

Present theoretical and experimental status of light vector mesons

Machavariani Alexander^{*a,b*}

^{*a*} VBLHEP JINR, Dubna
^{*b*} HEPI TSU, Tbilisi

CONTENT:

Short review of the observed properties and the theoretical approaches of the light mesons is given

We shall consider hadrons as quark clusters within the general and rigorous field-theoretical scheme (Huang-Weldon approach) of the bound states. Starting from this general formulation the quark-parton model of the inclusive particle production is exactly reproduced using the separable approximation.

This allows to extend the TMD parton model in the region, where four-momentum transfer $Q = \sqrt{Q^2}$ is same order or smaller than the transverse momentum p_T i.e. $p_T \simeq Q$ or $p_T > Q$.

Present model is applied for the numerical description of the experimental cross sections and spin density matrices of the inclusive ρ -meson production with the transverse momentum $\leq 1 - 2\text{GeV}/c$ in the energy region of NICA

Light vector mesons:

$$\begin{array}{lll}
 \rho^0(770) & \frac{1}{\sqrt{2}}[u\bar{u} - d\bar{d}] & \implies \pi^+ + \pi^- \\
 \omega(782) & \frac{1}{\sqrt{2}}[u\bar{u} + d\bar{d}] & \implies \pi^+ + \pi^- + \pi^0 \\
 \phi(1020) & s\bar{s} & \implies K^+ + K^- \text{ or } \pi^+ + \pi^- + \pi^0 \\
 K^*(892) & [u\bar{s} \text{ or } d\bar{s}], \dots &
 \end{array}$$

Light mesons are an excellent object for theoretical and experimental investigation of the simplest quark-antiquark system.

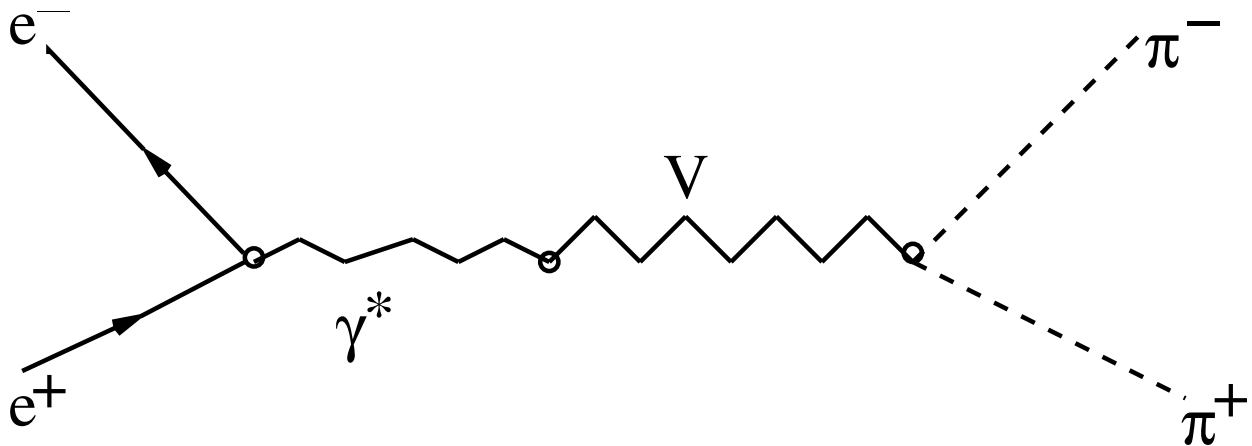
Almost theoretical models of the light vector mesons can be applied for the heavy vector mesons with heavy quarks.

Heavy vector mesons $M_V > 2000 \text{ MeV}$:
 $J/\psi(3097)$ $[c\bar{c}]$, $D(2010)$, $D(2007)$
 $[c\bar{d}]$ and $[c\bar{u}]$, $B(5325)$ -mesons $[u\bar{b}]$ and $[d\bar{b}]$ and $[s\bar{b}]$
 etc.

First unified model of the electromagnetic and strong interactions

$$\gamma^* \longleftrightarrow \rho^0 \text{ transition (mixing)}$$

of shell photon γ^* $E_{\gamma^*} = E_{e^+} + E_{e^-} = E_{\pi^+} + E_{\pi^-}$



Presently is considered

$$\gamma^* \longleftrightarrow \rho^0 \longleftrightarrow \omega \longleftrightarrow \phi \longleftrightarrow J/\psi \text{--mixing}$$

Experimental justification of $\gamma^* \longleftrightarrow \rho^0 \longleftrightarrow \omega$ mixing was performed for the el.-mag. form factor..

Vector Meson Dominance model (VMD).

In 1958–1995 were constructed gauge invariant and renormalizable Lagrangian's for the coupled $\gamma - \rho - \pi - N$ system (Review Paper: 1997, O'Connell, Pearce. Thomas and Williams), where

- 1) γ and ρ -meson were the gauge field
- 2) Photon appears to couple with ρ which vanishes at $q^2 = 0$ for on shell photon

Bando, Kugo et al (1985) have constructed a local gauge Lagrangian which reproduces VMD model starting from the non-linear σ -model. In this approach ρ is dynamical gauge boson in nonlinear, chiral Lagrangian.

ρ -meson is not QCD-particle in this approach, because the mass of ρ -meson is not generated by spontaneous symmetry breaking.

Crucial point of these approaches are the requirements

A) Universality of ρ -interaction

$$g_{\rho-\pi\pi} = g_{\rho-NN} = g_{rho-gauge}$$

B) El.-mag. form factor of pion satisfies condition

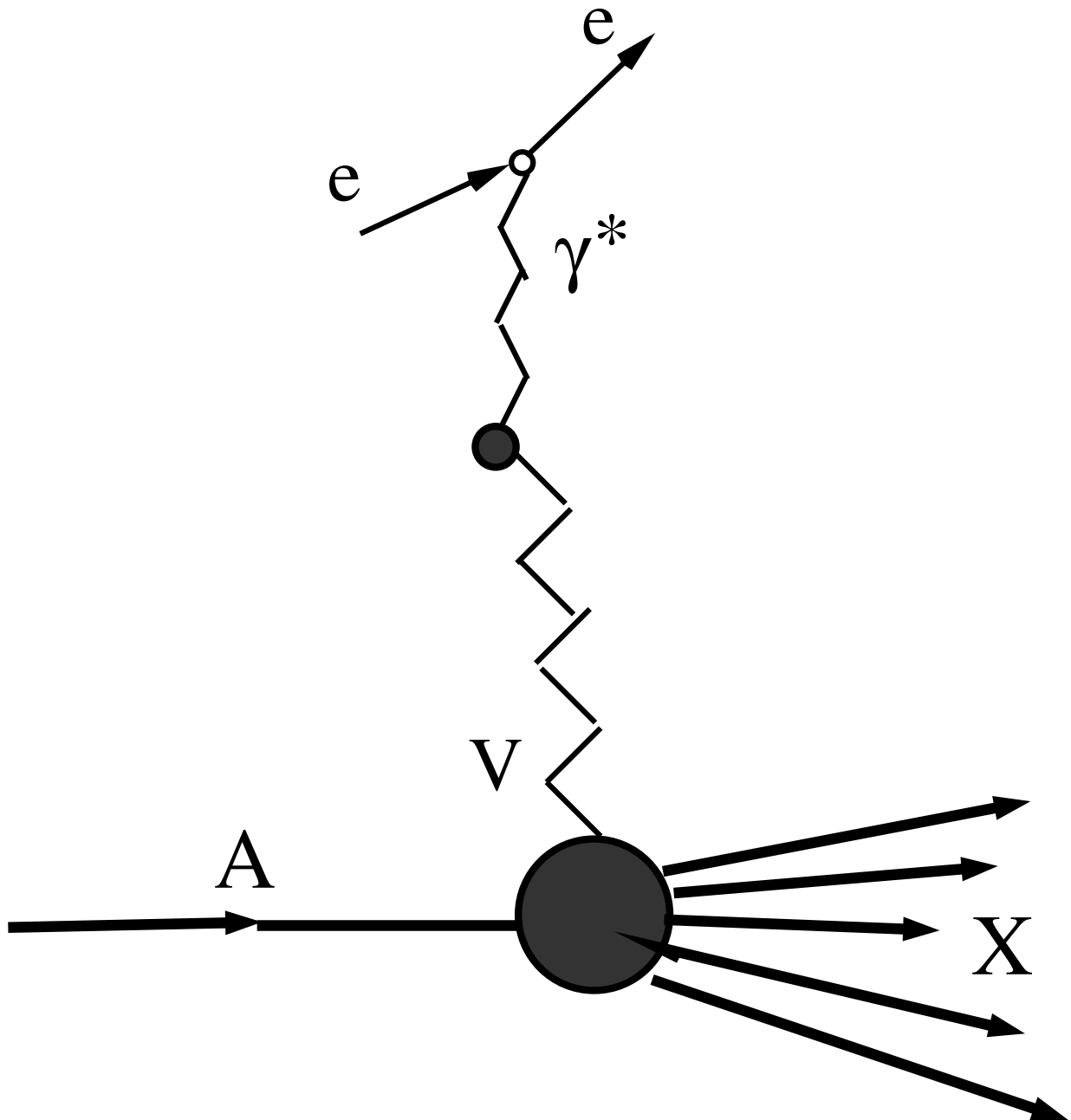
$$F_{\gamma^*\pi\pi}(q^2 = 0) = 1.$$

The realistic calculation of the $\pi\pi$, πN and NN scattering in the low energy region (up to $1 - 2GeV$) within One Boson Exchange model and also in the constituent quark model have required different $g_{\rho-\pi\pi}$ and $g_{\rho-NN}$, i.e. VMD model is violated.

Also in Effective Lagrangian theory the different $g_{\rho-\pi\pi}$ and $g_{\rho-NN}$ are used.

On the other hand the numerical calculations of $\pi\pi$, πN and NN scattering within above VMD-motivated formulations determine the limits for the coupling constants $g_{\rho-\pi\pi}$ and $g_{\rho-NN}$.

$\gamma^* - V$ mixing in the inclusive reactions



$\gamma - V$ mixing and VMD model is used for describing of inclusive reactions in the V -meson energy region,

One can not use parton model for inclusive reactions with creation of the light vector mesons, because parton model requires

$$Q^2 \gg P_T^2 \quad \textit{fragmentation theorem.}$$

for inclusive creation of V -mesons $Q^2 = m_V^2$
and $p_T \sim 1.2 \text{ GeV}$

Therefore we have reproduced parton-type model and its corrections from the nonlocal field-theoretical approach without fragmentation theorem.

Field-theoretical approach of hadrons as quark bound states (Huang-Weldon 1975)

I) Construction of the creation (annihilation) operators of the quark clusters (bound states).

$$A_{P_A}(X_o) = i \int d^3 X d^4 \rho U_A(x) \overleftrightarrow{\frac{\partial}{\partial X_o}} A(x)$$

$$A(x) \equiv T (q_1(x_1), \bar{q}_2(x_2))$$

Bethe-Salpeter wave function

$$U_A(x) \equiv \langle P_A || T (q_1(x_1), \bar{q}_2(x_2)) | 0 \rangle$$

$$\rho = x_1 - x_2; \quad X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2},$$

$$\lim_{X_o \rightarrow \pm \infty} A_{P_A}(X_o) = A_{P_A}^{(out)}$$

Main achievement of this approach

$$\text{quantization } [A_P^{(out)}, A_{P'}^{*(out)}]_- = \delta^3(\mathbf{p}' - \mathbf{p})$$

$$P \equiv (\sqrt{m^2 + \mathbf{p}^2}, \mathbf{p})$$

$$\text{But } [A_P(0), A_{P'}^*(0)]_- \neq \delta^3(\mathbf{p}' - \mathbf{p})$$

Generalization of S -matrix reduction
technique for nonlocal fields

$$S_{A\dots B,C\dots D} = \int d[x]\dots d[y]d[z]\dots d[v]$$

$$U^+_A(x) \frac{\mathcal{K}_A}{i} \dots U^+_B(y) \frac{\mathcal{K}_B}{i} \mathcal{G} \frac{\mathcal{K}_C}{i} U_C(z) \dots \frac{\mathcal{K}_D}{i} U_D(v)$$

$$\mathcal{K}_A \equiv \square_{X_A} + m_A^2; \quad d[x] \equiv d^4 X d^4 \rho = d^4 x_1 d^4 x_2$$

$$U_A(x) \equiv \langle P_A | T (q_1(x_1), \bar{q}_2(x_2)) | 0 \rangle$$

$$\mathcal{G} = \langle 0 | T (A(x) \dots B(y) C(z) \dots D(v)) | 0 \rangle$$

Relationship with the S -matrix reduction formula for local fields

$$U_A(x) \implies e^{-ip_A X} \text{ - plane wave}$$

$$A(x) = (q_1(x_1), \bar{q}_2(x_2)) \implies \Phi(x)$$

Similar formulas were used in non-relativistic scattering theory, nuclear physics etc.

We have obtain other representation for S -matrix

$$S_{A...B,C...D} = \int d^3[\mathbf{q}^A] \dots d^3[\mathbf{q}^B] d^3[\mathbf{q}^C] \dots d^3[\mathbf{q}_1^D]$$

$$\langle \mathbf{P}_A | \mathbf{q}_1^A \mathbf{q}_2^A; out \rangle \dots \langle \mathbf{P}_B | \mathbf{q}_1^B \mathbf{q}_2^B; out \rangle$$

$$\mathcal{S}_{quark} \langle in; \mathbf{q}_1^C \mathbf{q}_2^C | \mathbf{P}_C \rangle \dots \langle in; \mathbf{q}_1^D \mathbf{q}_2^D | \mathbf{P}_D \rangle$$

$$\mathcal{S}_{quark} = \langle out; \mathbf{q}_1^A \mathbf{q}_2^A \dots \mathbf{q}_1^B \mathbf{q}_2^B | \mathbf{q}_1^C \mathbf{q}_2^C \dots \mathbf{q}_1^D \mathbf{q}_2^D; in \rangle$$

$$d^3[\mathbf{q}] = \frac{d^3 \mathbf{q}_1}{2q_1^0} \frac{d^3 \mathbf{q}_2}{2q_2^0}; \quad q_{1,2}^0 = \sqrt{\mathbf{q}_{1,2}^2 + m_{1,2}^2}$$

Reproduction of the TMD parton model in
Huang-Weldon approach using the separable
approximation:

Inclusive creation of the V -meson

$$A(\mathbf{P}_A) + B(\mathbf{P}_B) \implies V(\mathbf{k}) + X$$

$$A + B \equiv p + p; \quad p + A; \quad A + A;$$

$$V = \rho, \phi, \quad \text{Vector mesons}$$

Using our $\mathcal{S}_{AB \rightarrow VX}$ reduction formulas with
reduction of the V -meson only

$$\mathcal{T}(AB - V_M X) = \sum_{n\bar{n}} \int \frac{d^3 \mathbf{q}_1}{2E_{\mathbf{q}_1}} \int \frac{d^3 \mathbf{q}_2}{2E_{\mathbf{q}_2}} \delta^4(k - q_1 - q_2)$$

$$\langle \mathbf{P}_A, \mathbf{P}_B | T | \mathbf{q}_1 n; \mathbf{q}_2, \bar{n} : X \rangle$$

$$\xi^\mu(\mathbf{k}, M) \langle \mathbf{q}_1 n, \mathbf{q}_2 \bar{n} | J_\mu(0) | 0 \rangle$$

In order to reproduce exactly the TMD parton model from above general expression, it is necessary to separate the V -meson-quark vertex

$$\begin{aligned} \langle \mathbf{q}_1 n; \mathbf{q}_2; \bar{n} | J_\mu(0) | 0 \rangle = & g_V^{n\bar{n}} \bar{v}(\mathbf{q}_2) \gamma_\mu u(\mathbf{q}_1) \\ & + g_T^{n\bar{n}} \bar{v}(\mathbf{q}_2) \frac{i\sigma_{\mu\nu} (q_1 + q_2)^\nu}{m_1 + m_2} u(\mathbf{q}_1) \end{aligned}$$

and

$$\mathbf{K} = \sum_X \mathcal{T}(AB - q_1 q_2; X) \mathcal{T}^*(AB - q_1 q_2; X)$$

with on shell quarks and $\delta^4(k - q_1 - q_2)$

Firstly similar separation of the V -meson and quark degrees of freedom was performed by A.V. Efremov and O. V. Teryaev. JINR Preprint 1982 (in Russian)

In this paper we have developed this approach.

Afterwards we have used separable approximation:

$$\mathcal{K}^{n'n} = F_{n/B}(\mathbf{q}_1)F_{\bar{n}/B}(\mathbf{q}_2)F_{n'/A}(\mathbf{q}'_1)F_{\bar{n}'/A}(\mathbf{q}'_2)$$

Special case: Ralson, Soper (1979), Boer (1999).

]]

Separable approximation of any smooth enough function $f(x, y)$

$$f(x, y) = \sum_{\bar{m}} g_m(x)\tilde{g}_m(y)$$

can be performed with any a priori given precision.

In practical calculations $g_m(x)$ is INPUT function and usually $m = 1$ or 2

In TMD PDF model: $\mathbf{f}_{n/A}$ is fully separated:

$$\mathbf{f}_{n/A}(x, \mathbf{q}_T) = f_{n/A}(x) \frac{e^{-\mathbf{q}_T^2/2b^2}}{2b^2}$$

$$x_{1,2} = x'_{1,2} = \frac{k_o \pm \mathbf{k}_Z}{2P}$$

Finally we get the same expression as in TMD PDF model

$$\frac{d\sigma_{A+B \rightarrow \rho^0+X}^{MM'}(TMD)}{d^3\mathbf{k}} \sim \sum_{\bar{n}} \int d^2\mathbf{q}_T$$

$$[f_{n/A}(x_A, \mathbf{q}_T) f_{\bar{n}/B}(x_B, \mathbf{k}_T - \mathbf{q}_T)]$$

$$\frac{d\sigma_{LO}^{MM'}(q\bar{q} \rightarrow V)}{d^3\mathbf{k}}$$

$$\frac{d\sigma_{LO}^{MM'}(q\bar{q} \rightarrow V)}{d^3\mathbf{k}} \sim \sum_{s_1 s_2}$$

$$\xi^\mu(\mathbf{k}, M) \langle \mathbf{q}_1 s_1, n; \mathbf{q}_2 s_2, \bar{n} | J_\mu(0) | 0 \rangle$$

$$\xi_\nu^*(\mathbf{k}, M') \langle 0 | J_\nu(0) | \mathbf{q}'_1 s_1, n; \mathbf{q}'_2 s_2, \bar{n} \rangle$$

Cross sections depends on $V - q\bar{q}$ coupling constants g_V and g_T and on the masses of quarks m_1, m_2 through the parameter

$$\delta = \frac{m_V^2 - (m_1 + m_2)^2}{m_V^2}$$

$$\left[\frac{g_T^2}{(g_V - g_T)^2} \frac{m_V^2 - (m_1 + m_2)^2}{(m_1 + m_2)^2} - \frac{2g_T}{(g_V - g_T)} - 1 \right]$$

I Curreant quark masses:

$$3MeV m_1 = m_2 < 25MeV$$

$$g_T = \alpha g_V \frac{m_1 + m_2}{m_V}$$

Magnetic constant $g_T \sim (m_1 + m_2)/m_V$ is small. Nevertheless we have degeneracy of δ through α .

In this model the same isotropic and anisotropic cross section are reproduced using different choice of α and g_V .

II Constituent quark masses:

$$200MeV m_1 = m_2 < m_V/2$$

$$g_T = \alpha g_V$$

In this case g_T is not small.

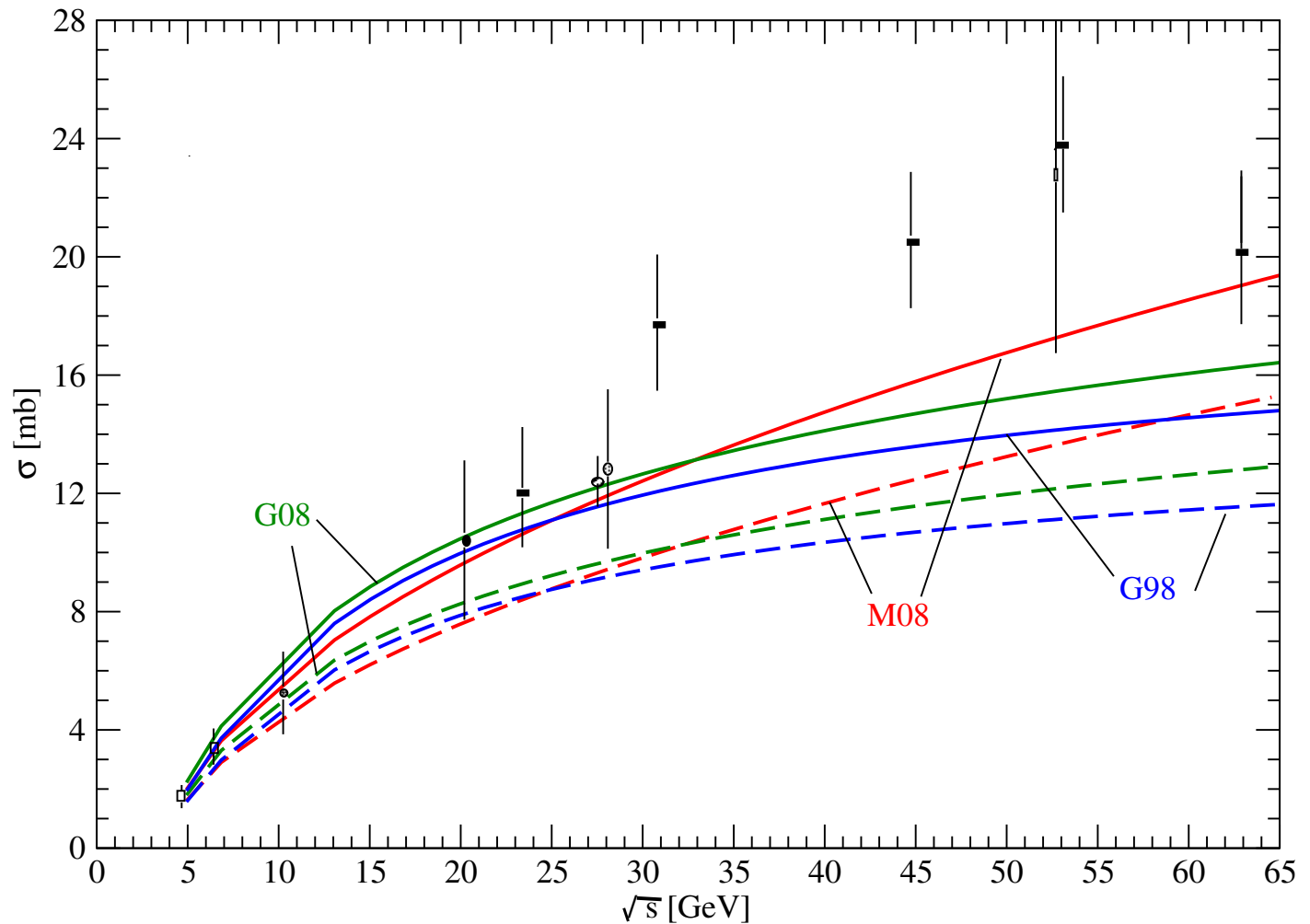
Strong degeneracy of δ through α and over the choice of the masses of quarks.

We have omitted the spin interaction between quarks.

Therefore present degeneracy is in analogue with the interaction of the two spin 1/2 particle through the central potential

$$\mathcal{V}_{tot}(r, s_1 s_2) = V(r) + V_1(r)(\mathbf{l}, \mathbf{s}_1 + \mathbf{s}_2) + V_2(r)(\mathbf{s}_1 \cdot \mathbf{s}_2) +$$

The interaction through the central potential only is degenerated.



$\sigma_{AB \rightarrow \rho^0 X}$ as a function of \sqrt{s} . Experimental data at 4.93, 6.84 GeV from Bloebel74, at 1.2 GeV from Ammosov76, at 26.8 GeV from Kiohmi78, Albrow79, at 23.6, 30.6, 44.6, 52.8, 63.0 GeV from Albrow79 at 52.5 GeV from Drijard81, and at 27.5 GeV from Anguilar91.

CONCLUSION:

Within general and rigorous field-theoretical scheme (Huang-Weldon approach) hadrons are constructed as quark bound states. Starting from this general formulation the quark-parton model of the inclusive particle production is exactly reproduced using the separable approximation.

The resulting 3D equations for the hadron scattering amplitudes and corresponding unitary conditions in the formulations with and without quark degrees of freedom have the same form. Thus propagation of quarks and gluons in the intermediate states does not change unitary condition for hadrons.

This allows to extend the TMD parton model in the region, where four-momentum transfer $Q = \sqrt{Q^2}$ is same order or smaller than the transverse momentum p_T i.e. $p_T \simeq Q$ or $p_T > Q$.

Present model is applied for the numerical description of the experimental cross sections and spin density matrices of the inclusive ρ -meson production with the transverse momentum $\leq 1 - 2\text{GeV}/c$ in the energy region of NICA