Form factors of heavy mesons from various versions of QCD sum rules

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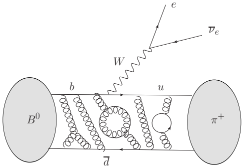
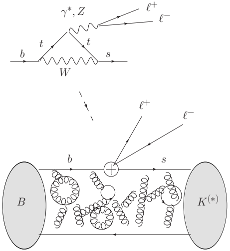
□ Why do we need heavy meson form factors ?

- exclusive semileptonic B-meson decays:



$B \to \pi \ell \nu$



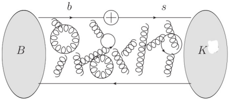
$B \to K \ell^+ \ell^-$

- used to probe Standard Model and beyond:
  - the CKM matrix elements: stil not precise enough

$$\boxed{D \to \pi \ell \nu_\ell,\, D \to K \ell \nu_\ell} \Rightarrow \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{array}{l} \Leftarrow \boxed{B \to \pi \ell \nu_\ell,\, B_s \to K \ell \nu_\ell} \\ \Leftarrow \boxed{B \to D \ell \nu_\ell} \end{array}$$

  - the FCNC $b \to s \ell^+ \ell^-$ transitions: tensions with SM
- need accurate estimates of hadronic matrix elements

□ Form factors of $B \to h$ transitions



- $B \to h$ matrix element ={ Lorentz structure}$\times$ form factor

  $$\langle h(p_h)|\bar{q}\Gamma_a b|\bar{B}(p_B)\rangle = L_a(p_h, p_B)F^{(B\to h)}(q^2), \quad q^2 = (p_B - p_h)^2$$

  $q = u, d, s, c, \qquad h = \pi, K, D, ...$

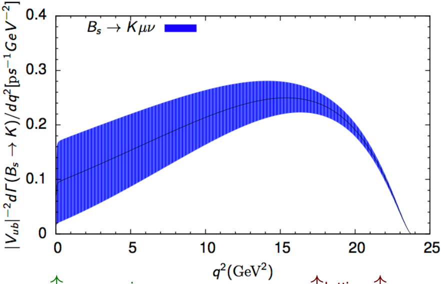
- $b \to c$, $D \to h$ form factors
- b, c - flavoured baryon transitions: $\Lambda_{b(c)} \to h$, $\quad h = N, \Lambda, ...$
- have to be calculated using QCD-based methods

  quark models are not sufficient
- continuous progress of form factor calculations in lattice QCD,
- do we still need continuum (non-lattice) QCD methods ?

□ Challenges for ongoing lattice calculations

- large recoil (small q^2)

  a recent example: from Fermilab-MILC Collab. [arXiv:1901.0255 [hep-lat]]



↑ z-expansion ↑lattice ↑

- two hadrons (resonance) in the final state of $B \to h$

  e.g., $h = \pi\pi(\rho), K\pi(K^*)$ with $J^P = 1^-$;

  $h = \pi\pi, K\pi$ with $J^P = 0^+, 2^+$

□ QCD sum rules

- the original (SVZ) method $\Rightarrow$ vacuum-to-vacuum correlators:

  OPE (condensates)$\oplus$ dispersion relation $\oplus$ quark-hadron duality

  [M. Shifman, A.Vainshtein, V. Zakharov, (1979)]

  - form factors accessible from three-point correlators [AK(1980)]

  -local quark condensate term $\rightarrow$ wrong large q^2 asymptotics [B. Ioffe, A. Smilga (1982)]

  -technically difficult to get NLO corrections

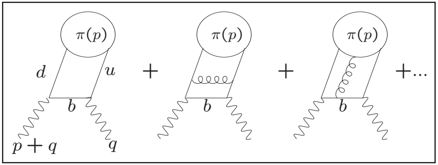
- Light-cone Sum Rules (LCSRs)$\Rightarrow$ vacuum-to-hadron correlators:

  OPE (LCDAs)$\oplus$ dispersion relation $\oplus$ quark-hadron duality

  [I.I.Balitsky, V.M.Braun, A.V. Kolesnichenko (1986); V.L.Chernyak, I.Zhitnisky (1990)]

- two different versions of LCSRs: valid at $q^2 \ll (m_B - m_h)^2$
  - with h- distribution amplitudes,
  - with B- meson distribution amplitudes,
- in what follows:
  - $B \rightarrow \pi, B_s \rightarrow K$ form factors from LCSRs with π DAs
  - $B \rightarrow (K\pi)_{J=1}$ beyond $B \rightarrow K^*$ from LCSRs with B DAs
- introductory reviews:, [V.Braun, hep-ph/9801222; P.Colangelo, AK, hep-ph/0010175

□ LCSR for $B \to \pi$ form factors



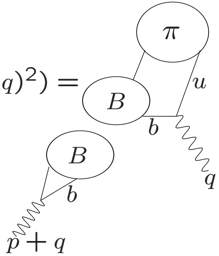
$\Leftarrow$ the correlation function

calculated from OPE in terms of pion distribution amplitudes

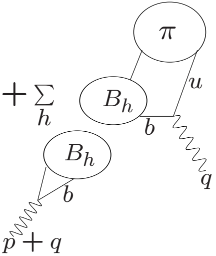
at $(p+q)^2, q^2 \ll m_b^2$

$\Downarrow$

hadronic dispersion relation $\Big\}$ $F(q^2,(p+q)^2) =$



$+\sum_h$



$f_B f^+_{B\pi}(q^2)$

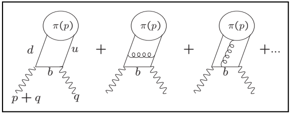
$\Uparrow$ QCD 2-point SR

$$\sum_{B_h} \Rightarrow \int_{s_0^B}^{\infty} ds \frac{\mathrm{Im}F(q^2,s)_{OPE}}{s-(p+q)^2}$$

quark-hadron duality

OPE calculation

- the correlation function $\quad q^2 \ll m_b^2$



$$[F(q^2,(p+q)^2)]_{OPE} =$$

$$= \sum_{t=2,3,4,..} \int_0^1 \mathcal{D}u \; T^{(t)}(\alpha_s, m_b, m_q; q^2, (p+q)^2, u, \mu)\, \varphi_\pi^{(t)}(u,\mu)$$

$$\Uparrow \qquad\qquad\qquad \Uparrow$$

{diagrams with b-propagator} $\otimes$ {pion Distribution Amplitudes}

- pion DA's, polynomial expansion:

$$\varphi_\pi^{(t)}(u,\mu) = f_\pi^{(t)}(\mu)\{C_0(u) + \sum_{n=1} a_n^{(t)}(\mu) C_n(u)\}$$

- accuracy of OPE
  - precision of the input: $m_b, m_q, \alpha_s, \quad f_\pi^{(t)}(\mu_0), \, a_n^{(t)}(\mu_0)$
  - truncation level: $O(\alpha_s), \quad t \leq 6, \quad n \leq 4$
  - variable scales: μ, $(p+q)^2 \to M^2 \sim m_b \chi, \quad m_b \gg \chi \gg \Lambda_{QCD}$

□ Hadronic dispersion relation

- based on analyticity $\oplus$ unitarity in QFT

$$[F(q^2,(p+q)^2)]_{OPE} = \frac{m_B^2 f_B f^+_{B\pi}(q^2)}{m_B^2-(p+q)^2} + \int\limits_{(m_{B^*}+m_\pi)^2}^{\infty} ds\, \frac{\rho_h(s)}{s-(p+q)^2}$$

$$\Uparrow$$

- quark-hadron "semilocal" duality

$$\boxed{\int\limits_{(m_{B^*}+m_\pi)^2}^{\infty} ds\, \frac{\rho_h(s)}{s-(p+q)^2} = \int\limits_{s_0}^{\infty} ds\, \frac{[\mathrm{Im} F(q^2,s)]_{OPE}]}{s-(p+q)^2}}$$

- Borel transformation , e.g. $\frac{1}{s-(p+q)^2} \to \exp(-s/M^2)$
- accuracy:
  - f_B calculated from 2-point QCD SR)
  - variable scale: $(p+q)^2 \to M^2 \sim m_b\chi \to$ optimal interval of M^2
  - duality approximation, s_0 (determined by calculating m_B^2)
- further improvements in OPE, LCDAs possible,

  the accuracy is largely limited by duality which is a "systematic" error'

LCSR Results for $B \to \pi \ell \nu_\ell$

- statistical (Bayesian) analysis of LCSR :

[I. S.Imsong, A.K., T. Mannel and D. van Dyk, 1409.7816]

inputs (assumed uncorrelated) taken as priors,
constructing theoretical likelihood by imposing $[m_B]_{SR}$ within 1% of m_B

- purely LCSR prediction (no parametrization/extrapolation involved)

$$\Delta\zeta\,(0, 12\text{GeV}^2) = \frac{1}{|V_{ub}|^2} \int_0^{12\,\text{GeV}^2} dq^2 \frac{d\Gamma}{dq^2}(B \to \pi \ell \nu_\ell)$$

$$\equiv \frac{G_F^2}{24\pi^3} \int_0^{12\,\text{GeV}^2} dq^2 p_\pi^3 |f^+_{B\pi}(q^2)|^2 = (5.25^{+0.68}_{-0.54})\,\text{ps}^{-1}\,,$$

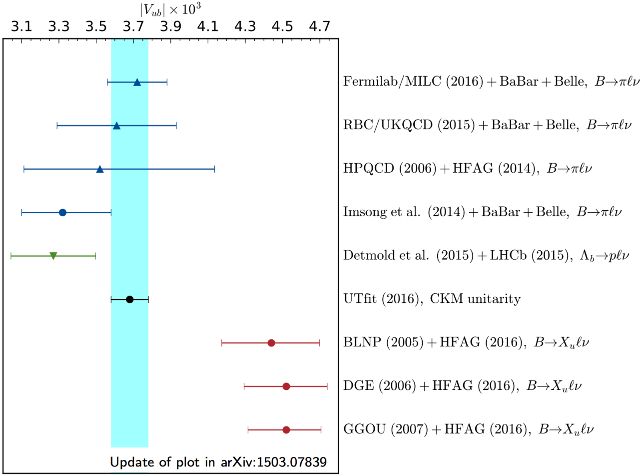
- z-series parameterization, including $f^+_{B\pi}(0)$ BCL-slighlty modified version

$$f^+_{B\pi}(q^2) = \frac{f^+_{B\pi}(0)}{1 - q^2/m^2_{B*}} \left\{ 1 + b_1^+ P_1(z(q^2)) + b_2^+ P_2(z(q^2)) \right\}$$

$f_{B\pi}(0) = 0.307 \pm 0.02$

Comparison of $|V_{ub}|$ determinations

A. Kronfeld @ Lattice 2017:



update of [J. A. Bailey *et al.* [Fermilab Lattice and MILC Collaborations], arXiv:1503.07839 [hep-lat].]

$\square$ $B_s \to K$ form factors

- LCSR prediction: $\Delta\zeta_{B_sK}\,(0, 12\,\mathrm{GeV}^2) = 7.03^{+0.67}_{-0.63}\ \mathrm{ps}^{-1}$ [AK, A.Rusov, 1703.04765]

$f^+_{B_sK}(0) = 0.336 \pm 0.023$ $b^+_{1\,(B_sK)} = -2.53 \pm 1.17$ Correlation=0.79
$f^T_{B_sK}(0) = 0.320 \pm 0.019$ $b^T_{1\,(B_sK)} = -1.08 \pm 1.53$ Correlation=0.74

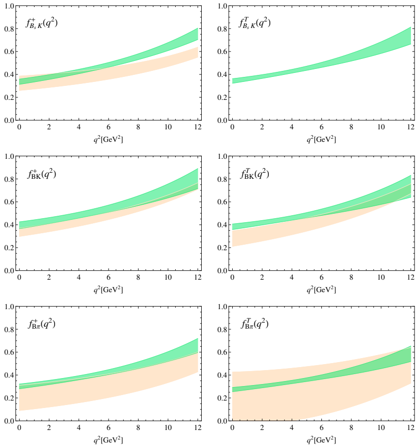


Figure 1. The vector (tensor) form factors of $B_s \to K$, $B \to K$ and $B \to \pi$ transitions calculated from LCSRs including estimated parametrical uncertainties are shown on the upper, middle and lower left (right) panels, respectively, with the dark-shaded (green) bands. Extrapolations of the lattice QCD results for $B_s \to K$ [Fermilab-MILC (2014)], $B \to K$ [HPQCD] and $B \to \pi$ [Fermilab=MILC (2015)] form factors are shown with the light-shaded (orange) bands.

□ $D \to \pi$ and $D \to K$ form factors from LCSRs

- byproduct: replace $b \to c$ and $B \to D$ in sum rules, adjust scales
- a useful cross-check of the LCSR method
- 2009 results [A.K., C. Klein, T. Mannel and N. Offen, 0907.2842 (2009)]

Form factor	$f^+_{D\pi}(0)$	$f^+_{DK}(0)$
	$0.67^{+0.10}_{-0.07}$	$0.75^{+0.11}_{-0.08}$
Twist-2 LO [1]	36.0 %	36.4 %
Twist-2 NLO [1]	6.3 %	6.0 %
Twist-3 LO [1]	66.3 %	68.2 %
Twist-3 NLO [1]	-9.5 %	-10.1 %
Twist-4 LO [1]	1.4 %	-0.07 %
Twist-5 LO factor.	-0.4 %	-0.4 %
Twist-6 LO factor.	-0.02 %	-0.03 %

preliminary estimate based on [A. Rusov, 1705.01929 (2017)]

- the recent lattice QCD determination:
  V. Lubicz *et al.* [ETM Collab.], 1706.03017 [hep-lat]]

$f_+^{D\to\pi}(0) = 0.612\ (35)$ and $f_+^{D\to K}(0) = 0.765\ (31)$. Using the experimental averages for $|V_{cd}| f_+^{D\to\pi}(0)$ and $|V_{cs}| f_+^{D\to K}(0)$, we extract $|V_{cd}| = 0.2330\ (137)$ and $|V_{cs}| = 0.945\ (38)$,

□ LCSRs with B-meson distribution amplitudes (DAs)

[A.K., N. Offen, Th. Mannel (2006)]

"SCET sum rules", [F. De Fazio, Th. Feldmann, T.Hurth (2006)]

- vacuum-to-B-correlation function:

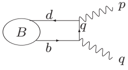
B on-shell state

$$F_{ab}(p,q) = i\int d^4x\, e^{ip\cdot x}\langle 0|\mathrm{T}\{\bar{d}(x)\Gamma_b q(x), \bar{q}(0)\Gamma_a b(0)\}|\bar{B}^0(q+p)\rangle = L_{ab}(p,q)F(p^2,q^2)$$

h-interpolating current

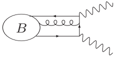
transition current

- OPE in terms of B-meson DA's
  defined in HQET,

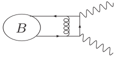


(a)

- valid at $0 < q^2 \ll m_B^2$
- dispersion relation:



(b)



(c)

$$F(p^2,q^2) = \frac{1}{\pi}\int\limits_{m_h^2}^{\infty}\frac{ds}{s-p^2}\mathrm{Im}F(s,q^2) = F_{OPE}(p^2,q^2)$$

$$\mathrm{Im}F_{ab} = \langle 0|\bar{d}\Gamma_b q|h\rangle\langle h|\bar{q}\Gamma_a b|\bar{B}^0(q+p)\rangle\pi\delta(m_h^2-s)+\ldots = L_{ab}\mathrm{Im}F(s,q^2)$$

$$\boxed{\frac{f_h}{m_h^2-p^2}F^{(B\to h)}(q^2) = \frac{1}{\pi}\int\limits_{m_h^2}^{s_0}\frac{ds}{s-p^2}\mathrm{Im}F_{OPE}(s,q^2)}$$

$\oplus$ Borel transf $p^2 \to M^2$

□ *B*-meson DAs

- definition of two-particle DA in HQET:

$$\langle 0|\bar{q}_{2\alpha}(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle$$
$$= -\frac{if_B m_B}{4}\int\limits_0^\infty d\omega e^{-i\omega v\cdot x}\left[(1+\not{v})\left\{\phi_+^B(\omega)-\frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\beta\alpha}$$

$\oplus$ higher twists

- key input parameter: the inverse moment

$$\frac{1}{\lambda_B(\mu)}=\int_0^\infty d\omega\frac{\phi_+^B(\omega,\mu)}{\omega}$$

  - possible to extract λ_B from $B\to\gamma\ell\nu_\ell$ using QCDF$\oplus$LCSR

    [M.Beneke, V.M. Braun, Y.Ji, Y.B. Wei (2018)]

  - current limit from Belle measurement (2018): $\lambda_B > 240$ MeV
  - QCD sum rules in HQET: $\lambda_B(1\text{ GeV}) = 460 \pm 110$ MeV

    [V.Braun, D.Ivanov, G.Korchemsky (2004)]

- higher twists DAs recently worked out [V. Braun, Y. Ji and A. Manashov (2017)]

□ $B \to K\pi\ell^+\ell^-$ beyond K^* from LCSRs

based on S. Descotes-Genon, AK and J.Virto, 1908.02267

- $B \to K^*\ell^+\ell^-$ - a major source of information on $b \to s\ell^+\ell^-$,
  - a rich set of observables
  - currently, tensions vs SM ("anomalies") ,
- $B \to K^*$ form factors
  - in lattice QCD, R.R.Horgan, Z.Liu, S.Meinel and M.Wingate, 1310.3722[hep-lat],
  - from QCD-based LCSRs, at large recoil

    A. Bharucha, D. M. Straub and R. Zwicky, 1503.05534 [hep-ph].

    N. Gubernari, A. Kokulu and D. van Dyk, 1811.00983 [hep-ph]].
- so far all calculations done in the limit of stable K^*
- in reality, K^* - unstable resonance in the $K\pi$ system,
  $\Gamma_{tot}(K^*) \simeq \Gamma_{tot}(K^* \to K\pi) \simeq 51$ MeV
- Can we go beyond the narrow K^* approximation in $B \to K\pi\ell\ell$?
  - the effect of the width
  - "nonresonant background" within and beyond the K^* window
- from the theory side we have to start from $B \to K\pi$ form factors
- Use of LCSRs with B-DAs for $B \to \pi\pi\ell\nu_\ell$ [S.Cheng, AK, J.Virto, 1701.01633]

□ $B \to K\pi$ form factors

- Definition:

$$\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma^\mu b|\bar{B}^0(q+k)\rangle = \frac{2\epsilon^{\mu\alpha\beta\gamma}}{\sqrt{k^2}\sqrt{\lambda}}\; q_\alpha\, k_\beta\, \bar{k}_\gamma F_\perp(q^2, k^2, q\cdot\bar{k})$$

$\oplus$ form factors of axial and tensor $b \to s$ currents

$k = k_1 + k_2$, $\bar{k} = k_1 - k_2 + \frac{m_K^2 - m_\pi^2}{k^2}(k_1 + k_2)$,
$q \cdot k \sim cos\theta$, θ- the angle between π and B in c.m. of $K\pi$.

- partial wave expansion, isolating $K\pi$ P-wave:

$$F_\perp(k^2, q^2, q\cdot\bar{k}) = \sum_{\ell=1}^{\infty} \sqrt{2\ell+1}\, F_\perp^{(\ell)}(k^2, q^2)\, \frac{P_\ell^{(1)}(\cos\theta_K)}{\sin\theta_K} = F_\perp^{(\ell=1)}(q^2, k^2)$$

$q^2 \lesssim 10$ GeV^2 (large recoil region), $k^2 \sim m_{K*}^2$ (resonance region)

- to reveal the K^* resonances:
  consider the dispersion relation for $F_\perp^{(\ell=1)}(q^2, k^2)$ in k^2

□ Dispersion relation in the $K\pi$ invariant mass

- a simpler object: the form factor in $\tau \to K\pi\nu_\tau$

$$\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma_\mu u|0\rangle = f_+(k^2)\,\overline{k}_\mu + ...$$

describes $K\pi$ dihadron in P-wave

- dispersion relation:

$$f_+(k^2) = \frac{1}{\pi}\int\limits_{(m_K+m_\pi)^2}^{\infty} ds\,\frac{\mathrm{Im} f^+(s)}{s-k^2}$$

no subtractions, cf. disp. relation for the pion FF

- unitarity:

$$\mathrm{Im} f^+(s)\bar{k}_\mu = \int d\tau_{K\pi}(s)\,\underbrace{\langle K\pi|K\pi\rangle^*}_{\mathcal{A}^*(K\pi\to K\pi)}\,\underbrace{\langle K\pi|\bar{u}\gamma_\mu s|0\rangle}_{f^+(s)\bar{k}_\mu}$$

$$+\sum_h \int d\tau_h(s)\langle K\pi|h\rangle^*\langle h|\bar{u}\gamma_\mu s|0\rangle$$

- dispersive approach: reconstructing $f^+(k^2)$ at low k^2

  elastic unitarity, Omnes representation, $\oplus$ $K\pi$ scattering data

- the $K^*(890)$ resonance appears in a model-independent way

$\square$ Resonance model of the $\tau \to K\pi\nu_\tau$ form factor

- $K\pi$ contribution to unitarity: approximate both components by intermediate K^*:

$$\langle K\pi|K\pi\rangle = \frac{\overbrace{\langle K\pi|K^*\rangle}^{g_{K^*K\pi}}\overbrace{\langle K^*|K\pi\rangle}^{g_{K^*K\pi}}}{m_{K^*}^2 - s} \qquad \langle K\pi|\bar{u}\gamma_\mu s|0\rangle = \frac{\overbrace{\langle K\pi|K^*\rangle}^{g_{K^*K\pi}}\overbrace{\langle K^*|\bar{u}\gamma_\mu s||0\rangle}^{f_{K^*}}}{m_{K^*}^2 - s}$$

  introducing decay constant f_{K^*} and strong coupling $g_{K^*K\pi}$.

- iterate, sum geometr.series, $\Rightarrow$ the energy dependent width in denominator:

$$\Gamma_{K^*}(s) = \frac{g_{K^*K\pi}^2 \lambda^{3/2}(s)}{32\pi s^{5/2}}\theta(s-(m_K+m_\pi)^2)\,, \quad \Gamma_{K^*}(m_K^*) = \Gamma_{tot}(K^*)$$

- resulting in the Breit-Wigner (Gounaris-Sakurai) formula
- add more resonances (motivation, QCD in the $N_c \to \infty$)
- the BW resonance formula

$$f_+(s) = \sum_{K^*} \frac{m_{K^*} f_{K^*} g_{K^*K\pi}}{m_{K^*}^2 - s - i\sqrt{s}\,\Gamma_{K^*}(s)} e^{i\phi_{K^*}(s)}$$

- relative phases reflect nondiagonal $K^* \to K^{*'}$ transitions
- more refined analysis demands coupled channel approach

□ Belle results on $\tau \to K\pi\nu_\tau$ form factor

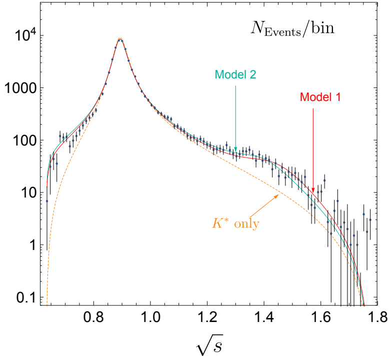
- Measurement of $\tau \to K_S \pi^- \nu_\tau$

  D.Epifanov et al. [Belle Collaboration], 0706.2231[hep-ex],

- $K\pi$ invariant mass distribution,
  fitted to the two-resonance model for $f^+(s)$ (Model 2):

$$f_+(s) = \sum_{K^*} \frac{m_{K^*}^2 \xi_{K^*}}{m_{K^*}^2 - s - i\sqrt{s}\,\Gamma_{K^*}(s)} e^{i\phi_{K^*}(s)}, ,$$

$$\xi_{K^*(892)} = 0.988\, e^{-i\,0.07}\ ,\ \xi_{K^*(1410)} = 0.074\, e^{i\,1.37}\ ,$$



□ Resonance model for the $B \to K\pi$ form factor

- in full analogy with the $\tau \to K\pi\nu_\tau$ form factor

$$F_\perp^{(\ell=1)}(s,q^2) = \sum_{K^*} \frac{c_{K^*,\perp}(s,q^2)\, g_{K^*K\pi}\, \mathcal{F}_{K^*\perp}(q^2)\, e^{i\phi_{K^*}(s)}}{m_{K^*}^2 - s - i\sqrt{s}\,\Gamma_{K^*}(s)}$$

$\mathcal{F}_{K^*\perp}(q^2) = V^{B\to K^*}(q^2)$ - the $B \to K^*$ form factor
(e.g., for $K^*(890)$, the one calculated on the lattice or with LCSR)

- having a possibility to calculate $F_\perp^{(\ell=1)}(s,q^2)$ we can try to:
  - neglect all excited K^* resonances and put $\Gamma_{K^*} \to 0$
  - restore $\Gamma_{K^*} \neq 0$
  - add more K^*
  - move outside the K^* window
- Can we calculate $B \to K\pi$ form factors using LCSRs?

□ Applying LCSRs with B-meson distribution amplitudes

- OPE diagrams ⇒ invariant amplitudes ⇒ dispersion form in k^2:

$$F^{\rm OPE}_{(\varepsilon)}(k^2,q^2) = f_B m_B \int_0^\infty d\sigma \, \frac{\phi_+^B(\sigma m_B)}{\bar\sigma(s-k^2)} + \{higher \;\; twists, 3-particle\ DAs\}$$

$s = s(\sigma,q^2) = \sigma m_B^2 - \sigma q^2/\bar\sigma$, $\bar\sigma \equiv 1-\sigma$, m_s included

- hadronic dispersion relation and unitarity:

$$F_{(\varepsilon)}(k^2,q^2) = \frac{1}{\pi} \int\limits_{4m_\pi^2}^{\infty} ds \, \frac{{\rm Im} F_{(\varepsilon)}(s,q^2)}{s-k^2} \, .$$

$$2\,{\rm Im} F_{\mu\nu}(k,q) = \int d\tau_{K\pi} \underbrace{\langle 0|\bar d\gamma_\mu s\,|K^-\pi^+\rangle}_{f_+(s)} \underbrace{\langle K^-\pi^+|\bar s\gamma_\nu b|\bar B^0(q+k)\rangle}_{B \to K\pi\ (\ell=1)\ \text{form factor}} + \cdots , e$$

Resulting sum rule for the form factor $F_\perp^{(\ell=1)}$

- obtained matching hadronic disp.relation and OPE $\oplus$ quark-hadron duality

$$\int_{(m_\pi+m_K)^2}^{s_0^{K\pi}} ds\, e^{-s/M^2} \frac{\sqrt{3}\,[\lambda_{K\pi}(s)]^{3/2}}{16\pi^2\sqrt{\lambda}\, s^{5/2}}\, f_+^\star(s)\, F_\perp^{(\ell=1)}(s,q^2)$$

$$= \quad f_B m_B \left[\int_0^{\sigma_0^{K\pi}} d\sigma\; e^{-s(\sigma,q^2)/M^2}\; \frac{\phi_+^B(\sigma m_B)}{\bar{\sigma}} + \{twist\ 3,4\} \right],$$

- determination of $s_0^{K\pi}$ in vector $K\pi$ channel,
- similar sum rules for all other P-wave $B\to K\pi$ form factors
- not a direct calculation, to fit a chosen ansatz of the $B\to K\pi$ form factors
- the complex phases of $B\to K\pi$ FF and $f_+(s)$ equal at low s:
- substitute resonance model for both $f_+(s)$ and $F_\perp(q^2,s)$, at $\Gamma_{tot}(K^*)\to 0$ restore the narrow limit for K^*.

$$\int_{(m_K+m_\pi)^2}^{s_0^{K\pi}} ds\, e^{-s/M^2}\{...\} f_{K^*} V^{B\to K^*}(q^2) \underbrace{\lim_{\Gamma\to 0}\left[\frac{1}{\pi}\frac{\sqrt{s}\,\Gamma_{K^*}(s)}{(m_{K^*}^2-s)^2+s\,\Gamma_{K^*}^2(s)}\right]}_{\delta(s-m_{K^*}^2)} = rhs\,.$$

□$B \to K^*$ form factors in the narrow width limit for K^*

Form Factor	This work	[1]	[2]	[3]
$V^{BK^*}(0)$	0.26(15)	0.39(11)	0.32(11)	0.34(4)
$A_1^{BK^*}(0)$	0.20(12)	0.30(8)	0.26(8)	0.27(3)
$A_2^{BK^*}(0)$	0.14(13)	0.26(8)	0.24(9)	0.23(5)
$A_0^{BK^*}(0)$	0.30(7)	–	0.31(7)	0.36(5)
$T_1^{BK^*}(0)$	0.22(13)	0.33(10)	0.29(10)	0.28(3)
$T_2^{BK^*}(0)$	0.22(13)	0.33(10)	0.29(10)	0.28(3)
$T_3^{BK^*}(0)$	0.13(12)	–	0.20(8)	0.18(3)

[1] AK, T. Mannel and N. Offen, hep/ph 0611193.

[2] N. Gubernari, A. Kokulu and D. van Dyk, 1811.00983 [hep-ph].

[3] A. Bharucha, D. M. Straub and R. Zwicky, 1503.05534 [hep-ph].

- [1] is the original calculation without new higher twist DAs
- [2] uses slightly different inputs including twist 5
- [3] is a different method using narrow K^* DAs

□ Dependence on the K^* width

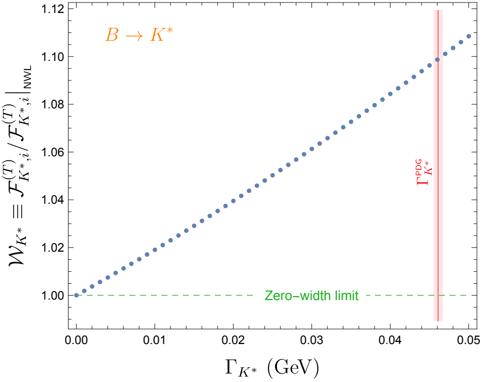
- effectively each sum rule is reduced to

$$\mathcal{F}_{K^*,i}(q^2)g(s_0, M^2, m_{K^*}, \Gamma_{K^*}) = P_i^{OPE}(q^2, \sigma_0, M^2)$$

where g in the single K^* case is independent on the type $i = \perp, \ldots$ of the FF

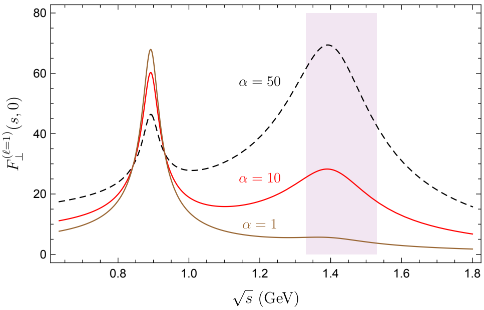
- Expanding g in the small ratio Γ_{K^*}/m_{K^*} yields for all form factors

$$\mathcal{W}_{K^*} = \frac{\mathcal{F}_{K^*,i}(q^2)}{\mathcal{F}_{K^*,i}(q^2)\big|_{NWL}} \simeq 1 + 1.9\,(\Gamma_{K^*}/m_{K^*}) \simeq 1.1$$



□ Probing the $K^{(*)}(1410)$ contribution

- the sum rule cannot separate contribuitons of $K^*(890)$ and $K^*(1410)$, we assume that the latter has a weight α:
  only marginally large α values influence the $K^*(890)$ peak

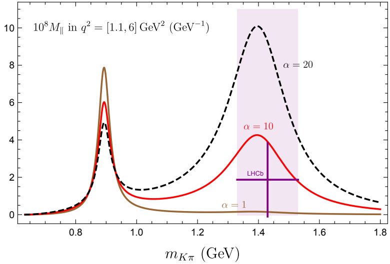


□ Constraining the $K^{(*)}(1410)$ contribution in the K^* window

- the differential decay width of $B \to K\pi\ell\ell$ can be expanded in angular moments

$$\frac{d\Gamma}{dq^2 dk^2 d\Omega} = \frac{1}{4\pi} \sum_{i=1}^{41} f_i(\Omega) \tilde{\Gamma}_i(q^2, k^2)$$

- forming a combination $M_{||}$ free from S and D wave contributions
- LHCb Collaboration in [1609.04736] has measured the moments in a large recoil q^2 bin and in the region of k^2 around the mass $K^{(*)}(1410)$
- we get an upper bound on the parameter α using these data and constrain the impact of $K^{(*)}(1410)$ in the $K^*(890$ region.



Conclusion

- LCSR results not discussed in this talk:
  - $B \to \pi\pi$ form factors in terms of dipion DAs

    [C. Hambrock, A.K., 1511.02509 (2015)]
  - $\Lambda_b \to p$, [AK, C.Klein, Th.Mannel, Y.M.Wang 1108.2971];
    $\Lambda_b \to \Lambda$, [T.Feldmann,M.Yip (2011)]
- at large recoil (small q^2) LCSRs with π, K LCDAs provide $B \to \pi, B_{(s)} \to K,\ D \to \pi, D \to K$ form factors
- more universal (yet less accurate) LCSRs with B-LCDAs provide access to $B \to K\pi\ell\ell$ form factors with $(K\pi)_{L=1}$ beyond K^*
- future plans: the S-wave of $h = \pi\pi, K\pi$ in $B \to h$ form factors
  combining dispersion theory in the low mass 2π and $K\pi$ with LCSRs.