meißner Phys. Rev. C 98, 044002 (2018)

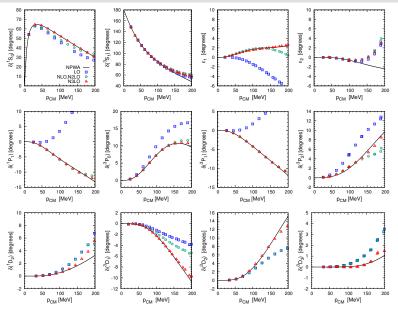
▲□▶▲□▶▲□▶▲□▶ □ のQ@

χEFT for nucleons: *NN* scattering phase shifts

- \square formulate the lattice action in the framework of chiral effective field theory
- ☐ fit the unknown coefficients of the short-range lattice interactions to empirical phase shifts



χ EFT for nucleons: *NN* scattering phase shifts



Li, SE, Epelbaum, Lee, Lu, Meißner Phys. Rev. C 98, 044002 (2018)

Many body quantum systems - ab initio nuclear theory



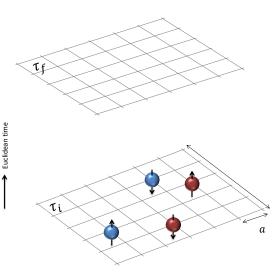






Lattice Monte Carlo calculations: Euclidean time projection

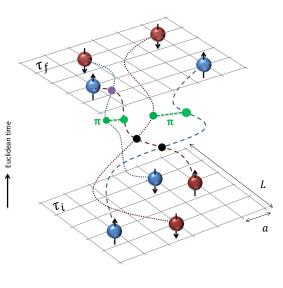
□ construct a trial state of nucleons, $|\psi_I\rangle$, as a Slater determinant of free-particle standing waves on the lattice.



▲□▶ ▲□▶ ▲□▶ ▲□▶ = 三 のへで

Lattice Monte Carlo calculations: Euclidean time projection

- □ construct a trial state of nucleons, $|\psi_I\rangle$, as a Slater determinant of free-particle standing waves on the lattice.
- The evolution in Euclidean time automatically incorporates the induced deformation, polarization and clustering.



◆□▶ ◆□▶ ◆□▶ ◆□▶ ● ● ● ●

Lattice EFT: (Euclidean time) projection Monte Carlo

Transfer matrix operator formalism $\mathbf{M} =: \exp(-H_{\text{LO}} a_t) :$

Microscopic Hamiltonian $H_{\rm LO} = H_{\rm free} + V_{\rm LO}$

$$Z(L_t) = \mathsf{Tr}(\mathbf{M}^{L_t}) = \int Dc Dc^* \exp[-S(c, c^*)]$$

Creutz, Found. Phys. 30 (2000) 487.

The exact equivalence of several different lattice formulations. Lee, PRC 78:024001, (2008); Prog.Part.Nucl.Phys., 63:117-154 (2009)

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ - 三 - のへぐ

Projection Monte Carlo uses a given initial state, $|\psi_I\rangle$, to evaluate a product of a string of transfer matrices **M**.

$$Z(L_t) = \langle \psi_I | \mathbf{M}(L_t - 1) \mathbf{M}(L_t - 2) \dots \mathbf{M}(1) \mathbf{M}(0) | \psi_I \rangle$$

In the limit of large Euclidean time the evolution operator $e^{-H_{\rm LO}\tau}$ suppress the signal beyond the low-lying states, and the ground state energy of our quantum system can be extracted by

$$\lim_{L_t\to\infty}\frac{Z(L_t+1)}{Z(L_t)}=e^{-E_0a_t}$$

(日) (日) (日) (日) (日) (日) (日)

These amplitudes are computed with the Hybrid Monte Carlo methods.

Phys. Lett. B195, 216-222 (1987), Phys. Rev. D35, 2531-2542 (1987).

perturbative higher order calculations

 $ho = NLO, NNLO, \cdots$

$$\mathbf{M}_{\mathsf{ho}} =: e^{-a_t(H_{\mathsf{LO}}+V_{\mathsf{ho}})} :$$

where the potential V_{ho} is treated perturbatively. Therefore, the higher order corrections to the ground state energy can be computed as,

$$e^{-\Delta E_{\mathsf{ho}} a_t} = \lim_{L_t o \infty} rac{\langle \psi_I | \, \mathbf{M}^{L_t/2} \, \mathbf{M}_{\mathsf{ho}} \, \mathbf{M}^{L_t/2} \, | \psi_I
angle}{\langle \psi_I | \, \mathbf{M}^{L_t} \, | \psi_I
angle}$$

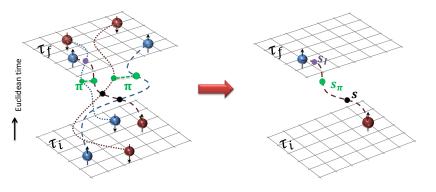
▲□▶▲□▶▲□▶▲□▶ □ ● ●

Auxiliary field Monte Carlo

Use a Gaussian integral identity

$$\exp\left[-\frac{C}{2}\left(N^{\dagger}N\right)^{2}\right] = \sqrt{\frac{1}{2\pi}}\int ds \,\exp\left[-\frac{s^{2}}{2} + \sqrt{-C}\,s\,\left(N^{\dagger}N\right)\right]$$

s is an auxiliary field coupled to particle density. Each nucleon evolves as if a single particle in a fluctuating background of pion fields and auxiliary fields.

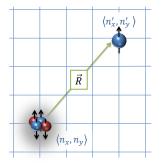


◆□▶ ◆□▶ ◆三▶ ◆三▶ ●□ ● ●

Scattering and reactions: Adiabatic projection method

The method constructs a low energy effective theory for the clusters

Use initial states parameterized by the relative spatial separation between clusters, and project them in Euclidean time.



$$|\vec{R}
angle = \sum_{\vec{r}} |\vec{r} + \vec{R}
angle_1 \otimes |\vec{r}
angle_2$$

$$|ec{R}
angle_{ au}=e^{-H\, au}\;|ec{R}
angle \quad {
m dressed}$$
 cluster state

The adiabatic projection in Euclidean time gives a systematically improvable description of the low-lying scattering cluster states.

In the limit of large Euclidean projection time the description becomes exact.

PRL 111 (2013) 032502; EPJA 49 (2013) 151; PRC 90, 064001 (2014); PRC 92,054612 (2015); EPJA 52: 174 (2016)

Scattering and reactions: Adiabatic projection method

 $ert ec{R}
angle_{ au} = e^{-H \, au} ert ec{R}
angle$ dressed cluster state (not orthogonal)

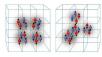
Hamiltonian matrixNorm matrix $[H_{\tau}]_{\vec{R},\vec{R}'} = \tau \langle \vec{R} | H | \vec{R}' \rangle_{\tau}$ $[N_{\tau}]_{\vec{R},\vec{R}'} = \tau \langle \vec{R} | \vec{R}' \rangle_{\tau}$

$$[H^a_{\tau}]_{\vec{R},\vec{R}'} = \sum_{\vec{R}'',\vec{R}'''} \left[N^{-1/2}_{\tau} \right]_{\vec{R}\vec{R}''} [H_{\tau}]_{\vec{R}''\vec{R}'''} \left[N^{-1/2}_{\tau} \right]_{\vec{R}'''\vec{R}'}$$

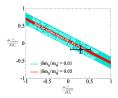
The structure of the adiabatic Hamiltonian, $[H^a_{\tau}]_{\vec{R},\vec{R}'}$, is similar to the Hamiltonian matrix used in calculations of ab initio no-core shell model/resonating group method (NCSM/RGM) for nuclear scattering and reactions.

Navratil, Quaglioni, PRC 83, 044609 (2011). Navratil, Roth, Quaglioni, PLB 704, 379 (2011). Navratil, Quaglioni, PRL 108, 042503 (2012).

Nuclear LEFT: *ab initio* nuclear structure and scattering theory

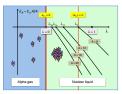


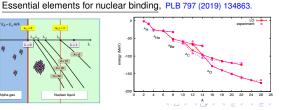




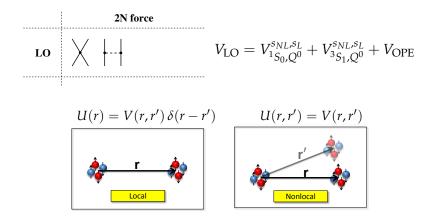


- Lattice EFT calculations for A = 3, 4, 6, 12 nuclei, PRL 104 (2010) 142501
- Ab initio calculation of the Hoyle state, PRL 106 (2011) 192501
- Structure and rotations of the Hoyle state, PRL 109 (2012) 252501
- Viability of Carbon-Based Life as a Function of the Light Quark Mass. PRL 110 (2013) 112502
- Radiative capture reactions in lattice effective field theory. PRL 111 (2013) 032502
- Ab initio calculation of the Spectrum and Structure of ¹⁶O, PRL 112 (2014) 102501
- Ab initio alpha-alpha scattering, Nature 528, 111-114 (2015).
- Nuclear Binding Near a Quantum Phase Transition, PRL 117, 132501 (2016).
- Ab initio calculations of the isotopic dependence of nuclear clustering. PRL 119, 222505 (2017).





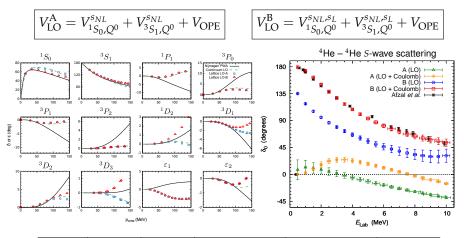
Degree of locality of nuclear forces



□ Does every *χ*EFT interaction give well controlled and reliable results for heavier systems?

□ Is the convergence of higher-order terms under control?

Degree of locality of nuclear forces - I

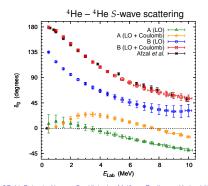


Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
³ Н	-7.82(5)	-7.78(12)	-7.82(5)	-7.78(12)	-8.482
³ He	-7.82(5)	-7.78(12)	-7.08(5)	-7.09(12)	-7.718
⁴ He	-29.36(4)	-29.19(6)	-28.62(4)	-28.45(6)	-28.296

SE, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähder Lee, Rupak, PRL 17, 132501 (2016) 🔿

Degree of locality of nuclear forces - I

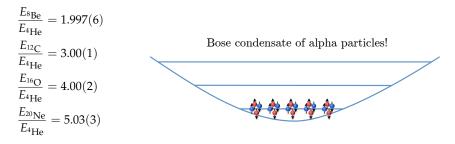
Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
⁴ He	-29.36(4)	-29.19(6)	-28.62(4)	-28.45(6)	-28.296
⁸ Be	-58.61(14)	-59.73(6)	-56.51(14)	-57.29(7)	-56.591
¹² C	-88.2(3)	-95.0(5)	-84.0(3)	-89.9(5)	-92.162
¹⁶ O	-117.5(6)	-135.4(7)	-110.5(6)	-126.0(7)	-127.619
²⁰ Ne	-148(1)	-178(1)	-137(1)	-164(1)	-160.645



SE, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, Lee, Rupak, PRL 117, 132501 (2016)

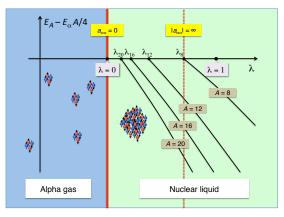
Degree of locality of nuclear forces - I

Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
⁴ He	-29.36(4)	-29.19(6)	-28.62(4)	-28.45(6)	-28.296
⁸ Be	-58.61(14)	-59.73(6)	-56.51(14)	-57.29(7)	-56.591
¹² C	-88.2(3)	-95.0(5)	-84.0(3)	-89.9(5)	-92.162
¹⁶ O	-117.5(6)	-135.4(7)	-110.5(6)	-126.0(7)	-127.619
²⁰ Ne	-148(1)	-178(1)	-137(1)	-164(1)	-160.645



Nuclear binding near a quantum phase transition

Consider a one-parameter family of interactions: $V = (1 - \lambda) V^A_{
m LO} + \lambda V^B_{
m LO}$



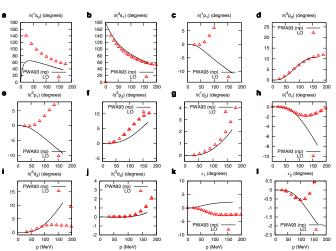
There is a quantum phase transition at the point where the α - α scattering length $a_{\alpha\alpha}$ vanishes, and it is a first-order transition from a Bose-condensed α -particle gas to a nuclear liquid.

SE, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, Lee, Rupak, PRL 117, 132501 (2016)

Degree of locality of nuclear forces - II

We can probe the degree of locality only by many-body calculations, and we consider an SU4-symmetric potential,

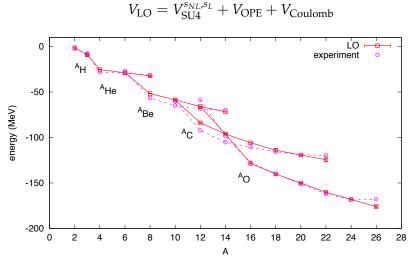
 $V_{\rm LO} = V_{\rm SU4}^{s_{\rm NL},s_{\rm L}} + V_{\rm OPE}$



(日)

) Q (

Ground state energies at LO



SE, Epelbaum, Krebs, Lähde, Lee, Li, Lu, Meißner, Rupak, PRL 119, 222505 (2017)

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □

Degree of locality of nuclear forces - III

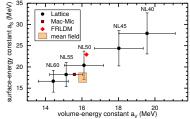
Consider the following potential in the framework of pionless effective field theory to probe the degree of locality from many-body calculations,

$$V_{\not \pi}^{\text{start}} = V_{\text{SU4}}^{C_2, s_{NL}, s_L} + V_{\text{SU4}}^{C_3} + V_{\text{Coulomb}}$$

- \Box C₂, s_L, and C₃ are tuned to get the few-body physics correct
- \Box For $A \ge 16$, the binding energies are well-parameterized with the Bethe-Weizsäcker mass formula;

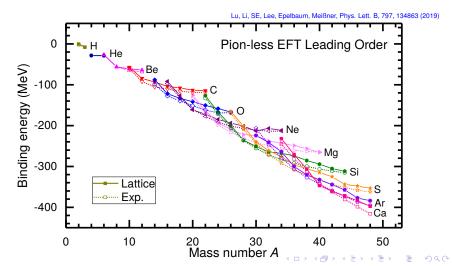
$$B(A) = a_V A - a_S A^{2/3} + E_{\text{Coulomb}} + (\text{symmetry} + \text{pairing} + \text{shellcorrection} + \dots)$$

□ Consider only N = Z even-even nuclei, and obtain C_2 , s_L , C_3 for various values of s_{NL}



Essential elements for nuclear binding

□ a lattice action with minimum number of parameters (four) which describes neutron matter up to saturation density and the ground state properties of nuclei up to calcium. a = 1.32 fm, $s_L = 0.0609$ (l.u.), and $s_{NL} = 0.5$ (l.u.)



Essential elements for nuclear binding

□ a lattice action with minimum number of parameters (four) which describes neutron matter up to saturation density and the ground state properties of nuclei up to calcium. a = 1.32 fm, $s_L = 0.0609$ (l.u.), and $s_{NL} = 0.5$ (l.u.)

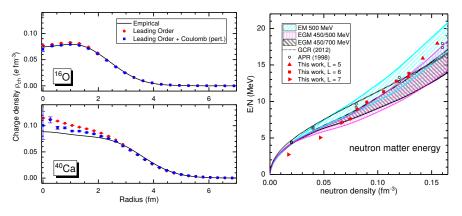
	В	Experiment	R _{ch}	Experiment
³ H	8.48(2)	8.48	1.90(1)	1.76
³ He	7.75(2)	7.72	1.99(1)	1.97
⁴ He	28.89(1)	28.3	1.72(1)	1.68
¹⁶ O	121.9(1)	127.6	2.74(1)	2.70
²⁰ Ne	161.6(1)	160.6	2.95(1)	3.01
²⁴ Mg	193.5(2)	198.3	3.13(1)	3.06
²⁸ Si	235.8(4)	236.5	3.26(1)	3.12
⁴⁰ Ca	346.8(6)	342.1	3.42(1)	3.48

Lu, Li, SE, Lee, Epelbaum, Meißner, Phys. Lett. B, 797, 134863 (2019)

Essential elements for nuclear binding

□ a lattice action with minimum number of parameters (four) which describes neutron matter up to saturation density and the ground state properties of nuclei up to calcium. a = 1.32 fm, $s_L = 0.0609$ (l.u.), and $s_{NL} = 0.5$ (l.u.)

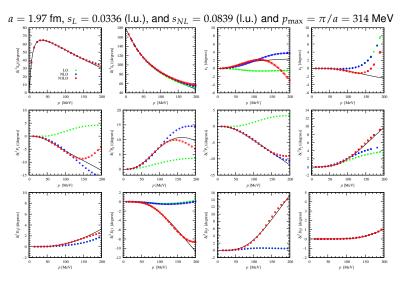




◆ロ▶★舂▶★≧▶★≧▶ 差 のなぐ

Ab initio nuclear structure: recent progress

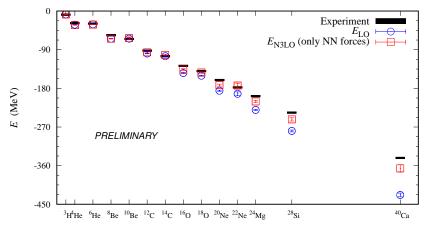
We have constructed a new lattice formulation of chiral effective field theory interactions with a simpler decomposition into spin channels. Li, SE, Epelbaum, Lee, Lu, Meißner, Phys. Rev. C 98, 044002 (2018)



NLEFT collaboration [work in progress]

Ab initio nuclear structure: recent progress

a = 1.97 fm, $s_L = 0.0336$ (l.u.), and $s_{NL} = 0.0839$ (l.u.) and $p_{max} = \pi/a = 314$ MeV Preliminary results for the ground state energies of light and medium-mass nuclei:



□ To determine the 3N forces, we consider several nuclear processes, such as triton beta-decay, neutron-alpha and alpha-alpha scattering, properties of ³He and ⁴He etc.

Triton beta-decay

a = 1.97 fm Preliminary results for the 3-nucleon systems:

L (fm)	$E_{\rm LO}$ (MeV)	$E_{\rm NLO}$ (MeV)	$E_{\rm N3LO}$ (MeV)
11.84	-10.03	-9.07	-9.10
15.78	-9.55	-8.43	-8.47
19.73	-9.51	-8.36	-8.39

Bovermann, SE, Epelbaum, Krebs, Lee, Meißner [work in progress]

 $\mathsf{MEC}_{\Box} \rightarrow \mathsf{A} \xrightarrow{\mathbb{Z}} \xrightarrow{\mathbb{Z}} \mathsf{A} \xrightarrow{\mathbb{Z}} \xrightarrow{\mathbb{Z}} \mathsf{A} \xrightarrow{\mathbb{Z}}$

L (fm)	$\langle \mathbf{F} \rangle_{\mathrm{LO}}$	$\langle \mathbf{F} \rangle_{\text{NLO}}$	$\langle \mathbf{F} \rangle_{\text{N3LO}}$	$\langle \mathbf{GT} \rangle_{\mathrm{LO}}$	$\langle \mathbf{GT} \rangle_{\mathrm{NLO}}$	$\langle \mathbf{GT} \rangle_{\mathrm{N3LO}}$
11.84	1.0000	0.9995	0.9995	1.7045	1.6714	1.6725
15.78	1.0000	0.9996	0.9996	1.7111	1.6789	1.6796
19.73	1.0000	0.9997	0.9997	1.7134	1.6839	1.6845

$$\begin{split} \left< \mathbf{F} \right>_{emp} &= 0.9998 \\ \left< \mathbf{GT} \right>_{emp} &= 1.6474(23) \end{split}$$

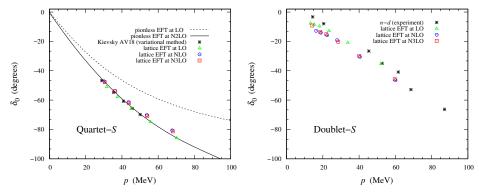
Phys. Rev. C 95, no. 5, 059902 (2017)



 C_{D}

n-d scattering

a = 1.97 fm Preliminary results for the 3-nucleon systems:



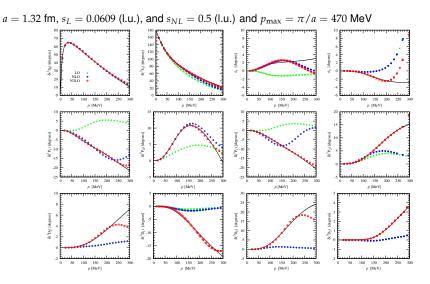
Bovermann, SE, Epelbaum, Krebs, Lee, Meißner [work in progress]

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

variational method: Nucl. Phys. A 607, 402 (1996) *n* – *d* (experiment): Phys. Lett. 24B, 562 (1967) pionless EFT: Nucl. Phys. A 675, 601 (2000)

Ab initio nuclear structure: recent progress

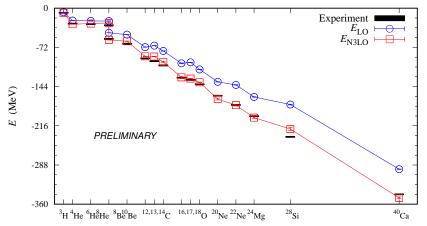
We have constructed a new lattice formulation of chiral effective field theory interactions with a simpler decomposition into spin channels. Li, SE, Epelbaum, Lee, Lu, Meißner, Phys. Rev. C 98, 044002 (2018)



▲□▶▲圖▶▲≧▶▲≧▶ ≧ のへで

Ab initio nuclear structure: recent progress

a = 1.32 fm, $s_L = 0.0609$ (l.u.), and $s_{NL} = 0.5$ (l.u.) and $p_{max} = \pi/a = 470$ MeV Preliminary results for the ground state energies of light and medium-mass nuclei:



□ To determine the 3N forces, we consider several nuclear processes, such as triton beta-decay, neutron-alpha and alpha-alpha scattering, properties of ³He and ⁴He etc.

Summary

- □ Nuclear forces in the framework of chiral effective field theory are well-established, and it is very important time for *ab initio* methods to make predictions in manynucleon system using these forces.
- □ A new lattice formulation of chiral effective field theory interactions has improved our *ab initio* nuclear theory which describes the nuclear structure successfully as well as nuclear scattering and reaction processes.
- □ Understanding of the connection between the degree of locality of nuclear forces and nuclear structure has led to a more efficient set of lattice chiral EFT interactions.
- □ Scattering and reaction processes involving alpha particle are in reach of *ab initio* methods and this has opened the door towards using experimental data from collisions of heavier nuclei as input to improve *ab initio* nuclear structure theory.



(日) (日) (日) (日) (日) (日) (日)





Quantum Phase Transtition

In physics, a quantum phase transition (QPT) is a phase transition between different quantum phases (phases of matter at zero temperature). Contrary to classical phase transitions, quantum phase transitions can only be accessed by varying a physical parameter-such as magnetic field or pressure-at absolute zero temperature. The transition describes an abrupt change in the ground state of a many-body system due to its quantum fluctuations.