STRONG CP PROBLEM
Aspects of Symmetry

Nov. 8-9, 2021 | Andreas Wirzba | Institute of Advanced Simulation

Cartoon 1

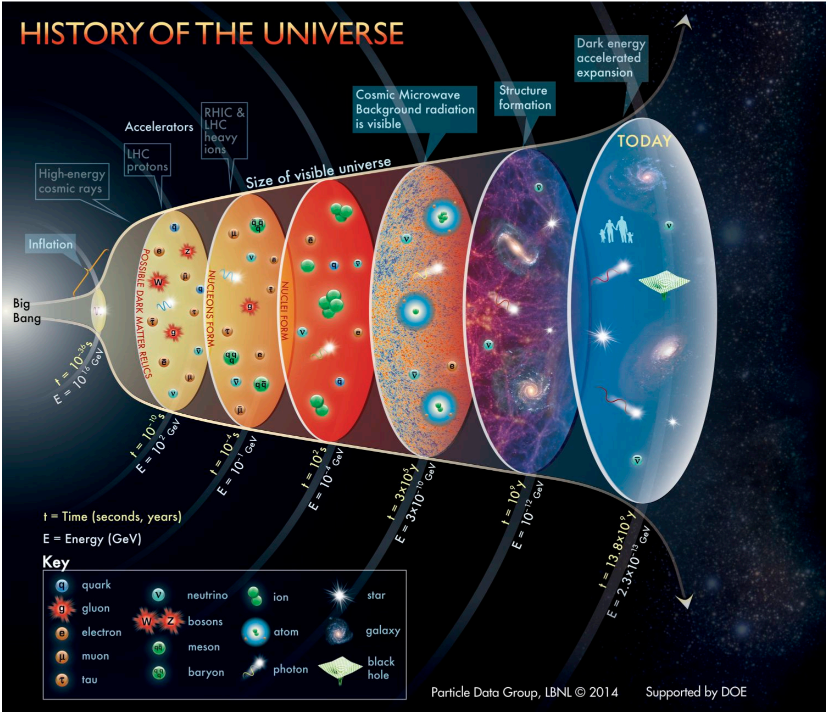
Let's start with a
Disclaimer

Cartoon 2 lecture 2

Outline of lecture 1:

1. Evolution of the Universe, matter vs. antimatter, Sakharov conditions
2. CP violation in the Standard Model
3. $U(1)_A$ problem
4. Instantons, topological charge and susceptibility
5. QCD vacuum structure and θ-angle
6. Strong CP problem
7. Peccei-Quinn mechanism → lecture 2
8. Invisible axions → lecture 2
9. Fine-tuning after all → lecture 2

HISTORY OF THE UNIVERSE AND CP-VIOLATION


HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
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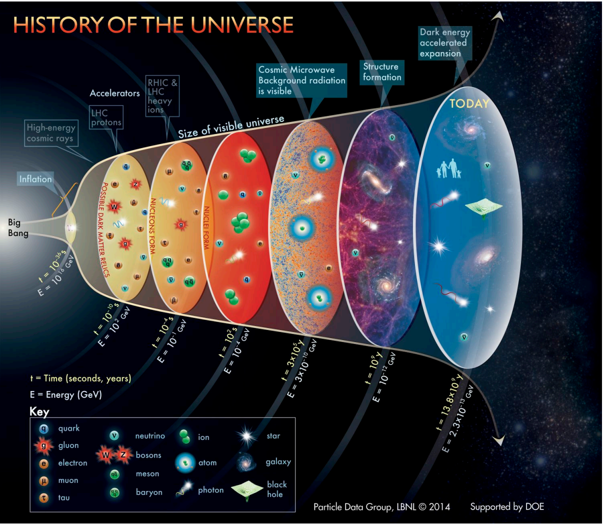
Matter Excess in the Universe



1. End of inflation: $n_B = n_{\bar{B}}$
2. Cosmic Microwave Bkgr.
   - SM(s) *prediction:* (*)
     $(n_B - \cancel{n_{\bar{B}}})/n_\gamma|_{\rm CMB} \sim 10^{-18}$
   - WMAP+PLANCK ('13):
     $n_B/n_\gamma|_{\rm CMB} = (6.05 \pm 0.07) 10^{-10}$

Sakharov conditions ('67)

for dyn. generation of net B:

1. B violation
   to depart from initial B=0
2. C & CP violation
   to distinguish B from $\bar{B}$ production rates
3. Either CPT violation or out of thermal equilibrium
   to distinguish B production from back reaction and to escape $\langle B \rangle$=0 if CPT holds

(*) $2J^{\rm CKM}_{\rm Jarlskog}(m_t^2-m_u^2)(m_t^2-m_c^2)(m_c^2-m_u^2)(m_b^2-m_d^2)(m_b^2-m_s^2)(m_s^2-m_d^2) \sim 10^{-18} M_{\rm EW}^{12}$

▸ in the SM?

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CP violation in the Standard Model

The conventional source: Kobayashi-Maskawa mechanism

Empirical facts: 3 generations of u/d quarks (& e/ν leptons)

- quarks & leptons in **mass** basis $\neq$ quarks & leptons in **weak**-int. basis
- $\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{gauge-fermion}} + \mathcal{L}_{\text{gauge-Higgs}} + \mathcal{L}_{\text{Higgs-fermion}}$ is CP inv.,
  - with the exception of the θ term of QCD (see later)

    and the **charged-weak-current** interaction ($\subset \mathcal{L}_{\text{gauge-fermion}}$)

$$\mathcal{L}_{\text{c-w-c}} = -\frac{g_w}{\sqrt{2}} \sum_{ij=1}^{3} \bar{d}_{Li} \gamma^\mu V_{ij}\, u_{Lj}\, W_\mu^- - \frac{g_w}{\sqrt{2}} \sum_{ij=1}^{3} \bar{\ell}_{Li} \gamma^\mu U_{ij}\, \nu_{Lj}\, W_\mu^- + \text{h.c.}$$

  - V: 3×3 unitary quark-mixing matrix (Cabibbo-Kobayashi-Maskawa matrix)

    3 angles + 1 ~~CP~~ phase δ_{KM}

    U: 3×3 unitary lepton-mixing matrix (Pontecorvo-Maki-Nakagawa-Sakata m.)

    3 angles +1(3) ~~CP~~ phase(s) for Dirac (Majorana) ν_i's

FAIRYTALE
OF
THE HARE
AND
THE HEDGEHOG

$U(1)_A$ problem: why only $N_F^2 - 1$ Pseudo-Goldstone Bosons ?

- GBs arise from **s**pontaneous **s**ymmetry **b**reaking (SSB) with one massless GB per broken symmetry generator (='charge')
- Pseudo-GBs acquire finite mass from small explicit SB
- In the **chiral limit**, the QCD **Lagrangian** is invariant under

$$U(N_F)_L \times U(N_F)_R = \underbrace{SU(N_F)_L \times SU(N_F)_R}_{\text{chiral group}} \times \underbrace{U(1)_V}_{\text{baryon \#}} \times \underbrace{U(1)_A}_{?}$$

$$\xrightarrow{SSB} \{SU(N_F)_L \times SU(N_F)_R / SU(N_F)_V\} \times U(1)_V \times U(1)_A$$

- What about the extra $U(1)_A$ symmetry? Spontaneous SB?

  Is there an extra "(P)GB" in addition to the $N_F^2 - 1$ ones? Not really:

$$N_F = 2: \qquad m_{\pi^0} \approx 135\,\text{MeV}\,,\ m_{\pi^\pm} \approx 139\,\text{MeV} \ll m_\eta \approx 548\,\text{MeV}$$

$$N_F = 3: \qquad m_{\pi^0} \lesssim m_{\pi^\pm} < m_{K^\pm} \lesssim m_{K^0,\bar{K}^0} < m_\eta \ll m_{\eta'} \approx 958\,\text{MeV}$$

  while for $N_F \geq 2$ there is the naive bound: $m_{\text{"}\eta\text{"}} \overset{!}{<} \sqrt{3} m_\pi \approx 240\,\text{MeV}$.

  $\hookrightarrow$ This is the $U(1)_A$ problem

  S. Weinberg, Phys. Rev. D 11 (1975) 3583

- Question rephrased:

  What happens to the *classical* $U(1)_A$ symmetry at quantum level ?

$U(1)_A$ anomaly – perturbative consideration

- Anomaly of the axial $U(1)_A$ current in **QCD** (in the chiral limit) :

$$\partial_\mu J^\mu_A = -\frac{g_s^2 N_F}{8\pi^2}\frac{1}{2} G^c_{\mu\nu}\tilde{G}^{c\,\mu\nu} = -\frac{g_s^2 N_F}{16\pi^2} G^c_{\mu\nu}\tilde{G}^c_{\mu\nu} \quad \left(\mathrm{Tr}_{\text{flavor}}[\mathbb{I}]=N_F,\ \mathrm{Tr}_{\text{color}}[t^c t^{c'}]=\tfrac{1}{2}\delta^{cc'}\right)$$

$\big($however, $SU(3)_A$: $\partial_\mu J^{a\,\mu}_A = 0,\ \forall a \neq 0$ since $\mathrm{Tr}_{\text{flavor}}[\tfrac{1}{2}\lambda^a] = 0.\big)$

- In the "path-integral language", the $U(1)_A$ anomaly arises due to the **Jacobian** in the fermion measure ($\mathcal{D}\psi'\mathcal{D}\bar{\psi}' = J^{-2}\mathcal{D}\psi\mathcal{D}\bar{\psi}$) resulting from the flavor-singlet axial transformation $\psi_f \to \psi'_f = e^{i\beta\gamma_5}\psi_f$:

$$\underbrace{\beta\int d^4x\,\partial_\mu j^\mu_A}_{\text{not zero, even in chiral limit}} \stackrel{!}{=} -i\ln\left(J^{-2}\right) = -\beta\, 2N_F\frac{g_s^2}{32\pi^2}\int d^4x G^c_{\mu\nu}\tilde{G}^{c\,\mu\nu}$$

- Note that $\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu} = \frac{g_s^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}(G_{\mu\nu}G_{\rho\sigma}) = \partial_\mu K^\mu$ is a total derivative with

$$K^\mu = \frac{g_s^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(A_\nu G_{\rho\sigma} + i\tfrac{2}{3}A_\nu A_\rho A_\sigma\right) \quad \text{(Chern-Simons current)}$$

↪ the $U(1)_A$ anomaly of *QCD* is irrelevant in perturbation theory !

$U(1)_A$ anomaly and large gauge transformations

- Since $G^c_{\mu\nu}\tilde{G}^{c\,\mu\nu} \propto \partial_\mu K^\mu$ is a total derivative, even a gauge-invariant, Lorentz-invariant, C- and still $P \times T$-invariant (although P & T breaking) θ-term,

$$\mathcal{L}^{\theta}_{\rm QCD} = -\bar{\theta}\,\frac{g_s^2}{32\pi^2}\,\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\,G^a_{\mu\nu}\,G^a_{\rho\sigma}\,,$$

  added to the usual QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}G^c_{\mu\nu}G^{c,\mu\nu} + \sum_f \bar{q}_f\Big(i\gamma^\mu\big(\partial_\mu - igA^c_\mu t^c\big) - m_f\Big)q_f\,,$$

  would be irrelevant as well – in perturbation theory !

- However, non-perturbative (*large*) gauge transformations (so-called *instantons*) exist in *E*uclidean space-time R^4, such that

$$\int_{R^4} d^4x_{\rm E}\,\frac{g_s^2}{32\pi^2}\,G^c_{\mu\nu}\tilde{G}^c_{\mu\nu} \;=\; n \in \mathbb{Z} \quad \text{(topologically protected and nonzero in general).}$$

Instantons in classical Yang-Mills theory

In **Euclidean** space-time ($t \to -i\tau$, $\partial_t \to i\partial_\tau$, $g_{\mu\nu} \to -\delta_{\mu\nu}$) the Yang-Mills action is positive:

$$S_E \equiv -iS_M(t \to -i\tau) = \frac{1}{2}\int d^4x_E \,\mathrm{Tr}\left(G^E_{\mu\nu}G^E_{\mu\nu}\right) \geq 0 \;(!)$$

(rescale $A^E_\mu \to A^E_\mu/g_s$ and drop index E from now on)

$$\hookrightarrow S_E = \underbrace{\frac{1}{4g_s^2}\int d^4x_E \,\mathrm{Tr}\left((G_{\mu\nu}\mp\tilde{G}_{\mu\nu})(G_{\mu\nu}\mp\tilde{G}_{\mu\nu})\right)}_{\geq 0} \pm \underbrace{\frac{1}{2g_s^2}\int d^4x_E \,\mathrm{Tr}\left(G_{\mu\nu}\tilde{G}_{\mu\nu}\right)}_{\equiv 8\pi^2 Q/g_s^2}$$

$$\hookrightarrow S_E \overset{!}{=} \frac{8\pi^2|Q|}{g_s^2} \quad \text{for} \left\{ \begin{array}{rl} \text{self-dual } G_{\mu\nu} = +\tilde{G}_{\mu\nu} & (Q \geq 0) \\ \text{anti-self-dual } G_{\mu\nu} = -\tilde{G}_{\mu\nu} & (Q < 0) \end{array} \right\} \quad \text{“instanton” configurations.}$$

Use $\frac{1}{16\pi^2}\mathrm{Tr}\left(G_{\mu\nu}\tilde{G}_{\mu\nu}\right) = \partial_\mu\left[\frac{1}{8\pi^2}\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\left(A_\nu(\partial_\alpha A_\beta - i\frac{2}{3}A_\alpha A_\beta)\right)\right] \equiv \partial_\mu K_\mu$

(with K_μ Chern-Simons current and $\epsilon_{0123} = -1$)

and $A_\mu \xrightarrow{|x|\to\infty} -i(\partial_\mu\Omega)\Omega^\dagger$ (pure gauge) such that $G_{\mu\nu} \xrightarrow{|x|\to\infty} 0$ and action is finite.

$\hookrightarrow$ **Topological charge** (= Pontryagin index = 2^{nd} Chern class)

$$Q = \frac{1}{16\pi^2}\int d^4x_E \,\mathrm{Tr}\left(G_{\mu\nu}\tilde{G}_{\mu\nu}\right) = \oint_{S^3} d\sigma_\mu \,\frac{-1}{24\pi^2}\,\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\left((\partial_\nu\Omega)\Omega^\dagger(\partial_\alpha\Omega)\Omega^\dagger(\partial_\beta\Omega)\Omega^\dagger\right)$$

is an integer determined by $\Omega(n_\mu)$ with n_μ ($n^2 = 1$) the direction in which $|x| \to \infty$.

Topological charge

$$Q = \frac{1}{16\pi^2}\int d^4x_E \,\mathrm{Tr}\left(G_{\mu\nu}\tilde{G}_{\mu\nu}\right) = \oint_{S^3} d\sigma_\mu \,\frac{-1}{24\pi^2}\,\epsilon_{\mu\nu\alpha\beta}\,\mathrm{Tr}\left((\partial_\nu\Omega)\Omega^\dagger(\partial_\alpha\Omega)\Omega^\dagger(\partial_\beta\Omega)\Omega^\dagger\right)$$

- Pure gauge condition $A_\mu \longrightarrow -i(\partial_\mu\Omega)\Omega^\dagger$ determined by $\Omega(n_\mu)$ where the unit 4-vector n_μ specifies the direction in which x approaches infinity

↪ Space-time at $|x| \to \infty$ isomorphic to S^3 sphere

↪ $\Omega(n_\mu)$: mapping $S^3 \to S^3$(group-valued) $\cong SU(2)$ (note: $2\pi^2 \times 3! \times \mathrm{Tr}[\mathrm{I}] = 24\pi^2$) in the two-color scenario (or for a color group $G = SU(N_c) \supseteq SU(2)$ in general).

- Space of mappings of $S^3 \to G$: **infinite set** of **isolated classes**, labeled by the **winding number** Q: mappings belonging to one class cannot be continuously deformed into those belonging to any other class

↪ **homotopy classes** $\Pi_3(G) = \mathbb{Z}$ (windings of mappings $S^3 \to G$)

– analog to $n \in \mathbb{Z}$ windings of $Q = \oint_{S^1} d\sigma_\mu \frac{1}{2\pi}\epsilon_{\mu\nu}A_\nu = \frac{1}{2\pi}\int_0^{2\pi} d\phi A_\phi$ for the maps $S_1 \to S_1$

Some examples:

$Q = +1$ mapping:	$\Omega_1(n_\mu) = n_0 + i\,\vec{n}\cdot\vec{\tau}$	with $n_\mu = x_\mu/\sqrt{x^2}$;
$Q = -1$ mapping:	$\Omega_{-1}(n_\mu) = \Omega_1^\dagger(n_\mu)$;	$\cdots$
$Q = +7$ mapping:	$(\Omega_1(nu_\mu))^7$	etc.

Instantons and the solution of the $U(1)_A$ problem

't Hooft, PRL 37 ('76), PRD 14 ('76), 18 ('78)

- Non-perturbative (*large*) gauge transformations (so-called *instantons*) exist in *E*uclidean space-time R^4, such that

$$\underbrace{Q}_{\text{topol. charge}} = \int_{R^4} d^4 x_E \underbrace{\frac{g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}}_{\text{topol. density } q(x)} = n \in \mathbb{Z} \quad \text{(protected by topology \& nonzero in general)}$$

∴ One-(anti-)*instanton amplitude* for QCD (after redefining $g_s A_\mu \longrightarrow A_\mu$)

$$\mathcal{A}_E^{\mathrm{I}/\bar{\mathrm{I}}} \propto e^{-\int d^4 x_E \left(\frac{1}{8g_s^2} (G^a_{\mu\nu} \mp \tilde{G}^a_{\mu\nu})^2 \pm \frac{8\pi^2}{g_s^2} \frac{1}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} \right)} \propto e^{-\frac{8\pi^2}{g_s^2(\mu)}} .$$

is nonzero and proportional to e^{-S_E}.

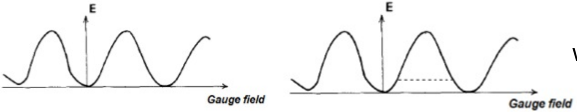
- Weak coupling ($g_s^2(\mu) \ll 1$): instanton amplitude exponentially small;
- but in the strong coupling case, $g_s^2(\mu) \sim (4\pi)^2$, no suppression!

↪ $\partial_\mu j_A^\mu \neq 0$ *non-perturbatively* $\rightsquigarrow$ $m^2_{\eta'} \gg m^2_{\pi,K,\eta}$ $\rightsquigarrow$ only $N_F^2 - 1$ Pseudo-GBs !

▸ Topological susceptibility

Instantons and non-trivial vacua in QCD

- Because of large gauge transformations $A_\mu \to A_\mu^{(n)} = A_\mu - i(\partial_\mu \Omega_n)\Omega_n^\dagger$, there are infinitely many homotopy classes $\Pi_3(SU(3) = \mathbb{Z}$ and QCD has a topologically **non-trivial vacuum structure**



with winding number n

- **instantons** ($\cong$ *large* gauge transformations) that induce $|n\rangle \to |n+1\rangle$ etc.
  $\hookrightarrow$ and solve the $U_A(1)$ problem

  't Hooft, PRL 37 ('76), PRD 14 ('76), 18 ('78)

- However, any **naively chosen vacuum** $|0\rangle_n \equiv |n\rangle$ (with n arbitrary, but fixed)
  1. is **unstable** under the one-instanton action, $\Omega_1 : |0\rangle_n \to |0\rangle_{n+1}$,
  2. is **not gauge invariant** under *large* gauge transformations,
  3. ***violates cluster decomposition:*** $\langle O_1 O_2 \rangle \overset{!}{=} \langle O_1 \rangle \langle O_2 \rangle$
     which can be traced back to causality, unitarity (and locality) of the underlying field theory, *e.g.:*
     let O_1 be the axial charge operator $Q^\dagger(t_E)$ and O_2 the corresponding operator $Q(0)$ at $t_E = 0$,
     then both $\langle n|O_1|n\rangle = 0$ and $\langle n|O_2|n\rangle = 0$ but $\langle n|O_1|n+2\rangle\langle n+2|O_2|n\rangle \neq 0$ even for $t_E \to \infty$.

θ vacua in strong interaction physics

Thus true vacuum must be a **superposition of all $|n\rangle$ vacua:**

$$|vac\rangle_\theta \equiv \sum_{n=-\infty}^{+\infty} e^{in\theta}|n\rangle \quad \text{with} \quad \Omega_1 : |vac\rangle_\theta \to e^{-i\theta}|vac\rangle_\theta \quad \text{(with a phase shift only)}$$

Note $\quad {}_{\theta'}\langle vac|e^{-iHt}|vac\rangle_\theta = \delta_{\theta-\theta'} \times {}_\theta\langle vac|e^{-iHt}|vac\rangle_\theta \quad$ such that θ is unique.

$\hookrightarrow$ θ is another parameter of strong interaction physics (as $m_u, m_d, \ldots$):

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}^{CP}_{\text{QCD}} + \mathcal{L}^{\not{CP}}_{\text{QCD}} = \mathcal{L}^{CP}_{\text{QCD}} - \theta \frac{g_s^2}{32\pi^2}\, \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}\,.$$

Under axial rotation of the quark fields $q_f \to \mathrm{e}^{\mathrm{i}\beta\gamma_5} q_f \approx (1 + \mathrm{i}\beta\gamma_5) q_f$

$$\mathcal{L}_{\text{QCD}} \quad \to \quad \mathcal{L}^{\text{CP}}_{\text{QCD}} - 2\beta \sum\nolimits_f m_f \bar{q} i \gamma_5 q - (\theta + 2N_f\beta) \frac{g_s^2}{32\pi^2} \tilde{G}^a_{\mu\nu} G^{a,\mu\nu}$$

$$\hookrightarrow \quad \mathcal{L}^{\text{str}\,\not{CP}}_{\text{SM}} = \mathcal{L}^{\text{CP}}_{\text{SM}} - \bar{\theta} \frac{g_s^2}{32\pi^2}\, \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \quad \text{with} \quad \bar{\theta} = \theta + \arg\det\mathcal{M}$$

Note:

- The $\bar{\theta}$ parameter is an *angle*, $\bar{\theta} \in [-\pi, \pi]$, since the one-instanton amplitude is $\propto e^{i\bar{\theta}Q}$.
- If any quark mass m_f were zero, then the $\bar{\theta}$ angle could be removed by a suitable axial rotation with $2\beta_f = -\bar{\theta}$.

Strong *CP* problem

The resolution of the $U(1)_A$ problem – via the complicated nature of the QCD vacuum – effectively adds an extra term to the QCD Lagrangian:

$$\mathcal{L}_{\bar{\theta}} = -\bar{\theta}\frac{g_s^2}{32\pi^2}\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}$$

Strong CP problem

The resolution of the $U(1)_A$ problem – via the complicated nature of the QCD vacuum – effectively adds an extra term to the QCD Lagrangian:

$$\mathcal{L}_{\bar\theta} = -\bar\theta \frac{g_s^2}{32\pi^2}\, \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}$$

- This term violates parity P and time-reflection invariance T

  (since only ϵ^{0123} and any of its permutations are non-zero)

  but conserves charge conjugation invariance C $\rightsquigarrow$ $\not{CP}$ $\rightsquigarrow$
- it induce an electric dipole moment (EDM) for the neutron:

$$|d_n| \simeq |\bar\theta| \cdot \frac{m_q^*}{\Lambda_{\rm QCD}} \cdot \frac{e}{2m_n} \sim |\bar\theta| \cdot 10^{-2} \cdot 10^{-14}\, e\,\text{cm} \sim |\bar\theta| \cdot 10^{-16}\, e\,\text{cm}$$

compared with $|d_n^{\text{exp.}}| < 1.8 \cdot 10^{-26}\, e\,\text{cm}$ Abel et al. [*nEDM Coll.*] (2020).

$\hookrightarrow |\bar\theta| \lesssim 10^{-10}$, while NDA (naive dim. analysis) predicts $|\bar\theta| \sim \mathcal{O}(1)$.

(Note that the other CP-violating phase of the SM, δ_{KM}, is indeed of $\mathcal{O}(1)$).

This mismatch is called **the strong CP problem.**

Resolution(s) of the Strong *CP* problem

- **Fine-tuning**
  – motivated by many-worlds scenarios, anthropic principle (?) etc.
- or **spontaneously broken** *CP* such that $\bar{\theta} := 0$ at the Lagrangian level
  – but $\bar{\theta} \neq 0$ reintroduced at the loop level
  – and the CKM mechanism predicts *CP*-breaking of *explicit* nature and *not* as SSB
- or an **additional chiral symmetry**
  (i) by a **vanishing** (u-)quark mass (?)
  – excluded by Lattice QCD: $m_u = 2.16^{+0.49}_{-0.26}$ MeV Particle Data Group (2020)
  (ii) or by an **additional global** chiral $U_{\rm PQ}(1)$ symmetry of the SM
  – Peccei-Quinn (PQ) mechanism Peccei & Quinn, PRL 38 & PRD 16 (1977)
  – including *axions* Weinberg, PRL 40 (1978), Wilczek PRL (40) (1978)
  ⋮
  (iii) however, the *"Empire strikes back":* fine-tuning *may be back*
  – reintroduced by Planck-scale explicit PQ-symmetry breaking terms

Kiwoon Choi (Daejeon, Korea), Bethe-Lectures, Bonn, March 2015

End of Lecture1

Some leftovers from lecture 1

1. EW Baryogenesis in the Standard Model
2. Physics of Electric Dipole Moments (EDMs)
3. Dimensional analysis of the nucleon EDM

Remaining outline for lecture 2:

4. Peccei-Quinn mechanism and axions
5. Invisible axions
6. Fine-tuning after all
7. EDM roadmap and bounds *(if time permits)*
8. Oscillating EDMs, axions and Axion-Like Particles (ALPs) *(if time permits)*

CP violation and Electric Dipol Moments (EDMs)

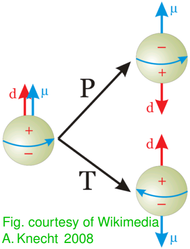


Fig. courtesy of Wikimedia
A. Knecht 2008

EDM: $\vec{d} = \sum_i \vec{r}_i e_i \xrightarrow[\text{particles}]{\text{subatomic}} d \cdot \vec{S}/|\vec{S}|$

(polar) (axial)

$$\mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} - d \frac{\vec{S}}{S} \cdot \vec{E}$$

P: $\mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} + d \frac{\vec{S}}{S} \cdot \vec{E}$

T: $\mathcal{H} = -\mu \frac{\vec{S}}{S} \cdot \vec{B} + d \frac{\vec{S}}{S} \cdot \vec{E}$

Any *non-zero EDM* of **P-non-degenerate**, **finite** (e.g. subatomic) particle requires **explicit** breaking of **P** & **T** in **quantum mechanics**

- Assuming CPT to hold, CP violated as well (diagonally in flavor !)
  ↪ subatomic EDMs: "rear window" to CP violation in early universe
- Strongly suppressed in SM (CKM-matrix): $|d_n| \sim 10^{-31-33} e\,\text{cm}$, $|d_e| \sim 10^{-44} e\,\text{cm}$
- Current bounds: $|d_n| < 1.8^{\diamond}/1.6^{*} \cdot 10^{-26} e\,\text{cm}$, $|d_p| < 2 \cdot 10^{-25} e\,\text{cm}$, $|d_e| < 1.1 \cdot 10^{-29} e\,\text{cm}$

n: Abel et al. [*nEDM*] (2020)$^{\diamond}$, *p* prediction*: Dimitriev&Sen'kov (2003), *e*: Andreev et al. [*ACME*] (2018)†

* indirect from $|d_{^{199}\text{Hg}}| < 7.4 \cdot 10^{-30} e\,\text{cm}$ bound of Graner et al. (2016), † indirect from polar ThO

back

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Naive nucleon-EDM estimate from known physics

(apart from measured bound $|d_n^{\mathrm{exp}}| < 10^{-26}\,e\,\mathrm{cm}$) Khriplovich & Lamoreaux (1997); Kolya Nikolaev (2012)

- CP & P conserving (magnetic) moment ~ nuclear magneton μ_N

$$\mu_N = \frac{e}{2m_p} \sim 10^{-14}\,e\,\mathrm{cm}\,.$$

- Nonzero EDM requires

  parity P violation: price to pay $\sim 10^{-7}$

  $(\,G_F \cdot F_\pi^2 \sim 10^{-7}$ with $G_F \approx 1.166 \cdot 10^{-5}\mathrm{GeV}^{-2}\,)$,

  and *additionally* **CP violation**: price to pay $\sim 10^{-3}$

  $(\,|\eta_{+-}| \equiv |\mathcal{A}(K_L^0 \to \pi^+\pi^-)|\,/\,|\mathcal{A}(K_S^0 \to \pi^+\pi^-)| = (2.232 \pm 0.011) \cdot 10^{-3}\,)$.

- In summary: $|d_N| \lesssim 10^{-7} \times 10^{-3} \times \mu_N \sim 10^{-24}\,e\,\mathrm{cm}$

- In SM (without θ): extra $G_F F_\pi^2$ factor to *undo* flavor change of CKM-matrix

$$\hookrightarrow \quad \left|d_N^{\mathrm{SM}}\right| \lesssim 10^{-7} \times 10^{-24}\,e\,\mathrm{cm} \sim 10^{-31}\,e\,\mathrm{cm}$$

$\hookrightarrow$ BSM *window* for physics search beyond SM @θ:=0

$$10^{-24}\,e\,\mathrm{cm} \gtrsim |d_N| \gtrsim 10^{-30}\,e\,\mathrm{cm}$$

Hare Theta

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Estimate of strong CP-violating parameter $\bar{\theta}$

Another source of **CP**- (i.e. **P**- & **T**-) violation in SM: QCD θ-term (of dimension 4)

$$-\theta \frac{g_s^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \xrightarrow[\text{rotation}]{\text{chiral } U_A(1)} \bar{\theta} m_q^* \sum_f \bar{q}_f i\gamma_5 q_f$$

with $\bar{\theta} = \theta + \arg\det\mathcal{M}_{\text{quark}}$ the *physical* parameter in QCD for **CP** violation

$$\hookrightarrow \left|d_N^{\bar{\theta}}\right| \sim \left|\bar{\theta}\right| \cdot \frac{m_q^*}{\Lambda_{\text{QCD}}} \cdot \frac{e}{2m_N} \sim \left|\bar{\theta}\right| \cdot 10^{-2} \cdot 10^{-14}\, e\,\text{cm} \sim \left|\bar{\theta}\right| \cdot 10^{-16}\, e\,\text{cm}$$

- $m_q^*/\Lambda_{\text{QCD}}$ suppression factor from $U_A(1)$ rotation with $m_q^* = \frac{m_u m_d m_s}{m_u m_d + m_s m_u + m_s m_d} \sim \frac{m_u m_d}{m_u + m_d}$ reduced quark mass.
- From empirical nEDM limit, $|d_n^{\text{emp}}| < 1.8 \cdot 10^{-26}\, e\,\text{cm}$, and naive CKM-SM estimate, $|d_N^{\text{SM}}| \lesssim 10^{-31}\, e\,\text{cm}$
  $\hookrightarrow$ **window of opportunity for determining $\bar{\theta}$**:

  $$\text{SM/CKM} \longrightarrow 10^{-14} \lesssim |\bar{\theta}| \lesssim 10^{-10} \longleftarrow \textit{nEDM} \text{ of Abel et al.}$$

  cannot explain cosmic matter surplus, since $\Lambda_{\chi\text{SB}} \ll \Lambda_{\text{EWSB}}$,
  $\hookrightarrow$ **CP**-violating (dimension≥ 6) sources from BSM physics needed

▸ Hare ◂ KM

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Rough EDM-scale estimate in BSM scenario

solely based on dimensional considerations

EDM d_i of quark or lepton i of mass m_i and charge e_i

scales as $$d_i \simeq \frac{1}{16\pi^2} \frac{m_i}{\Lambda^2_{\rm BSM}} e_i \sin\phi$$ where

- $\Lambda_{\rm BSM}$ mass scale of underlying BSM physics,
- $d_i \propto m_i$ (helicity flip from Higgs interaction) $\rightsquigarrow$ dimension-6 source terms
  (Solely existing dimension-5 CP-violating operator: Majorana mass term in neutrino physics)
- $\sin\phi$ results from the **CP**-violating BSM phases,
- $g^2/16\pi^2 \sim 10^{-2}$ (if $g \sim 1$) one-loop suppression factor (as in SUSY extensions)
  (10^{-4} suppression factor for two-loop (Barr-Zee) processes in, e.g., multi-Higgs scenarios,
  while no suppression factor for loop-free particle exchanges as, e.g., in leptoquark scenarios)

Thus

$$|d_N| \sim 10^{-24} \left(\frac{1\,\text{TeV}}{\Lambda_{\rm BSM}}\right)^2 |\sin\phi|\, e\,\text{cm}$$ if $m_q \sim 5\,\text{MeV}$

compatible with naive estimate $10^{-24}\,e\,\text{cm}$ if $\Lambda_{\rm BSM} \gtrsim 1\,\text{TeV}$ and $\sin\phi \sim 1$,

while a $10^{-29}\,e\,\text{cm}$ sensitivity would allow testing down to $\phi \gtrsim 10^{-5}$ @1TeV scale

or up to $\Lambda_{\rm BSM} \lesssim 300\,\text{TeV}$ @$\phi \sim 1$ (in the one-loop scenario)

◂ BSM

END OF LEFTOVERS

RESUMING
THE FAIRYTALE
OF THE
HARE AND HEDGEHOG

From the $U(1)_A$ problem to the strong CP problem

The Hare and the Hedgehog – a fairy tale

- $U(1)_A$ problem of QCD: $m_{\eta,\eta'} > \sqrt{3}m_\pi \approx 240\,\text{MeV}$
  – proposed solution: $U(1)_A$ anomaly
- Problem: resulting current proportional to a total derivative in perturbation theory
  – solution: non-perturbative QCD vacuum including instantons
- Problem: vacuum $|n\rangle$ not unique, not gauge inv., cluster decomposition viol.
  – solution: θ vacuum (superposition of all $|n\rangle$ vacua $\times e^{i\theta n}$)
- Problem: neutron EDM bound $\rightsquigarrow$ strong CP problem
  – proposed solution: Peccei-Quinn mechanism $\cdots$

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Peccei-Quinn symmetry and the axion

Peccei & Quinn (1977): imposed on the SM
a global chiral $U(1)_{\rm PQ}$ symmetry that is non-linearly realized (*i.e.* SSB)

Peccei & Quinn, PRL 38 & PRD 16 (1977)

Weinberg & Wilczek (1978): introduced the corresponding Nambu–Goldstone boson, the so-called axion

Weinberg, PRL 40 (1978), Wilczek PRL (40) (1978)

The static angular parameter $\bar\theta$ (mod 2π) is replaced by a dynamical pseudoscalar field $a(x)$ which transforms under PQ as

$$U(1)_{\rm PQ}: \; f_a^{-1} a(x) \to f_a^{-1} a(x) + \alpha_{\rm PQ}$$

where f_a is the order parameter associated with spont. breaking of $U(1)_{PQ}$ symmetry.

The SM Lagrangian is augmented by axion interactions

$$\mathcal{L}_{\rm total} = \mathcal{L}_{\rm SM} - \bar\theta \frac{g_s^2}{32\pi^2} G^c_{\mu\nu}\tilde G^{c\,\mu\nu} + \frac{1}{2}\partial_\mu a\, \partial^\mu a + \mathcal{L}_{\rm int}[\partial^\mu a/f_a, \psi, \bar\psi] + \xi \frac{a}{f_a}\frac{g_s^2}{32\pi^2} G^c_{\mu\nu}\tilde G^{c\,\mu\nu}$$

$\hookrightarrow$ the PQ current $J^\mu_{\rm PQ} = \partial^\mu a + \frac{\partial \mathcal{L}_{\rm int}}{\partial\, \partial_\mu a}$ is anomalous:

$$\partial_\mu J^\mu_{\rm PQ} \equiv \partial_\mu \left(\partial^\mu a + \frac{1}{f_a} \frac{\partial \mathcal{L}_{\rm int}}{\partial\, \partial_\mu a/f_a} \right) = \frac{\xi}{f_a} \frac{g_s^2}{32\pi^2} G^c_{\mu\nu}\tilde G^{c\,\mu\nu}$$

The effective potential for the axion field

The minimum of this effective potential occurs at $\langle a \rangle = \bar{\theta} f_a/\xi$:

$$\left\langle \frac{\partial V_{\rm eff}}{\partial a} \right\rangle = -\frac{\xi}{f_a} \left\langle \frac{g_s^2}{32\pi^2} G^c_{\mu\nu} \tilde{G}^{c\,\mu\nu} \right\rangle \Bigg|_{\langle a \rangle = \bar{\theta} f_a/\xi} = 0$$

such that the $\bar{\theta}$ term is canceled out at this minimum.
Without the QCD anomaly, the $U(1)_{\rm PQ}$ symmetry is compatible with all values

$$0 \le \xi \frac{\langle a \rangle}{f_a} < 2\pi \,.$$

With the QCD anomaly, the axion potential has to be periodic and even in the *effective* vacuum angle $-\bar{\theta} + \langle a \rangle \xi / f_a \;\equiv\; \theta_a$:

- rotate θ_a via chiral rotation $q \to e^{i\theta_a\gamma_5/2}$ into the quark mass term, $-m_q \bar{q} q \to -m_q \bar{q} e^{i\gamma_5\theta_a} q$,
- then in one-instanton approximation

$$\langle V_{\rm eff} \rangle \simeq \tfrac{1}{2} \sum_q m_q \left\langle \bar{q} e^{i\gamma_5\theta_a} q + \bar{q} e^{-i\gamma_5\theta_a} q \right\rangle \;\; \text{and with} \;\; \lim_{m_q\to 0} \lim_{V_4\to\infty} \langle \bar{q} q \rangle < 0 \;\;\&\;\; \lim_{m_q\to 0} \lim_{V_4\to\infty} \langle \bar{q} i\gamma_5 q \rangle = 0$$

$$\hookrightarrow \; \langle V_{\rm eff} \rangle \approx \cos\left(-\bar{\theta} + \langle a \rangle \xi / f_a\right) \left(m_u \langle \bar{u} u \rangle + m_d \langle \bar{d} d \rangle\right) \quad \text{with the minimum at } \langle a \rangle = \frac{f_a}{\xi} \bar{\theta}$$

$$\text{and} \quad m_a^2 = \left\langle \frac{\partial^2 V_{\rm eff}}{\partial a^2} \right\rangle \Bigg|_{\langle a \rangle = f_a \bar{\theta}/\xi} \approx \left(\xi \frac{m_\pi f_\pi}{f_a} \right)^2 \quad \text{as axion mass}^2$$

The road to the invisible axion models

The $U(1)_{PQ}$ order parameter f_a of the axion interaction Lagrangian

$$\mathcal{L}_{\rm int}(\partial^\mu a/f_a, \psi_f) + \xi \frac{a}{f_a}\frac{g_s^2}{32\pi^2} G^c_{\mu\nu}\tilde{G}^{c\,\mu\nu}$$

- associated with the scale of the spontaneous breaking of the PQ symmetry.
- Original PQ-model (with two Higgs) had $f_a \sim v_F \equiv \sqrt{v_1^2 + v_2^2} \approx 246\,\text{GeV}$ and predicted $\mathcal{BR}(K^+ \to \pi^+ + a) < 3 \cdot 10^{-5} \cdot (v_2/v_1 + v_1/v_2)$
- however $\mathcal{BR}_{\rm exp}(K^+ \to \pi^+ \text{nothing}) < 3.8 \cdot 10^{-8}$ Asano et al. (KEK), PLB 107 (1981)

  such that $f_a \gg v_F \rightsquigarrow$ basically two classes of invisible axion models:

(1) **KSVZ model:** scalar field σ with $f_a = \langle\sigma\rangle \gg v_F$ and super-heavy quark with PQ charge and $M_Q \sim f_a$ Kim, PRL 43 ('79); Shifman, Vainshtein, Zakharov, NPB 166 ('80)

(2) **DFSZ model:** adds to original PQ model a scalar field with PQ charge and $f_a = \langle\phi\rangle \gg v_F$ Dine, Fischler, Srednicki, PLB 104 ('81), Zhitnitsky, Sov.J.NP 31 ('80)

Photon couplings to axions

QCD anomaly induces an anomalous axion-coupling to 2 photons, *e.g.:*

$$\mathcal{L}^{\rm KSVZ}_{\rm axion} = \frac{a}{f_a}\left(\xi \frac{g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu} + 3e_Q^2\frac{\alpha_{\rm EM}}{4\pi}F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

$\hookrightarrow$ $a\gamma\gamma$ coupling corrected by axion mixing with the lowest pseudoscalars:

$$3e_Q^2 \to 3e_Q^2 - \frac{4m_d+m_u}{3(m_u+m_d)}$$

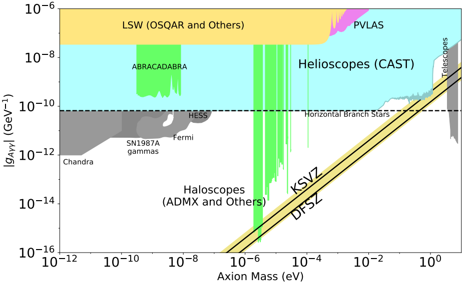
$\mathcal{L}_{a\gamma\gamma} = \frac{G_{a\gamma\gamma}}{4} aF_{\mu\nu}\tilde{F}^{\mu\nu}$ in general:

$$G_{a\gamma\gamma} = \frac{\alpha_{\rm EM}}{\pi f_a}\left[\frac{E}{2N} - \frac{4m_d+m_u}{3(m_d+m_u)}\right]$$

E & N strength of em / strong anomaly, respectively:

DSFZ: $E/N = 8/3$
$= 3\times\left[(\tfrac{2}{3})^2+(-\tfrac{1}{3})^2\right]+(-1)^2$,

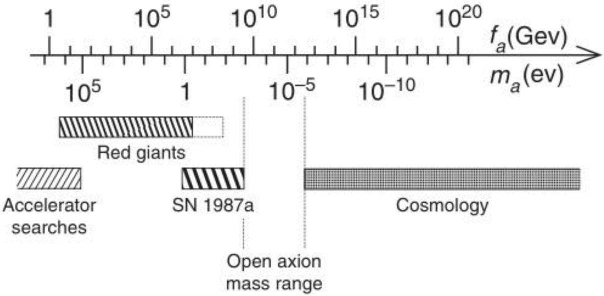
KSVZ: $E/N = 0$ (if $e_Q := 0$)



A. Ringwald et al., PDG (2019)

Window for axion searches

S.Asztalos, ed. G.Bertone, Cambridge Univ. Press ('10)



Axion searches

- in Labs (Colliders, Lasers) – light-shining-through-the-wall (*e.g.* ALPSII)
- for Astro-sources –Helioscopes (*e.g.* CAST, IAXO)
- for galactic axions – Halioscopes/microwave cavities (*e.g.* ADMX)
- indirect constraints: – from Astrophysics (red giants, SN 1987a)
  – and from Cosmology: DM bounds ($\Omega_{CDM} \approx 0.22$) on axion oscillations

$$\rightsquigarrow \quad f_a^{\max} \leftrightarrow m_a^{\min}$$

Preliminary Summary: Axions

- predicted as a / the resolution of the *Strong CP problem*:
  to escape the fine-tuning problem $|\bar{\theta}| < 10^{-10}$ while $\delta_{KM} \sim \mathcal{O}(1)$
- extendible to ALPS: axion-like particles with f_a and $g_{alps\gamma\gamma}$ decoupled
- couple feebly ($\sim 1/f_a$) and gravitationally to matter and radiation
- can be candidates for Cold Dark Matter

  i.e. with a well-determined and narrow window for searches:

however fine-tuning may back ...

THE EMPIRE STRIKES BACK

Axions and EDMs: generic effective Lagrangian of the axion

Kiwoon Choi (Daejeon, Korea), Bethe-Lectures, Bonn, March 2015

$$\mathcal{L}_{\rm eff}(a) = \underbrace{\mathcal{L}_0}_{\text{indep. of } a} + \underbrace{\tfrac{1}{2}(\partial_\mu a)^2 + \frac{\partial_\mu a}{f_a}\tilde{J}^\mu(\bar\psi\ldots\psi,\phi)}_{\text{PQ-invariant}} + \underbrace{\frac{a}{f_a}\frac{N}{32\pi^2}G\tilde{G}}_{\text{expl. PQ-breaking by QCD anomaly}}$$
$$+ \underbrace{\Delta\mathcal{L}_{\rm UV}\left(=\epsilon m_{\rm UV}^4\cos(a/f_a+\delta_{\rm UV})\right)}_{a \text{ coupling from expl. PQ breaking at UV scale}}$$

$\bar\theta = \langle a\rangle/f_a$ is calculable in terms of the $\not{CP}$ phases (in the presence of axion!):

- $\delta_{\rm KM}$ = Kobayashi-Maskawa phase in the PQ-invariant SM
- $\delta_{\rm BSM}$ = $\not{CP}$ phase in PQ-invariant Beyond SM at the scale $m_{\rm BSM}$
- $\delta_{\rm UV}$ = $\not{CP}$ phase in explicit PQ-breaking sector at $m_{\rm UV} \sim M_{\rm Planck}$, applying

$$V(a) = V_{\rm QCD} + V_{\rm KM} + V_{\rm BSM} + V_{\rm UV}$$

$$V_{\rm QCD} \sim -f_\pi^2 m_\pi^2 \cos(a/f_a) \quad \text{(expl. PQ-breaking by low-energy QCD, min. at } \langle a\rangle = 0)$$

$$V_{\rm KM} \sim f_\pi^2 m_\pi^2 \times \overbrace{G_F^2 f_\pi^4}^{10^{-14}} \times \overbrace{10^{-5}\sin\delta_{\rm KM}}^{\text{Jarlskog inv.}} \times \sin(a/f_a)$$

$$V_{\rm BSM} \sim f_\pi^2 m_\pi^2 \times \underbrace{\left(10^{-2}-10^{-3}\right)}_{\text{loop suppression}} \times \frac{f_\pi^2}{m_{\rm BSM}^2}\sin\delta_{\rm BSM} \times \sin(a/f_a)$$

$$V_{\rm UV} \sim \epsilon m_{\rm UV}^4 \sin\delta_{\rm UV}\sin(a/f_a) \qquad [\text{from } \Delta\mathcal{L}_{\rm UV}]$$

$\bar\theta = \langle a\rangle/f_a$ and contributions to the nucleon EDM

$$\bar\theta \;\sim\; 10^{-19}\sin\delta_{\rm KM} + \overbrace{\left(10^{-10}-10^{-11}\right)}^{(10^{-2}\text{–}10^{-3})\times f_\pi^2/\text{TeV}^2} \times \left(\frac{\text{TeV}}{m_{\rm BSM}}\right)^2 \sin\delta_{\rm BSM}$$
$$+\;\epsilon\,\frac{m_{\rm UV}^4}{f_\pi^2 m_\pi^2}\,\sin\delta_{\rm UV} \quad \left(\text{with } \epsilon < 10^{-10}\, f_\pi^2 m_\pi^2/m_{\rm UV}^4 \;\sim\; 10^{-88} \text{ for } m_{\rm UV}\sim M_{\rm Pl}\right)$$

↪ Regardless of the existence of BSM physics near the TeV scale, $\bar\theta = \langle a\rangle/f_a$ can have *any value* below the present bound $\sim 10^{-10}$.

$$d_N \;\sim\; \frac{e}{2m_N}\left[\frac{m^*}{\Lambda_{\rm QCD}}\bar\theta \;+\; G_F^2 f_\pi^4 \times 10^{-5}\sin\delta_{\rm KM} + \left(10^{-2}-10^{-3}\right)\times\frac{f_\pi^2}{m_{\rm BSM}^2}\sin\delta_{\rm BSM} + \left(10^{-2}-10^{-3}\right)\times\frac{f_\pi^2}{m_{\rm UV}^2}\sin\delta_{\rm UV}\right]$$

$$\sim\; \frac{e}{2m_N}\left[\overbrace{\frac{m^*}{\Lambda_{\rm QCD}}}^{\sim 10^{-2}} \times \overbrace{\bar\theta_{\rm UV}}^{\frac{\epsilon m_{\rm UV}^4 \sin\delta_{\rm UV}}{f_\pi^2 m_\pi^2}} \;+\; \left(10^{-2}-10^{-3}\right)\times\frac{f_\pi^2}{m_{\rm BSM}^2}\sin\delta_{\rm BSM}\right]$$

▸ $\bar\theta$ ▸ KM ▸ BSM

likely dominated by $\bar\theta_{\rm UV}$ induced by $\not{CP}$ in the $\not{PQ}$ sector @ $m_{\rm UV}(\sim M_{\rm Pl})$, and/or by the BSM contribution near the TeV scale.

Dialectics: from the $U(1)_A$ problem to axions

The Hare and the Hedgehog **– a fairy tale**

- $U(1)_A$ problem of QCD: $m_{\eta,\eta'} > \sqrt{3} m_\pi \approx 240\,\text{MeV}$
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  – solution: θ vacuum (superposition of all $|n\rangle$ vacua $\times e^{i\theta n}$)
- Problem: neutron EDM bound $\rightsquigarrow$ strong CP problem
  – proposed solution: Peccei-Quinn mechanism and axions
- Problem: original Peccei-Quinn model w. $f_a = v_F$ excluded by exp.
  – solution: invisible axions with $f_a \gg v_F$
- Problem: how to detect an (invisible) axion
  – possible solution: direct/indirect searches in rather narrow window
- Problem: fine-tuning back from explicit PQ-breaking at the UV scale
  – possible solution: check several EDMs (*e.g.* $d_n, d_p, d_D, d_{^3\text{He}}, \ldots$)

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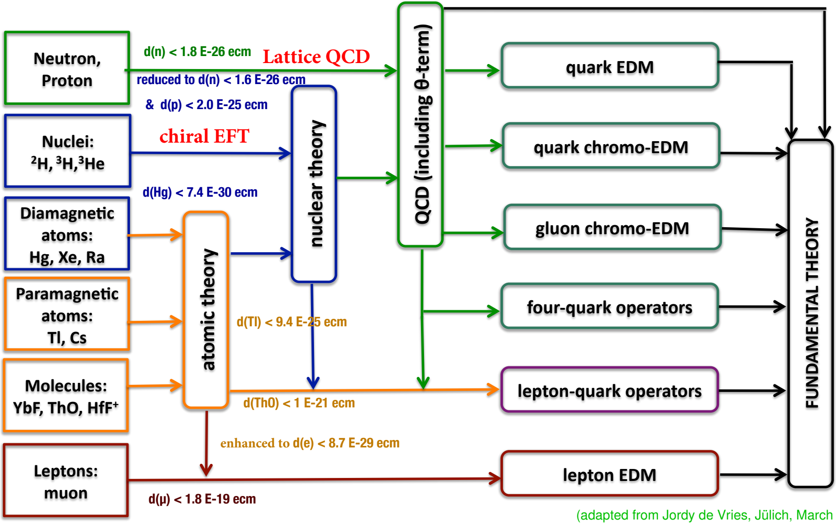
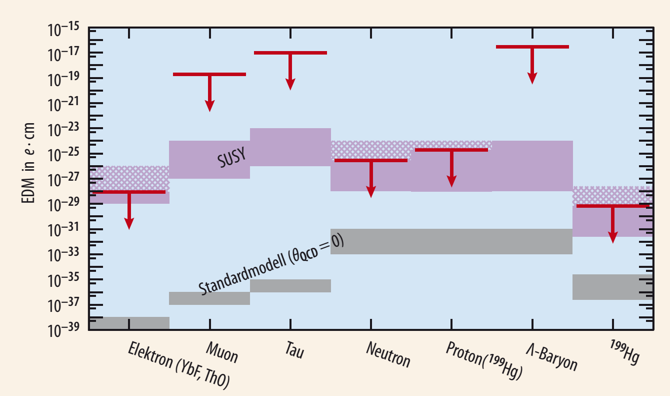
Road map from EDM measurements to the sources

Experimentalist's point of view → ← Theorist's point of view



(adapted from Jordy de Vries, Jülich, March 14, 2013)

Measured upper bounds for EDMs



K. Kirch, J. Pretz & A.W., Physik Journal 16 (2017) Nr. 11

Note: $d_e^{\rm SM}$, $d_\mu^{\rm SM}$, and $d_\tau^{\rm SM}$ should be 10^{-6} times smaller as indicated above,
see M. Pospelov & A. Ritz, PRD 89 (2014)

1st goal: measurement of any non-zero permanent EDM to establish CP violation beyond the SM

2nd goal: measurements of several non-zero EDMs to narrow down the underlying mechanism

Oscillating EDMs: back to axions/ALPs

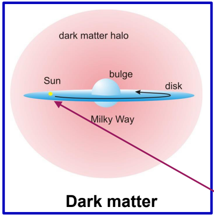
P.W. Graham & S. Rajendran, PRD 84 (2011) & 88 (2013)

a test of hypothesis that Dark Matter (DM) in our Galaxy is saturated by classical oscillating field of axions (or ALPs) of mass $10^{-22}\text{eV} \lesssim m_a \lesssim 10^{-7}\text{eV}$

$$\mathcal{L}_{axion} = C_G \frac{a}{f_a} \frac{g_s^2}{16\pi^2} \mathrm{tr}\, G\tilde{G} + \tfrac{1}{2}\partial_\mu a \partial^\mu a - \tfrac{1}{2} m_a^2 a^2 \qquad \text{(for axions: } C_G := 1\text{)}$$

(axion-mass range $\longleftrightarrow$ $10^{29}\text{GeV} \gtrsim f_a \gtrsim 10^{14}\text{GeV}$ if $m_a \approx 0.5\, m_\pi f_\pi / f_a$ in QCD epoch)

- In our Galaxy, axion/ALP DM-field $a(t,\vec{x})$ *spatially* constant ($\ell_{\rm coh} \approx \hbar/(m_a v_{\rm DM})$) over range of $\sim 0.5\,\text{km} \times 10^{-7}\text{eV}/m_a$



$\rho_{\rm DM} \approx 0.4$ GeV/cm^3

$v_{\rm DM} \sim 300$ km/s

$\hookrightarrow$ Use lab ansatz

$$a(t,\not{\vec{x}}) \approx a_0 \cos\left(m_a\left(1+\frac{v_{\rm DM}^2}{2c^2}\right)(t-t_0)+\phi_0\right)$$
$$\approx a_0 \cos(m_a(t-t_0)+\phi_0)$$

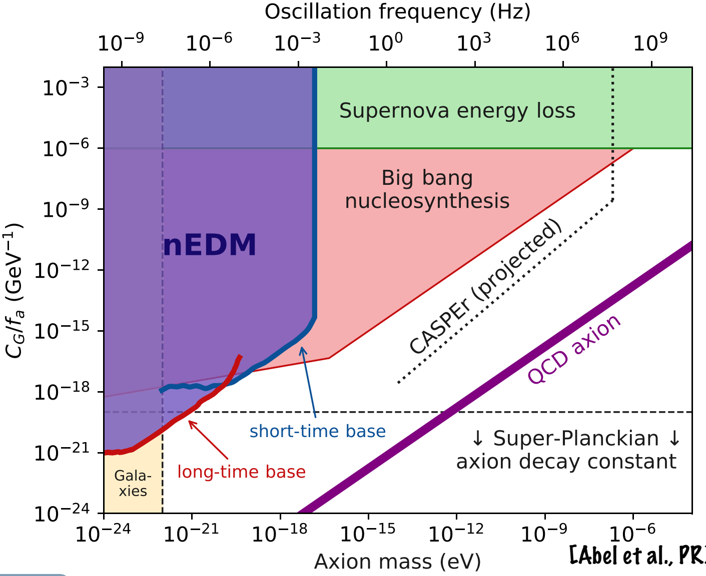
valid if $|t-t_0| < \tau_{\rm coh} \approx \hbar/(m_a v_{DM}^2)$

Equating $\frac{1}{2} m_a^2 a_0^2 \sim \rho_{\rm DM}^{\rm local} \approx (0.4 \pm 0.1)\,\text{GeV/cm}^3$ gives axion amplitude

$$\theta_a \equiv \frac{a_0}{f_a} \sim \frac{\sqrt{2\rho^{\rm local}}}{m_a f_a} \sim \frac{\sqrt{2\rho_{\rm DM}^{\rm local}}}{0.5\, m_\pi f_\pi} \sim 3 \times 10^{-19} \quad \xrightarrow[\text{independently of } f_a]{d_n \approx 10^{-16}\theta_a e\,\text{cm}} \quad d_n \approx 5 \times 10^{-35} \cos(m_a(t-t_0)+\phi_0)\, e\,\text{cm}$$

Oscillating Axions? But bounds on oscillating ALPs possible, since $f_a \overset{!}{<} 0.5 m_\pi f_\pi / m_a$

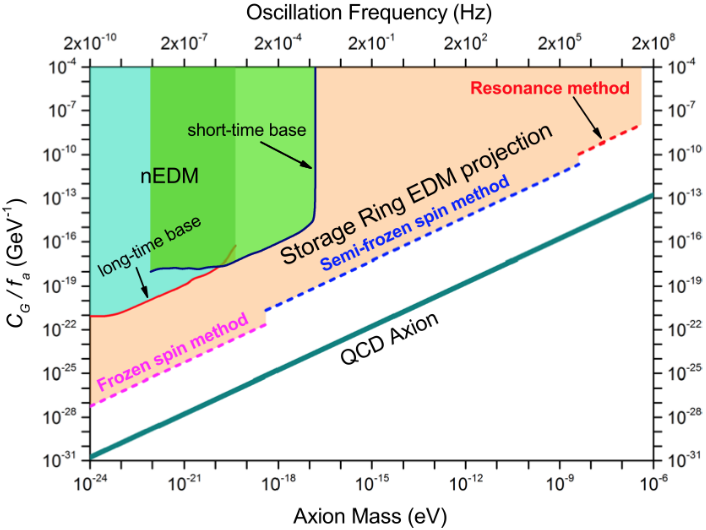
Bounds on oscillating ALPs from Astrophysics and nEDM searches?



[Abel et al., PRX, 2017]

▸ short cut (final)

Bounds on oscillating ALPs from storage ring EDM searches?

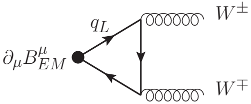


S.P. Chang et al., Phys. Rev. D **99** (2019)

Jump slides

EW Baryogenesis: Standard Model

Conservation of the EM current under weak ($L - R$) interactions:



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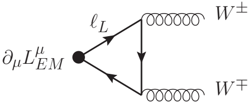
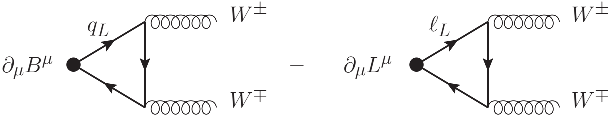


$\propto \; N_c \cdot (Q_u + Q_d) \, + \, (0-1) \, = \, 1 - 1 \, = \, 0$

$\hookrightarrow \Delta(Q_B + Q_L) = 0$ (charge conservation!)

EW Baryogenesis: Standard Model

Conservation of the Baryon–Lepton current under $(L-R)$ interactions:



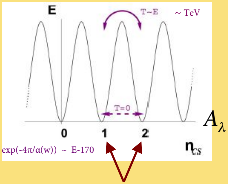
$\propto N_c \cdot 1/3 - 1 = 1 - 1 = 0$

$\hookrightarrow \Delta(B-L) = 0$ but $\Delta(B+L) \neq 0$!

Sakharov criteria

1. B violation ✓
   ($\Delta(B+L) \neq 0$ sphaleron transitions)
2. C & CP violation *x*
   (CKM determinant)
3. Nonequilibrium dynamics *x*
   (only fast cross over for $\mu_{chem} = 0$)

Anomalous Processes



Different vacua: $\Delta(B+L) = \Delta N_{CS}$

Sphaleron Transitions

Construction of the CKM matrix

Since weak interactions do not respect the global flavor symmetry, there is mixing within the groups of quarks with the same charge:

$$U \equiv \begin{pmatrix} u \\ c \\ t \end{pmatrix} \to \tilde{U} = M_U U\,, \qquad D \equiv \begin{pmatrix} d \\ s \\ b \end{pmatrix} \to \tilde{D} = M_D D\,,$$

where M_U & M_D are 3×3 unitary matrices

$$\hookrightarrow \text{charged weak current: } J_\mu = \bar{\tilde{U}}^\mu \gamma_\mu (1-\gamma_5) \tilde{D}^\mu = \bar{U} \gamma_\mu (1-\gamma_5) \underbrace{M_U^\dagger M_D}_{\text{CKM matrix } M} D\,.$$

- *M unitary* $n_G \times n_G$ matrix for n_G quark generations $\rightsquigarrow$ n_G^2 *real* parameters.
- $2n_G - 1$ of these can be absorbed by the relative phases of the quark wave functions $\rightsquigarrow$ $(n_G - 1)^2$ remaining parameters:

  $n_G = 2$: one remaining real parameter: *Cabibbo angle*
  $n_G = 3$: 4 real parameters: $O(3)$ matrix with $\frac{1}{2} 3 \cdot (3-1) = 3$ angles *plus* 1 ~~CP~~ phase
- Lepton case: neutrinos may be Majoranas: $\rightsquigarrow$ 3 angles plus 3 ~~CP~~ phases
- If phase(s) present, M complex matrix, whereas CP invariance $\rightsquigarrow$ $M^* = M$!

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Hidden Symmetry and Goldstone Bosons

$$[Q_V^a, H] = 0, \quad \text{and} \quad e^{-iQ_V^a}|0\rangle = |0\rangle \Leftrightarrow Q_V^a|0\rangle = 0 \quad \text{(Wigner-Weyl realization)}$$

$$[Q_A^a, H] = 0, \quad \text{but} \quad e^{-iQ_A^a}|0\rangle \neq |0\rangle \Leftrightarrow Q_A^a|0\rangle \neq 0 \quad \text{(Nambu-Goldstone realiz.)}$$

- Consequence: $e^{-iQ_A^a}|0\rangle \neq |0\rangle$ is not the vacuum, but

$$H\, e^{-iQ_A^a}|0\rangle = e^{-iQ_A^a} H|0\rangle = 0 \quad \text{is a massless state!}$$

Goldstone theorem: for *continuous* global symmetry that does *not* leave the ground state invariant ('hidden' or 'spontaneously broken' symmetry)

- mass- and spinless particles, "Goldstone bosons" (GBs)
- number of (relativistic) GBs = number of broken symmetry generators
- axial generators broken ⇒ GBs should be pseudoscalars
- finite masses via (small) quark masses
  ↪ 8 lightest hadrons: $\pi^\pm$, π^0, $K^\pm$, K^0, $\bar{K}^0$, η (not η')
- Goldstone bosons decouple (non-interacting) at vanishing energy & momentum

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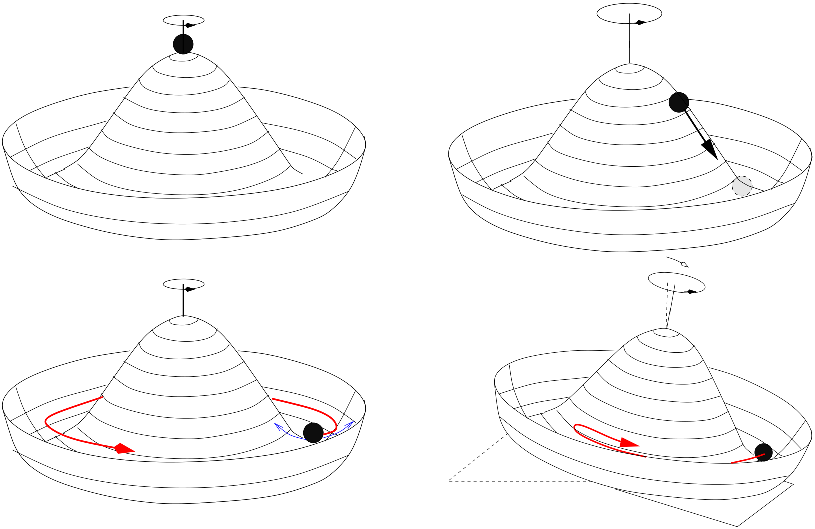
Illustration: spontaneous symmetry breaking



The symmetries of QCD

$$\mathcal{L}_{\rm QCD} = -\tfrac{1}{2}\mathrm{Tr}\,(G_{\mu\nu}G^{\mu\nu}) + \sum_f \bar{q}_f(\mathrm{i}\not{D} - m_f)q_f + \ldots$$

$$D_\mu = \partial_\mu - igA_\mu \equiv \partial_\mu - igA^a_\mu\frac{\lambda^a}{2}\,, \qquad G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

- Lorentz-invariance, P, C, T invariance, SU(3)$_c$ gauge invariance
- The masses of the u, d, s quarks are small: $m_{u,d,s} \ll 1\,\mathrm{GeV} \approx \Lambda_{\rm hadron}$.
- Chiral decomposition of quark fields:

$$q = \tfrac{1}{2}(1-\gamma_5)q + \tfrac{1}{2}(1+\gamma_5)q = q_L + q_R\,.$$

- For massless fermions: left-/right-handed fields do not interact

$$\mathcal{L}[q_L, q_R] = i\bar{q}_L\not{D}q_L + i\bar{q}_R\not{D}q_R - m\,(\bar{q}_Lq_R + \bar{q}_Rq_L)$$

and $\mathcal{L}^0_{QCD}$ invariant under (global) chiral $\mathrm{U}(3)_L \times \mathrm{U}(3)_R$ transformations:

↪ rewrite $\mathrm{U}(3)_L \times \mathrm{U}(3)_R = \mathrm{SU}(3)_V \times \mathrm{SU}(3)_A \times \mathrm{U}(1)_V \times \mathrm{U}(1)_A$.

- $\mathrm{SU}(3)_V = \mathrm{SU}(3)_{R+L}$: still conserved for $m_u = m_d = m_s > 0$
- $\mathrm{U}(1)_V = \mathrm{U}(1)_{R+L}$: quark or baryon number is conserved
- $\mathrm{U}(1)_A = \mathrm{U}(1)_{R-L}$: broken by quantum effects ($\mathrm{U}(1)_A$ anomaly + instantons)

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Mass term of U(3) pseudo-Goldstone bosons

S. Weinberg, Phys. Rev. D 11 (1975) 3583 & *The Quantum Theory of Fields*, Vol. II, Ch. 19.10 (1996)

$$\frac{F_\pi^2(2B_0)}{4}\mathrm{Tr}\Big(\mathcal{M}\big(U+U^\dagger\big)\Big) \quad \text{with} \quad U=\exp\Big(i\sum_{a=1}^{8}\lambda^a\phi^a/F_\pi + i\lambda^0\eta^0/F_s\Big)\equiv e^{i\tilde\phi/F_\pi}$$

where
$$\tilde\phi=\begin{bmatrix}\pi^0+\frac{1}{\sqrt3}\eta^8 & \sqrt2\pi^+ & \sqrt2K^+\\ \sqrt2\pi^- & -\pi^0+\frac{1}{\sqrt3}\eta^8 & \sqrt2K^0\\ sqrt2K^- & \sqrt2\bar K^0 & -\frac{2}{\sqrt3}\eta^8\end{bmatrix}+\sqrt{\frac23}\frac{F_\pi}{F_s}\begin{bmatrix}\eta^0&0&0\\0&\eta^0&0\\0&0&\eta^0\end{bmatrix} \;\&\; \mathcal{M}=\begin{bmatrix}m_u&0&0\\0&m_d&0\\0&0&m_s\end{bmatrix}$$

For flavor-neutral and flavor-charged pGBs:

$$B_0\mathrm{Tr}\left(\mathcal{M}\tilde\phi^2\right) = B_0\Big[m_u\Big(\pi^0+\frac{1}{\sqrt3}\eta^8+\sqrt{\frac23}\frac{F_\pi}{F_s}\eta^0\Big)^2+m_d\Big(-\pi^0+\frac{1}{\sqrt3}\eta^8+\sqrt{\frac23}\frac{F_\pi}{F_s}\eta^0\Big)^2+2(m_u+m_d)\pi^+\pi^-$$
$$+\,m_s\Big(-\frac{2}{\sqrt3}\eta^8+\frac23\frac{F_\pi}{F_s}\eta^0\Big)^2+2(m_u+m_s)K^+K^-+2(m_d+m_s)K^0\bar K^0\Big]$$

$\hookrightarrow$ the mass-mixing matrix of the flavor-neutrals has two pseudo-zero modes for $m_{u,d}\ll m_s$ fixed:

$$B_0\begin{pmatrix}\pi^0\\ \eta^8\\ \eta^0\end{pmatrix}^{\mathrm T}\begin{bmatrix} m_u+m_d & \frac{1}{\sqrt3}(m_u-m_d) & \sqrt{\frac23}\frac{F_\pi}{F_s}(m_u-m_d)\\ \frac{1}{\sqrt3}(m_u-m_d) & \frac13(m_u+m_d+4m_s) & \frac{\sqrt2F_\pi}{3F_s}(m_u+m_d-2m_s)\\ \sqrt{\frac23}\frac{F_\pi}{F_s}(m_u-m_d) & \frac{\sqrt2F_\pi}{3F_s}(m_u+m_d-2m_s) & \frac{2F_\pi^2}{3F_s^2}(m_u+m_d+m_s)\end{bmatrix}\begin{pmatrix}\pi^0\\ \eta^8\\ \eta^0\end{pmatrix}$$

$\hookrightarrow$ $u_{\pi^0}=(1,0,0)^{\mathrm T}$ with $B_0u^{\mathrm T}_{\pi^0}[\cdots]u_{\pi^0}=B_0(m_u+m_d)\equiv m^2_{\pi^\pm}$

& $u_{\text{"}\eta\text{"}}=(0,1,\frac{\sqrt2F_s}{F_\pi})^{\mathrm T}/\sqrt{1+2F_s^2/F_\pi^2}$ with $B_0u^{\mathrm T}_{\text{"}\eta\text{"}}[\cdots]u_{\text{"}\eta\text{"}}=(1+2)\frac{B_0(m_u+m_d)}{1+2F_s^2/F_\pi^2}\le 3m^2_{\pi^\pm}$

$\rightsquigarrow$ $m_{\text{"}\eta\text{"}}\overset{!}{<}\sqrt3m_\pi\approx 240\,\mathrm{MeV}$

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’t Hooft’s explicit instanton solution for $SU(2)$

G. ’t Hooft, Phys. Rev. D **14** (1976) 3432-3450

In terms of the anti-symmetric *’t Hooft symbols*

$$\overset{(-)}{\eta}{}^a_{00} = 0, \quad \overset{(-)}{\eta}{}^a_{ij} = \epsilon_{aij}, \quad \overset{(-)}{\eta}{}^a_{i0} = -\overset{(-)}{\eta}{}^a_{0i} = \overset{+}{(-)}\delta_{ai} \quad \left(\text{with } \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}\overset{(-)}{\eta}{}^a_{\alpha\beta} = \overset{+}{(-)}\overset{(-)}{\eta}{}^a_{\mu\nu} \text{ and } \overset{(-)}{\eta}{}^a_{\mu\nu}\overset{(-)}{\eta}{}^a_{\mu\nu} = 3!+2\cdot 3 = 12\right)$$

the **one**-(anti)**instanton** [=(anti-)**self-dual**] ***solution*** reads

$$A^a_\mu(x) = \overset{(-)}{\eta}{}^a_{\mu\nu}\,\partial_\nu \ln\left(1 + \frac{(x-\bar{x})^2}{\rho^2}\right) = \frac{2\overset{(-)}{\eta}{}^a_{\mu\nu}(x_\nu - \bar{x}_\nu)}{(x-\bar{x})^2+\rho^2} \;\rightsquigarrow\; G^a_{\mu\nu} = \frac{-4\rho^2\overset{(-)}{\eta}{}^a_{\mu\nu}}{[(x-\bar{x})^2+\rho^2]^2}\,.$$

The pertinent **Yang-Mills action $S_E = 8\pi^2/g_s^2$** itself is **independent** of the instanton **position** $\bar{x}_\mu$, **scale** (size) ρ, and (gauge) **rotations**.

Instanton solutions for bigger unitary unimodular groups (e.g. $SU(3)$) can be obtained by ***natural* embedding** $SU(2) \subset SU(N)$ from the $SU(2)$ solution: $\begin{pmatrix} \multicolumn{2}{c}{SU(2)} & 0 & \cdots & 0 \\ & & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$.

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Topological charge and susceptibility

- Topological density $q(x) = \frac{g_s^2}{32\pi^2} G^a_{\mu\nu}(x)\tilde{G}^{a\,\mu\nu}(x)$
- Topological charge $Q = \int d^4x\, q(x)$
- Partition function (Ω is space-time volume)

$$Z(\theta) = \int \mathcal{D}A e^{-S_{\rm YM}[A]+i\theta Q} \equiv e^{-\Omega F(\theta)} \quad \text{such that} \quad Q = \frac{1}{i}\frac{\partial}{\partial\theta} \ln Z(\theta)\Big|_{\theta=0}.$$

$$\hookrightarrow \mathcal{A}_\theta^{\rm I=1} \propto e^{-\int d^4x_E\left(\frac{1}{8g_s^2}(G\pm\tilde{G})^2 \mp \left(\frac{8\pi^2}{g_s^2}\mp i\theta\right)\frac{1}{32\pi^2}G\tilde{G}\right)} \propto e^{-\frac{8\pi^2}{g_s^2(\mu)}\pm i\theta} \quad (\text{note } \theta\in[-\pi,\pi])$$

one-instanton amplitude.

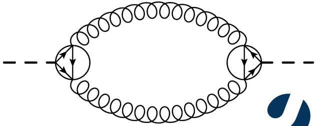
- Topological susceptibility $\chi = \frac{\partial^2 F(\theta)}{\partial\theta^2}\Big|_{\theta=0} = \int d^4x_E \langle q(x)q(0)\rangle = \lim_{\Omega\to\infty}\frac{\langle Q^2\rangle}{\Omega}$
- Note: $2N_F$ from $\partial_\mu J^\mu_A = 2N_F q(x)$ & $\chi = \mathcal{O}(N_c^0)$ since $g_s \sim 1/\sqrt{N_c}$ and $\langle G\tilde{G}(x)G\tilde{G}(0)\rangle \sim N_c^2$:

$$\rightsquigarrow \frac{F^2_{\eta'}, m^2_{\eta'}}{2N_F} = \int d^4x\,\langle q(x)q(0)\rangle = \left(\frac{g_s^2}{32\pi^2}\right)^2 \int d^4x \left\langle G^a_{\mu\nu}(x)\tilde{G}^{a\,\mu\nu}(x)G^b_{\rho\sigma}(0)\tilde{G}^{b\,\rho\sigma}(0)\right\rangle = \mathcal{O}(N_c^0)$$

$\hookrightarrow m_{\eta'} = \mathcal{O}(N_c^{-1/2})$

since

$F_{\eta'} = \mathcal{O}(N_c^{1/2})$



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Instanton amplitudes

- Since $G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu} = \partial_\mu K^\mu$ is a total derivative,

$$\mathcal{L}_{\rm QCD} = -\bar{\theta}\,\frac{g_s^2}{32\pi^2}\,\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\,G^a_{\mu\nu}\,G^a_{\rho\sigma}$$

  is irrelevant in perturbation theory.
- Non-perturbatively, large gauge transformations (instantons) exist:

$$\int_{R^4} d^4x_E\,\frac{1}{32\pi^2}\,G\tilde{G} \;=\; \text{integer}$$

- Some amplitudes depend on the periodic angle parameter

$$\bar{\theta} = \bar{\theta} + 2\pi\,:$$

$$\mathcal{A}_\theta \propto e^{-S_E} \propto e^{-\int d^4x_E\left(\frac{1}{8g_s^2}(G\pm\tilde{G})^2 \mp\left(\frac{8\pi^2}{g_s^2}\mp i\bar{\theta}\right)\frac{1}{32\pi^2}\,G\tilde{G}\right)} \propto e^{-\frac{8\pi^2}{g_s^2(\mu)}\pm i\bar{\theta}}$$

- Weak coupling $g_s^2(\mu) \ll 1$: instanton amplitudes exponentially small.
- For strong coupling $g_s^2(\mu) \sim 8\pi^2$, no suppression

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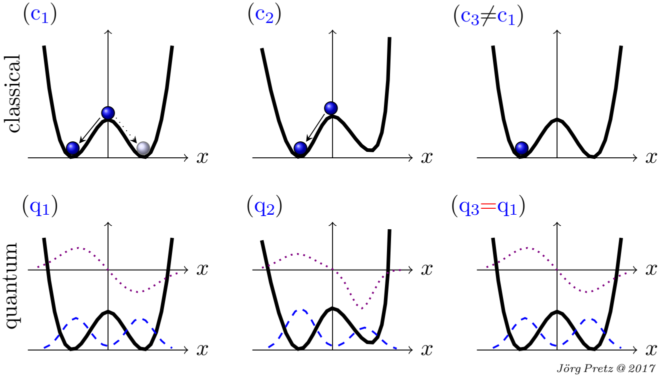
Double Well Potential and Spontaneous Symmetry Breaking

Comparison of classical with quantum scenario for sequence:
(1) symmetric case → (2) explicit perturbation of a symmetry → (3) restored symmetry



Message:

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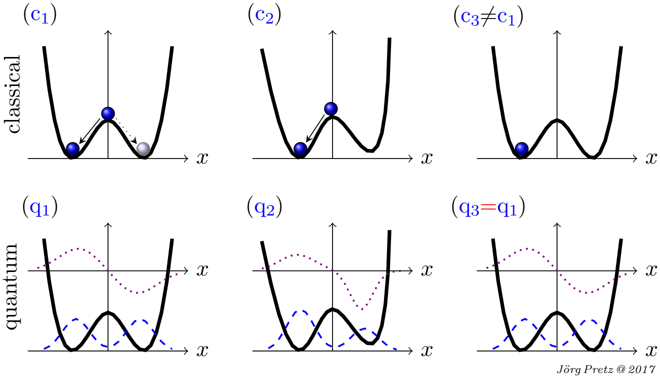

Message:

In quantum mechanics *asymmetric* stationary states of finite system only in presence of an *explicitly* broken symmetry (tunneling!)

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essence

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Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j \neq 0$

Let $\langle j^{\mathrm{P}} | \vec{d} \, | j^{\mathrm{P}} \rangle = d \, \langle j^{\mathrm{P}} | \vec{J} \, | j^{\mathrm{P}} \rangle$ with $\vec{d} \equiv \int \vec{r} \rho(\vec{r}) \mathrm{d}^3 r$ be an EDM operator in a stationary state $|j^{\mathrm{P}}\rangle$ of definite parity P and nonzero spin j, such that

$$\langle j^{\mathrm{P}} | \vec{d} \, | j^{\mathrm{P}} \rangle \to \mp \langle j^{\mathrm{P}} | \vec{d} \, | j^{\mathrm{P}} \rangle \quad \& \quad \langle j^{\mathrm{P}} | \vec{J} \, | j^{\mathrm{P}} \rangle \to \pm \langle j^{\mathrm{P}} | \vec{J} \, | j^{\mathrm{P}} \rangle \quad \text{under} \quad \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

If $d \neq 0$ and $|j^{\mathrm{P}}\rangle$ has *no* degeneracy (besides rotational), *then* ~~P~~ & ~~T~~.

* *non*-selfconjugate particle is *not* its own antiparticle ⇒ at least one "charge" *non*-zero

Werner Bernreuther (2012)

JÜLICH Forschungszentrum

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It can be interpreted as a special case of the theorem:

Any *finite* quantum system without *explicit* symmetry breaking cannot have a spontaneously broken groundstate.

Keywords: *symmetric double-well* potential and *quantum tunneling* (instantons)

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‘Isn’t an elementary particle a point-particle without structure?
How can such a particle be polarized and support an EDM?’

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'Isn't an elementary particle a point-particle without structure?
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There are always vacuum polarizations with rich short-distance structure

($g-2$ of the electron and muon aren't exactly zero either)

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‘What about the huge EDMs of H_2O or NH_3 molecules?’

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'What about the huge EDMs of H_2O or NH_3 molecules?'

The ground states of these molecules at non-zero temperatures or strong E-fields are mixtures of at least 2 opposite parity states:

The theorem doesn't apply for degenerate states: neither $\not{T}$ nor $\not{P}$!

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State $|j^P\rangle$ can be *‘elementary’* particle (quark, charged lepton, $W^{\pm}$ boson, Dirac neutrino, ...) or a *‘composite’* neutron, proton, nucleus, atom, molecule.

‘But what about the induced EDM (polarization)?’

Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j \neq 0$

Let $\langle j^{\rm P} | \vec{d} \,| j^{\rm P} \rangle = d \,\langle j^{\rm P} | \vec{J} \,| j^{\rm P} \rangle$ with $\vec{d} \equiv \int \vec{r} \rho(\vec{r}) {\rm d}^3 r$ be an EDM operator in a stationary state $| j^{\rm P} \rangle$ of definite parity P and nonzero spin j, such that

$$\langle j^{\rm P} | \vec{d} \,| j^{\rm P} \rangle \to \mp \langle j^{\rm P} | \vec{d} \,| j^{\rm P} \rangle \quad \& \quad \langle j^{\rm P} | \vec{J} \,| j^{\rm P} \rangle \to \pm \langle j^{\rm P} | \vec{J} \,| j^{\rm P} \rangle \quad \text{under} \quad \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

If $d \neq 0$ and $| j^{\rm P} \rangle$ has *no* degeneracy (besides rotational), *then* $\not{\rm P}$ & $\not{\rm T}$.

* *non*-selfconjugate particle is *not* its own antiparticle ⇒ at least one “charge” *non*-zero

State $| j^{\rm P} \rangle$ can be *‘elementary’* particle (quark, charged lepton, $W^{\pm}$ boson, Dirac neutrino, ...) or a *‘composite’* neutron, proton, nucleus, atom, molecule.

‘But what about the induced EDM (polarization)?’

The induced EDM is *quadratic* in the electric field and *neither* $\not{\rm P}$ nor $\not{\rm T}$

induced EDM	⟷	quadratic Stark effect ($\propto E^2$)
permanent EDM	⟷	linear Stark effect ($\propto E$)

Theorem: Permanent EDMs of *non*-selfconjugate* particles *with* spin $j \neq 0$

Let $\langle j^P | \vec{d} | j^P \rangle = d \, \langle j^P | \vec{J} | j^P \rangle$ with $\vec{d} \equiv \int \vec{r} \rho(\vec{r}) d^3 r$ be an EDM operator in a stationary state $|j^P\rangle$ of definite parity P and nonzero spin j, such that

$$\langle j^P | \vec{d} | j^P \rangle \to \mp \langle j^P | \vec{d} | j^P \rangle \quad \& \quad \langle j^P | \vec{J} | j^P \rangle \to \pm \langle j^P | \vec{J} | j^P \rangle \quad \text{under} \begin{cases} \text{space reflection,} \\ \text{time reversal.} \end{cases}$$

If $d \neq 0$ and $|j^P\rangle$ has *no* degeneracy (besides rotational), *then* ~~P~~ & ~~T~~.

* *non*-selfconjugate particle is *not* its own antiparticle ⇒ at least one "charge" *non*-zero

State $|j^P\rangle$ can be *'elementary'* particle (quark, charged lepton, $W^{\pm}$ boson, Dirac neutrino, ...) or a *'composite'* neutron, proton, nucleus, atom, molecule.

If the interactions are described by an action which is

local, Lorentz-invariant, and hermitian

then CPT invariance holds: thus ~~T~~ $\Longleftrightarrow$ ~~CP~~

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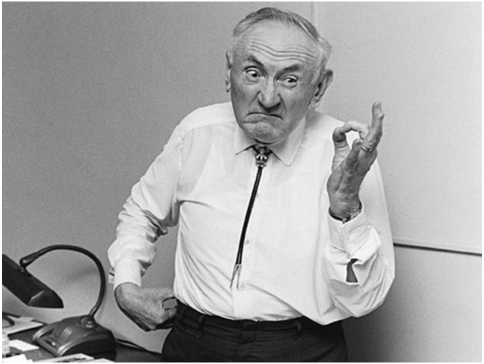
The essence of electric dipole moments (EDMs)

A spherical cow has no EDM

The essence of electric dipole moments (EDMs)

A spherical cow has no EDM and

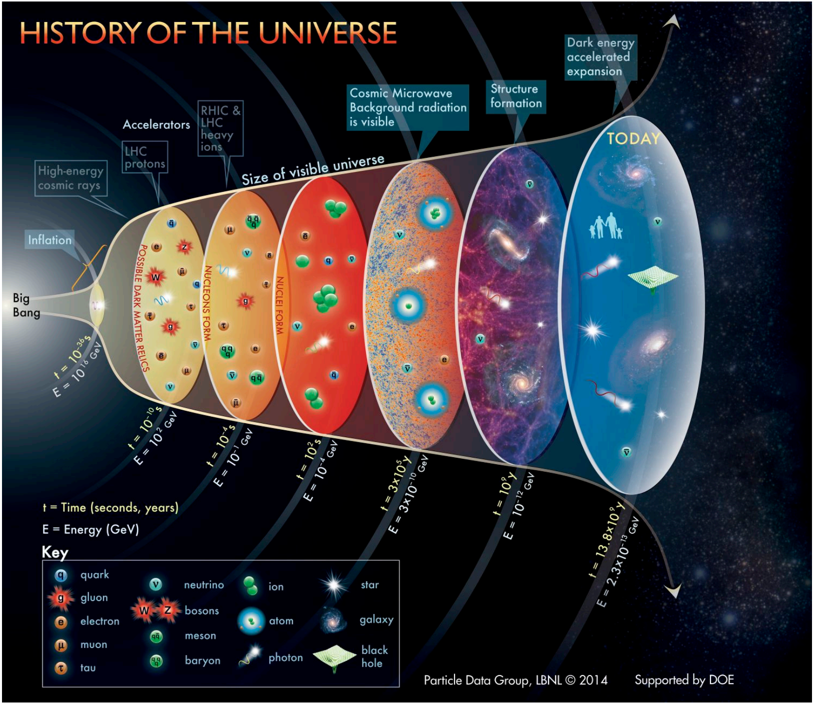
a spherical bastard has no EDM either*



* according to Fritz Zwicky *a person who is a bastard no matter from which direction you look at him or her* – even worse than a blockhead !

back

JÜLICH Forschungszentrum



HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Cosmic Microwave Background radiation is visible
Structure formation
Accelerators
RHIC & LHC heavy ions
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
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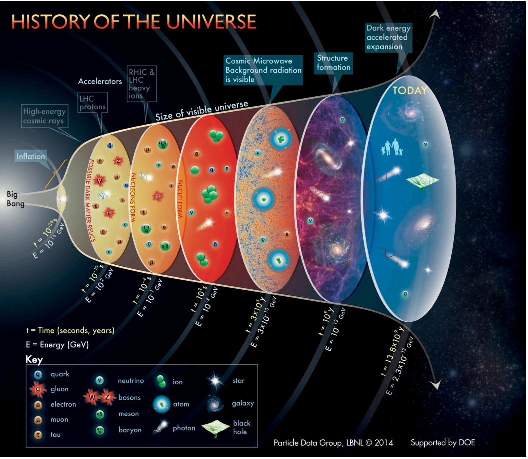
Matter Excess in the Universe



1. End of inflation: $n_B = n_{\bar{B}}$
2. Cosmic Microwave Bkgr.
   - SM(s) *prediction:*
     $(n_B - \cancel{n_{\bar{B}}})/n_\gamma|_{\rm CMB} \sim 10^{-18}$
   - WMAP+PLANCK ('13):
     $n_B/n_\gamma|_{\rm CMB}=(6.05\pm0.07)10^{-10}$

Sakharov conditions ('67)

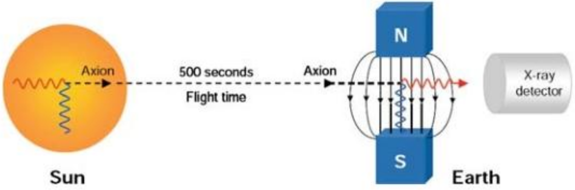
for dyn. generation of net B:

1. B violation
   to depart from initial B=0
2. C & CP violation
   to distinguish B from $\bar{B}$ prod. rates
3. Either CPT violation or
   out-of-thermal equilibrium
   to distinguish B production from back reaction and to escape $\langle B \rangle$=0 if CPT holds

back

Helioscopy

R. Battesti et al., Springer Lect. Notes Phys. 741 (2008)



- Time-reversed *Primakoff effect*: $a + \gamma_{\text{virtual}} \to \gamma$
- most sensitive for $10^{-5}\,\text{eV} \le m_a \le 1\,\text{eV}$
- depends on field B, length L, transferred momentum $q = m_a/2E$
- and solar models

CAST experiment (CERN Axion Solar Telescope)

- $m_a < 1.17\,\text{eV}$ (intersecting the KSVZ band)
- Next generation: IAXO (International Axion Oberservatory)@CERN

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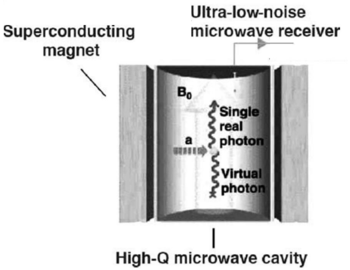
Halioscopy

R. Battesti et al., Springer Lect. Notes Phys. 741 (2008)



- Search for galactic axions via Primakoff effect: $a + \gamma_{\text{virtual}} \to \gamma$
- Tunable cavity search for microwave resonances
- Most sensitive detectors for CDM axions (μeV $\lesssim m_a \lesssim$ meV)

ADMX (Axion Dark Matter eXperiment) @University of Washington

- sensitivity to KVSZ axions between 1.9 μeV $\lesssim m_a \lesssim$ 3.3, μeV
- still on-going (ADMX II)

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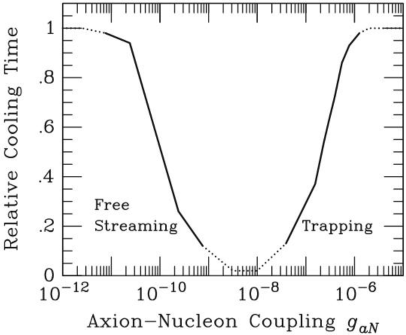
Supernovae (SN1987a)

G.G. Raffelt, Springer Lect. Notes Phys. 741 (2008)



- Axions emitted by nucleon Bremsstrahlung $NN \to NNa$

  – depends therefore on g_{aNN}

- Constraints:
  - energy loss rate $\epsilon_{\text{axion}} \lesssim 10^{19} \text{erg}\, \text{g}^{-1} \text{s}^{-1}$
  - Neutrino burst duration

back

Resonance method for oscillating ALPs searches at storage rings

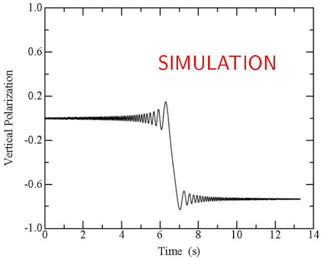
- Spin precession in magnetic field in particle rest frame:

$$\frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{B} + \vec{d} \times \vec{E} = \left(\vec{\Omega}_{g-2} + \vec{\Omega}_{EDM}\right) \times \vec{S}$$

$$= \underbrace{-\frac{e}{m}\frac{g-2}{2}\vec{B} \times \vec{S}}_{\text{in ring plane}} + \underbrace{\frac{-e}{2m}\eta(t)\left(\vec{\beta} \times \vec{B}\right) \times \vec{S}}_{\text{perp. to ring plane}} \quad \text{with } \eta(t) = \eta_{\text{stat}} + \eta_{\text{osc}} \cos(m_a t + \phi)$$

- Idea: measure vertical spin polarization for different $g-2$ frequencies
- off resonance: averaging to zero / on resonance: accumulation $\rightsquigarrow$ jump
- t-scan of $g-2$ frequencies:

Ed Stephenson et al., 2018



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