PhD school and Workshop Aspects of Symmetry (November 8–12, 2021)

Axions in Baryon Chiral Perturbation Theory

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November 10, 2021

Contents

Introduction

From the QCD θ-vacuum to the QCD Axion

Introduction: The QCD θ-vacuum

- Consider Quantum Chromodynamics (QCD) with θ-term Callan, Dashen, and Gross, Phys. Lett. B **63** (1976) 334–340

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{QCD},0} + \mathcal{L}_{\mathcal{M}} + \theta \left(\frac{g}{4\pi}\right)^2 \mathrm{Tr}\left[G_{\mu\nu}\tilde{G}^{\mu\nu}\right]$$

  dual field strength tensor $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$

  $\hookrightarrow$ consequence of instanton solution Belavin et al., Phys. Lett. B **59** (1975) 85–87

  $\hookrightarrow$ recall A. Wirzba, *Strong CP-problem* (Lecture)

- $\mathcal{L}_{\text{QCD}}$ **violates** *CP*!

  $\hookrightarrow$ except if:

  - $\bar{\theta} = \theta + \mathrm{Arg}\det\mathcal{M} = 0$ (or $= \pi$, but this point is special) Smilga, Phys. Rev. D **59** (1999) 114021
  - or any of the quarks is massless

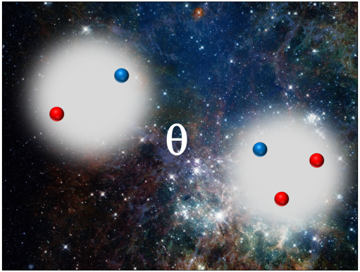
Introduction: Strong CP-problem

- notable consequence: neutron electric dipole moment $\propto \bar{\theta}$ Baluni, Phys. Rev. D **19** (1979) 2227–2230
- somewhere between $|d_n| \approx 10^{-16}\bar{\theta}\, e\,\mathrm{cm}$ and $|d_n| \approx 10^{-15}\bar{\theta}\, e\,\mathrm{cm}$ Kim and Carosi, Rev. Mod. Phys. **82** (2010) 557–602, Guo et al., Phys. Rev. Lett. **115** (2015) 062001
- recent measurements: $|d_n^{\mathrm{exp}}| < 1.8 \times 10^{-26}\, e\,\mathrm{cm}$ (90 % C.L.) Abel et al., Phys. Rev. Lett. **124** (2020) 081803

Strong CP-problem

Why $\bar{\theta} \lesssim 10^{-11}$, whereas naively it is expected to be $\mathcal{O}(1)$?

Introduction: Solutions to the Strong CP-problem



- anthropic principle? $\rightarrow$ probably **not** Ubaldi, Phys. Rev. D **81** (2010) 025011, Lee et al., Phys. Rev. Res. **2** (2020) 033392
- Peccei–Quinn mechanism Peccei and Quinn, Phys. Rev. Lett. **38** (1977) 1440–1443, Peccei and Quinn, Phys. Rev. D **16** (1977) 1791–1797
- new global chiral symmetry: $U(1)_{\mathrm{PQ}}$
  $\hookrightarrow$ leads to a cancellation of the θ-term
- but also a new, very light pseudoscalar Nambu–Goldstone boson with zero bare mass Weinberg, Phys. Rev. Lett. **40** (1978) 223–226, Wilczek, Phys. Rev. Lett. **40** (1978) 279–282
  $\hookrightarrow$ **Axion** (LO mass: $m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2} \frac{M_\pi^2 F_\pi^2}{f_a^2}$)
- Dark matter? Preskill, Wise, and Wilczek, Phys. Lett. B **120** (1983) 127–132, Abbott and Sikivie, Phys. Lett. B **120** (1983) 133–136, Dine and Fischler, Phys. Lett. B **120** (1983) 137–141

Motivating our work

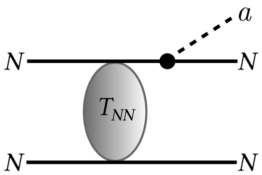
- Study the coupling of QCD axion to nucleons and other octet baryons
  $\hookrightarrow$ Why?

- axion-nucleon coupling plays crucial role in determining the **axion window** Preskill, Wise, and Wilczek, Phys. Lett. B **120** (1983) 127–132, Abbott and Sikivie, Phys. Lett. B **120** (1983) 133–136, Kim, Phys. Rept. **150** (1987) 1–177

$$10^9\ \text{GeV} \lesssim f_a \lesssim 10^{12}\ \text{GeV}$$

  $\hookrightarrow$ **nuclear bremsstrahlung** processes in massive stellar objects Raffelt, Phys. Rept. **198** (1990) 1–113, Turner, Phys. Rept. **197** (1990) 67–97, Di Luzio et al., Phys. Rept. **870** (2020) 1–117

- What about **hyperons** in neutron stars? Tolos and Fabbietti, Prog. Part. Nucl. Phys. **112** (2020) 103770



Motivating our work

- Axion-nucleon coupling to the leading order in chiral power counting:
  - Donnelly et al., Phys. Rev. D **18** (1978) 1607
  - Kaplan, Nucl. Phys. B **260** (1985) 215–226
  - Srednicki, Nucl. Phys. B **260** (1985) 689–700
  - Georgi, Kaplan, and Randall, Phys. Lett. B **169** (1986) 73–78
  - Grilli di Cortona et al., JHEP **01** (2016) 034
  - ...
- Axion-nucleon coupling **up to one-loop order** in SU(2) HBCHPT

  Vonk, Guo, and Meißner, JHEP **03** (2020) 138
- Axion-baryon coupling **up to one-loop order** in **SU(3)** HBCHPT

  Vonk, Guo, and Meißner, JHEP **08** (2021) 024

Restrict to canonical “invisible” axion models

KSVZ axion model Kim, Phys. Rev. Lett. **43** (1979) 103, Shifman, Vainshtein, and Zakharov, Nucl. Phys. B **166** (1980) 493–506

DFSZ axion model Dine, Fischler, and Srednicki, Phys. Lett. B **104** (1981) 199–202, Zhitnitsky, Sov. J. Nucl. Phys. **31** (1980) 260

QCD with Axions

QCD with Axions

Consider QCD Lagrangian with axions

$$\mathcal{L}_{QCD} = \mathcal{L}_{\mathrm{QCD},0} - \bar{q}\mathcal{M}q + \frac{a}{f_a}\left(\frac{g}{4\pi}\right)^2 \mathrm{Tr}\left[G_{\mu\nu}\tilde{G}^{\mu\nu}\right] + \bar{q}\gamma^\mu\gamma_5\frac{\partial_\mu a}{2f_a}\mathcal{X}_q q$$

here:

- quark fields $q = (u, d, s, c, b, t)^{\mathrm{T}}$
- axion field a
- 6×6 mass matrix $\mathcal{M} = \mathrm{diag}\{m_q\}$
- 6×6 axion-quark coupling matrix $\mathcal{X}_q = \mathrm{diag}\{X_q\}$
  - $X_q^{\mathrm{KSVZ}} = 0$
  - $X_{u,c,t}^{\mathrm{DFSZ}} = \frac{1}{3}\frac{x^{-1}}{x+x^{-1}} = \frac{1}{3}\sin^2\beta$

    $X_{d,s,b}^{\mathrm{DFSZ}} = \frac{1}{3}\frac{x}{x+x^{-1}} = \frac{1}{3}\cos^2\beta = \frac{1}{3} - X_{u,c,t}^{\mathrm{DFSZ}}$

QCD with Axions

$$\mathcal{L}_{QCD} = \mathcal{L}_{\text{QCD},0} - \bar{q}\mathcal{M}q + \underbrace{\frac{a}{f_a}\left(\frac{g}{4\pi}\right)^2 \mathrm{Tr}\left[G_{\mu\nu}\tilde{G}^{\mu\nu}\right]}_{\text{remove by suitable axial rotation}} + \bar{q}\gamma^\mu\gamma_5\frac{\partial_\mu a}{2f_a}\mathcal{X}_q q$$

Apply ($z = m_u/m_d$ and $w = m_u/m_s$)

$$q \to \exp\left(i\gamma_5\frac{a}{2f_a}\mathcal{Q}_a\right) q$$

with

$$\mathcal{Q}_a = \frac{\mathcal{M}_q^{-1}}{\langle\mathcal{M}_q^{-1}\rangle} \approx \frac{1}{1+z+w}\,\mathrm{diag}\,(1, z, w, 0, 0, 0)$$

$\hookrightarrow$ this particular $\mathcal{Q}_a$ chosen in order to avoid mixing between axion and the neutral Nambu–Goldstone bosons of chiral symmetry breaking (π^0, η).

QCD with Axions

- Now the axion-quark interaction Lagrangian is given by

$$\mathcal{L}_{a-q} = -\left(\bar{q}_L \mathcal{M}_a q_R + \text{h.c.}\right) + \bar{q}\gamma^\mu\gamma_5 \frac{\partial_\mu a}{2f_a}\left(\mathcal{X}_q - \mathcal{Q}_a\right) q$$

with

$$\mathcal{M}_a = \exp\left(i\frac{a}{f_a}\mathcal{Q}_a\right)\mathcal{M}_q$$

$\hookrightarrow$ need match to external fields $s, a_\mu, a_\mu^{(s)}$ in order to include the interaction in chiral perturbation theory (recall Ulf-G. Meißner's Lecture)

- e.g. in SU(2)

$$\mathcal{L}_{a-q} = -\left(\bar{q}_L \mathcal{M}_a q_R + \text{h.c.}\right) + \left(\bar{q}\gamma^\mu\gamma_5\left(c_{u-d}\frac{\partial_\mu a}{2f_a}\tau_3 + c_{u+d}\frac{\partial_\mu a}{2f_a}\mathbb{1}\right) q\right)_{q=(u,d)^{\mathrm{T}}} + \left(\bar{q}\gamma^\mu\gamma_5 c_q \frac{\partial_\mu a}{2f_a} q\right)_{q=(s,c,b,t)^{\mathrm{T}}}$$

QCD with Axions

$$\mathcal{L}_{a\text{-}q} = -\left(\bar{q}_L \mathcal{M}_a q_R + \text{h.c.}\right) + \left(\bar{q}\gamma^\mu\gamma_5 \left(\underbrace{c_{u-d}\frac{\partial_\mu a}{2f_a}\tau_3}_{a_\mu} + \underbrace{c_{u+d}\frac{\partial_\mu a}{2f_a}\mathbb{1}}_{a^{(s)}_{\mu,u+d}} \right) q \right)_{q=(u,d)^{\mathrm{T}}}$$

$$+ \left(\bar{q}\gamma^\mu\gamma_5 \underbrace{c_q \frac{\partial_\mu a}{2f_a}\mathbb{1}}_{a^{(s)}_{\mu,q}} q \right)_{q=(s,c,b,t)^{\mathrm{T}}}$$

with

$$c_{u\pm d} = \frac{1}{2}\left(X_u \pm X_d - \frac{1 \pm z}{1+z+w}\right)$$

$$c_s = X_s - \frac{w}{1+z+w}\,, \qquad c_{c,b,t} = X_{c,b,t}$$

$\hookrightarrow$ SU(3) case: later!

Heavy baryon chiral perturbation theory with Axions I

General structure of the coupling

General meson-baryon Lagrangian

- Chiral Lagrangians are organized with respect to chiral orders

$$\mathcal{L}_{MB} = \mathcal{L}_{MB}^{(1)} + \mathcal{L}_{MB}^{(2)} + \mathcal{L}_{MB}^{(3)} + \cdots + \mathcal{L}_{M}^{(2)} + \mathcal{L}_{M}^{(4)} + \ldots$$

- baryon fields B, e.g. in SU(2) $B = N = \binom{p}{n}$

  meson fields in $u = \sqrt{U} = \exp\left(i\frac{\Phi}{2F_p}\right)$, e.g. in SU(2) $\Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$

- note: in HB limit, $\mathcal{L}_{MB}$ contains expansion in $1/m_B$. Dirac bilinears entirely expressed by means of v_μ and $S_\mu = \frac{i}{2}\gamma_5\sigma_{\mu\nu}v^\nu$. Properties:

$$\{S_\mu, S_\nu\} = \frac{1}{2}(v_\mu v_\nu - g_{\mu\nu}) \qquad [S_\mu, S_\nu] = i\epsilon_{\mu\nu\rho\sigma}v^\rho S^\sigma$$

$$v^2 = 1 \qquad (S \cdot v) = 0 \qquad S^2 = \frac{1-d}{4}$$

$\hookrightarrow$ again: recall Ulf-G. Meißner's Lecture

General meson-baryon Lagrangian

- Chiral Lagrangians are organized with respect to chiral orders

$$\mathcal{L}_{MB} = \mathcal{L}_{MB}^{(1)} + \mathcal{L}_{MB}^{(2)} + \mathcal{L}_{MB}^{(3)} + \cdots + \mathcal{L}_{M}^{(2)} + \mathcal{L}_{M}^{(4)} + \ldots$$

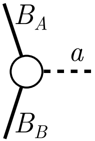
- axion in the following objects:
  - $u_\mu = i\left[u^\dagger \partial_\mu u - u \partial_\mu u^\dagger - i u^\dagger a_\mu u - i u a_\mu u^\dagger\right]$
  - $u_{\mu,i} = 2 a_{\mu,i}^{(s)}$
  - $\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$, with $\chi = 2 B_0 \mathcal{M}_a$
  - $\Gamma_\mu = \frac{1}{2}\left[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger - i u^\dagger a_\mu u + i u a_\mu u^\dagger\right]$
    $\hookrightarrow$ in SU(2): $\mathcal{D}_\mu N = \partial_\mu N + \Gamma_\mu N$
    $\hookrightarrow$ in SU(3): $[\mathcal{D}_\mu, B] = \partial_\mu B + [\Gamma_\mu, B]$

- recall:
  - $a_\mu,\ a_{\mu,i}^{(s)} = \mathcal{O}(1/f_a)$
  - $\mathcal{M}_a = \exp\left(i \frac{a}{f_a} \mathcal{Q}_a\right) \mathcal{M}_q = \mathcal{M}_q + i \frac{a}{f_a} \frac{1}{\left\langle \mathcal{M}_q^{-1} \right\rangle} + \mathcal{O}(1/f_a^2)$

$\hookrightarrow$ axion-baryon coupling contains expansion in $1/f_a$

General form of the axion-baryon coupling

- Conclusion:



$$= G_{aAB}\ (S \cdot q)$$

with

$$G_{aAB} = -\frac{1}{f_a} g_{aAB} + \mathcal{O}\left(\frac{1}{f_a^2}\right)$$

- expansion in chiral power counting

$$g_{aAB} = \underbrace{g_{aAB}^{(1)}}_{\text{LO,tree}} + \underbrace{g_{aAB}^{(2)}}_{\text{NLO},1/m_B} + \underbrace{g_{aAB}^{(3)}}_{\text{NNLO},1/m_B^2,\text{one-loop}} + \ldots$$

- in SU(2): $G_{aAB},\ g_{aAB} \to G_{aNN},\ g_{aNN}$
- in SU(3): $G_{aAB},\ g_{aAB}$ with SU(3) indices A, B in physical basis

Heavy baryon chiral perturbation theory with Axions II

SU(2)

Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\pi N} = \bar{N}\Big\{ & i\,(v\cdot\mathcal{D}) + g_A\,(S\cdot u) + g_0^i\,(S\cdot u_i) \\
& + \mathcal{L}_{1/m_N} + \mathcal{L}_{1/m_N^2} \\
& + d_{16}(\lambda)\,(S\cdot u)\,\langle\chi_+\rangle + d_{16}^i(\lambda)\,(S\cdot u_i)\,\langle\chi_+\rangle + d_{17}S^\mu\,\langle u_\mu\chi_+\rangle \\
& + id_{18}S^\mu\,[\mathcal{D}_\mu,\chi_-] + id_{19}S^\mu\,[\mathcal{D}_\mu,\langle\chi_-\rangle] \\
& + \tilde{d}_{25}(\lambda)\left(v\cdot\overleftarrow{\mathcal{D}}\right)(S\cdot u)\,(v\cdot\mathcal{D}) + \tilde{d}_{25}^i(\lambda)\left(v\cdot\overleftarrow{\mathcal{D}}\right)(S\cdot u_i)\,(v\cdot\mathcal{D}) \\
& + \tilde{d}_{29}(\lambda)\,(S^\mu\,[(v\cdot\mathcal{D})\,,u_\mu]\,(v\cdot\mathcal{D}) + \text{h.c.}) \\
& + \tilde{d}_{29}^i(\lambda)\,(S^\mu\,[(v\cdot\mathcal{D})\,,u_{\mu,i}]\,(v\cdot\mathcal{D}) + \text{h.c.})\Big\}N
\end{aligned}
$$

Note: scale-dependent LECs needed for renormalization

Leading order and $1/m_N$ expansion

- we set

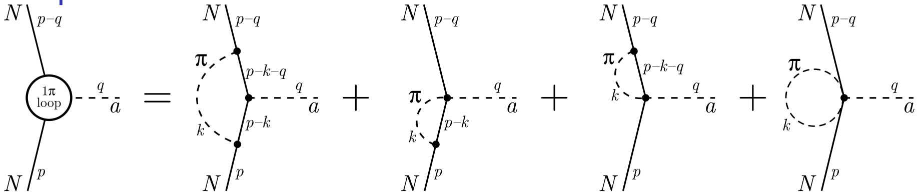
$$g_a = g_A c_{u-d} \tau_3 + g_0^i c_i \mathbb{1}$$

- then we get the leading order result including $1/m_N$ expansion

$$g_{aNN}^{\text{LO, tree}} + g_{aNN}^{1/m_N} = g_a \left(1 + \frac{q_0}{2m_N} + \frac{q_0^2}{4m_N^2} \right)$$

- note: given in nucleon rest frame, i.e. $v = (1, 0, 0, 0)^{\mathsf{T}}$ and $q_0 = (v \cdot q) \ll m_N$

Loop contributions



(a) (b$_1$) (b$_2$) (c)

$\hookrightarrow$ diagrams (b$_1$) and (b$_2$) vanish because of $(S \cdot v) = 0$, e.g.

$$(\mathrm{b}_1) \propto S^\mu \frac{1}{i} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{k_\mu}{(M_\pi^2 - k^2 + i\eta)(v \cdot (p-k) + i\eta)} = (S \cdot v)\ J_1(\omega) = 0$$

$\hookrightarrow$ only diagrams (a) and (c) contribute

$\hookrightarrow$ Divergences and scale dependencies (picked up by LECs)

$$L(\lambda) = \frac{\lambda^{d-4}}{(4\pi)^2} \left(\frac{1}{d-4} - \frac{1}{2} \left[\ln(4\pi) + \Gamma'(1) + 1 \right] \right),$$

Results: Axion-Nucleon coupling at $\mathcal{O}(p^3)$

$$
\begin{aligned}
g_{aNN} =& g_a \left(1 + \frac{q_0}{2m_N} + \frac{q_0^2}{4m_N^2}\right) + \frac{\hat{g}_a}{6}\left(\frac{g_A M_\pi}{4\pi F_\pi}\right)^2 \\
& \times \left[-1 + \left(\frac{q_0}{M_\pi}\right)^2 + \frac{2}{q_0 M_\pi^2}\left(\frac{\pi M_\pi^3}{2} - \left(M_\pi^2 - q_0^2\right)^{\frac{3}{2}} \arccos\frac{q_0}{M_\pi}\right)\right] \\
& + 4M_\pi^2\left[\left(\bar{d}_{16}\tau_3 + d_{17}\frac{m_u - m_d}{m_u + m_d}\right)c_{u-d} + \bar{d}_{16}^i c_i - (d_{18} + 2d_{19})\frac{m_u m_d}{(m_u + m_d)^2}\right]
\end{aligned}
$$

Recall:

$$g_a = g_A c_{u-d}\tau_3 + g_0^i c_i \mathbb{1}$$

and here also

$$\hat{g}_a = \tau_i g_a \tau_j \delta_{ij} = -g_A c_{u-d}\tau_3 + 3g_0^i c_i \mathbb{1}\ .$$

Results: Closer look at leading axion-nucleon coupling

- We saw that

$$g_{aNN}^{(1)} = g_a = g_A c_{u-d} \tau_3 + g_0^i c_i \mathbb{1}$$

where the index $i = (u+d, s, c, b, t)$ runs over all isoscalar quark combinations.

- g_A and the g_0^i's can be matched to the nucleon matrix elements, i. e.

$$\begin{aligned} g_A &= \Delta u - \Delta d \\ g_0^{u+d} &= \Delta u + \Delta d \\ g_0^q &= \Delta q \ , \ \text{for } q = s, c, b, t \end{aligned}$$

where $s^\mu \Delta q = \langle p | \bar{q} \gamma^\mu \gamma_5 q | p \rangle$, with s^μ the spin of the proton.

- determined from Lattice QCD Aoki et al., Eur. Phys. J. C **80** (2020) 113

$$\Delta u = 0.847(50) \qquad \Delta d = -0.407(34) \qquad \Delta s = -0.035(13)$$
$$z = 0.485(19) \qquad w = 0.025(1)$$

Results: Axion-Nucleon coupling

- We then get

$$g_{app}^{(1)} = -\frac{\Delta u + z\Delta d + w\Delta s}{1+z+w} + \Delta u X_u + \Delta d X_d + \sum_{q=\{s,c,b,t\}} \Delta q X_q$$

$$g_{ann}^{(1)} = -\frac{z\Delta u + \Delta d + w\Delta s}{1+z+w} + \Delta d X_u + \Delta u X_d + \sum_{q=\{s,c,b,t\}} \Delta q X_q$$

- or inserting the numerical values

$$g_{app}^{(1)} = -0.430(36) + 0.847(50)X_u - 0.407(34)X_d - 0.035(13)X_s$$
$$g_{ann}^{(1)} = -0.002(30) - 0.407(34)X_u + 0.847(50)X_d - 0.035(13)X_s$$

- now including loops and NNLO contributions:

$$g_{app} = -0.430(50) + 0.862(75)X_u - 0.417(66)X_d - 0.035(54)X_s$$
$$g_{ann} = +0.007(46) - 0.417(66)X_u + 0.862(75)X_d - 0.035(54)X_s$$

$\hookrightarrow$ remember: the X_q are model dependent!

Heavy baryon chiral perturbation theory with Axions III

Brief look at SU(3)

Axion-quark Lagrangian

- Recall:

$$\mathcal{L}_{a\text{–}q} = -\left(\bar{q}_L \mathcal{M}_a q_R + \text{h.c.}\right) + \bar{q}\gamma^\mu\gamma_5 \frac{\partial_\mu a}{2f_a}\left(\mathcal{X}_q - \mathcal{Q}_a\right) q$$

- This time we write

$$\begin{aligned}\mathcal{L}_{a\text{–}q} = &-\left(\bar{q}_L \mathcal{M}_a q_R + \text{h.c.}\right)\\ &+ \left(\bar{q}\gamma^\mu\gamma_5 \frac{\partial_\mu a}{2f_a}\left(c^{(1)}\mathbb{1} + c^{(3)}\lambda_3 + c^{(8)}\lambda_8\right) q\right)_{q=(u,d,s)^\mathsf{T}}\\ &+ \sum_{q=\{c,b,t\}} \left(\bar{q}\gamma^\mu\gamma_5 \frac{\partial_\mu a}{2f_a} X_q q\right)\end{aligned}$$

with

$$\begin{aligned}c^{(1)} &= \frac{1}{3}\left(X_u + X_d + X_s - 1\right)\\ c^{(3)} &= \frac{1}{2}\left(X_u - X_d - \frac{1-z}{1+z+w}\right)\\ c^{(8)} &= \frac{1}{2\sqrt{3}}\left(X_u + X_d - 2X_s - \frac{1+z-2w}{1+z+w}\right)\end{aligned}$$

Leading order HBCHPT

$$\mathcal{L}_{MB}^{(1)} = \left\langle i\bar{B}v^{\mu}\left[\mathcal{D}_{\mu}, B\right]\right\rangle + D\left\langle \bar{B}S^{\mu}\left\{u_{\mu}, B\right\}\right\rangle + F\left\langle \bar{B}S^{\mu}\left[u_{\mu}, B\right]\right\rangle + D^{i}\left\langle \bar{B}S^{\mu}u_{\mu,i}B\right\rangle$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma_3 + \frac{1}{\sqrt{6}}\Lambda_8 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma_3 + \frac{1}{\sqrt{6}}\Lambda_8 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda_8 \end{pmatrix}$$

$$\Phi = \sqrt{2}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

$\hookrightarrow$ Note: Physical Σ^0, Λ, π^0, and η are mixed states parameterized by the mixing angle ϵ with

$$\tan 2\epsilon = \frac{\langle\lambda_3\mathcal{M}_q\rangle}{\langle\lambda_8\mathcal{M}_q\rangle} \; .$$

Leading order coupling

$$\mathcal{L}_{MB}^{(1)} = \left\langle i\bar{B}v^{\mu}\left[\mathcal{D}_{\mu}, B\right]\right\rangle + D\left\langle \bar{B}S^{\mu}\left\{u_{\mu}, B\right\}\right\rangle + F\left\langle \bar{B}S^{\mu}\left[u_{\mu}, B\right]\right\rangle + D^{i}\left\langle \bar{B}S^{\mu}u_{\mu,i}B\right\rangle$$

This Lagrangian directly leads to the leading order tree level coupling:

$$g_{aAB}^{(1)} = \frac{1}{2}\Big\{D\left(c^{(3)}\left\langle\tilde{\lambda}_A^{\dagger}\left\{\lambda_3, \tilde{\lambda}_B\right\}\right\rangle + c^{(8)}\left\langle\tilde{\lambda}_A^{\dagger}\left\{\lambda_8, \tilde{\lambda}_B\right\}\right\rangle\right)$$
$$+ F\left(c^{(3)}\left\langle\tilde{\lambda}_A^{\dagger}\left[\lambda_3, \tilde{\lambda}_B\right]\right\rangle + c^{(8)}\left\langle\tilde{\lambda}_A^{\dagger}\left[\lambda_8, \tilde{\lambda}_B\right]\right\rangle\right) + 2c_i D^i \delta_{AB}\Big\}$$

Note: here physical basis based on a set of traceless, non-Hermitian matrices $\tilde{\lambda}_A$, $A = \{1, \ldots, 8\}$:

$$B = \frac{1}{\sqrt{2}}\sum_A \tilde{\lambda}_A B_A$$

such that

$$B_1 = \Sigma^+ \qquad B_2 = \Sigma^- \qquad B_3 = \Sigma^0 \qquad B_4 = p$$
$$B_5 = \Xi^- \qquad B_6 = n \qquad B_7 = \Xi^0 \qquad B_8 = \Lambda$$

Results in SU(3) HBCHPT

$$
\begin{aligned}
g^{(1)}_{a\Sigma^+\Sigma^+} &= -0.543(34) + 0.847(50)X_u - 0.035(13)X_d - 0.407(34)X_s \\
g^{(1)}_{a\Sigma^-\Sigma^-} &= -0.242(21) - 0.035(13)X_u + 0.847(50)X_d - 0.407(34)X_s \\
g^{(1)}_{a\Sigma^0\Sigma^0} &= -0.396(25) + 0.417(25)X_u + 0.395(25)X_d - 0.407(35)X_s \\
g^{(1)}_{app} &= -0.430(36) + 0.847(50)X_u - 0.407(34)X_d - 0.035(13)X_s \\
g^{(1)}_{a\Xi^-\Xi^-} &= 0.140(15) - 0.035(13)X_u - 0.407(34)X_d + 0.847(50)X_s \\
g^{(1)}_{ann} &= -0.002(30) - 0.407(34)X_u + 0.847(50)X_d - 0.035(13)X_s \\
g^{(1)}_{a\Xi^0\Xi^0} &= 0.267(23) - 0.407(34)X_u - 0.035(13)X_d + 0.847(50)X_s \\
g^{(1)}_{a\Lambda\Lambda} &= 0.126(25) - 0.147(25)X_u - 0.125(25)X_d + 0.677(35)X_s \\
g^{(1)}_{a\Sigma^0\Lambda} &= -0.153(10) + 0.463(25)X_u - 0.476(25)X_d + 0.013(1)X_s
\end{aligned}
$$

Results in SU(3) HBCHPT

Process	$g_{aAB}^{(1)}$				
	KSVZ	DFSZ			
		general	$\sin^2\beta = 0$	$\sin^2\beta = \frac{2}{3}$	$\sin^2\beta = 1$
$\Sigma^+ \to \Sigma^+ + a$	$-0.543(34)$	$-0.690(36) + 0.430(21)\sin^2\beta$	$-0.690(36)$	$-0.404(36)$	$-0.261(38)$
$\Sigma^- \to \Sigma^- + a$	$-0.242(21)$	$-0.095(29) - 0.158(21)\sin^2\beta$	$-0.095(29)$	$-0.201(23)$	$-0.254(22)$
$\Sigma^0 \to \Sigma^0 + a$	$-0.396(25)$	$-0.400(29) + 0.143(12)\sin^2\beta$	$-0.400(29)$	$-0.305(27)$	$-0.257(27)$
$p \to p + a$	$-0.430(36)$	$-0.577(38) + 0.430(21)\sin^2\beta$	$-0.577(38)$	$-0.291(38)$	$-0.147(39)$
$\Xi^- \to \Xi^- + a$	$0.140(15)$	$0.287(25) - 0.158(21)\sin^2\beta$	$0.287(25)$	$0.181(17)$	$0.128(16)$
$n \to n + a$	$-0.002(30)$	$0.269(34) - 0.406(21)\sin^2\beta$	$0.269(34)$	$-0.002(31)$	$-0.138(22)$
$\Xi^0 \to \Xi^0 + a$	$0.267(23)$	$0.531(29) - 0.406(21)\sin^2\beta$	$0.531(29)$	$0.267(25)$	$0.131(26)$
$\Lambda \to \Lambda + a$	$0.126(25)$	$0.310(29) - 0.233(12)\sin^2\beta$	$0.310(29)$	$0.155(27)$	$0.077(26)$
$\Sigma^0 \to \Lambda + a$ $\Lambda \to \Sigma^0 + a$	$-0.153(10)$	$-0.308(13) + 0.309(16)\sin^2\beta$	$-0.308(13)$	$-0.102(11)$	$0.000(13)$

Summary

Summary

In this talk:

- Have determined g_{app} and g_{ann} up to one-loop order in SU(2) HBCHPT
- Have determined g_{aAB} for $A, B \in$ baryon octet: coupling to hyperons of similar strength as to nucleons
- in the most unfavorable case g_{ann} might vanish
- strong model dependence of g_{aAB} due to model dependent axion-quark coupling
- remember: f_a is still the biggest unknown! $\rightarrow$ axion-baryon coupling is $G_{aAB} = -\frac{1}{f_a} g_{aAB} + \mathcal{O}\left(\frac{1}{f_a^2}\right)$, not only g_{aAB}
- in any case: uncertainties at NNLO increase due to unknown LECs