Workshop “Frontiers of QCD”
Tbilisi, September 26-28, 2019

Neutron-Antineutron Oscillations:
Discrete Symmetries and Effective Operators

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[arXiv:1809.00997] Phys.Lett. B788 (2019)

Search for the neutron-antineutron oscillations was suggested by Vadim Kuzmin in 1970, and such experiments are under active discussion now, see D. G. Phillips, II *et al.*, Phys. Rept. **612**, 1 (2016) Particular interesting perspectives are due to European Spallation Source currently under construction in Lund — the most powerful pulsed neutron source.

This is a transition where the baryon charge $\mathcal{B}$ is changed by two units. The observation of the transition besides demonstration of the baryon charge non-conservation could be also important for explanation of baryogengesis. Of course, following Sakharov conditions, it should be also accompanied by CP non-conservation.

Thus, discrete symmetries associated with neutron-antineutron mixing are of real interest.

C, P and T symmetries in $|\Delta\mathcal{B}| = 2$ transitions

In our 2015 text Zurab Berezhiani, AV, arXiv:1506.05096

we noted that the parity $\mathbf{P}$, defined in such a way that $\mathbf{P}^2 = 1$, is broken in n-nbar transition as well as $\mathbf{CP}$. Indeed, eigenvalues of parity $\mathbf{P}$ are ± 1 and opposite for neutron and antineutron. So, with this definition n-nbar mixing breaks $\mathbf{P}$ and $\mathbf{CP}$.

We noted, however, that it does not automatically imply an existence of CP breaking in absence of interaction. In September of the same 2015 we presented at the INT workshop in Seattle a modified definition of parity $\mathbf{P}_z$, such that $\mathbf{P}_z^2 = -1$, and parities $\mathbf{P}_z$ are i for both, neutron and antineutron. With this modification all discrete symmetries are preserved in n-nbar transition.

The subject has an interesting history which goes back to the famous paper by Ettore Majorana in 1937 where he introduced the notion of Majorana fermions. In the footnote there he noted that the parity for Majorana fermion is i. In the same issue of Nuovo Cimento this was discussed in more detail by Gulio Racah.

Wolfenstein and Kaiser followed this definition in their applications to neutrino. But it took some time to apply it to neutron-antineutron mixing.

Dirac Lagrangian for neutron

$$\mathcal{L}_D = i\bar{n}\gamma^\mu \partial_\mu n - m\,\bar{n}n$$

describes free neutron and antineutron and preserves the baryon charge, $\mathcal{B} = 1$ for n, $\mathcal{B} = -1$ for $\bar{n}$.

Continuous $\mathrm{U}(1)_\mathcal{B}$ symmetry:

$$n \to \mathrm{e}^{i\alpha} n, \quad \bar{n} \to \mathrm{e}^{-i\alpha} \bar{n}$$

Another term $-im'\bar{n}\gamma_5 n$ consistent with $\mathcal{B}$ conservation can be rotated away by the chiral rotation, $n \to \mathrm{e}^{i\beta\gamma_5} n$.

Four degenerate states: two spin doublets differ by $\mathcal{B}$.

How does baryon number non-conservation shows up?

At the level of free particles it could be only bilinear $|\Delta\mathcal{B}| = 2$ mass terms:

$$C = i\gamma^2\gamma^0$$

$$n^T C n\,, \quad n^T C\gamma_5 n\,, \quad \bar{n} C \bar{n}^T, \quad \bar{n} C\gamma_5 \bar{n}^T$$

At these bilinear in fields the most generic Lorentz invariant modifications reduce by field redefinitions to the only one term, breaking baryon charge by two units,

$$\Delta\mathcal{L}_{\not{B}} = -\frac{1}{2}\,\epsilon\left[n^T C n + \bar{n} C \bar{n}^T\right] \qquad C = i\gamma^2\gamma^0$$

where ϵ is a real positive parameter. Redefinitions are due to U(2) symmetry of the kinetic term $i\bar{n}\gamma^\mu\partial_\mu n$

What is the status of $\mathbf{C}$, $\mathbf{P}$ and $\mathbf{T}$ discrete symmetries? Let us start with the charge conjugation $\mathbf{C}$:

$$\mathbf{C}: \quad n \longleftrightarrow n^c = C\bar{n}^T$$

Kind of Z_2 symmetry, $\mathbf{C}^2 = 1$. Most simple in the Majorana representation

$$n^c = n^*\,.$$

Lagrangians can be rewritten as

$$\mathcal{L}_D = \frac{i}{2}\left[\bar{n}\gamma^\mu\partial_\mu n + \overline{n^c}\gamma^\mu\partial_\mu n^c\right] - \frac{m}{2}\left[\bar{n}n + \overline{n^c}n^c\right],$$

$$\Delta\mathcal{L}_{\not{B}} = -\frac{1}{2}\,\epsilon\left[\overline{n^c}n + \bar{n}\,n^c\right],$$

what makes C-invariance explicit.

Lagrangians are diagonalized in terms of Majorana fields $n_{1,2}$

$$n_{1,2} = \frac{n \pm n^c}{\sqrt{2}}, \qquad \mathrm{C}n_{1,2} = \pm\, n_{1,2}.$$

$$\mathcal{L}_D = \frac{1}{2}\sum_{k=1,2}\left[\bar{n}_k\gamma^\mu\partial_\mu n_k - m\,\bar{n}_k n_k\right],$$

$$\Delta\mathcal{L}_{\not{B}} = -\frac{1}{2}\,\epsilon\left[\bar{n}_1\,n_1 - \bar{n}_2\,n_2\right].$$

Splitting into two Majorana spin doublets with masses

$$M_1 = m + \epsilon \qquad M_2 = m - \epsilon$$

The parity transformation $\mathbf{P}$ involves, besides reflection of space coordinates, the substitution

$$\mathbf{P}: \qquad n \to \gamma^0 n\,, \qquad n^c \to -\gamma^0 n^c\,.$$

We use $\gamma^0 C \gamma^0 = -C$. The opposite signs reflect the opposite parities of fermion and antifermion C.N. Yang '50 V.B. Berestetsky '51

The definition satisfies $\mathbf{P}^2 = 1$ so eigenvalues of $\mathbf{P}$ are ± 1, opposite parities for fermion and antifermion. Different parities of neutron and antineutron implies that their mixing breaks $\mathbf{P}$ parity. Indeed, $\mathbf{P}$ -transformation changes $\Delta\mathcal{L}_{\not B}$ to $(-\Delta\mathcal{L}_{\not B})$. With $\mathbf{C}$-invariance it implies that $\Delta\mathcal{L}_{\not B}$ is also $\mathbf{CP}$ odd.

This CP-oddness, however, does not translates immediately into observable CP-breaking effects.
To get them one needs an interference of amplitudes provided only by interaction.

This subtlety is discussed in number of textbooks, see e.g.
V. B. Berestetsky, E. M. Lifshitz and L. P. Pitaevsky,
Let's remind it.

When $\mathcal{B}$ is conserved there is there is no transition between sectors with different $\mathcal{B}$. One can combine $\mathbf{P}$ with a $\mathrm{U}(1)_{\mathcal{B}}$ phase rotation and define $\mathbf{P}_\alpha$

$$\mathbf{P}_\alpha = \mathbf{P}\, \mathrm{e}^{i\mathcal{B}\alpha} : \quad n \to \mathrm{e}^{i\alpha}\gamma^0 n\,, \quad n^c \to -\mathrm{e}^{-i\alpha}\gamma^0 n^c$$

Of course, then $\mathbf{P}_\alpha^2 = \mathrm{e}^{2i\mathcal{B}\alpha} \neq 1$ but the phase is unobservable while $\mathcal{B}$ is is conserved.

When $\mathcal{B}$ is its not conserved the only remnant of $\mathrm{U}(1)_{\mathcal{B}}$ rotations is Z_2 symmetry, $n \to -n.$ It means that we can consider a different parity definition P_z, such that $\mathrm{P}_z^2 = -1$.

Thus, choosing $\alpha = \pi/2$ we come to

$$\mathrm{P}_z = \mathrm{P}e^{iB\pi/2}: \qquad n \to i\gamma^0 n\,, \qquad n^c \to i\gamma^0 n^c\,.$$

Moreover, in case of Majorana fermions it is the only possible choice. Indeed, in Majorana representation where

$$\gamma^0 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}$$

only $i\gamma^0$ preserves reality of the Majorana spinor. Also P_z preserves the Majorana structure of $n_{1,2}$ fields, $n_{1,2} = \dfrac{n \pm n^c}{\sqrt{2}}$,

This was derived by Ettore Majorana and Giulio Racah in 1937.

Free neutron oscillations

If we start at t=0 from neutron state then $\Delta B = 2$ transition leads for probability to find antineutron

$$P_{\bar{n}} = \sin^2 \epsilon t$$

Here we implying that time t is much smaller than the neutron lifetime. For such time intervals

$$P_{\bar{n}} = (\epsilon t)^2 = (t/\tau_{n\bar{n}})^2 \qquad \tau_{n\bar{n}} = 1/\epsilon$$

Free neutron ILL experiment (1994) gives the bound

. $$\tau_{n\bar{n}} > 0.86 \times 10^8 \text{ s}$$

Six-quarks operators: discrete symmetries

New physics beyond the Standard Model, leading to $|\Delta B| = 2$ transitions, induces the effective six-quark interaction,

$$\mathcal{L}\,(\Delta\mathcal{B} = -2) = \frac{1}{M^5}\sum c_i \mathcal{O}^i\,,$$
$$\mathcal{O}^i = T^i_{A_1A_2A_3A_4A_5A_6} q^{A_1} q^{A_2} q^{A_3} q^{A_4} q^{A_5} q^{A_6}\,,$$

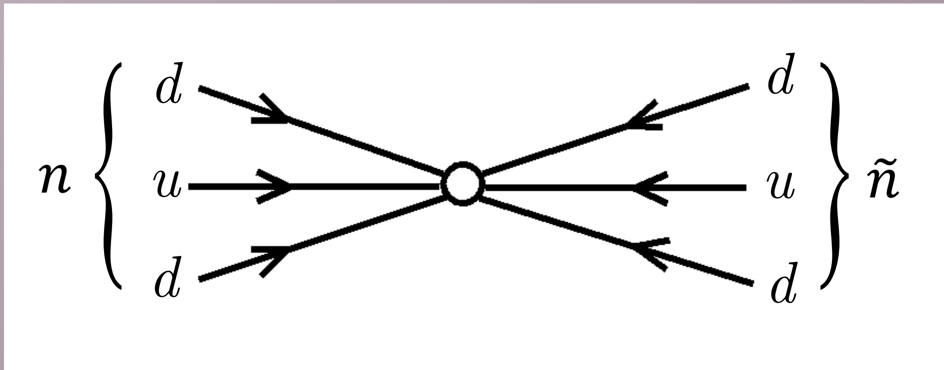
where coefficients T^i account for color, flavor and spinor structures.

In particular, for n-nbar mixing

$$\langle\bar{n}|\,\mathcal{L}\,(\Delta\mathcal{B} = -2)\,|n\rangle = -\frac{1}{2}\,\epsilon\, v^T_{\bar{n}} C\, u_n$$

it lead to an estimate

$$\epsilon = \frac{1}{\tau_{n\bar{n}}} \sim \frac{\Lambda^6_{\mathrm{QCD}}}{M^5}\,.$$



This implies that $M > 10^3$ TeV

For u and d quarks of the first generation the full list of operators was determined S. Rao and R. Shrock, W. E. Caswell, J. Milutinovic and G. Senjanovic

$$\mathcal{O}^{1}_{\chi_1\chi_2\chi_3} = u^{iT}_{\chi_1} C u^{j}_{\chi_1} d^{kT}_{\chi_2} C d^{l}_{\chi_2} d^{mT}_{\chi_3} C d^{n}_{\chi_3} \big[\epsilon_{ikm}\epsilon_{jln} + \epsilon_{ikn}\epsilon_{jlm} + \epsilon_{jkm}\epsilon_{nil} + \epsilon_{jkn}\epsilon_{ilm}\big],$$

$$\mathcal{O}^{2}_{\chi_1\chi_2\chi_3} = u^{iT}_{\chi_1} C d^{j}_{\chi_1} u^{kT}_{\chi_2} C d^{l}_{\chi_2} d^{mT}_{\chi_3} C d^{n}_{\chi_3} \big[\epsilon_{ikm}\epsilon_{jln} + \epsilon_{ikn}\epsilon_{jlm} + \epsilon_{jkm}\epsilon_{nil} + \epsilon_{jkn}\epsilon_{ilm}\big],$$

$$\mathcal{O}^{3}_{\chi_1\chi_2\chi_3} = u^{iT}_{\chi_1} C d^{j}_{\chi_1} u^{kT}_{\chi_2} C d^{l}_{\chi_2} d^{mT}_{\chi_3} C d^{n}_{\chi_3} \big[\epsilon_{ijm}\epsilon_{kln} + \epsilon_{ijn}\epsilon_{klm}\big].$$

Here χ_i stand for L or R quark chirality. Accounting for relations

$$\mathcal{O}^{1}_{\chi LR} = \mathcal{O}^{1}_{\chi RL}\,, \quad \mathcal{O}^{2,3}_{LR\chi} = \mathcal{O}^{2,3}_{RL\chi}\,,$$

$$\mathcal{O}^{2}_{\chi\chi\chi'} - \mathcal{O}^{1}_{\chi\chi\chi'} = 3\mathcal{O}^{3}_{\chi\chi\chi'}\,,$$

we deal with 14 operators for $\Delta\mathcal{B} = -2$ transitions.

The P_z reflection interchanges L and R chiralities χ_i in the operators $O^i_{\chi_1\chi_2\chi_3}$. Thus, we can divide operators into P_z even and P_z odd ones,

$$O^i_{\chi_1\chi_2\chi_3} \pm L \leftrightarrow R$$

The charge conjugation C transforms operators $O^i_{\chi_1\chi_2\chi_3}$ into Hermitian conjugated $[O^i_{\chi_1\chi_2\chi_3}]^\dagger$. So, we have 14 C-even operators, $O^i_{\chi_1\chi_2\chi_3} + \mathrm{H.c.}$, and 14 C-odd ones, $O^i_{\chi_1\chi_2\chi_3} - \mathrm{H.c.}$

In total, we break all 28 operators in four sevens with different P_z, C and CP_z features,

$$[O^i_{\chi_1\chi_2\chi_3} + L \leftrightarrow R] + \mathrm{H.c.}, \quad \mathrm{P}_z = +, \quad \mathrm{C} = +, \quad \mathrm{CP}_z = +$$

$$[O^i_{\chi_1\chi_2\chi_3} + L \leftrightarrow R] - \mathrm{H.c.}, \quad \mathrm{P}_z = +, \quad \mathrm{C} = -, \quad \mathrm{CP}_z = -$$

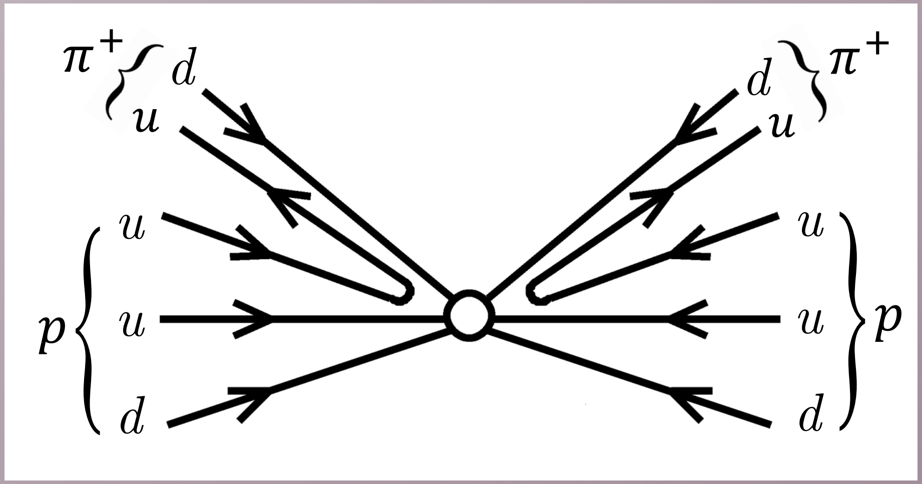
$$[O^i_{\chi_1\chi_2\chi_3} - L \leftrightarrow R] + \mathrm{H.c.}, \quad \mathrm{P}_z = -, \quad \mathrm{C} = +, \quad \mathrm{CP}_z = -$$

$$[O^i_{\chi_1\chi_2\chi_3} - L \leftrightarrow R] - \mathrm{H.c.}, \quad \mathrm{P}_z = -, \quad \mathrm{C} = -, \quad \mathrm{CP}_z = +$$

Only the first seven operators which are both P_z and C even contributes to n-nbar mixing. What about remaining 21 operators which are odd under P_z or C ? Although they do not contribute to nnbar oscillations they show up in instability of nuclei.

Diagram shows

$$p + p \to \pi^+ + \pi^+$$



Relation between nnbar oscillations and nuclear instability:

$$\Gamma_A = 4\,\frac{\epsilon^2}{\Gamma_{\bar{n}}}$$

Here $\Gamma_A = 1/T_A$ is the width associated with the nuclei instability (per one neutron), and $\Gamma_{\bar{n}}$ is the width, associated with absorption of antineutron in nucleus.

Friedman and Gal in their 2008 paper made more refined calculations relating the lower bound

$$T_A > 1.77 \times 10^{32}\,\mathrm{yr}$$

from the oxygen lifetime measured in Super-Kamiokande to get the lower bound for the oscillation time

$$\tau_{n\bar{n}} > 3.3 \times 10^{8}\ \mathrm{s}$$

Their consideration does not account for annihilation processes with $|\Delta B| = 2$ such as

$$N + N \to n\pi$$

we discussed above.

These two-particle contributions are suppressed as compared to the one-particle nnbar part due to smallness of ratio of the nucleon size over distance between nucleons in in the nucleus.

However, the two-particle part grows with nucleon number A as A^2 while nnbar part is linear in A. Thus, the nucleus lifetime is more sensitive to $|\Delta B| = 2$ transitions and its relation to the oscillation time $\tau_{n\bar{n}}$ should be reconsidered.

Conclusions

We demonstrate that the free neutron oscillation preserves all discrete symmetries, C, P and T. The subtlety is that P should be defined as $\mathbf{P}_z$ with $\mathbf{P}_z^{\mathbf{2}} = -\mathbf{1}$. Then, parities of both, neutron and and antineutron, are the same i, and their mixing is consistent with conservation of parity.

Our classification of $|\Delta B| = 2$ operators coming from new physics could be useful in association with Sakharov conditions for theory of baryogenisis.

Relation between the nucleus instability and nnbar oscillations could be more subtle due to $|\Delta B| = 2$ two-particle annihilation processes. It calls for additional studies.