Precision tests of fundamental symmetries with η and η' decays

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Outline

η and η': properties, symmetries, quantum numbers

***P*- and *CP*-violation**

- It's the EDMs, stupid! Gan, BK, Passemar, Tulin, arXiv:2007.00664

***C*- and *CP*-violation**

- examples
- *C*-odd Dalitz plot asymmetries Akdag, Isken, BK, arXiv:2111.02417

Summary / Outlook

η and η' properties

- quantum numbers $I^G\,J^{PC} = 0^+0^{-+}$

  $\longrightarrow$ *C*, *P* eigenstates, can be used to test discrete symmetries
- η: (largely) (pseudo-)Goldstone boson

  $M_\eta = 547.86$ MeV, $\Gamma_\eta = 1.31$ keV

  $\longrightarrow$ tiny width, all decay modes forbidden at leading order by symmetries (*C*, *P*, angular momentum, isospin/*G*-parity. . .)
- η': no Goldstone boson due to $U(1)_A$ anomaly

  $M_{\eta'} = 957.78$ MeV, $\Gamma_{\eta'} = 196$ keV

  $\longrightarrow$ still much narrower than e.g. ω, ϕ
- all additive quantum numbers are zero: flavour-conserving lab for symmetry tests and BSM physics searches, little SM background
- theoretical methods:
  - ▷ (large-N_c) chiral perturbation theory
  - ▷ dispersion theory (final-state interactions)
  - ▷ (sometimes) vector-meson dominance

η (and η') physics

Channel	Expt. branching ratio	Discussion
$\eta \to 2\gamma$	39.41(20)%	chiral anomaly, η–η' mixing
$\eta \to 3\pi^0$	32.68(23)%	$m_u - m_d$
$\eta \to \pi^0\gamma\gamma$	$2.56(22) \times 10^{-4}$	χPT at $O(p^6)$, leptophobic B boson, light Higgs scalars
$\eta \to \pi^0\pi^0\gamma\gamma$	$< 1.2 \times 10^{-3}$	χPT, axion-like particles (ALPs)
$\eta \to 4\gamma$	$< 2.8 \times 10^{-4}$	$< 10^{-11}$ [54]
$\eta \to \pi^+\pi^-\pi^0$	22.92(28)%	$m_u - m_d$, C/CP violation, light Higgs scalars
$\eta \to \pi^+\pi^-\gamma$	4.22(8)%	chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta \to \pi^+\pi^-\gamma\gamma$	$< 2.1 \times 10^{-3}$	χPT, ALPs
$\eta \to e^+e^-\gamma$	$6.9(4) \times 10^{-3}$	theory input for $(g-2)_\mu$, dark photon, protophobic X boson
$\eta \to \mu^+\mu^-\gamma$	$3.1(4) \times 10^{-4}$	theory input for $(g-2)_\mu$, dark photon
$\eta \to e^+e^-$	$< 7 \times 10^{-7}$	theory input for $(g-2)_\mu$, BSM weak decays
$\eta \to \mu^+\mu^-$	$5.8(8) \times 10^{-6}$	theory input for $(g-2)_\mu$, BSM weak decays, P/CP violation
$\eta \to \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
$\eta \to \pi^+\pi^-e^+e^-$	$2.68(11) \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \to \pi^+\pi^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \to e^+e^-e^+e^-$	$2.40(22) \times 10^{-5}$	theory input for $(g-2)_\mu$
$\eta \to e^+e^-\mu^+\mu^-$	$< 1.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \to \mu^+\mu^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \to \pi^+\pi^-\pi^0\gamma$	$< 5 \times 10^{-4}$	direct emission only
$\eta \to \pi^\pm e^\mp \nu_e$	$< 1.7 \times 10^{-4}$	second-class current
$\eta \to \pi^+\pi^-$	$< 4.4 \times 10^{-6}$ [55]	P/CP violation
$\eta \to 2\pi^0$	$< 3.5 \times 10^{-4}$	P/CP violation
$\eta \to 4\pi^0$	$< 6.9 \times 10^{-7}$	P/CP violation

- Standard Model tests
  - WZW/chiral anomaly
  - quark masses $m_u - m_d$
  - theory input for $(g-2)_\mu$
  - scalar resonance dynamics $f_0(500)$, $a_0(980)$
- BSM physics
  - new sources of *P*-/*C*- and *CP*-violation
  - dark photons, protophobic X-, leptophobic $U(1)_B$ bosons
  - light Higgs-like scalars
  - axion-like particles (ALPs)

Gan, BK, Passemar, Tulin 2020

Patterns of discrete symmetry breaking

Class	Violated	Conserved	Interaction
0		C, P, T, CP, CT, PT, CPT	strong, electromagnetic
I	C, P, CT, PT	T, CP, CPT	(weak, with no KM phase or flavor-mixing)
II	P, T, CP, CT	C, PT, CPT	
III	C, T, PT, CP	P, CT, CPT	
IV	C, P, T, CP, CT, PT	CPT	weak

- class II: *P*-, *CP*-violation
  - ▷ QCD θ-term; in general: electric dipole moments
  - ▷ $\eta^{(\prime)}$ decay examples: $\eta^{(\prime)} \to 2\pi$, $\eta^{(\prime)} \to \pi^+\pi^-\gamma^{(*)}$
- class III: *C*-, *CP*-violation
  - ▷ far less discussed; in SMEFT, start at dimension 8 only
  - ▷ $\eta^{(\prime)}$ decay examples: $\eta^{(\prime)} \to 3\gamma$, $\eta^{(\prime)} \to \pi^0\gamma^* \dots$

The paradigm: θ-term and $\eta \to \pi\pi$

- $\eta \to \pi\pi$: class II, *P*-, *CP*-violating
  ($\pi\pi$ *S*-wave: $J^{PC} = 0^{++}$, η: $J^{PC} = 0^{-+}$)
- θ-term induces such a decay:

$$\mathcal{B}(\eta \to \pi^+\pi^-) = \frac{\bar{g}_{\eta\pi\pi}^2}{16\pi M_\eta \Gamma_\eta}\sqrt{1 - \frac{4M_{\pi^\pm}^2}{M_\eta^2}}, \qquad \bar{g}_{\eta\pi\pi} = \frac{2\bar{\theta}M_\pi^2 m_u m_d}{\sqrt{3}F_\pi(m_u + m_d)^2}$$

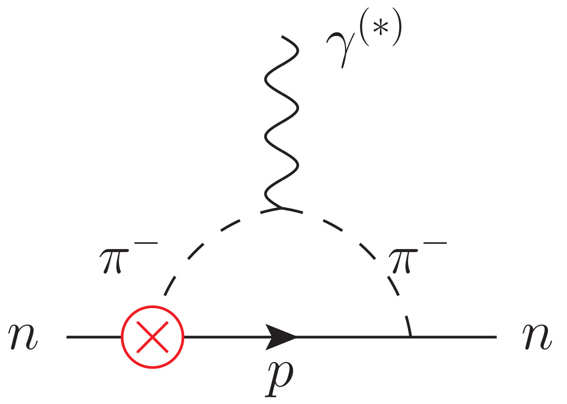
Crewther et al. 1979, Pich, de Rafael 1991

- experimental limit $\mathcal{B}(\eta \to \pi^+\pi^-) < 4.4 \times 10^{-6}$ KLOE 2020
  implies $\bar{\theta} < 4 \times 10^{-4}$
- problem: neutron EDM constraint much stronger, $|\bar{\theta}| \lesssim 10^{-10}$
  $\eta \to \pi\pi$ via θ-term suppressed beyond all experimental reach
- what if it's not the θ-term, but quark(-chromo) EDMs, gluon-chromo EDMs, four-quark operators. . . that cause strong CPV?
  $\longrightarrow$ can't $\eta \to \pi\pi$ still be there?

Reversing the argument: $\eta \to \pi\pi$ always induce EDMs!

Gan et al. 2020; cf. also Gorchtein 2008, Gutsche et al. 2017...

- θ-term induces nEDM via
  CPV pion–nucleon coupling



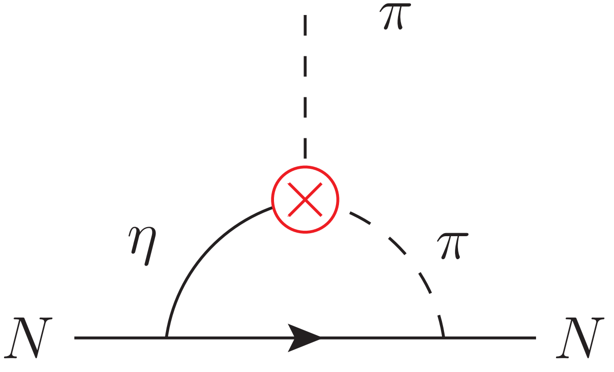
- any $\bar{g}_{\eta\pi\pi}$, no matter what source, induces CPV πN coupling at 1-loop:

$$d_n \approx 7 \times 10^{-16} \left(\frac{\bar{g}_{\eta\pi\pi}}{\text{GeV}}\right) e\,\text{cm}$$

- resulting constraint, independent of θ-term:

$$\mathcal{B}(\eta \to \pi^+\pi^-) < 2 \times 10^{-17}$$

  and similar for $\eta' \to \pi\pi$



- perspectives to observe $\eta^{(\prime)} \to \pi\pi$: pretty bleak

CP-violation in $\eta \to \pi^+\pi^-\gamma^{(*)}$?

- *CP*-violation: interference of magnetic and electric transitions:

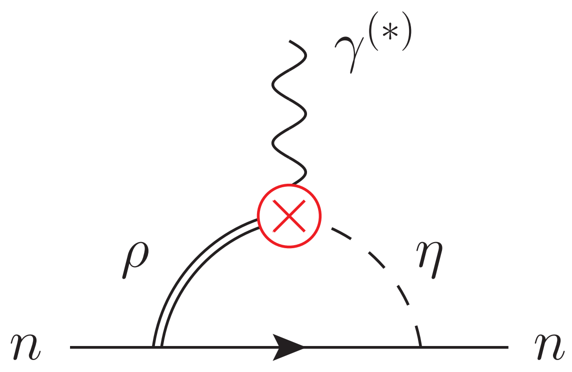
$$\begin{aligned}\mathcal{A}(\eta \to \pi^+\pi^-\gamma^{(*)}) &= M\, \epsilon_{\mu\nu\alpha\beta} p_+^\mu p_-^\nu k^\alpha \epsilon^\beta \\ &+ E\, \big((\epsilon \cdot p_+)(k \cdot p_-) - (\epsilon \cdot p_-)(k \cdot p_+)\big)\end{aligned}$$

  e.g., asymmetry in angle between $\pi^+\pi^-$ and $\ell^+\ell^-$ decay planes

- Standard Model: $M \propto e/F_\pi^3$ (chiral anomaly), $E = 0$
- E generated by four-quark operator Geng et al. 2002; Gao 2002

$$\propto \underbrace{\bar{s}\, i\sigma_{\mu\nu}\gamma_5 (p-k)^\nu s}_{\eta\to\gamma}\ \underbrace{\bar{q}\gamma^\mu q}_{\to\pi^+\pi^-}$$

- automatically generates
  effective CPV $\eta\gamma\rho$ vertex
  $\longrightarrow$ neutron EDM at one loop
  $\longrightarrow$ order-of-magnitude $E/M \lesssim 10^{-11}$
  $\longrightarrow$ not observable anytime soon



Gan, BK, Passemar, Tulin 2020

A loophole: scalar quark–lepton operators

- new class of *CP*-tests in Sánchez-Puertas 2019

$$\eta \to \mu^+\mu^- \,, \quad \eta \to \mu^+\mu^-\gamma \,, \quad \eta \to \mu^+\mu^- e^+ e^-$$

CP-odd observables for the first two require muon polarisation

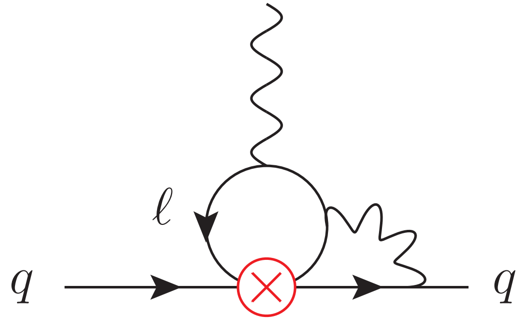
- *CP*-odd $\eta\gamma^*\gamma^*$ couplings ruled out as before
- quark–lepton four-fermion operators (scalar–pseudoscalar):

$$\mathcal{L}_{\text{eff}} = \frac{1}{2v^2} \operatorname{Im} c_{\ell edq}^{2222} \Big[(\bar{\mu}\mu)\big(\bar{s}i\gamma^5 s\big) - \big(\bar{\mu}i\gamma^5\mu\big)(\bar{s}s) \Big] + [u\text{-},\ d\text{-quarks}]$$

- EDMs only generated at two loops

constraint for strange quarks weakest:

$$|\operatorname{Im} c_{\ell edq}^{2222}| < 0.04$$



- asymmetries in $\eta \to \mu^+\mu^-$ may be within reach at REDTOP
- testing asymmetries in $\eta \to \pi^+\pi^-\mu^+\mu^-$ Zillinger, BK, Sánchez-Puertas

C- and CP-violation

- $\eta^{(\prime)}$ are $C = +1$ eigenstates: opportunity to test C-violation!

Channel	Branching ratio	Note
$\eta \to 3\gamma$	$< 1.6 \times 10^{-5}$	
$\eta \to \pi^0\gamma$	$< 9 \times 10^{-5}$	Violates angular momentum conservation or gauge invariance
$\eta \to \pi^0 e^+ e^-$	$< 7.5 \times 10^{-6}$	C, CP-violating as single-γ process
$\eta \to \pi^0 \mu^+ \mu^-$	$< 5 \times 10^{-6}$	C, CP-violating as single-γ process
$\eta \to 2\pi^0\gamma$	$< 5 \times 10^{-4}$	
$\eta \to 3\pi^0\gamma$	$< 6 \times 10^{-5}$	

- example ops.: Khriplovich 1991; Ramsey-Musolf 1999; Kurylov et al. 2001

$$\frac{1}{\Lambda^3}\bar{\psi}_f \gamma_5 D_\mu \psi_f \, \bar{\psi}_{f'} \gamma^\mu \gamma_5 \psi_{f'} + \text{h.c.}\,, \qquad \frac{1}{\Lambda^3}\bar{\psi}_f \sigma_{\mu\nu} \psi_f F^{\mu\lambda} Z^\nu_\lambda$$

$\longrightarrow$ require helicity flip, actually dimension-8 in SMEFT

- electroweak radiative corrections mix class II and class III
  still weaker EDM constraints

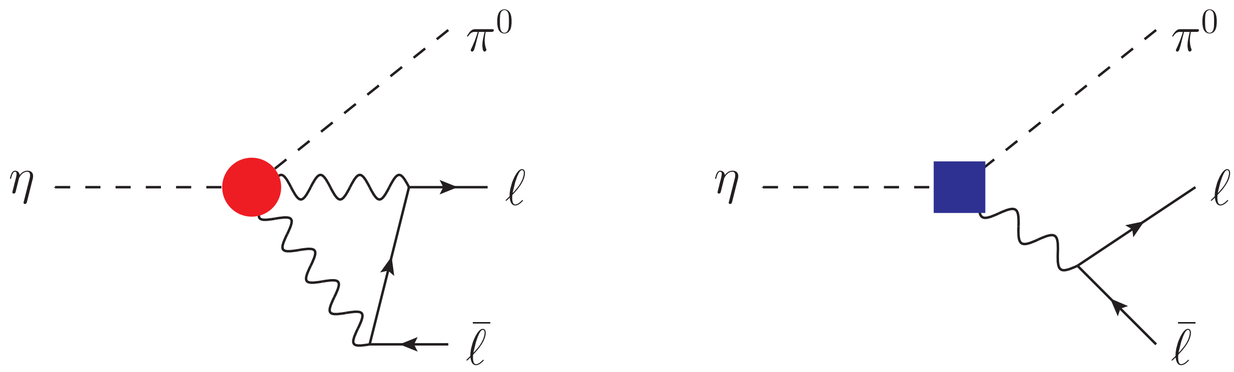
Example: $\eta \to \pi^0 \ell^+ \ell^-$

- gauge invariance: $\eta(p) \to \pi^0(k)\gamma(q)$ vanishes for real photon:

$$\langle \pi^0(k)|j_\mu(0)|\eta(p)\rangle = e\big[q^2(p+k)_\mu - (M_\eta^2 - M_\pi^2)q_\mu\big]F_{\eta\pi^0}(q^2)$$

$\longrightarrow$ can only measure dilepton decays

- *C*-even two-photon decay as Standard Model background:



$$\mathcal{B}(\eta \to \pi^0 e^+ e^-) = 2.1(5)\times 10^{-9}\,, \quad \mathcal{B}(\eta \to \pi^0 \mu^+ \mu^-) = 1.2(3)\times 10^{-9}$$

based on VMD model for $\eta \to \pi^0\gamma\gamma$ Escribano, Royo 2020

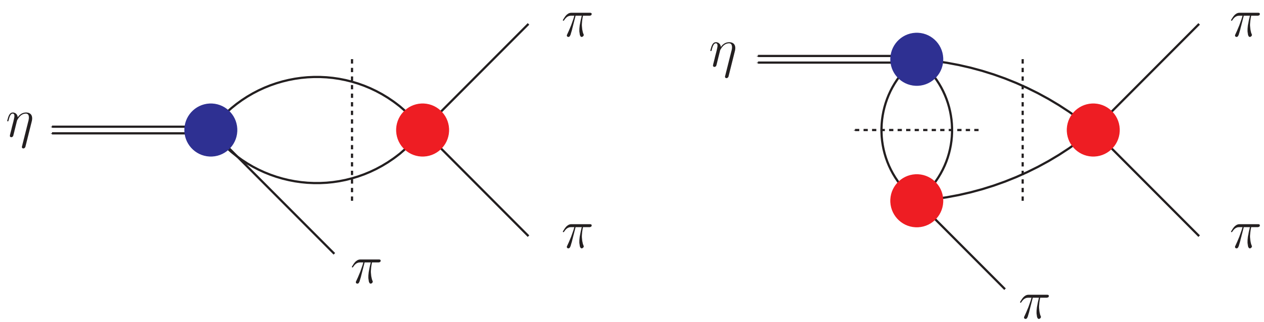
$\longrightarrow$ 3 orders of magnitude below current experimental limits

A new old proposal: Dalitz plot asymmetries

- $\eta(I^G = 0^+) \to 3\pi(I^G = 1^-)$ breaks *G*-parity:
  - ▷ SM: *C* conserved, isospin broken (& el.magn. suppressed)
    $\longrightarrow$ ideal process to extract $m_u - m_d$
    see e.g. Colangelo, Lanz, Leutwyler, Passemar 2018 and refs. therein
  - ▷ BSM: *C* broken, isospin either conserved or broken

$$\mathcal{M}(s,t,u) = \mathcal{M}_1^{C}(s,t,u) + \mathcal{M}_0^{\not{C}}(s,t,u) + \mathcal{M}_2^{\not{C}}(s,t,u)$$

- interference: $\pi^+ \leftrightarrow \pi^-$ asymmetries linear in BSM couplings
  Gardner, Shi 2019
- follow SM strategy for hadronic amplitudes: Akdag, Isken, BK 2021
  analyse $\mathcal{M}_{0,2}^{\not{C}}(s,t,u)$ using dispersive Khuri–Treiman framework



$\eta \to \pi^+\pi^-\pi^0$: amplitude decomposition

- Bose symm.: even (odd) $\pi\pi$ isospin $\leftrightarrow$ even (odd) partial waves
- "reconstruction theorem": symmetrised partial-wave expansion

$$\mathcal{M}_1^{C}(s,t,u) = \mathcal{F}_0(s) + (s-u)\,\mathcal{F}_1(t) + (s-t)\,\mathcal{F}_1(u) + \mathcal{F}_2(t) + \mathcal{F}_2(u) - \frac{2}{3}\mathcal{F}_2(s)$$

$$\mathcal{M}_0^{\not C}(s,t,u) = (t-u)\,\mathcal{G}_1(s) + (u-s)\,\mathcal{G}_1(t) + (s-t)\,\mathcal{G}_1(u)$$

$$\mathcal{M}_2^{\not C}(s,t,u) = 2(u-t)\,\mathcal{H}_1(s) + (u-s)\,\mathcal{H}_1(t) + (s-t)\,\mathcal{H}_1(u) - \mathcal{H}_2(t) + \mathcal{H}_2(u)$$

$\longrightarrow$ rescattering for *S*- and *P*-waves

- note: *C*-even/odd $\leftrightarrow$ even/odd under $t \leftrightarrow u$
- inhomogeneous Omnès solutions ($\mathcal{A}_I = \mathcal{F}_I, \mathcal{G}_I, \mathcal{H}_I$):

$$\mathcal{A}_I(s) = \Omega_I(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{dx}{x^n}\,\frac{\sin\delta_I(x)\,\hat{\mathcal{A}}_I(x)}{|\Omega_I(x)|\,(x-s)}\right)$$

  - ▷ Omnès fns. $\Omega_I(s)$ given in terms of known phase shifts $\delta_I(s)$
  - ▷ $P_{n-1}(s)$: subtraction polynomial, free parameters

$\eta \to \pi^+\pi^-\pi^0$: parameters, data

SM amplitude $\mathcal{M}_1^C$

- minimal subtraction scheme: 3 (real) constants
- "data" fit to
  - ▷ KLOE Dalitz plot $\eta \to \pi^+\pi^-\pi^0$ — KLOE 2016
  - ▷ A2 Dalitz plot $\eta \to 3\pi^0$ — A2 2018
  - ▷ chiral constraints [at $\mathcal{O}(p^4)$] — Colangelo et al. 2018

$\longrightarrow$ χ^2/dof ≈ 1.054, works very well!

BSM amplitude $\mathcal{M}_1^C + \mathcal{M}_0^{\not C} + \mathcal{M}_2^{\not C}$

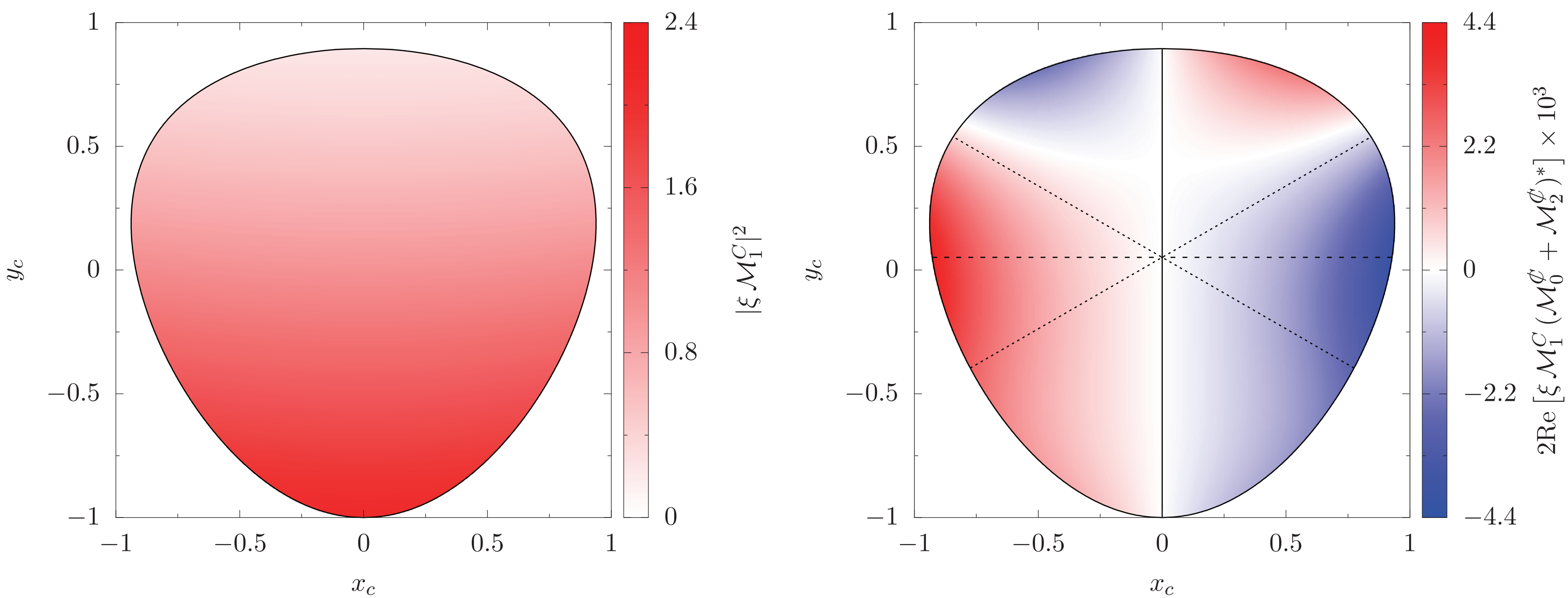
- by same assumptions: 1 complex subtraction each for $\mathcal{M}_{0,2}^{\not C}$

  act as overall normalisation constants $\longrightarrow$ χ^2/dof ≈ 1.048
- all *C*-/*CP*-violating signals vanish within $(1-2)\sigma$

$\eta \to \pi^+\pi^-\pi^0$: Dalitz plot asymmetries

- Dalitz plot decomposition (central fit result)

$$\left|\mathcal{M}_c\right|^2 \approx \left|\mathcal{M}_1^{C}\right|^2 + 2\mathrm{Re}\left[\mathcal{M}_1^{C}\left(\mathcal{M}_0^{\not C}\right)^*\right] + 2\mathrm{Re}\left[\mathcal{M}_1^{C}\left(\mathcal{M}_2^{\not C}\right)^*\right]$$



- asymmetries constrained to the permille level

$\eta \to \pi^+\pi^-\pi^0$: Dalitz plot asymmetries

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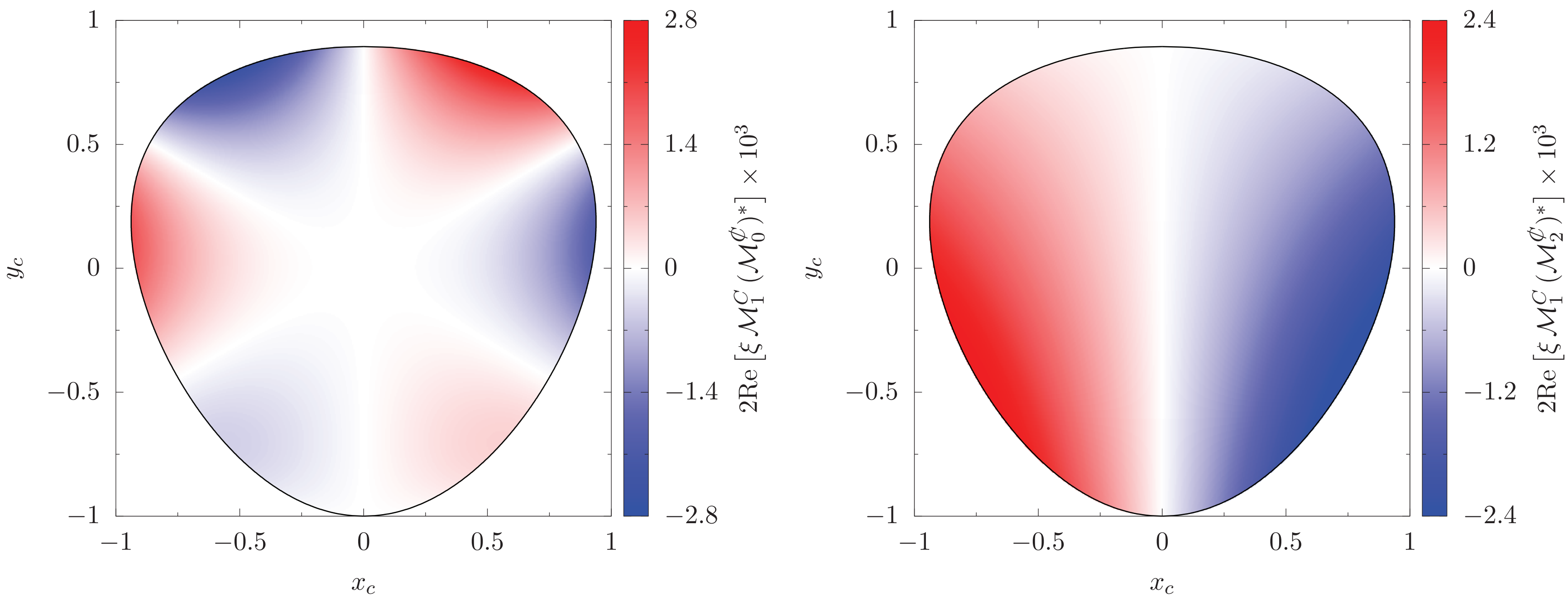


- asymmetries constrained to the permille level
- $\mathcal{M}_0^{\not C}$ and $\mathcal{M}_2^{\not C}$ lead to different interference patterns

Effective BSM couplings

Akdag, Isken, BK 2021

- subtractions no good observables $\longrightarrow$ polynomial ambiguities
- define unambigous Taylor invariants & match to these:

$$\mathcal{M}_0^{\not{C}}(s,t,u) = g_0\,(s-t)(u-s)(t-u) + \mathcal{O}(p^8)$$
$$\mathcal{M}_2^{\not{C}}(s,t,u) = g_2\,(t-u) + \mathcal{O}(p^4)$$

- fit corresponds to

$$g_0 = \big[-3(4)+7(13)i\big]\,\mathrm{GeV}^{-6}\,, \quad g_2 = \big[-1(15)+7(42)i\big]10^{-3}\,\mathrm{GeV}^{-2}$$

$\longrightarrow$ sensitivity $|g_0/g_2| \sim 10^3\,\mathrm{GeV}^{-4} = \mathcal{O}(M_\pi^{-4})$

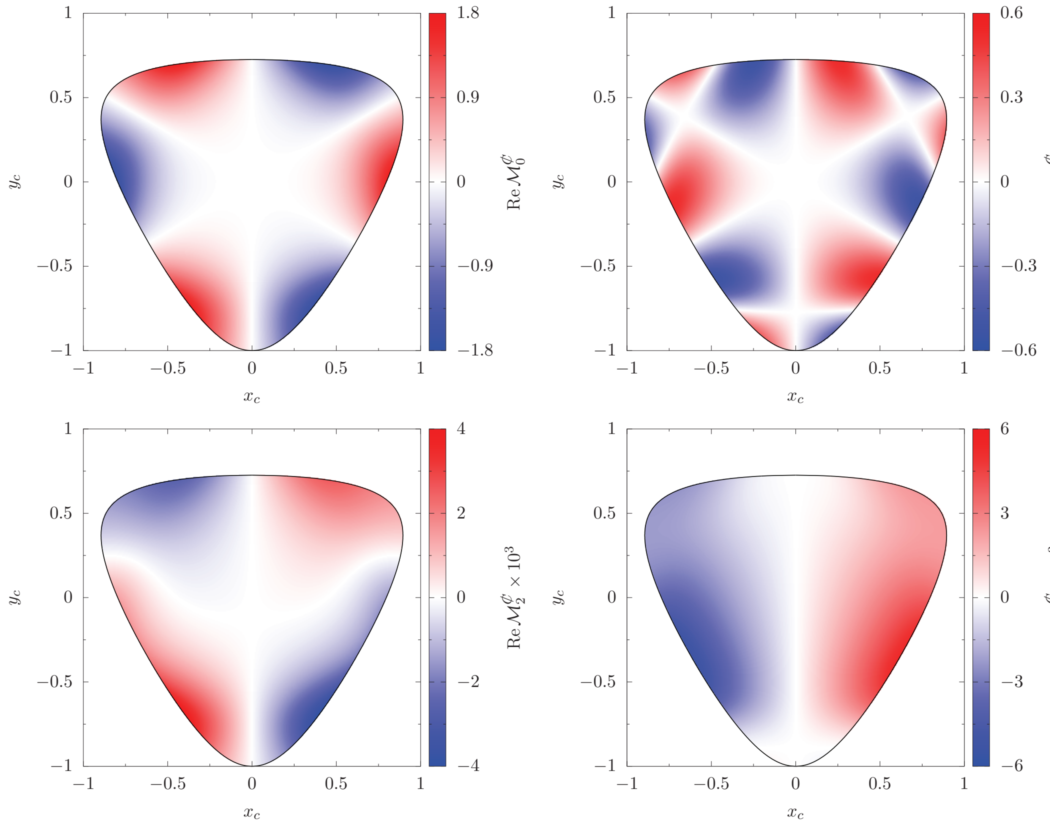
$\longrightarrow$ theoretical/chiral expectation: $|g_0/g_2| \sim \mathrm{GeV}^{-4}$

- small phase space ($M_\eta - 3M_\pi \sim M_\pi$) reduces sensitivity to $\mathcal{M}_0^{\not{C}}$

Generalisation to η' decays

$\eta' \to \pi^+\pi^-\pi^0$

- rather rare, $\mathcal{B} \sim 3.6 \times 10^{-3} \longrightarrow$ data not so precise BESIII 2016
- rescale $\eta \to \pi^+\pi^-\pi^0$ with same $g_{0,2} \longrightarrow$ more sensitive to g_0 by factor ~ 100



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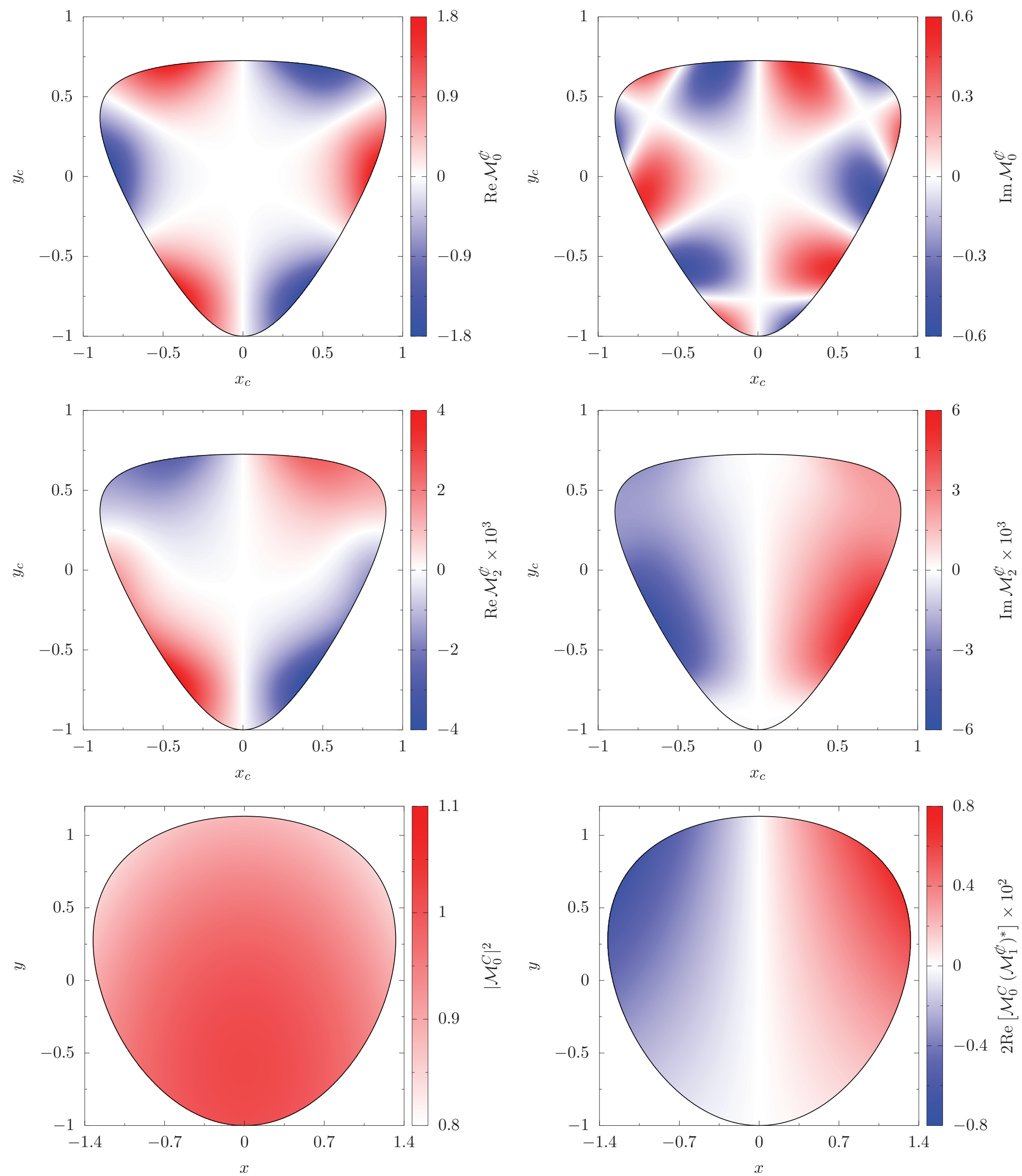


$\eta' \to \eta\pi^+\pi^-$

- SM conserves isospin
- *C*-odd op. $\Delta I = 1 \longrightarrow$ constrained at 10^{-2} BESIII 2017

Summary & Outlook

Class II, *P*- and *CP*-violation

- strong EDM bounds constrain almost all candidate decays beyond experimental reach
- exception: strange-quark–muon dimension-6 operators may be testable in $\eta \to \mu^+\mu^-$

Class III, *C*- and *CP*-violation

- very little theory work done to date
- dispersive formalism for amplitude analysis of Dalitz plots $\eta^{(\prime)} \to \pi^+\pi^-\pi^0$, $\eta' \to \eta\pi^+\pi^-$ established
- matching to fundamental (SMEFT) operators to be done

Experiments

- JLab Eta Factory (JEF)
- Rare Eta Decays with a TPC for Optical Photons (REDTOP)

What else? — highlights in η and η' physics

Our (personal) recommended highlights selection:

Decay channel	Standard Model	Discrete symmetries	Light BSM particles
$\eta \to \pi^+\pi^-\pi^0$	light quark masses	C/CP violation	scalar bosons (also η')
$\eta^{(\prime)} \to \gamma\gamma$	η–η' mixing, precision partial widths		
$\eta^{(\prime)} \to \ell^+\ell^-\gamma$	$(g-2)_\mu$		Z' bosons, dark photon
$\eta \to \pi^0\gamma\gamma$	higher-order χPT, scalar dynamics		$U(1)_B$ boson, scalar bosons
$\eta^{(\prime)} \to \mu^+\mu^-$	$(g-2)_\mu$, precision tests	CP violation	
$\eta \to \pi^0\ell^+\ell^-$		C violation	scalar bosons
$\eta^{(\prime)} \to \pi^+\pi^-\ell^+\ell^-$	$(g-2)_\mu$		ALPs, dark photon
$\eta^{(\prime)} \to \pi^0\pi^0\ell^+\ell^-$		C violation	ALPs

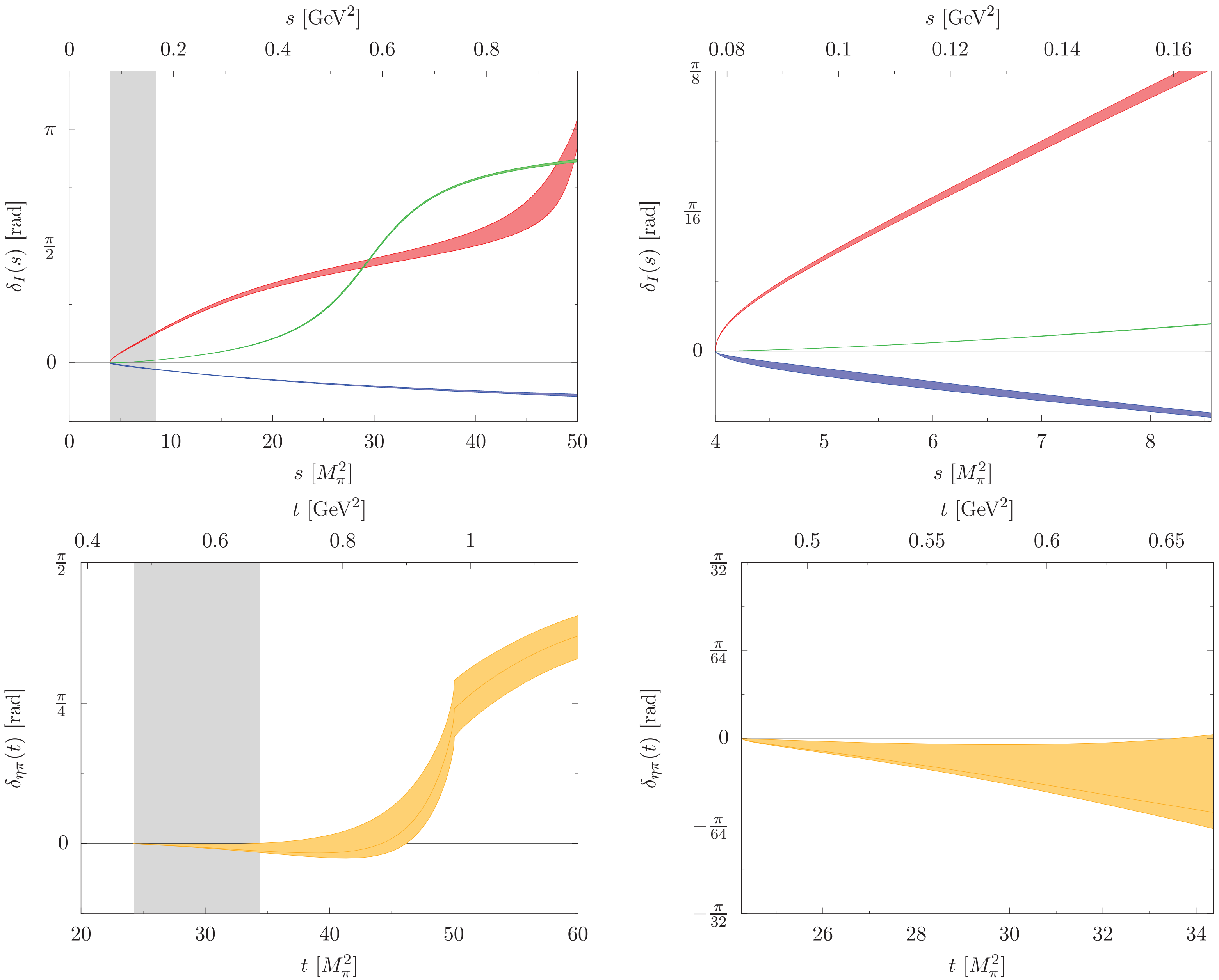
$\longrightarrow$ decay channels that allow for synergies between

- Standard Model precision analyses
- discrete symmetry tests
- searches for light BSM particles

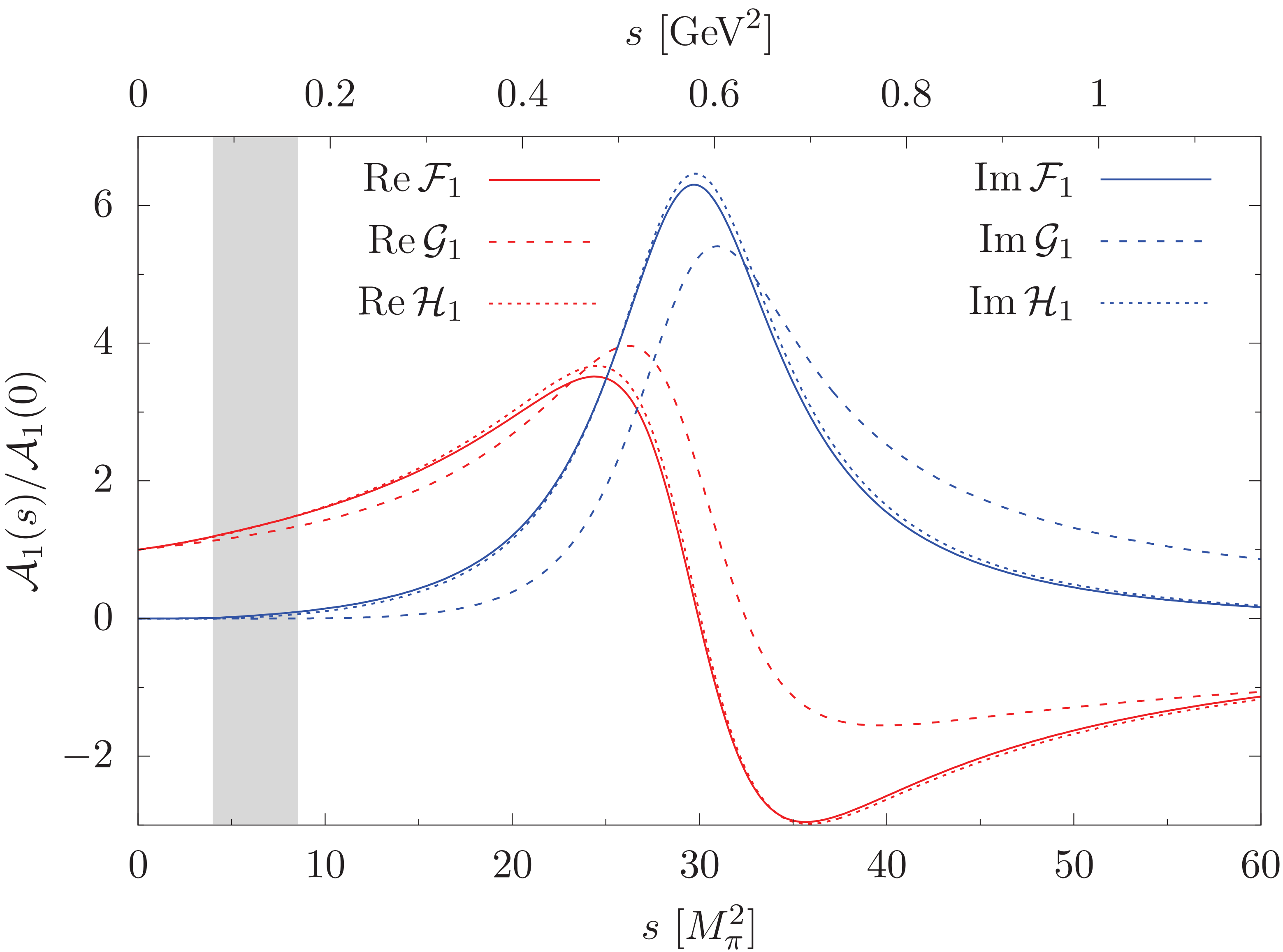
Gan, BK, Passemar, Tulin 2020

Spares

Phase shift input

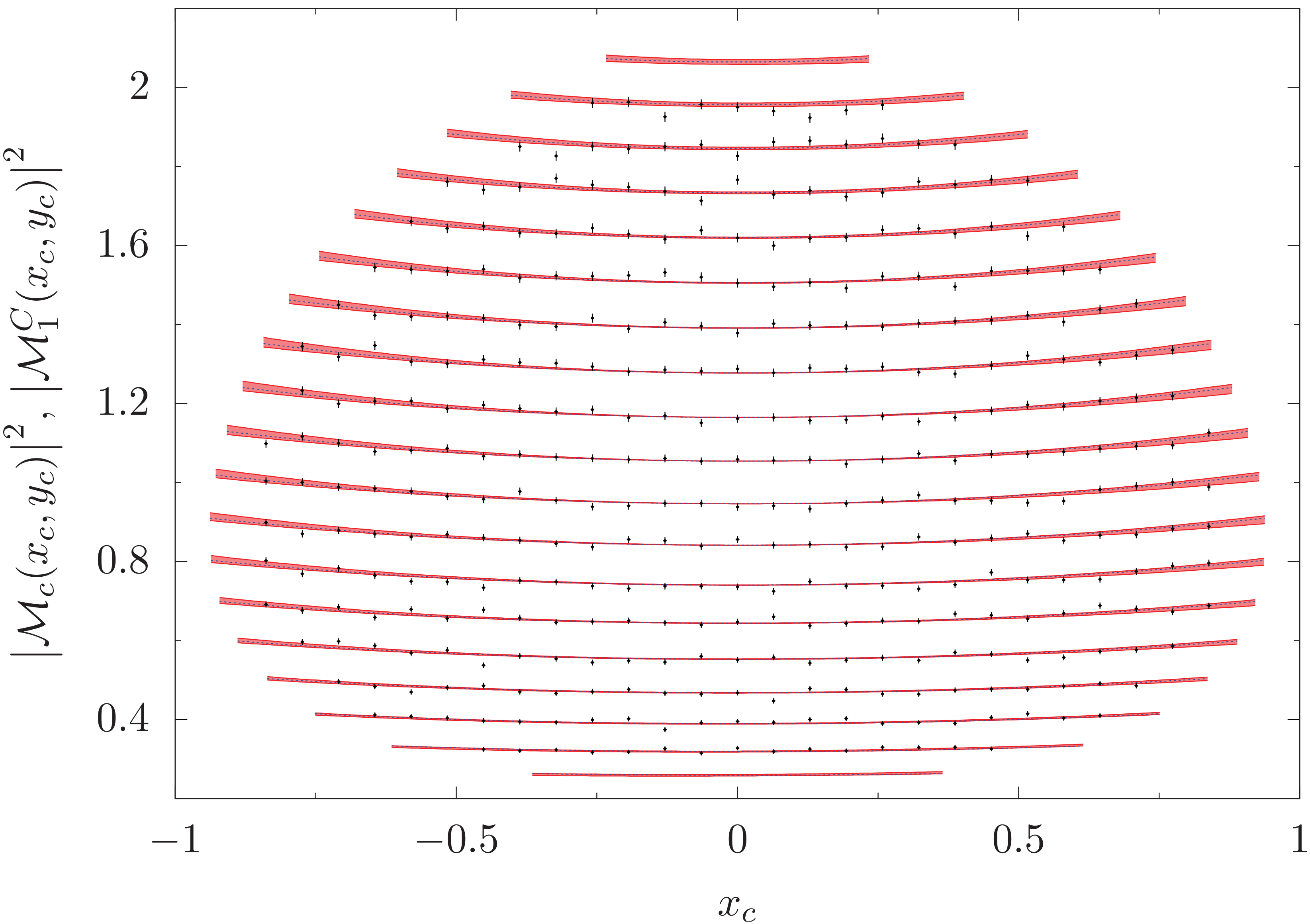


Not all *P*-waves are equal!



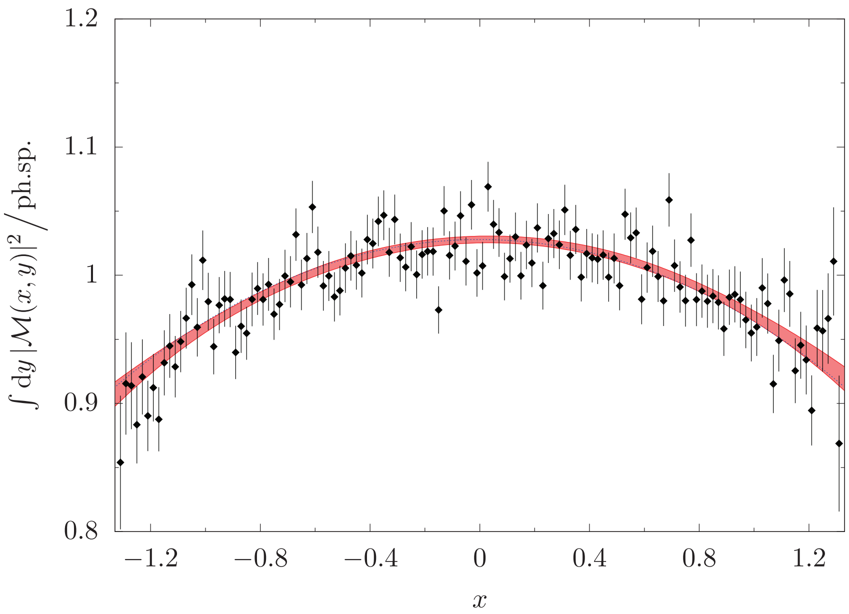
Akdag, Isken, BK 2021

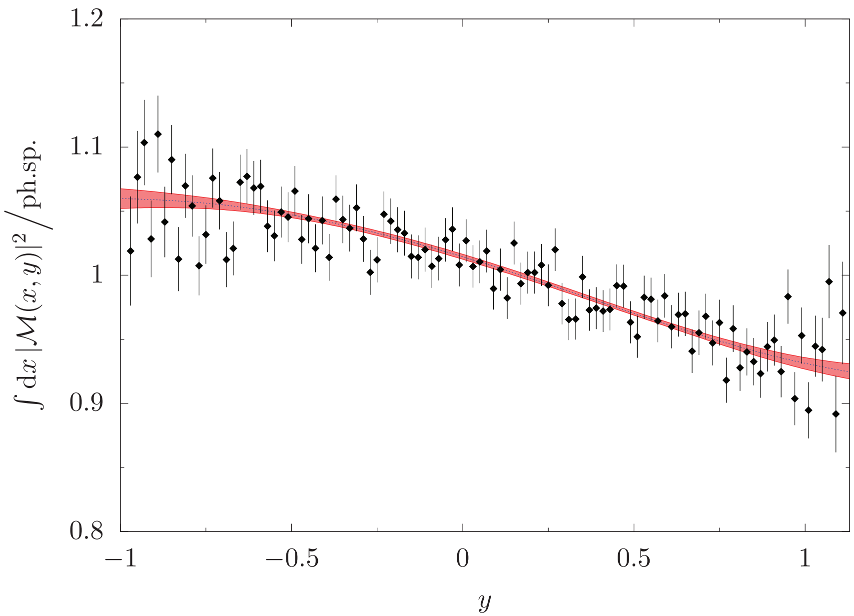
Fit results KLOE Dalitz plot $\eta \to \pi^+\pi^-\pi^0$



KLOE2016 vs. Akdag, Isken, BK 2021

Fit results BESIII (projected) Dalitz plot $\eta' \to \eta\pi^+\pi^-$





BESIII 2017 vs. Akdag, Isken, BK 2021