Symmetries at Null Boundaries: 3-dimensional gravity

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Outline

- Solution phase space near a null boundary in 3-dimensional Einstein gravity
- Null boundary symmetries, surface charges and algebra
- Change of slicing on solution phase space
- Einstein gravity in higher dimensions

Gauge Theories in Presence of Boundaries

Gauge transformations in presence of **boundaries** fall in two classes

- Trivial gauge transformations $\rightarrow$ vanishing charge
- Physical or Large gauge transformations $\rightarrow$ non-vanishing charge

Motivations:

- Black hole horizons are null.
- Boundaries of asymptotically flat spacetimes are null.
- Null hypersurfaces are one-way membranes.

Solution Phase Space

Einstein gravity in 3-dimensions:

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\,\sqrt{-g}(R-2\Lambda)$$

The line-element:

$$ds^2 = -V dv^2 + 2\eta dv dr + g\,(d\phi + U dv)^2$$

Null boundary: $r=0$.

Expansion around $r=0$:

$$\begin{aligned}
V &= 2\left(\eta\kappa - \partial_v\eta + \mathcal{U}\partial_\phi\eta\right) r + \mathcal{O}(r^2)\,,\\
U &= \mathcal{U} - \frac{\eta}{\Omega^2}\Upsilon r + \mathcal{O}(r^2)\,,\\
g &= \Omega - 2\eta\lambda\, r + \mathcal{O}(r^2)\,.
\end{aligned}$$

Solution phase space: $\{\eta, \Omega, \Upsilon\}$.

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Null Boundary Symmetry

Null Boundary Symmetries:

$$\xi = T\,\partial_v + r\,(\partial_v T - \mathcal{U}\partial_\phi T - W)\,\partial_r + \left(Y - r\frac{\eta}{\Omega}\partial_\phi T\right)\partial_\phi + \mathcal{O}(r^2)$$

where $T = T(v,\phi)$, $W = W(v,\phi)$ and $Y = Y(v,\phi)$.

Variation of Fields:

$$\begin{aligned}
\delta_\xi \Omega &= T\partial_v\Omega + \mathcal{U}\Omega\partial_\phi T + \partial_\phi(Y\Omega)\,,\\
\delta_\xi \eta &= 2\eta\partial_v T + T\partial_v\eta - 2\eta\mathcal{U}\partial_\phi T - W\eta + Y\partial_\phi\eta\,,\\
\delta_\xi \Upsilon &= T\partial_v\Upsilon + 2\Upsilon\mathcal{U}\partial_\phi T + 2\Upsilon\partial_\phi Y + Y\partial_\phi\Upsilon + \Omega(\partial_\phi W - \Gamma\partial_\phi T)\,.
\end{aligned}$$

Algebra of Null Boundary Symmetries (NBS):

$$\text{Diff}(\mathcal{N}) \in \text{Weyl}(\mathcal{N})$$

Surface Charges

Using the covariant phase space method (CPSM), surface charges become

$$\not{\delta} Q_\xi = \frac{1}{16\pi G}\int_{\mathcal{N}_v} d\phi\left(W\delta\Omega + Y\delta\Upsilon + T\not{\delta}\mathcal{A}\right)\,,$$

with

$$\not{\delta}\mathcal{A} = -2\Omega\delta\Theta + \Omega\Theta\frac{\delta\eta}{\eta} - \Gamma\delta\Omega + \mathcal{U}\delta\Upsilon\,.$$

where

$$\Theta := \frac{1}{\Omega}\left(\partial_v\Omega - \partial_\phi(\mathcal{U}\Omega)\right), \qquad \Gamma := -2\kappa + 2\Theta + \frac{\partial_v\eta}{\eta} - \frac{\mathcal{U}\partial_\phi\eta}{\eta}$$

Charge algebra: The surface charge algebra is exactly the same as NBS algebra without any central extension term.

Change of Slicing and Heisenberg algebra

Change of slicing:

$$\tilde{W} = W - \Gamma T - (Y + T\mathcal{U})\,\partial_\phi \mathcal{P}\,,$$
$$\tilde{T} = \Omega\Theta T + \partial_\phi[\Omega(Y + T\mathcal{U})]\,,$$
$$\tilde{Y} = Y + T\mathcal{U}\,.$$

Charge expression in new slicing:

$$\delta\tilde{Q} = \frac{1}{16\pi G}\int_{\mathcal{N}_v} d\phi \left(\tilde{W}\delta\Omega + \tilde{Y}\delta\mathcal{J} + \tilde{T}\delta\mathcal{P}\right)$$

where

$$\mathcal{J} = \Upsilon + \Omega\partial_\phi\mathcal{P}, \qquad \mathcal{P} = \ln\frac{\eta}{\Theta^2}\,.$$

Surface Charges:

- Ω: **entropy aspect charge**
- $\mathcal{J}$: **angular momentum aspect charge**
- $\mathcal{P}$: **expansion aspect charge**

Charge Algebra

Transformation laws are as

$$\delta_\xi \Omega = \tilde{T}\,,$$
$$\delta_\xi \mathcal{P} \approx -\tilde{W}\,,$$
$$\delta_\xi \mathcal{J} \approx 2\mathcal{J}\partial_\phi \tilde{Y} + \tilde{Y}\partial_\phi \mathcal{J}\,.$$

The charge algebra is the **Heisenberg $\oplus$ Witt algebra**

$$\{\Omega(v,\phi),\Omega(v,\phi')\} = \{\mathcal{P}(v,\phi),\mathcal{P}(v,\phi')\} = 0,$$
$$\{\Omega(v,\phi),\mathcal{P}(v,\phi')\} = 16\pi G\delta\left(\phi-\phi'\right),$$
$$\{\mathcal{J}(v,\phi),\Omega(v,\phi')\} = \{\mathcal{J}(v,\phi),\mathcal{P}(v,\phi')\} = 0,$$
$$\{\mathcal{J}(v,\phi),\mathcal{J}(v,\phi')\} = 16\pi G\left(\mathcal{J}(v,\phi')\partial_\phi - \mathcal{J}(v,\phi)\partial'_\phi\right)\delta\left(\phi-\phi'\right).$$

Non-uniqueness and Existence of Genuine Slicings!

Non-uniqueness:

The genuine (integrable) slicings are not ***unique****.*

Existence:

Integrability Conjecture*: In the absence of genuine flux passing through the boundary, there are specific slicings,* **genuine slicings***, such that the charge variation becomes integrable.*

3**-dimensional Einstein gravity**:

No bulk propagating d.o.f $\rightarrow$ *Integrable charge expression.*

Higher Dimensions

Solution Phase Space: **boundary $\oplus$ bulk d.o.fs**

- **boundary phase space**:
  1. entropy aspect charge: 1
  2. angular momentum aspect charges: $D-2$
  3. expansion aspect charge: 1
- **bulk phase space**: $D(D-3)$ bulk degrees of freedom

Balance equation: $\frac{d}{dv}Q \sim -F$

Charge algebra: **Heisenberg $\oplus$ Diff($\mathcal{N}_v$)**

Thank You!