Higher order QCD corrections for neutral B-meson oscillations

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In the work present new contributions to the decay matrix element $\Gamma^{q}{}_{12}$ of the $\mathrm{B_q}$-$\overline{\mathrm{B_q}}$ mixing complex, where q = d or s. This is the first step to the next to next leading order (NNLO) calculation of quantum chromodynamic corrections to the lead within the Standard Model for the CP violation of the $\mathrm{B_q}$-$\overline{\mathrm{B_q}}$ (q=s,d) system. We calculate a class of threeloop Feynman penguin diagrams with non-zero charm quark mass, Our results show an importance of charm quark mass in penguin sector for $\Delta\Gamma_{s}$.

The oscillations between the flavor eigenstates $\mathrm{B_q}$ and $\overline{\mathrm{B_q}}$, where q = d or s, are governed by two 2 × 2 matrices, the mass matrix M and the decay matrix Γ.

$$\frac{\Delta\Gamma_q}{\Delta M_q} = - \mathrm{Re}\frac{\Gamma^q{}_{12}}{M_{12}^q}$$

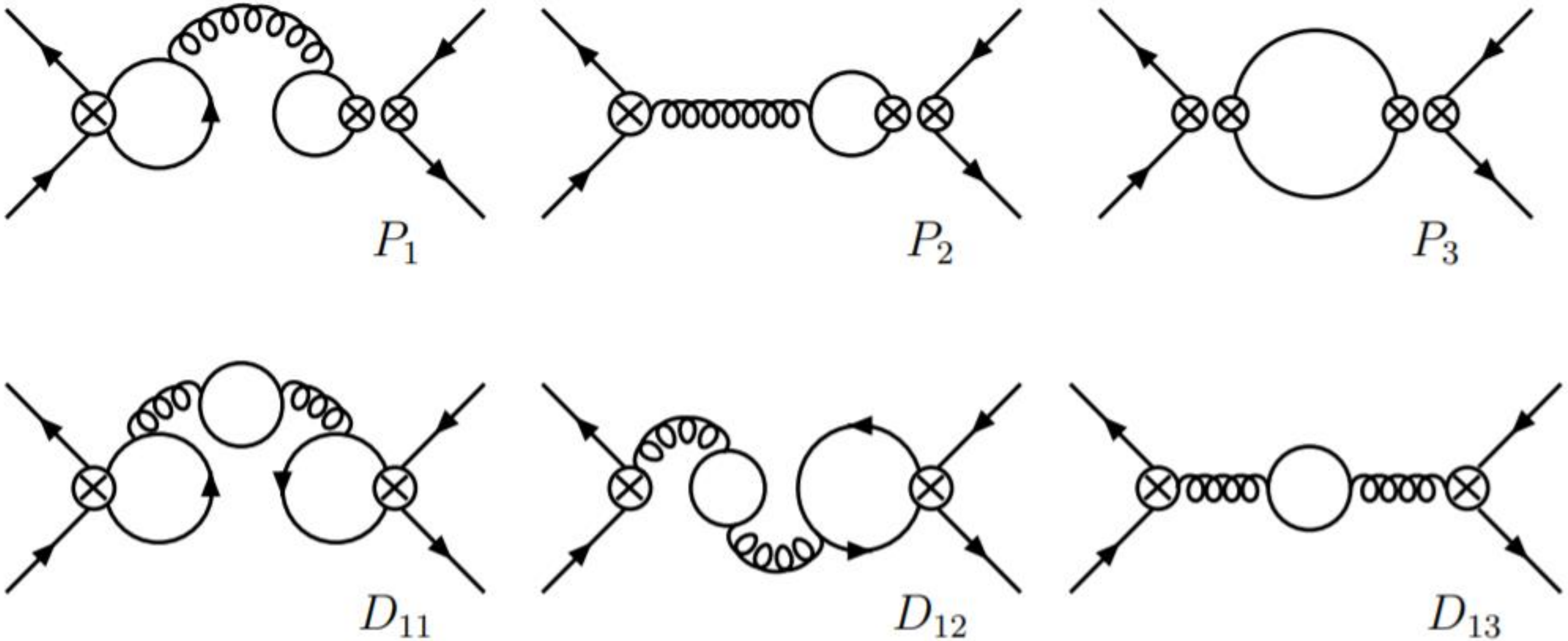
$$\Delta M_s = (17.757 \pm 0.021)\, pc^{-1}$$

$$\Delta M_d = (0.5064 \pm 0.0019)\, pc^{-1}$$

$$\Delta\Gamma_s^{Exp} = (8.9 \pm 0.6) \cdot 10^{-2} pc^{-1}$$

$$\Delta\Gamma_d^{Exp} = (-1.32 \pm 6.58) \cdot 10^{-3} pc^{-1}$$

CALCULATION OF IMAGINARY PARTS OF ONE-LOOP DIAGRAM



Diagrams for the penguin contribution at O($a_s{}^2 N_f$). The small Wilson coefficients C_{3-6} are counted as O(a_s). P_1, P_2 are diagrams with one insertion of a penguin operator O_3, ..., O_6, depicted as two circles with crosses, and one insertion of $O_2{}^{u,c}$or O_8, shown as a single circle with cross. P_3 denotes a one-loop diagram with two insertions of penguin operators O_3, ..., O_6. D_{11}, D_{12} and D_{13} are diagrams with insertions of operators $O_2{}^{u,c}$ or O8.

Here, in particular, we will consider diagram D_{12}. (FIG.2) In general, quarks b, d, s, c, u can contribute to the loop. However, the contribution of the u, d, s quarks is the same, because the masses are very small.

According to Feynman's rules we will have the following

$$\int \frac{d^d k_1}{(2\pi)^d} \bar{b}(-i)\frac{1}{{p_b}^2} m_b(\Upsilon^{\alpha}(p^{\beta},\gamma_{\beta})-(p^{\beta},\gamma_{\beta})\,\Upsilon^{\alpha})(1-\gamma_5)s\Upsilon^{\mu}(1-\gamma_5)\,\frac{{k_1}^{\beta}\gamma_{\beta}-p_{b_{\alpha}}\Upsilon^{\alpha}-m_c}{({k_1}^{\beta}\gamma_{\beta}-p_b)^2-{m_c}^2}\,\Upsilon^{\nu}\,\frac{{k_1}^{\beta}\gamma_{\beta}-m_c}{({k_1}^{\beta}\gamma_{\beta})^2-{m_c}^2}\,\Upsilon^{\mu}(1-\gamma_5)\times$$

$$(-i)\frac{1}{{p_b}^2}\Upsilon^{\nu}\,\frac{{k_2}^{\beta}\gamma_{\beta}-p_{b_{\alpha}}\Upsilon^{\alpha}-m_i}{({k_2}^{\beta}\gamma_{\beta}-p_b)^2-{m_i}^2}\,\Upsilon^{\nu}\,\frac{{k_2}^{\beta}\gamma_{\beta}-m_i}{({k_2}^{\beta}\gamma_{\beta})^2-{m_i}^2}$$

d=->4-2ε

Imaginary part of diagram, which contributes b quark, will be

$$\left(\frac{\alpha s}{(4\pi)}\right)^2 \sqrt{1-4z}(\mathrm{bs}[1]+\mathrm{bs}[2]) * \left(\frac{(1+2z)}{9\pi\epsilon} - \frac{(-19+3\sqrt{3}\pi-44z+6\sqrt{3}\pi z+18(1+2z)\mathrm{Log}(m_b))}{27\pi} - \frac{3(1+2z)\mathrm{Log}({m_b}^2)+3\mathrm{Log}(1-4z)+6z\mathrm{Log}(1-4z)}{27\pi}\right),$$

Where $\frac{{m_c}^2}{{m_b}^2}$ = z , the product (bs [1] + bs [2]) is a matrix element that contains color factors.

(bs [1] + bs [2]) we will do the calculation at the end

Imaginary part of diagram, which contributes c quark, will be

$$\left(\frac{\mathrm{a_s}}{(4\pi)}\right)^2 * \frac{1}{9\pi}(\mathrm{bs}[1]+\mathrm{bs}[2])\left(\frac{\sqrt{1-4z}(1+2z)+\sqrt{1-4z_2}(1+2z_2)}{\epsilon} + \sqrt{1-4z_2}\left(\frac{7}{3}+4z+\frac{20z2}{3}+8zz_2+(-2-4z_2)\mathrm{Log}(m_b{}^2) + (-1-2z_2)\mathrm{Log}(z)+(-1-2z_2)\mathrm{Log}(1-4z_2)\right) + \sqrt{1-4z}\left(\frac{7}{3}+4z_2+\frac{4}{3}z(5+6z_2)+(-2-4z)\mathrm{Log}(m_b{}^2)+(-1-2z)\mathrm{Log}(1-4z) + (-1-2z)\mathrm{Log}(z_2\right)\right) + \sqrt{1-4z}\sqrt{1-4z_2}\left((1+2z)(1+2z_2)\mathrm{Log}(\sigma)+(1+2z)(1+2z_2)\mathrm{Log}(\sigma_2)\right)\Big)$$

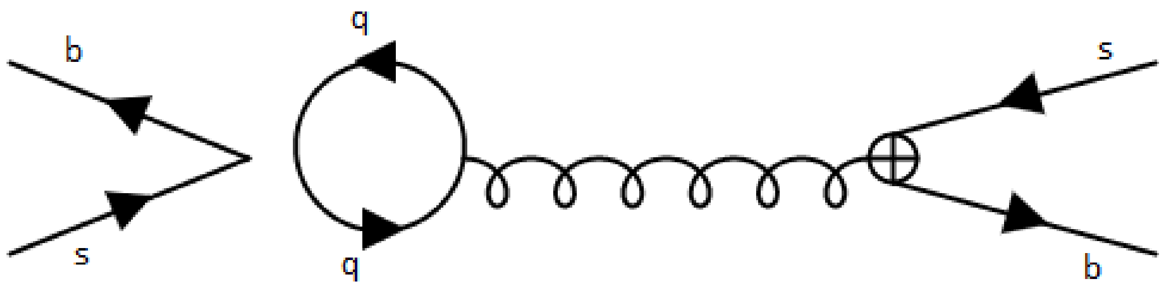
Where z_2 k is the ratio $\frac{{m_c}^2}{{m_b}^2}$ due to the c quark investing in the loop with the operator O_2 , z is the ratio $\frac{{m_c}^2}{{m_b}^2}$ in a fermion loop with c quark $\sigma = \frac{1-\sqrt{1-4z}}{1+\sqrt{1-4z}}$, $\sigma_2 = \frac{1-\sqrt{1-4z_2}}{1+\sqrt{1-4z_2}}$

RENORMALIZATION

Here we have two types of renormalization.
1.a strong interaction constant of g_s
2.Operators, in this case the O_2 operator.

Diagrams needed to renormalization of g_s

$$M(g_s)=\frac{\alpha_s}{4\pi}\frac{4{m_b}^2(\mathrm{bs}[1]+\mathrm{bs}[2])(-\frac{27}{16}\sqrt{1-4z}(1+2z)+\sqrt{1-4z}\epsilon(-\frac{9}{8}(1+5z)+}{81\pi}+\frac{\frac{27}{16}(1+2z)\mathrm{Log}({m_b}^2)+\frac{27}{16}(1+2z)\mathrm{Log}(1-4z)))}{81\pi}$$



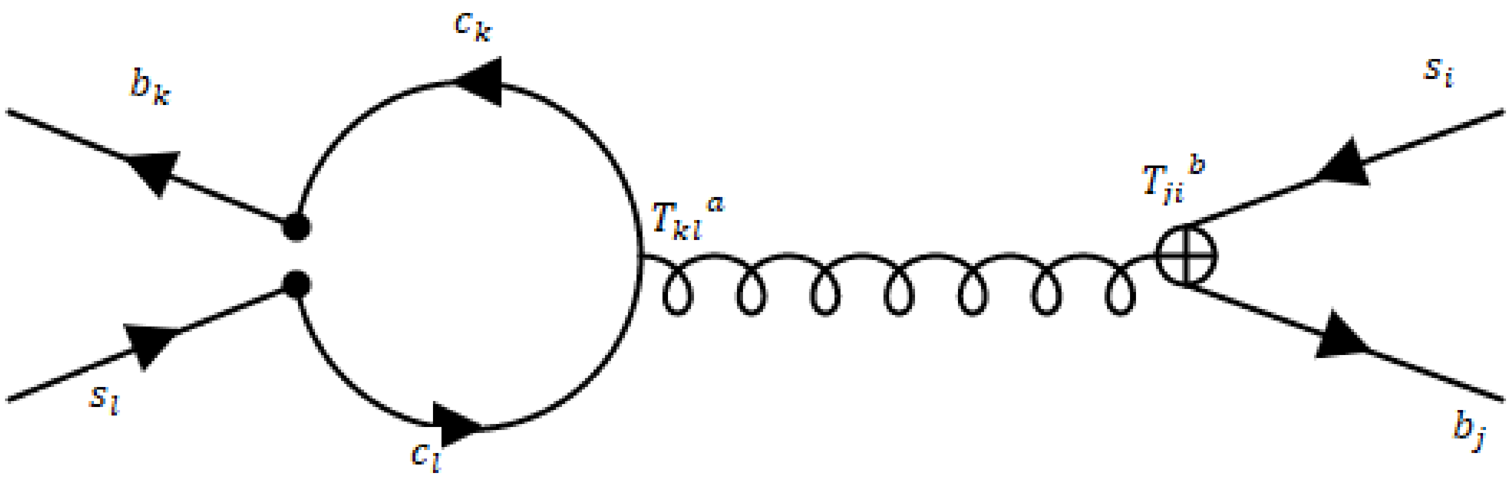
Here is the diagrams needed to renormalization of the O_2 operator. The operators O_3, O_4, O_5, O_6invest here. q = u, d, s, c.

$$O_3=\overline{(s_i}b_i)_{(V-A)}\ \overline{(q_j}q_j)_{(V-A)}, \qquad O_4=\overline{(s_i}b_j)_{(V-A)}\ \overline{(q_j}q_i)_{(V-A)},$$

$$O_5=\overline{(s_i}b_i)_{(V-A)}\ \overline{(q_j}q_j)_{(V+A)}, \qquad O_6=\overline{(s_i}b_j)_{(V-A)}\ \overline{(q_j}q_i)_{(V+A)}),$$

ResNLOO4=

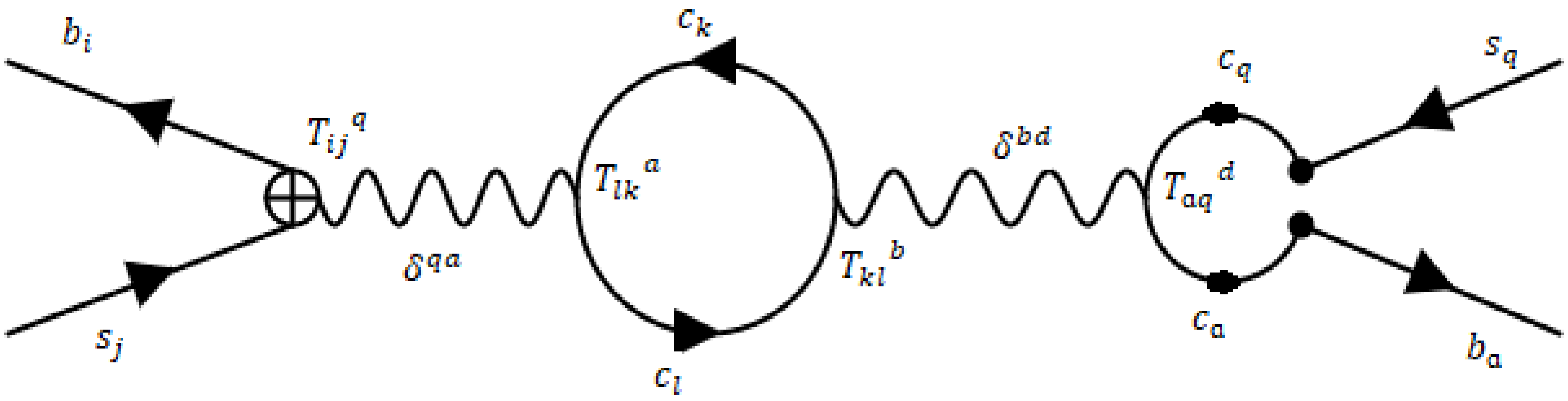
$$\frac{\alpha_s N_f}{4\pi}\frac{m_b{}^2(\mathrm{bs}[1]+\mathrm{bs}[2])(\sqrt{1-4z}(1+2z)+\sqrt{1-4z}\epsilon(\frac{1}{3}(5+16z)}{12\pi}+\frac{(-1-2z)\mathrm{Log}(m_b{}^2)+(-1-2z)\mathrm{Log}(1-4z)))}{12\pi}=\mathrm{ResNLOO6}$$



$$\text{bs[1]}=\frac{1}{2}\,\bar{b}_i L s_l * \overline{b_l{}'} L s_i{}' - \frac{1}{2N}\,\bar{b}_l L s_l \overline{b_i{}'} L s_i{}' = \frac{1}{2}\langle\widetilde{Q_s}\rangle - \frac{1}{2N}\langle Q_s\rangle$$

$$\text{bs[2]}=\frac{1}{2}\,\bar{b}_i \gamma^{\mu} L s_l * \overline{b_l{}'} \gamma^{\mu} L s_i{}' - \frac{1}{2N}\,\bar{b}_l \gamma^{\mu} L s_l \overline{b_i{}'} \gamma^{\mu} L s_i{}' = \frac{1}{2}\langle\tilde{Q}\rangle - \frac{1}{2N}\langle Q\rangle$$

$$\text{bs[1]}+\text{bs[2]}=\frac{1}{6}\langle Q\rangle - \frac{2}{3}\langle Q_s\rangle$$



bs[1]+ bs[2]=$\frac{1}{3}\langle Q\rangle$-$\frac{4}{3}\langle Q_s\rangle$

renormalization constants

$$Z_{g_s}^{(1),N_f} = \frac{a_s}{6\pi\epsilon} N_f T_R, \quad \text{where } T_R = \frac{1}{2}$$

$$Z_{42}^{(1)} = Z_{62}^{(1)} = \frac{a_s}{12\pi\epsilon}$$

For a diagram, that contains b quark in loop, finally we get

$$-\frac{\mathrm{a}_s^2}{4\pi}\frac{2\sqrt{1-4z}(1+2z)\left(-17+3\sqrt{3}\pi+18\mathrm{Log}(m_b)-6\mathrm{Log}\left(m_b^2\right)\right)}{81}$$

For a diagram, that contains c quark in loop, we get .

$$\frac{\alpha_s{}^2}{4\pi} 2 \frac{((1+2z_2)(2+12z-3\mathrm{Log}\left(\frac{\mu_1}{m_b}\right)-3\mathrm{Log}(z))}{27} + \frac{\sqrt{1-4z}(1+2z)(5-3\mathrm{Log}\left(\frac{\mu_1}{m_b}\right)-3\mathrm{Log}(z_2))}{27} + \frac{3\sqrt{1-4z}(1+2z)\sqrt{1-4z_2}(1+2\mathrm{z}2)(\mathrm{Log}(\sigma)+\mathrm{Log}(\sigma_2)))}{27}$$

μ_1 is a renormalization scale. μ_1=$[m_b/2, 2m_b]$

PHENOMENOLOGY

i	$C_i^{(0)}(\mu_b)$	$C_i^{(1)}(\mu_b)$	$C_i^{(2)}(\mu_b)$
1	-0.2687	4.332	50.142
2	1.1179	-2.024	-17.114
3	0.0121	0.090	—
4	-0.0274	-0.465	—
5	0.0079	0.041	—
6	-0.0343	-0.434	—
8	-0.1508	-1.0006	—

For the parameters of CKM matrix we have.

λ = 0,22453$\pm$0,00044, A=0,836 $\pm$0,015, $\bar{\rho}$=0,,$122^{+0,018}_{-0,017}$, $\bar{\eta}$=0,$355^{+0,012}_{-0,011}$

In NLO order $\bar{z} = m_c^2(m_b)/\, m_b^2(m_b)$= 0,049

in NNLO order $\bar{z} = m_c^2(m_b)/\, m_b^2(m_b)$= 0,045

$$
\begin{aligned}
\bar{m}_b(\bar{m}_b) &= (4.18 \pm 0.03)\ \mathrm{GeV} \\
\bar{m}_s(\bar{m}_b) &= (0.0786 \pm 0.0006)\ \mathrm{GeV} \\
m_b^{\mathrm{pow}} &= 4.7\ \mathrm{GeV} \\
M_{B_s} &= 5366.88\,\mathrm{MeV} \\
B_{B_s} &= 0.813 \pm 0.034 \\
\widetilde{B}'_{S,B_s} &= 1.31 \pm 0.09 \\
B^s_{R_0} &= 1.27 \pm 0.52 \\
B^s_{\tilde{R}_2} &= 0.89 \pm 0.35 \\
B^s_{\tilde{R}_3} &= 1.14 \pm 0.39 \\
B^q_{R_2} &= -B^q_{\tilde{R}_2} \\
f_{B_s} &= (0.2307 \pm 0.0013)\,\mathrm{GeV} \\
\sin(2\beta) &= 0.7083^{+0.0127}_{-0.0098} \\
|V_{us}| &= 0.22483^{+0.00025}_{-0.00006}
\end{aligned}
$$

$$
\begin{aligned}
\bar{m}_c(\bar{m}_c) &= (1.2982 \pm 0.0013_{\mathrm{stat}} \pm 0.0120_{\mathrm{syst}})\ \mathrm{GeV} \\
\bar{m}_t(m_t) &= (165.26 \pm 0.11_{\mathrm{stat}} \pm 0.30_{\mathrm{syst}})\ \mathrm{GeV} \\
\alpha_s(M_Z) &= 0.1181(11) \\
M_{B_d} &= 5279.64\,\mathrm{MeV} \\
B_{B_d} &= 0.806 \pm 0.041 \\
\widetilde{B}'_{S,B_d} &= 1.20 \pm 0.09 \\
B^d_{R_0} &= 1.02 \pm 0.55 \\
B^d_{\tilde{R}_2} &= B^s_{\tilde{R}_2} \\
B^d_{\tilde{R}_3} &= B^s_{\tilde{R}_3} \\
B^q_{R_3} &= \tfrac{5}{7} B^q_{\tilde{R}_3} + \tfrac{2}{7} B^q_{\tilde{R}_2} \\
f_{B_d} &= (0.1905 \pm 0.0013)\,\mathrm{GeV} \\
R_t &= 0.9124^{+0.0064}_{-0.0100}
\end{aligned}
$$

The penguin contribution at order a_s evaluates to

$$\frac{\delta\Delta\,\Gamma^{1,\mathrm{p}}{}_{s}(\mathrm{z})}{\Delta\,\Gamma^{\mathrm{NLO}}{}_{s}(\mathrm{z})}\text{=-14.5\% (pole)} \qquad \frac{\delta\Delta\,\Gamma^{1,\mathrm{p}}{}_{s}(\mathrm{z})}{\Delta\,\Gamma^{\mathrm{NLO}}{}_{s}(\mathrm{z})} = \text{-11.2\% } (\overline{MS})$$

and the new $a_s^2\ N_f$ corrections are

$$\frac{\delta\Delta\,\Gamma^{2,N_f\,.\mathrm{p}}{}_{s}(\mathrm{z})}{\Delta\,\Gamma^{\mathrm{NLO}}{}_{s}(\mathrm{z})}= \text{2.4\% (pole)} \qquad \frac{\delta\Delta\,\Gamma^{2,N_f\,.\mathrm{p}}{}_{s}(\mathrm{z})}{\Delta\,\Gamma^{\mathrm{NLO}}{}_{s}(\mathrm{z})} = \text{1.8\% } (\overline{MS})$$

updated NLO SM values for $\Delta\Gamma_q/\Delta\mathrm{M}_q$

$$\frac{\Delta\Gamma_s}{\Delta\mathrm{M}_s} = = (4.33 \pm 0.83_{\mathrm{scale}} \pm 0.11_{B,\widetilde{B_s}} \pm 0.94_{\Lambda_{ACD}/m_b}) \times 10{-}3 \text{ (pole)}$$

$$\frac{\Delta\Gamma_s}{\Delta\mathrm{M}_s} = = (4.97 \pm 0.62_{\mathrm{scale}} \pm 0.13_{B,\widetilde{B_s}} \pm 0.80_{\Lambda_{ACD}/m_b}) \times 10{-}3 \ (\overline{MS})$$

$$\frac{\Delta\Gamma_d}{\Delta\mathrm{M}_d} = = (4.48 \pm 0.82_{\mathrm{scale}} \pm 0.12_{B,\widetilde{B_s}} \pm 0.86_{\Lambda_{ACD}/m_b}) \times 10{-}3 \text{ (pole)}$$

$$\frac{\Delta\Gamma_d}{\Delta\mathrm{M}_d} = = (5.07 \pm 0.61_{\mathrm{scale}} \pm 0.14_{B,\widetilde{B_s}} \pm 0.73_{\Lambda_{ACD}/m_b}) \times 10{-}3 \ (\overline{MS})$$

We have calculated the penguin contributions of order $a_s^2 N_f$ to the width difference $\Delta\Gamma_q$.
$\Delta\Gamma_q$ is a fundamental quantity characterizing the $\mathrm{B_q}$-$\overline{\mathrm{B_q}}$ mixing complex. $\Delta\Gamma_q$ terms have signs opposite to the NLO corrections. The calculated partial NNLO corrections are smaller than the corresponding NLO terms by factors of roughly 6 for $\Delta\Gamma_q$ respectively, indicating a good convergence of the perturbative series.

Thank You