$N = 1$ Linearized Supergravities in Various Dimensions

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Plan of the Talk

- Motivation
- $N = 1$ Supergravities in various dimensions
- BRST construction: Free Lagrangians. Supersymmetry
- BRST construction: Cubic Interactions. Supersymmetry.
- Open problems

Based on

- D.Sorokin, M.T.,
  Nucl. Phys. **B 929**, 216, (2018), arXiv: 1801.04615
- I.L.Buchbinder, V.A.Krykhtin, M.T., D.Weissman,
  Nucl. Phys. **B 967**, 115427, (2021); arXiv: 2103.08231

- The ultimate goal is to consider interactions between Higher Spin $s > 2$ fields.
- Understanding properties of Higher Spin fields can provide further insights into Holography, String Theory, Quantum Gravity, Cosmology
- Supersymmetric Higher Spin theories are very interesting but relatively less explored, especially in higher dimensions
- We use linearized Supergravities as models to be generalized for Higher Spins.
- To reproduce the cubic vertices in SUGRAs we use a particular formulation of Open Superstring Field Theory as a hint

$N = 1$ Supergravities

- $N = 1$ SUSY algebra without central charges

$$\{Q_a, Q_b\} = (\gamma^\mu)_{ab} P_\mu$$

- SUGRA multiplets are irreducible representations of this algebra. They contain maximal spin 2 (graviton)
- In $D = 4$: graviton $g_{\mu\nu}$, gravitino $\tilde{\Psi}^a_\mu$
- In $D = 6$: graviton $g_{\mu\nu}$, self-dual two form $B^+_{\mu\nu}$, and gravitino $\tilde{\Psi}^a_\mu$
- In $D = 10$: graviton $g_{\mu\nu}$, two form $B_{\mu\nu}$, dilaton φ, gravitino $\tilde{\Psi}^a_\mu$, dilatino Ξ_a

Lower spin supermultiplets

- In $D = 4$: complex scalar ϕ, fermion Ξ_a
- In $D = 6$: anti self-dual two form $B^-_{\mu\nu}$, fermion Ξ_a

- $D = 10$ $N = 1$ Lagrangian

$$
\begin{aligned}
L &= -\frac{1}{2}R - \frac{1}{2}\bar{\tilde{\Psi}}_\mu \gamma^{\mu\nu\rho}\mathcal{D}_\nu \tilde{\Psi}_\rho - \frac{3}{4}\phi^{-\frac{3}{2}} H^{\mu\nu\rho} H_{\mu\nu\rho} - \\
&- \frac{1}{2}\bar{\Xi}\gamma^\mu \mathcal{D}_\mu \Xi - \frac{9}{16}\frac{\partial^\mu\phi\,\partial_\mu\phi}{\phi^2} - \frac{3\sqrt{2}}{8}\bar{\tilde{\Psi}}_\mu \gamma^\nu\gamma^\mu \Xi\,\frac{\partial_\nu\phi}{\phi} + \\
&+ \frac{\sqrt{2}}{16}\phi^{-\frac{3}{4}} H_{\nu\rho\tau}(\bar{\tilde{\Psi}}_\mu \gamma^{\mu\nu\rho\tau\lambda}\tilde{\Psi}_\lambda + 6\bar{\tilde{\Psi}}^\nu \gamma^\rho \tilde{\Psi}^\tau - \sqrt{2}\bar{\tilde{\Psi}}_\mu \gamma^{\nu\rho\tau}\gamma^\mu \Xi) + \\
&+ (fermion)^4
\end{aligned}
$$

  where $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$ and $\mathcal{D}_\mu$ is a covariant derivative
- We linearize around the flat background

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

  and consider a cubic Lagrangian with global SUSY

Free action for $s=2$ and $s=0$

- A gauge invariant Klein-Gordon equation

$$\Box G_{\mu\nu}(x) = \partial_\mu C_\nu(x) + \partial_\nu C_\mu(x)$$

with an extra field $C_\mu(x)$. Gauge transformations are

$$\delta G_{\mu\nu}(x) = \partial_\mu \lambda_\nu(x) + \partial_\nu \lambda_\mu(x), \quad \delta C_\mu(x) = \Box \lambda_\mu(x)$$

- A gauge invariant transversality equation

$$\partial^\nu G_{\mu\nu}(x) - \partial_\mu D(x) = C_\mu(x)$$

with an extra field $D(x)$, with $\delta D(x) = \partial^\mu \lambda_\mu(x)$

- A gauge invariant field equation for $D(x)$ is

$$\Box D(x) = \partial^\mu C_\mu(x)$$

- The corresponding Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial^\mu G^{\nu\rho})(\partial_\mu G_{\nu\rho}) + 2C^\mu \partial^\nu G_{\mu\nu} - C^\mu C_\mu + (\partial^\mu D)(\partial_\mu D) + 2D\partial^\mu C_\mu$$

Describes spin 2 field $g_{\mu\nu}(x)$ and a scalar $\phi(x)$ contained in $G_{\mu\nu}(x)$

Free action for $s = 3/2$ and $s = 1/2$

- A spin-vector field $\Psi^a_\mu(x)$, where a is a spinorial index.
- Gauge invariant transversality condition

$$\partial^\mu \Psi_\mu(x) + \gamma^\nu \partial_\nu \chi(x) = 0$$

- Introduced an extra field $\chi^a(x)$

$$\delta\Psi_\mu(x) = \partial_\mu \tilde{\lambda}(x), \quad \delta\chi(x) = -\gamma^\nu \partial_\nu \tilde{\lambda}(x)$$

- The gauge invariant Dirac equation

$$\gamma^\nu \partial_\nu \Psi_\mu(x) + \partial_\mu \chi(x) = 0$$

- The equations are again Lagrangian

$$L_F = -i\bar{\Psi}^\nu \gamma^\mu \partial_\mu \Psi_\nu - i\bar{\Psi}^\mu \partial_\mu \chi + i\bar{\chi}\partial^\mu \Psi_\mu + i\bar{\chi}\gamma^\mu \partial_\mu \chi$$

- Describes spin $\frac{3}{2}$ field $\tilde{\Psi}^a(x)$ and spin $\frac{1}{2}$ field $\Xi_a(x)$ - gamma trace

- Use open superstring field theory as a hint It is possible to obtain the free systems above by taking a formal limit $\alpha' \to \infty$ in the free field equations for the Open Superstring
- Introduce an auxiliary Fock space

$$[\alpha_\mu, \alpha_\nu^+] = \eta_{\mu\nu}, \quad \{\psi_\mu, \psi_\nu^+\} = \eta_{\mu\nu}$$

- Introducing divergence, gradient and Dirac operators

$$l^\pm = p \cdot \alpha^\pm, \quad g^\pm = p \cdot \psi^\pm, \quad g_0 = p \cdot \gamma, \quad l_0 = g_0^2, \quad p_\mu = -i\partial_\mu$$

- Introduce (b, c) ghosts and (β, γ) ghosts with

$$\{b, c^+\} = \{b^+, c\} = \{b_0, c_0\} = 1, \quad [\gamma_0, \beta_0] = [\gamma, \beta^+] = [\gamma^+, \beta] = i$$

and construct nilpotent BRST charges

$$Q_B^2 = Q_F^2 = 0$$

- A Lagrangian (schematically)

$$L_{tot.} = \langle \Phi_B | Q_B | \Phi_B \rangle + \langle \Phi_F | Q_F | \Phi_F \rangle$$

- Invariant under supersymmetry transformations

$$\delta \langle \Phi_B | = \langle \Phi_F | \epsilon \mathcal{Q}, \qquad \delta | \Phi_F \rangle = \epsilon \mathcal{Q} | \Phi_B \rangle .$$

provided the SUSY generator $\mathcal{Q}$ satisfies

$$Q_F \, \mathcal{Q} = \mathcal{Q} \, Q_B$$

- Here: a solution (finite number of fields, no pictures)

$$\mathcal{Q} \;= {}_B\langle 0| \;\exp\left(\frac{1}{\sqrt{2}} \gamma \cdot \psi + (\beta\gamma \quad ghosts) \right) |0\rangle_F .$$

- SUSY closes on-shell in $D = 4, 6, 10$ in both sectors.
- Consideration in OSFT: Y.Kazama, A.Neveu, H.Nicolai, P.West, Nucl.Phys. **B 278**, 833 (1986). Contains an infinite number of oscillators and fields. Presence of pictures

Field Content

- Requirement that the fields $|\Phi_B\rangle$ and $|\Psi_F\rangle$ have zero ghost number fixes the field content
- The bosonic sector contains mixed symmetry field $\phi_{\mu,\nu}(x)$

$$
\begin{aligned}
|\Phi_B\rangle &= (\phi_{\mu,\nu}(x)\alpha^{\mu,+}\psi^{\nu,+} + A(x)c^+\beta^+ + B(x)\gamma^+ b^+ \\
&+ c_0 b^+ C_\mu(x)\psi^{\mu,+} + c_0\beta^+ E_\mu(x)\alpha^{\mu,+})|0\rangle_B.
\end{aligned}
$$

- The fermionic sector (a bit tricky)

$$
|\Psi_F\rangle = \left(\Psi_\mu(x)\alpha^{\mu,+} + (\gamma_0 + c_0 g_0)b^+\chi(x)\right)|0\rangle_F
$$

- The Lagrangians, gauge and SUSY transformations can be written in terms of these fields using explicit forms of Q_B, Q_F and $\mathcal{Q}$.
- Super-Maxwell (a comment)
  Bosons: $\phi_\mu(x)$, $E(x)$-auxiliary. Fermion $\Psi(x)$

- The Lagrangian in the bosonic sector ($\phi_{\mu,\nu}(x)$ is physical)

$$\begin{aligned} L_B \quad &= -\phi^{\mu,\nu}\Box\phi_{\mu,\nu} + B\Box A + A\Box B \\ &+E^\mu\partial_\mu B + C^\nu\partial^\mu\phi_{\nu,\mu} + C^\nu\partial_\nu A + E^\mu\partial^\nu\phi_{\nu,\mu} \\ &-B\partial_\mu E^\mu - \phi^{\nu,\mu}\partial_\mu C_\nu - A\partial_\mu C^\mu - \phi^{\mu,\nu}\partial_\mu E_\nu \\ &+C^\mu C_\mu + E^\mu E_\mu \,. \end{aligned}$$

- The Lagrangian in the fermionic sector ($\Psi^\mu(x)$ is physical)

$$L_F = -i\bar\Psi^\mu\gamma^\nu\partial_\nu\Psi_\mu - i\bar\Psi^\mu\partial_\mu\chi + i\bar\chi\partial_\mu\Psi^\mu + i\bar\chi\gamma^\nu\partial_\nu\chi,$$

- SUSY transformations

$$\delta\phi_{\nu,\mu}(x) = i\bar\Psi_\mu(x)\gamma_\nu\,\epsilon, \quad \delta C_\nu(x) = -i\,(\partial_\mu\bar\chi(x))\gamma^\mu\gamma_\nu\,\epsilon, \quad \delta B(x) = -i\,\bar\chi(x)\,\epsilon,$$

$$\delta\Psi_\mu(x) = -\gamma^\nu\gamma^\rho\epsilon\,\partial_\nu\phi_{\rho,\mu}(x) - \epsilon E_\mu(x), \quad \delta\chi(x) = -\gamma^\nu\epsilon\,C_\nu(x)\,.$$

- Taking three copies of these fields we get for a Lagrangian

$$\mathcal{L}_{3B} \sim \sum_{i=1}^{3} \langle \Phi_{(i)} | Q_{(i)} | \Phi_{(i)} \rangle + g \langle \Phi_{(3)} | \langle \Phi_{(2)} | \langle \Phi_{(1)} || V \rangle$$

- Nonlinear gauge transformations

$$\delta_{cub.} |\Phi_{(1)}\rangle \sim Q_{(1)} |\Lambda_{(1)}\rangle - g(\langle \Phi_{(2)} | \langle \Lambda_{(3)} | + \langle \Phi_{(3)} | \langle \Lambda_{(2)} |) | V \rangle)$$

- The invariance of $\mathcal{L}_{3B}$:

$$\begin{aligned} g^0 : &\qquad Q^2_{(1)} = Q^2_{(2)} = Q^2_{(3)} = 0 \\ g^1 : &\qquad (Q_{(1)} + Q_{(2)} + Q_{(3)}) | V \rangle = 0 \end{aligned}$$

- It is easier to consider only physical fields
- Impose an off-shell gauge fixing condition

$$p\cdot\alpha|\Phi_B\rangle = p\cdot\psi|\Phi_B\rangle = 0 \quad and \quad p\cdot\alpha|\Psi_F\rangle = 0$$

- Take two copies of the fermionic field and one copy of the bosonic field

$$\mathcal{L}_{FFB} \sim \sum_{i=1}^{2}\langle\Psi_{(i)}|p_{(i)}\cdot\gamma|\Psi_{(i)}\rangle + \langle\Phi_{(3)}|p_{(3)}\cdot p_{(3)}|\Phi_{(3)}\rangle + g\langle\Phi_{(3)}|\langle\Psi_{(2)}|\langle\Psi_{(1)}||\mathcal{V}\rangle$$

- Nonlinear gauge transformations

$$\begin{aligned}
\delta_{cub.}|\Psi_{(1)}\rangle &\sim Q_{(1)}|\Lambda_{(1)}\rangle - \\
&- g(\langle\Psi_{(2)}|\langle\Lambda_{(3)}||\mathcal{W}^1_{2,3}\rangle + \langle\Phi_{(3)}|\langle\Lambda_{(2)}|)|\mathcal{W}^1_{3,2}\rangle) \\
\delta_{cub.}|\Phi_{(3)}\rangle &\sim Q_{(3)}|\Lambda_{(3)}\rangle - \\
&- g(\langle\Psi_{(1)}|\langle\Lambda_{(2)}||\mathcal{W}^3_{1,2}\rangle + \langle\Psi_{(2)}|\langle\Lambda_{(1)}|)|\mathcal{W}^3_{2,1}\rangle)
\end{aligned}$$

- Requirement of invariance of the Lagrangian $\mathcal{L}_{FFB}$ and of the associativity imposes conditions on the vertices $|\mathcal{V}\rangle$ and $|\mathcal{W}^k_{ij}\rangle$

- The "universal" ones

$$|\mathcal{V}\rangle = \gamma \cdot \psi^{+}_{(3)} \mathcal{Z}_{\alpha} |0\rangle_{FFB}, \quad |V\rangle = \mathcal{Z}_{\psi} \mathcal{Z}_{\alpha} |0\rangle_{3B}$$

where

$$\mathcal{Z}_{\psi} = (\psi^{+}_{(1)} \cdot \psi^{+}_{(2)})(\psi^{+}_{(3)} \cdot (p_{(1)} - p_{(2)})) + cyclic$$

$$\mathcal{Z}_{\alpha} = (\alpha^{+}_{(1)} \cdot \alpha^{+}_{(2)})(\alpha^{+}_{(3)} \cdot (p_{(1)} - p_{(2)})) + cyclic$$

- In this gauge the supersymmetry transformations

$$\delta|\Phi_B\rangle = \bar{\epsilon}\,(\psi^{+} \cdot \gamma)|\Psi_F\rangle, \quad \delta|\Psi_F\rangle = -2(p \cdot \gamma)\,(\psi \cdot \gamma)\,\epsilon\,|\Phi_B\rangle$$

put the fields completely on-shell

- Super Yang-Mills (a comment)

$$|\mathcal{V}\rangle_{ABC} = f_{ABC}(\gamma \cdot \psi^{+}_{(3)})|0\rangle_{FFB}, \quad |V\rangle_{ABC} = f_{ABC} \mathcal{Z}_{\psi} |0\rangle_{3B}$$

- "Non-universal" ones, specific for $D = 10$, $N = 1$
- Split $|\Psi_F\rangle$ into irreducible modes $|\tilde{\Psi}\rangle$ and $|\Xi\rangle$
- We have interactions of the type

$$\langle\Phi_{(3)}|\, \langle\tilde{\Psi}_{(1)}|\, \langle\tilde{\Psi}_{(2)}||\mathcal{V}_{\mathcal{J}}\rangle, \quad \textit{and} \quad \langle\Phi_{(3)}|\, \langle\tilde{\Psi}_{(1)}|\, \langle\Xi_{(2)}||\mathcal{V}_{\mathcal{L}}\rangle,$$

with

$$\begin{aligned}
\mathcal{V}_{\mathcal{J}} &= \gamma_{\mu\tau\sigma\lambda\nu}\, \alpha^{\mu(1),+}\alpha^{\nu(2),+}p^{\tau,(3)}\alpha^{\sigma(3),+}\psi^{\lambda(3),+} + \\
&- [(\alpha^{(3),+}\cdot\gamma)((p^{(3)}\cdot\alpha^{(1),+})(\alpha^{(2),+}\cdot\psi^{(3),+})) - \\
&- (\psi^{(3),+}\cdot\gamma)((p^{(3)}\cdot\alpha^{(1),+})(\alpha^{(2),+}\cdot\alpha^{(3),+}) + \\
&+ (p^{(3)}\cdot\gamma)((\alpha^{(1),+}\cdot\psi^{(3),+})(\alpha^{(2),+}\cdot\alpha^{(3),+}) - \\
&- (\alpha^{(1),+}\leftrightarrow\alpha^{(2),+})]
\end{aligned}$$

$$\begin{aligned}
\mathcal{V}_{\mathcal{L}} &= \gamma_{\mu\nu\tau}(\alpha_1^{(1),+}\cdot\gamma)p^{\mu,(3)}\alpha^{\nu(3),+}\psi^{\tau(3),+} + \\
&+ (\alpha^{(3),+}\cdot\psi^{(3),+})(\alpha^{(1),+}\cdot p^{(3)})
\end{aligned}$$

Conclusions

- $D = 6$ $N = 1$ SUGRA with $(1, 0)$ tensor supermultiplet can be handled in a similar way
- One can promote the nonlinear solutions to completely unconstrained, off-shell systems
- Deformation to curved backgrounds
- Can be generalized for Higher Spins, at least some of the vertices
- Helpful for studies of higher order interactions
- Consider systems with massive fields
- Many other questions

THANK YOU!!!