Null Surface Thermodynamics

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Based on my recent and upcoming papers with
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Gravity & Thermodynamics

- Distinctive feature of gravity is its universality.

- Thermodynamics has a similar universality.

- These two universal theories seem to be deeply related:

  - Black holes [Carter-Bardeen-Hawking & Hawking, Bekenstein (early 1970's)], [Wald (1993,4)];

  - Accelerated observers see a thermal bath [Unruh (1976)];

  - Einstein equations from thermodynamics [Jacobson (1995)];

  - Gravity as entropic force [E. Verlide (2010)];

  - Holographic principle & AdS/CFT.

Boundary symmetries and d.o.f.

- Presence of boundaries in spacetime brings in boundary d.o.f.

- This boundary may be asymptotic boundary or any arbitrary codimension one surface in spacetime.

- For gauge or diff. inv. theories the boundary d.o.f. may be labeled by non-trivial gauge/diff. transf..

- These boundary/asymptotic d.of. have been proposed to be relevant to BH information problem [Hawking-Perry-Strominger (2016)].

- We discuss the relation between thermodynamic description of gravity & the boundary d.o.f.......

Outline

- Einstein GR and equivalence principle in presence of boundaries
- Null surfaces and boundaries as models for BH horizons
- Null boundary symmetries and charges, D dimensional example
- Null Surface Thermodynamics
- Summary and Outlook

Gauge theories in presence of boundaries

- Consider a gauge theory with generic fields Φ_α described by the action

$$S[\Phi_\alpha] = \int_{\mathcal{M}} d^D x \; \mathbf{L}(\Phi_\alpha)$$

where $\mathbf{L}$ is the Lagrangian which is a D-form.

- Φ_α belong to representation $\mathcal{R}_\alpha$ of the gauge Lie algebra $\mathcal{A}$,

$$\Phi_\alpha \to \tilde{\Phi}_\alpha = \mathcal{R}_\alpha \cdot \Phi_\alpha .$$

- In the above $\mathcal{R}_\alpha$ is a function over the spacetime and

$$S[\Phi_\alpha] = S[\tilde{\Phi}_\alpha]$$

- In gauge theories fields are defined up to gauge equivalence classes and physical observables are gauge invariant quantities.

- Gauge symmetry is in fact a redundancy of description which should be removed by gauge fixing, but yet, there may be nontrivial gauge transformations in presence of boundary $\partial\mathcal{M}$ in spacetime $\mathcal{M}$.

- In a different viewpoint, we may define our boundary/initial value problem by specifying the behavior of Φ_α at the boundary:

$$\boldsymbol{\Phi}_\alpha\Big|_{\partial\mathcal{M}} := \varphi_\alpha, \qquad \delta\boldsymbol{\Phi}_\alpha\Big|_{\partial\mathcal{M}} := \delta\varphi_\alpha$$

- φ_α may be non-invariant under a certain measure-zero subset of gauge transformations at $\partial\mathcal{M}$. These may be called boundary non-trivial, physical gauge transformations.

- boundary d.o.f may be labelled through φ_α.

- The nontrivial boundary gauge transformations are a handy and powerful method to identify and formulate boundary d.o.f without invoking addition of extra d.o.f by hand.

▶ As an example one may consider Maxwell theory in a box,

- Besides the photons in the box we have b.o.d.f.

- Their response to the EM fields in the box is the boundary currents.

- Boundary currents are specified, choosing boundary conditions.

- This gives a macroscopic formulation of b.d.o.f and fixes the boundary/bulk interactions.

■ Einstein GR and its local (gauge) symmetry

- Einstein GR is a generally (in/co)variant theory.
- Physical observables in the Einstein GR are all defined through local diffeomorphism invariant quantities.
- In particular, any two metric tensors related by diffeomorphisms are physically equivalent:

$$x^\mu \to x^\mu + \xi^\mu(x), \quad g_{\eta\nu} \to g_{\mu\nu} + \delta g_{\mu\nu},\ \delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

- We typically fix diffeomorphisms through choice of observers.

■ Einstein GR, generic structure of d.o.f & EoM

- In a D dimensional spacetime, metric has $D(D+1)/2$ components:

  $D(D-3)/2$ propagating gravitons,
  D diffeos.

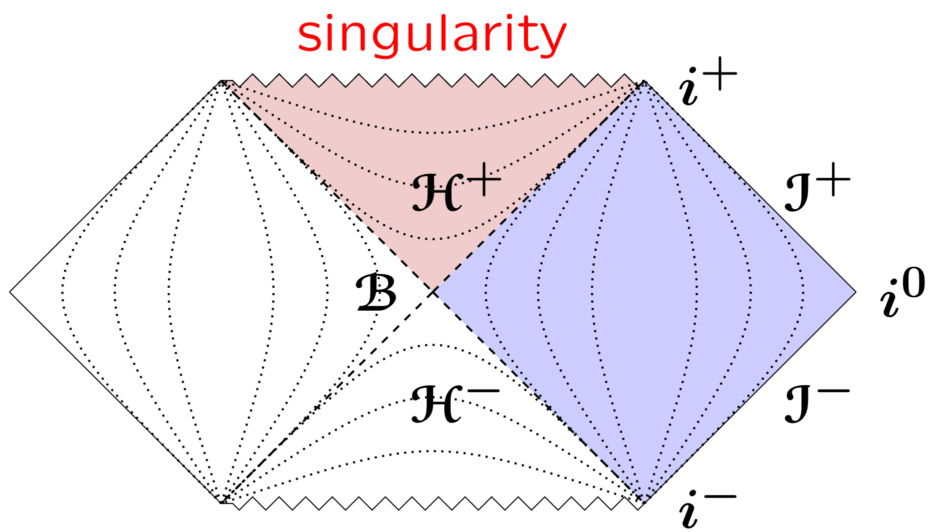
- Out of $D(D+1)/2$ field equations, $G_{\mu\nu} = 8\pi G T_{\mu\nu}$,

  $D(D-3)/2$ are second order diff.eq.,
  D constraints ($\nabla^\mu G_{\mu\nu} = 0$) and D first order equations.

- Solutions are fully specified specifies boundary and/or initial data, which in the most general case involves $2D$ functions over codimension one boundary.

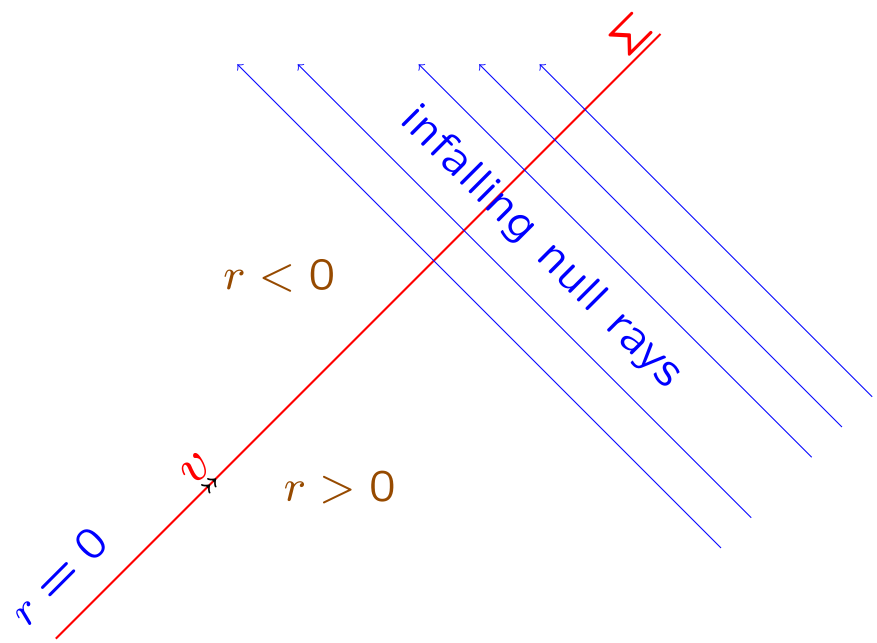
Null boundaries as models of horizons

- In a stationary black hole setup, horizon is the boundary of outside observers.



- Horizons are tpyically one way surfaces.

Depiction of a null surface



b.d.of. are residing on Σ.

b.d.o.f. interact with themselves and with infalling flux. Interaction with infalling flux is fixed by diff invariance (generalized Bondi news/balance equation).

- Motivated by problems in BHs, we choose Σ to be a null surface, sitting at $r = 0$:

$$ds^2 = -F dv^2 + 2\eta dr dv + h_{ij}(dx^i + g^i dv)(dx^j + g^j dv) \qquad (1)$$

F, g^i, h_{ij} are functions of r, v, x^i, $i = 1, 2, \cdots, D-2$ and $\eta = \eta(v, x^i)$,

$$g^{rr}\Big|_{r=0} = 0 \quad \Longrightarrow \quad (Fh + g^2)\Big|_{r=0} = 0,$$

where $h := \det h_{ij}, g^2 := h^{ij} g_i g_j$.

- We choose $r = 0$ to be the boundary of our spacetime and restrict ourselves to $r \geq 0$.

Solution space

- Metric (1) has $1 + 1 + (D-2) + (D-1)(D-2)/2$ functions in it.

- These may be decomposed into

  - three scalars $(F, h; \eta)$,
  - one vector g_i and
  - one symmetric-traceless tensor $H_{ij} := h_{ij}/h^{1/(D-2)}$,

  from the viewpoint of codimension two surface Σ_v, (constant v slice on Σ).

- These functions are subject to field equations, which here we take Einstein vacuum equations, determine their r dependence.

- The r dependence of the tensor mode H_{ij} is determined through $H_{ij}^{(0)}(v, x^i) := H_{ij}(r = 0; v, x^i)$, $H_{ij}^{\prime(0)}(v, x^i) := \partial_r H_{ij}(r = 0; v, x^i)$.

- The r dependence of the vector mode obeys first order eq. in r and is completely specified by $\mathcal{G}_i(v, x^i) := g_i(r = 0; v, x^i)$.

- Raychaudhuri equation + the condition that Σ is null, allows for solving F in terms of $\mathcal{G}_i, h, \eta$.

- The r dependence of the other two are determined in terms of $\eta := \eta(v, x^i)$, $\Omega(v, x^i) := \sqrt{h(r = 0; v, x^i)}$.

- Solution phase space is determined by
  - "Tensor modes" (gravitons) $H_{ij}^{(0)}, H_{ij}^{\prime(0)}$,
  - Vector mode $\mathcal{G}_i$,
  - Scalars modes Ω, η,

- These are respectively, $D(D-3)$, $D-2, 2$ functions of v, x^i.

- We have only assumed smoothness of metric at $r=0$,

- but no particular behavior (falloff condition), around $r=0$.

- The boundary $r = 0$ is not a special place in spacetime and can be any given (null) $D-1$ dimensional hypersurface.

- By construction there can't be any solution geometry which is smooth around $r = 0$ and is not in the form (1) and

$$\begin{aligned} F &= -\eta\left(\Gamma - \frac{2}{D-2}\frac{\mathcal{D}_v\Omega}{\Omega} + \frac{\mathcal{D}_v\eta}{\eta}\right) r + \mathcal{O}(r^2) \\ g^i &= \mathcal{G}^i - r\frac{\eta}{\Omega}\mathcal{J}^i + \mathcal{O}(r^2) \\ g_{ij} &= \Omega_{ij} + \mathcal{O}(r) \end{aligned} \tag{2}$$

  where all the fields are functions of v, x^i

$$\Omega_{ij} = \Omega^{2/(D-2)}\gamma_{ij}, \qquad \Omega := \sqrt{\det \Omega_{ij}}, \qquad \det \gamma_{ij} = 1.$$

Ω^{ij} and Ω_{ij} raise and lower capital Latin indices.

$$\mathcal{D}_v := \partial_v - \mathcal{L}_{\mathcal{G}},$$

where $\mathcal{L}_{\mathcal{G}}$ is the Lie derivative along $\mathcal{G}^i$ direction.

Θ expansion of vector field generating the null surface $\mathcal{N}$: $\Theta := \mathcal{D}_v \ln \Omega$,

$\mathcal{N}_{ij}$ the *news tensor* associated with flux of gravitons through $\mathcal{N}$:

$$\mathcal{N}_{ij} := \frac{1}{2}\Omega^{2/(D-2)}\mathcal{D}_v\gamma_{ij}$$

$\mathcal{N}_{ij}$ as defined above is a symmetric-traceless tensor.

Einstein Field Equations at $r = 0$

$$\mathcal{D}_v \Omega = \Theta\ \Omega, \tag{3a}$$

$$\mathcal{D}_v \mathcal{P} = \Gamma + \frac{2}{\Theta} N_{ij} N^{ij}, \tag{3b}$$

$$\mathcal{D}_v \mathcal{J}_i + \Theta \Omega \partial_i \mathcal{P} + \Omega \partial_i \Gamma + 2\Omega \bar{\nabla}^j N_{ij} = 0. \tag{3c}$$

where

$$\mathcal{P} := \ln \frac{\eta}{\Theta^2},$$

and $\bar{\nabla}_i$ is covariant derivative w.r.t Ω_{ij}.

So the solution space may be parametrized by

$$\Omega, \mathcal{P}, \mathcal{J}_i, N_{ij} \text{ and } H'_{ij}(r = 0).$$

and Einstein equations may be used to solve for $\Gamma, \mathcal{G}^i$ in terms of these.

Residual diffeos over the null surface Σ

- We have used diffeos to fix the null surface Σ at $r = 0$.

- There is a measure zero subset of them that keep $r = 0$ intact remained unfixed:

$$\begin{aligned} v &\to v + T(v, x^i) + \mathcal{O}(r) \\ r &\to \Big(\partial_v T(v, x^i) - W(v, x^i)\Big) r + \mathcal{O}(r^2) \\ x^i &\to x^i + Y^i(v, x^i) + \mathcal{O}(r) \end{aligned} \tag{4}$$

- Subleading terms in r may be fixed order-by-order requiring that (4) keep the form of metric in solution space (1).

- Residual diffeos are specified by two scalar functions $T(v, x^i), W(v, x^i)$ and one vector $Y^i(v, x^i)$ over $r = 0$ null surface.

Symmetries of the solution space

- Upon (4) metric (1) keep its form but with transformed functions:

$$\begin{gathered}\mathcal{G}_i \to \mathcal{G}_i + \delta\mathcal{G}_i, \qquad \eta \to \eta + \delta\eta, \qquad \Omega \to \Omega + \delta\Omega, \\ H_{ij}^{(0)} \to H_{ij}^{(0)} + \delta H_{ij}^{(0)}, \qquad H_{ij}^{'(0)} \to H_{ij}^{'(0)} + \delta H_{ij}^{'(0)},\end{gathered} \tag{5}$$

  where δX are linear in residual diffeo functions T, W, Y^i.

- Besides dynamical, propagating gravitons, there are $2 + (D-2)$ functions over Σ in our solution space.

- There are $2 + (D-2)$ functions over Σ in our residual diffeos.

- Residual diffeos rotate us within the solution space. They are hence symmetry generators in the usual classical Noether sense.

- There are two classes of fields/states in our solution space:
  - $D(D-3)$ propagating tensor modes $H_{ij}^{(0)}, H_{ij}^{\prime(0)}$, one may call them bulk modes,
  - D scalar and vector modes, one may call them boundary modes.
- Boundary modes only reside on $D-1$ dimensional hypersurface Σ and do not propagate into the bulk (away from $r=0$).
- In our example we have chosen Σ to be null surface, like future horizon of a BH.

Symmetries of the solution phase space

- One may use Covariant Phase Space Formalism (CPSF) to show solution space is a phase space and there is a charge (Hamiltonian generator) associated with the boundary symmetries.

- These surface charges are given by integrals over codimension-2 compact spacelike surfaces, constant v slices on Σ, Σ_v.

- Surface charges are linear in symmetry generators $T(v,x^i), W(v,x^i)$ and $Y^i(v,x^j)$, but may have different field/states dependence, i.e.

- integrands of the surface charge integrals may have different functional dependence on $\Omega, \eta, \mathcal{G}_i$ as well as $H_{ij}^{(0)}, H_{ij}^{\prime(0)}$.

Surface charges and their algebra

- Standard computations yields the following surface charge variations associated with the symmetry generators ξ

$$\delta\!\!\!/ Q_\xi = \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x \Big[(W - \Gamma T)\,\delta\Omega + (Y^i + \mathcal{G}^i T)\delta\mathcal{J}_A + T\Omega\Theta\delta\mathcal{P} - T\Omega\Omega^{ij}\delta N_{ij}\Big], \tag{6}$$

- Charge variation is an integral over $\sum_{A=1}^{4} \mathcal{C}_A\ \delta\boldsymbol{\mathcal{Q}}_A$,

- $\boldsymbol{\mathcal{Q}}_A$ parameterize the solution phase space:
  N_{ij} corresponds to the bulk degrees of freedom,
  $\Omega, \mathcal{J}_i, \mathcal{P}$ parameterize boundary information.
  $\Gamma, \mathcal{G}^i$ functions which appear in $\mathcal{C}_A$ are subject to field equations.

- Charge variation may be split into Noether (integrable) part Q^{N} and the 'flux' part F:

$$\not\delta Q_\xi = \delta {Q^{\mathsf{N}}}_\xi + F_\xi(\delta g; g).$$

- Q^{N} may be computed for the Einstein-Hilbert action using the standard Noether procedure, yielding

$${Q^{\mathsf{N}}}_\xi = \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x \left[W\,\Omega + Y^i\,\mathcal{J}_i + T\,\left(-\Gamma\Omega + \mathcal{G}^i\mathcal{J}_i\right)\right], \qquad (7)$$

- Non-integrable flux part

$$F_\xi(\delta g; g) = \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x\, T\left(\Omega\delta\Gamma - \mathcal{J}_i\delta\mathcal{G}^i + \Omega\Theta\delta\mathcal{P} - \Omega\Omega^{ij}\delta N_{ij}\right). \quad (8)$$

- Symmetry generators T, W, Y^i are assumed to be field-independent, i.e. $\delta T = \delta W = 0 = \delta Y^i$.

- Note that $\mathcal{P}$ and N_{ij} only appear in the flux and not in the Noether charges.

- The zero mode Noether charges,

$$
\begin{aligned}
Q^{\mathrm{N}}_{-r\partial_r} &= \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x\ \Omega, \\
Q^{\mathrm{N}}_{\partial_i} &= \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x\ \mathcal{J}_i, \\
Q^{\mathrm{N}}_{\partial_v} := \mathbf{E} &= \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x\ \left(-\Gamma\Omega + \mathcal{G}^i\mathcal{J}_i\right),
\end{aligned}
\tag{9}
$$

- Note that the charge variation associated with ∂_v is

$$
\not{\delta} Q_{\partial_v} := \not{\delta}\mathbf{H} = \frac{1}{16\pi G}\int_{\mathcal{N}_v} \mathrm{d}^{D-2}x\ \left(-\Gamma\delta\Omega + \mathcal{G}^i\delta\mathcal{J}_i + \Omega\Theta\delta\mathcal{P} - \Omega\Omega^{ij}\delta N_{ij}\right).
$$

- Balance equation

$$\frac{\mathrm{d}}{\mathrm{d}v} Q^{\mathrm{N}}_{\xi} \approx -F_{\partial_v}(\delta_\xi g; g), \tag{10}$$

where $\approx$ denotes on-shell equality.

- The abov Eq. is
  - a manifestation of the boundary EoM written in terms of charges;
  - a generalized charge conservation equation as it relates time dependence, or non-conservation, of the charge (as viewed by the null boundary observer) to the flux passing through the boundary;
  - and shows how passage of flux through the null boundary is 'balanced' by the rearrangements in the charges.

Review of Thermodynamics

- Consider a thermodynamical system with
  - chemical potentials μ_a $(a = 1, 2, \cdots, N)$ and temperature T,
  - charges Q_i, the entropy S and the energy E;
- There are $N + 2$ charges and $N + 1$ chemical potentials.
- In microcanonical ensemble (which we assume), the first law takes the form

$$\mathrm{d}E = T\,\mathrm{d}S + \sum_{i=a}^{N} \mu_a\,\mathrm{d}Q_a. \tag{11}$$

- The LHS is an exact one-form over the thermodynamic space.

- Chemical potentials and the charges are related by the Gibbs-Duhem relation

$$S \; \mathrm{d}T + \sum_{i=a}^{N} Q_a \; \mathrm{d}\mu_a = 0. \tag{12}$$

- Together with the first law, this yields $E = TS + \sum_a \mu_a Q_a$.

- This equation relates E to the other charges and chemical potentials, e.g. $E = E(S, Q_a)$.

- $N+1$ number of chemical potentials and/or charges may be taken to be 'independent' variables parameterizing the thermodynamical configuration space and the rest of $N+1$ of them as functions of the former $N+1$ variables.

Null Boundary Thermodynamical Phase Space

I. Null boundary thermodynamics consists of three parts:

I.1) $(D-1)$ dimensional 'thermodynamic sector' parametrized by $(\Gamma, \mathcal{G}^i)$ and conjugate charges $(\Omega, \mathcal{J}_i)$;

I.2) $\mathcal{P}$, which only appears in the flux and not in the Noether charge;

I.3) the bulk mode parameterized by determinant free part of Ω^{ij} and its 'conjugate charge' N_{ij} which appear in the flux.

II. N_{ij} parameterizes effects of the bulk and how they take the boundary system out-of-thermal-equilibrium (OTE) whereas $\mathcal{P}$ parameterizes OTE within the boundary dynamics.
Put differently, OTE may come from inner boundary dynamics and/or from the gravity-waves passing through the null boundary.

III. Expansion parameter Θ is a measure of OTE, from both bulk and boundary viewpoints. When $\Theta = 0$ the system is completely specified by the $D-1$ dimensional thermodynamic phase space.

IV. The rest of the in-falling graviton modes parameterized through $\mathcal{O}(r)$ terms in H_{ij}, do not enter in the boundary/thermo dynamics, as of course expected from usual causality and that the boundary is a null surface.

Below we give local first law, then local Gibbs-Duhem equation and come to local zeroth law, specifying the subsectors which can be brought to a (local) equilibrium.

Notation: $\boldsymbol{\mathcal{X}}$ we will denote the density of the quantity $\mathbf{X}$,

$$\mathbf{X} := \int_{\mathcal{N}_v} \mathrm{d}^{D-2}x \ \boldsymbol{\mathcal{X}}.$$

Local First Law at Null Boundary

- Defining $\boldsymbol{\mathcal{P}} := \mathcal{P}/(16\pi G)$ and $\boldsymbol{\mathcal{N}}_{ij} := (16\pi G)^{-1} N_{ij}$,

$$\not{\delta}\boldsymbol{\mathcal{H}} = T_{\mathcal{N}}\ \delta\boldsymbol{\mathcal{S}} + \mathcal{G}^i\ \delta\boldsymbol{\mathcal{J}}_i + \Omega\Theta\delta\boldsymbol{\mathcal{P}} - \Omega\Omega^{ij}\ \delta\boldsymbol{\mathcal{N}}_{ij}, \qquad T_{\mathcal{N}} := -\frac{\Gamma}{4\pi}$$

- The above is true at each v, x^i over the null surface and represents the local null boundary first law.

- LHS, unlike the usual first law , is not a complete variation; the system is describing an open thermodynamic system due to the existence of the expansion and the flux.

- The above reduces to a usual first law for closed systems when $N_{ij} = 0$ or in the non-expanding $\Theta = 0$ case.

Local Extended Gibbs-Duhem Equation at Null Boundary

- For the densities in the our notation, we have

$$\boxed{\mathcal{E} = T_{\mathcal{N}}\boldsymbol{\mathcal{S}} + \mathcal{G}^i \mathcal{J}_i}$$

which is an analogue of the Gibbs-Duhem equation if $\mathcal{E}$ is viewed as energy, $\boldsymbol{\mathcal{S}}$ as entropy and $\mathcal{J}_i$ as other conserved charges and $\Gamma, \mathcal{G}^i$ as the respective chemical potentials.

- Note that it is a local equation at the null boundary, unlike its usual thermodynamic counterpart.

- This equation also holds for non-stationary/non-adiabatic cases when the system is out-of-thermal-equilibrium (OTE) it is 'local extended Gibbs-Duhem' (LEGD) equation at the null boundary.

- LEGD equation, like the local first law, is a manifestation of diffeomorphism invariance of the theory.

- We expect them to be universally true for any diff-invariant theory of gravity in any dimension.

- This equation is on par with the first law of thermodynamics but extends it in two important ways:

  it is a local equation in v, x^i and holds also for OTE.

- Since the integrable parts of the charge are (by definition) independent of the bulk flux N_{ij} and also of $\mathcal{P}$, the LEGD also do not involve $\mathcal{P}$ and N_{ij}.

- The chemical potentials, Γ and $\mathcal{G}^i$, implicitly depend on N_{ij} and $\mathcal{P}$ through Raychaudhuri and Damour equations.

Local Zeroth Law

- Zeroth law is a statement of thermal equilibrium: as a consequence of the zeroth law, two (sub)systems with the same temperature and chemical potentials are in thermal equilibrium.

- Flow of charges is proportional to the gradient of associated chemical potentials and hence the absence of such fluxes can be taken as a statement of the zeroth law.

- Here the system is parameterized by chemical potentials $\Gamma, \mathcal{G}^i$ and γ^{ij} which are functions of charges $Q_\alpha \in \{\Omega, \mathcal{P}, \mathcal{J}_i, N_{ij}\}$.

- This system is not in general in equilibrium but there could be special subsectors which are. The zeroth law is to specify such subsectors.

- Zeroth law requires existence of $\mathcal{G} = \mathcal{G}(\Omega, \mathcal{P}, \mathcal{J}_i, N_{ij})$ such that,

$$\delta\mathcal{G} = -\mathcal{S}\ (\delta T_{\mathcal{N}} - 4G\Theta\delta\mathcal{P}) - \mathcal{J}_i\ \delta\mathcal{G}^i + \Omega\mathcal{N}_{ij}\delta\Omega^{ij} \tag{13}$$

admits non-zero solutions.

- Integrability condition for the zeroth law is $\delta(\delta\mathcal{G}) = 0$, yielding an equation like

$$\sum_{\alpha,\beta} C_{\alpha\beta}\delta Q_\alpha \wedge \delta Q_\beta = 0,$$

where Q_α are generic charges and $C_{\alpha\beta}$ is skew-symmetric. This equation is satisfied only for $C_{\alpha\beta} = 0$.

- One can immediately see $N_{ij} = 0 = \delta N_{ij}$ is a necessary (but not sufficient) condition for the zeroth law to have non-trivial solutions.

- Let us note that when zeroth law is fulfilled the charge $\boldsymbol{\mathcal{H}}$, which appears in the LHS of the local first law, becomes integrable and we obtain

$$\boxed{\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{G}} + T_{\mathcal{N}}\boldsymbol{\mathcal{S}} + \mathcal{G}^i \boldsymbol{\mathcal{J}}_i}$$

- Besides $N_{ij} = 0$, in terms of $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}(\boldsymbol{\mathcal{S}}, \boldsymbol{\mathcal{J}}_A, \boldsymbol{\mathcal{P}})$ local zeroth law implies,

$$\boxed{T_{\mathcal{N}} = \frac{\delta \boldsymbol{\mathcal{H}}}{\delta \boldsymbol{\mathcal{S}}}, \qquad \mathcal{G}^i = \frac{\delta \boldsymbol{\mathcal{H}}}{\delta \boldsymbol{\mathcal{J}}_i}, \qquad \mathcal{D}_v \boldsymbol{\mathcal{S}} = \boldsymbol{\mathcal{S}}\Theta = \frac{1}{4G}\,\frac{\delta \boldsymbol{\mathcal{H}}}{\delta \boldsymbol{\mathcal{P}}}}$$

  last equation may be seen as the *equation of state*.

- For the special case of $\Theta = 0$, one simply deduces that $\boldsymbol{\mathcal{H}}$ does not depend on $\boldsymbol{\mathcal{P}}$.

- It ensures that total energy and angular momentum are conserved on-shell.

Generic $\Theta \neq 0$ case.

Zeroth law requires $N_{ij} = 0$ and we have Einstein boundary field equations

$$T_{\mathcal{N}} = -4G\mathcal{D}_v\mathcal{P}, \qquad \mathcal{D}_v\left[\mathcal{J}_i + 4G\bar{\nabla}_i(\mathcal{S}\mathcal{P})\right] = 0.$$

Zeroth law is satisfied for any $\mathcal{H} = \mathcal{H}(\mathcal{S}, \mathcal{P}, \mathcal{J}_i)$, when $\mathcal{S}, \mathcal{P}$ and $\mathcal{J}_i$ have the following basic Poisson brackets:

$$\{\mathcal{S}(x,v), \mathcal{P}(y,v)\} = \frac{1}{4G}\delta^{D-2}(x-y),$$

$$\{\mathcal{S}(x,v), \mathcal{S}(y,v)\} = \{\mathcal{P}(x,v), \mathcal{P}(y,v)\} = 0,$$

$$\{\mathcal{S}(x,v), \mathcal{J}_i(y,v)\} = \mathcal{S}(y,v)\frac{\partial}{\partial x^i}\delta^{D-2}(x-y),$$

$$\{\mathcal{P}(x,v), \mathcal{J}_i(y,v)\} = \left(\mathcal{P}(y,v)\frac{\partial}{\partial x^i} + \mathcal{P}(x,v)\frac{\partial}{\partial y^i}\right)\delta^{D-2}(x-y),$$

$$\{\mathcal{J}_i(x,v), \mathcal{J}_j(y,v)\} = \frac{1}{16\pi G}\left(\mathcal{J}_i(y,v)\frac{\partial}{\partial x^j} - \mathcal{J}_j(x,v)\frac{\partial}{\partial y^i}\right)\delta^{D-2}(x-y)$$

- The above Poisson brackets imply

$$\partial_v \mathcal{X} = \{\mathcal{H}, \mathcal{X}\}.$$

- That is, $\mathcal{H}$ is the Hamiltonian over the thermodynamic phase space.

- $\Theta = 0$ **case.** may be worked out similarly

  - in this case $\mathcal{P} = 0 = N_{ij}$ and the thermodynamic phase space is described by $\mathcal{S}, \mathcal{J}_i$ and their chemical potentials.

  - Local zeroth law is satisfied by any scalar Hamiltonian $\mathcal{H} = \mathcal{H}(\mathcal{S}, \mathcal{J}_i)$, together with basic Poisson brackets given above, but with $\mathcal{P}$ dropped and again with $\partial_v \mathcal{X} = \{\mathcal{H}, \mathcal{X}\}$.

- Zeroth law is just defining the Poisson bracket structure over the thermodynamic phase space and existence of Hamiltonian dynamics, but does not specify a Hamiltonian.

- Choice of Hamiltonian fixes a boundary Lagrangian and the boundary dynamical equations which in turn specifies local dynamics of charges on the null boundary $\mathcal{N}$.

- In analogy with isolated horizon of black holes, if the zeroth law holds the null surface may be called an 'isolated null surface'.

- Our zeroth law is a weaker condition than stationarity as ∂_v of the chemical potentials need not vanish.

Discussion, Concluding Remarks and Outlook

⊛ Presence of Boundaries brings in new 'boundary d.o.f.'.

- The b.d.o.f. may be classified and labelled by nontrivial diffeos.

- Using CPSM one can construct the boundary phase space which govern b.d.o.f.

- Motivated by identification and formulation of BH microstates we discussed null boundaries Σ.

- $\Sigma \sim R_v \times \Sigma_v$, where Σ_v is a codim. two compact surface.

- Σ may be viewed as the null limit of the stretched horizon.

- Physics in the outside horizon region is then described by

$$\text{b.d.o.f} \oplus \text{bulk d.o.f.}$$

- The Hilbert space of b.d.o.f, $\mathcal{H}_{bdof}$ may be labeled by the surface charges associated with nontrivial diffeos on Σ.

Boundary d.o.f interact with bulk d.of. through the *Bondi news* through the horizon. The balance equation equates time derivative of boundary charges to the flux through the boundary.

- We identified null surface thermodynamic phase space, which in general describes an open system.

- The thermodynamics phase space is described by $D-1$ charges and associated chemical potentials as well as the flux.

- We showed that local laws of thermodynamics governs the thermodynamics phase space.

- Zeroth law of thermodynamics ensures that we have a phase space by specifying the Poisson bracket structure.

- Our laws of thermodynamics are nothing but a manifestation of diffeomorphism invariance of the theory at the boundary.

- Einstein field equations then appear as boundary Hamilton equations, but boundary Hamiltonian is still free to be chosen.

- Second law of thermodynamics and how it can be realized in our setting is an important problem that should be tackled. Focusing theorem may be of use.

- Our analysis can provide a new framework to formulate a general memory effect, especially a horizon memory effect.

- The analysis so far is classical and we should quantize the system.

- It should be possible to perform a semiclassical analysis in which the boundary d.o.f are quantized while the bulk is classical.

Formulating quantum dynamics of the boundary thermodynamic phase space will hopefully shed light on BH micorstate & information puzzle.

Thank You For Your Attention