Collective treatment of the Isovector and Isoscalar pair correlations. Pairing vibrations. Boson representation.

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Our consideration is based on the Hamiltonian with a constant pairing

$$H = H_0 + H_{\text{int}}$$

Where

$$H_0 = \sum_{j,m,\tau} (E_j - \lambda) a^+_{jm\tau} a_{jm\tau},$$

$$H_{\text{int}} = -\sum_{JMT\tau} G^J_T A^{+JM}_{T\tau} A^{JM}_{T\tau}$$

The pair creation operator $(A^{JM}_{T\tau})^+$ looks as

$$(A^{JM}_{T\tau})^+ = \sum_j \sqrt{j+1/2}(A^{JM}_{T\tau}(j))^+$$

$$(A^{JM}_{T\tau}(j))^+ = \frac{1}{\sqrt{2}} \sum_{m,m',t,t'} C^{JM}_{jmjm'} C^{T\tau}_{1/2t1/2t'} a^+_{jmt} a^+_{jm't'}$$

It is convenient to distinguish single particle levels located below and above Fermi level. The former ones are denoted as j_- and the later ones as j_+ . Thus

$$A^{+JM}_{T\tau} = \sum_{j_+} \sqrt{j_+ + 1/2} A^{+JM}_{T\tau}(j_+) + \sum_{j_-} \sqrt{j_- + 1/2} A^{+JM}_{T\tau}(j_-)$$

After introduction of the particle and hole creation and annihilation operators

$$a^+_{jm\tau} = \begin{cases} c^+_{jm\tau}, & j \in j_+ \\ (-1)^{j-m+1/2-\tau} c_{j-m-\tau} \equiv \tilde{c}_{jm\tau}, & j \in j_- \end{cases}$$

We obtain that

$$A^{+JM}_{T\tau} = \sum_{j_+} \sqrt{j_+ + 1/2} A^{+JM}_{T\tau}(j_+) + \sum_{j_-} \sqrt{j_- + 1/2} \tilde{A}^{JM}_{T\tau}(j_-)$$

We use below Dyson type boson representation of the bifermion operators. This boson representation is finite. Thus, there is no problem which appears if boson expansion are used.

$$
\begin{aligned}
c^{+}_{j_+mt}c^{+}_{j_+m't'} &\to b^{+}_{mt,m't'}(j_+) - \sum_{m_1m_2t_1t_2} b^{+}_{mt,m_1t_1}(j_+)b^{+}_{m't',m_2t_2}(j_+)b_{m_1t_1,m_2t_2}(j_+),\\
c_{j_+m't'}c_{j_+mt} &\to b_{mt,m't'}(j_+),\\
c^{+}_{j_-mt}c^{+}_{j_-m't'} &\to b^{+}_{mt,m't'}(j_-),\\
c_{j_-m't'}c_{j_-mt} &\to b_{mt,m't'}(j_-) - \sum_{m_1m_2t_1t_2} b^{+}_{m_1t_1,m_2t_2}(j_-)b_{m't',m_2t_2}(j_-)b_{mt,m_1t_1}(j_-).\\
c^{+}_{j_\pm mt}c_{j_\pm mt} &\to 2\sum_{m_1t_1} b^{+}_{mt,m_1t_1}(j_\pm)b_{mt,m_1t_1}(j_\pm),
\end{aligned}
$$

Here boson operators $b^{+}_{m\tau,m'\tau'}(j)$ and $b_{m\tau,m'\tau'}(j)$ satisfy the following commutation relations

$$
[b_{m\tau,m'\tau'}(j), b^{+}_{m_1\tau_1,m_2\tau_2}(j)] = \delta_{mm_1}\delta_{\tau\tau_1}\delta_{m'm_2}\delta_{\tau'\tau_2} - \delta_{mm_2}\delta_{\tau\tau_2}\delta_{m'm_1}\delta_{\tau'\tau_1}
$$

Using the angular momentum algebra we obtain

$$
\begin{aligned}
A^{+JM}_{T\tau}(j_+) &= b^{+JM}_{T\tau}(j_+) - 2\Pi_{JJ_1J_2J_3}\Pi_{TT_1T_2T_3} \sum_{J_1M_1T_1\tau_1} \sum_{J_2M_2T_2\tau_2} \sum_{J_3M_3T_3\tau_3} \sum_{J',M',T',\tau'} \times \\
&\times C^{J'M'}_{JMJ_3M_3} C^{J'M'}_{J_1M_1J_2M_2} C^{T'\tau'}_{T\tau T_3\tau_3} C^{T'\tau'}_{T_1\tau_1T_2\tau_2} b^{+J_1M_1}_{T_1\tau_1}(j_+) b^{+J_2M_2}_{T_2\tau_2}(j_+) b^{J_3M_3}_{T_3\tau_3}(j_+) \times \\
&\times \begin{Bmatrix} j_+ & j_+ & J_2 \\ j_+ & j_+ & J_1 \\ J_3 & J & J' \end{Bmatrix} \begin{Bmatrix} 1/2 & 1/2 & T_2 \\ 1/2 & 1/2 & T_1 \\ T_3 & T & T' \end{Bmatrix} \\
A^{JM}_{T\tau}(j_+) &= b^{JM}_{T\tau}(j_+) \\
A^{+JM}_{T\tau}(j_-) &= b^{+JM}_{T\tau}(j_-) \\
A^{JM}_{T\tau}(j_-) &= b^{JM}_{T\tau}(j_-) - 2\Pi_{JJ_1J_2J_3}\Pi_{TT_1T_2T_3} \sum_{J_1M_1T_1\tau_1} \sum_{J_2M_2T_2\tau_2} \sum_{J_3M_3T_3\tau_3} \sum_{J',M',T',\tau'} \times \\
&\times C^{J'M'}_{JMJ_3M_3} C^{J'M'}_{J_1M_1J_2M_2} C^{T'\tau'}_{T\tau T_3\tau_3} C^{T'\tau'}_{T_1\tau_1T_2\tau_2} b^{+J_3M_3}_{T_3\tau_3}(j_-) b^{J_2M_2}_{T_2\tau_2}(j_-) b^{J_1M_1}_{T_1\tau_1}(j_-) \times \\
&\times \begin{Bmatrix} j_- & j_- & J_2 \\ j_- & j_- & J_1 \\ J_3 & J & J' \end{Bmatrix} \begin{Bmatrix} 1/2 & 1/2 & T_2 \\ 1/2 & 1/2 & T_1 \\ T_3 & T & T' \end{Bmatrix}
\end{aligned}
$$

The operator $b^{+JM}_{T\tau}(j)$ is given as

$$b^{+JM}_{T\tau}(j) = \frac{1}{\sqrt{2}} \sum C^{JM}_{jmjm'} C^{T\tau}_{1/2\tau_1 1/2\tau_2} b^{+}_{m\tau_1, m'\tau_2}(j)$$

In terms of these operators the Hamiltonian takes the form

$$\begin{aligned}
H = &\sum_{j_+} 2(E_{j_+} - \lambda) \sum_{JMT\tau} b^{+JM}_{T\tau}(j_+) b^{JM}_{T\tau}(j_+) + \sum_{j_-} 2(\lambda - E_{j_-}) \sum_{JMT\tau} b^{+JM}_{T\tau}(j_-) b^{JM}_{T\tau}(j_-) \\
&- \sum_{JMT\tau} G^J_T \left(\sum_{j_+} \sqrt{j_+ + 1/2}\, b^{+JM}_{T\tau}(j_+) + \sum_{j_-} \sqrt{j_- + 1/2}\, \tilde{b}^{JM}_{T\tau}(j_-) \right) \\
&\times \left(\sum_{j'_+} \sqrt{j'_+ + 1/2}\, b^{JM}_{T\tau}(j'_+) + \sum_{j'_-} \sqrt{j'_- + 1/2}\, \tilde{b}^{+JM}_{T\tau}(j'_-) \right) \\
&+2 \sum_{JMT\tau} G^J_T \left(F^{JM}_{T\tau}(+) + F^{JM}_{T\tau}(-) \right) \left(\sum_{j_+} \sqrt{j_+ + 1/2}\, b^{JM}_{T\tau}(j_+) + \sum_{j_-} \sqrt{j_- + 1/2}\, \tilde{b}^{+JM}_{T\tau}(j_-) \right)
\end{aligned}$$

Where

$$F^{JM}_{T\tau}(+) = \sum P^{J_1J_2J_3J'}_{T_1T_2T_3T'}\sqrt{j_+ + \frac{1}{2}}\begin{Bmatrix} j_+ & j_+ & J \\ j_+ & j_+ & J_3 \\ J_1 & J_2 & J' \end{Bmatrix}\begin{Bmatrix} 1/2 & 1/2 & T \\ 1/2 & 1/2 & T_3 \\ T_1 & T_2 & T' \end{Bmatrix}$$

$$\times \left((b^{+J_1}_{T_1}(j_+) b^{+J_2}_{T_2}(j_+))^{J'}_{T'}\ \tilde{b}^{J_3}_{T_3}(j_+) \right)^{JM}_{T\tau},$$

$$F^{JM}_{T\tau}(-) = \sum P^{J_1J_2J_3J'}_{T_1T_2T_3T'}\sqrt{j_- + \frac{1}{2}} \times \begin{Bmatrix} j_- & j_- & J \\ j_- & j_- & J_3 \\ J_1 & J_2 & J' \end{Bmatrix}\begin{Bmatrix} 1/2 & 1/2 & T \\ 1/2 & 1/2 & T_3 \\ T_1 & T_2 & T' \end{Bmatrix}$$

$$\times \left(b^{+J_3}_{T_3}(j_-)\ (\tilde{b}^{J_1}_{T_1}(j_-) \tilde{b}^{J_2}_{T_2}(j_-))^{J'}_{T'} \right)^{JM}_{T\tau}$$

And

$$P^{J_1J_2J_3J'}_{T_1T_2T_3T'} \equiv \sqrt{(2J_1+1)(2J_2+1)(2J_3+1)(2J'+1)(2T_1+1)(2T_2+1)(2T_3+1)(2T'+1)}.$$

Collective Hamiltonian

The boson image of the total Hamiltonian presented above contains several boson creation and annihilation operators for every set of the angular momentum and isospin quantum numbers J, T

Our task is to extract from the variety of the boson operators for every set of J and T the collective boson operators describing the softest modes in the case of pair vibrations, or the modes corresponding to the more rapid descent to a minimum of the potential energy in the case of static pair correlations.

We assume that the collective boson operators can be presented as the linear combination of the boson operators introduced above.

$$\beta^{+JM}_{T\tau}(k_\pm) = \sum_{j_\pm} \tilde{u}^{JT}_{k_\pm, j_\pm} b^{+JM}_{T\tau}(j_\pm) - \sum_{j_\mp} u^{JT}_{k_\pm, j_\mp} \tilde{b}^{JM}_{T\tau}(j_\mp),$$

$$\tilde{\beta}^{JM}_{T\tau}(k_\pm) = \sum_{j_\pm} u^{JT}_{k_\pm, j_\pm} \tilde{b}^{JM}_{T\tau}(j_\pm) - \sum_{j_\mp} \tilde{u}^{JT}_{k_\pm, j_\mp} b^{+JM}_{T\tau}(j_\mp)$$

The amplitudes $u^{JT}_{k_\pm,j_\mp}$ and $\tilde{u}^{JT}_{k_\pm,j_\pm}$ satisfy the following normalization conditions

$$\delta_{j_\pm j'_\pm} = \sum_k \left(\tilde{u}^{JT}_{k_\pm,j_\pm} u^{JT}_{k_\pm,j'_\mp} - \tilde{u}^{JT}_{k_\mp,j_\pm} u^{JT}_{k_\mp,j'_\pm} \right)$$

$H^{(2)}$ takes the form

$$H^{(2)} = \sum_{J,T,k_+} \omega^{JT}_{k_+} \beta^{+JM}_{T\tau}(k_+) \beta^{JM}_{T\tau}(k_+) + \sum_{J,T,k_-} \omega^{JT}_{k_-} \beta^{+JM}_{T\tau}(k_-) \beta^{JM}_{T\tau}(k_-)$$

The set of nonlinear equations determining $\omega^{JT}_{k_\pm}$ is given below

$$1 = G_T^J\left(\sum_{j_\pm}\frac{(1-\rho_{j_\pm})}{D_{j_\pm}-\omega_{k_\pm}^{JT}} + \sum_{j_\mp}\frac{(1-\rho_{j_\mp})}{D_{j_\mp}+\omega_{k_\pm}^{JT}}\right),$$

$$\rho_{j_\pm} = (1-\rho_{j_\pm})\sum_{J,T,k}(2J+1)(2T+1)\frac{(G_T^J W_{k_\mp}^{JT})^2}{(D_{j_\pm}+\omega_{k_\mp}^{JT})^2},$$

$$D_{j_\pm} = 2|E_{j_\pm}-\lambda| + \sum_{J,T,k}(2J+1)(2T+1)\frac{(G_T^J W_{k_\mp}^{JT})^2}{(D_{j_\pm}+\omega_{k_\mp}^{JT})},$$

$$(G_T^J W_{k_\pm}^{JT})^{-2} = \sum_{j_\pm}\frac{(1-\rho_{j_\pm})(j_\pm+1/2)}{(D_{j_\pm}-\omega_{k_\pm}^{JT})^2} - \sum_{j_\mp}\frac{(1-\rho_{j_\mp})(j_\mp+1/2)}{(D_{j_\mp}+\omega_{k_\pm}^{JT})^2}$$

These equations have a solution for any value of G_T^J .

Potential Energy

$$\beta^{JM}_{T\tau}(1_+) = \frac{1}{\sqrt{2}}(z^{JM}_{T\tau} + ip^{+JM}_{T\tau}),$$

$$\tilde{\beta}^{JM}_{T\tau}(1_-) = \frac{1}{\sqrt{2}}\left(z^{+JM}_{T\tau} + ip^{JM}_{T\tau}\right)$$

Where

$$[z^{JM}_{T\tau}, p^{J'M'}_{T'\tau'}] = i\delta_{JJ'}\delta_{MM'}\delta_{TT'}\delta_{\tau\tau'}$$

Separating terms depending on the coordinates only we obtain the potential energy.

Now we consider the collective Hamiltonian containing only Isovector and Isoscalar modes.

$$z^{*00}_{1\tau} = \Delta^0_1 e^{-i\varphi}\left(D^1_{\tau 0}(\vec{\Omega}_{\text{iso}})\cos\theta^0_1 + \frac{1}{\sqrt{2}}(D^1_{\tau 1}(\vec{\Omega}_{\text{iso}}) + D^1_{\tau -1}(\vec{\Omega}_{\text{iso}}))\sin\theta^0_1\right),$$
$$z^{*1M}_{00} = \Delta^1_0 e^{-i\varphi}\left(D^1_{M0}(\vec{\Omega}_{\text{space}})\cos\theta^1_0 + \frac{1}{\sqrt{2}}(D^1_{M1}(\vec{\Omega}_{\text{space}}) + D^1_{M-1}(\vec{\Omega}_{\text{space}}))\sin\theta^1_0\right)$$

Here $D^1_{MM'}$ are Wigner functions. Angle φ is related to the particle number conservation.

Two level model. ($j \gg 1$)

$$V = 4D_j(1-\rho_j)\Big(\frac{|E_j-\lambda| - G_1^0}{\omega^{01}}(\Delta_1^0)^2 + \frac{|E_j-\lambda| - G_0^1}{\omega^{10}}(\Delta_0^1)^2\Big)$$
$$+ \frac{G_1^0}{(2j+1)}\Big(\frac{D_j(1-\rho_j)}{\omega^{01}}\Big)^2(\Delta_1^0)^4 + \frac{3}{5}\frac{G_0^1}{(2j+1)}\Big(\frac{D_j(1-\rho_j)}{\omega^{10}}\Big)^2(\Delta_0^1)^4 +$$

$$+ \frac{(G_1^0+G_0^1)}{(2j+1)}(D_j(1-\rho_j))^2\frac{1}{\omega^{01}\omega^{10}}(\Delta_1^0\Delta_0^1)^2$$
$$- \frac{1}{2}\frac{G_1^0}{(2j+1)}\Big(\frac{D_j(1-\rho_j)}{\omega^{01}}\Big)^2\Big((\Delta_1^0)^2\cos(2\theta_1^0)\Big)^2$$
$$+ \frac{3}{10}\frac{G_0^1}{(2j+1)}\Big(\frac{D_j(1-\rho_j)}{\omega^{10}}\Big)^2\Big((\Delta_0^1)^2\cos(2\theta_0^1)\Big)^2$$
$$+ \frac{1}{2}\frac{(G_1^0+G_0^1)}{(2j+1)}(D_j(1-\rho_j))^2\frac{1}{\omega^{01}\omega^{10}}(\Delta_1^0\Delta_0^1)^2\cos(2\theta_1^0)\cos(2\theta_0^1)$$

Above $D_{j\pm} \equiv D_j,\ \rho_{j\pm} \equiv \rho_j$

Let us analyze the expression for the potential energy concentrating mainly on the position of the minimum. Potential energy has a minimum at $\cos(2\theta_1^0) = 1$ and $\cos(2\theta_0^1) = 1$. With this results for $(\theta_1^0)_{\min}$ and $(\theta_0^1)_{\min}$ after introduction of the new variables $x^{JT} = (\Delta_T^J)^2/\omega^{JT}$ and the new notations

$$c_1^0 = 8(2G_1^0 - |E_j - \lambda|)D_j(1-\rho_j),$$
$$c_0^1 = 8(2G_0^1 - |E_j - \lambda|)D_j(1-\rho_j),$$
$$d_1^0 = \frac{1}{2(2j+1)}\left(D_j(1-\rho_j)\right)^2 G_1^0,$$
$$d_0^1 = \frac{1}{2(2j+1)}\left(D_j(1-\rho_j)\right)^2 G_0^1,$$
$$d_{\text{mix}} = \frac{1}{(2j+1)}\left(D_j(1-\rho_j)\right)^2 (G_1^0 + G_0^1)$$

The potential energy can be presented as a sum of two terms each of them depends only on one combination of the variables x^{01} and x^{10}

$$V = \alpha_x x + \beta_x x^2 + \alpha_y y + \beta_y y^2$$

Where

$$
\begin{aligned}
x &= \cos(\varphi)x^{01} + \sin(\varphi)x^{10},\\
y &= -\sin(\varphi)x^{01} + \cos(\varphi)x^{10},\\
\cos(2\varphi) &= (d_1^0 - d_0^1)/\sqrt{(d_1^0 - d_0^1)^2 + d_{\mathrm{mix}}^2},\\
\sin(2\varphi) &= d_{mix}/\sqrt{(d_1^0 - d_0^1)^2 + d_{\mathrm{mix}}^2},\\
\alpha_x &= -(c_1^0\cos(\varphi) + c_0^1\sin(\varphi)),\\
\alpha_y &= -(-c_1^0\sin(\varphi) + c_0^1\cos(\varphi)),\\
\beta_x &= \frac{1}{2}\left(d_1^0 + d_0^1 + \sqrt{(d_1^0 - d_0^1)^2 + d_{\mathrm{mix}}^2}\right),\\
\beta_y &= \frac{1}{2}\left(d_1^0 + d_0^1 - \sqrt{(d_1^0 - d_0^1)^2 + d_{\mathrm{mix}}^2}\right).
\end{aligned}
$$

Several cases should be considered.

If both α_x and α_y are positive then $x_{\min}^{01} = x_{\min}^{10} = 0$. This means an absence of the static pair correlations of both types, isovector and isoscalar.

Consider the case when both α_x and α_y are negative. Then

$$x_{\min}^{01} = \frac{(2j+1)(1+\rho)}{D} \frac{4(G_0^1)^2 - 0.8G_1^0G_0^1 - 2|E_j - \lambda|(G_1^0 - 0.2G_0^1)}{0.25(G_1^0)^2 + 0.25(G_0^1)^2 + 0.2G_1^0G_0^1}$$

$$x_{\min}^{10} = \frac{(2j+1)(1+\rho)}{D} \frac{4(G_1^0)^2 - 2|E_j - \lambda|G_0^1}{0.25(G_1^0)^2 + 0.25(G_0^1)^2 + 0.2G_1^0G_0^1}.$$

Conclusions

In the present work we apply the method of the finite boson representation of the bifermion operators to nuclear Hamiltonian with isovector and isoscalar pair interaction. The collective pair addition and pair removal modes are separated from the variety of nuclear degrees of freedom. In contrast to RPA this can be done for any value of the interaction constant. The collective Hamiltonian for description of dynamics of the pairing modes is constructed and the collective potential energy is considered. It is shown that the situation of the mixed spin pairing is quite probable if the system is described by the considered Hamiltonian.

THANK YOU