

THE 2021 RDP SCHOOL & WORKSHOP ON
“ASPECTS OF SYMMETRY”

Lattice QCD, Fundamental Symmetries, and Neutrinoless Double Beta Decay


ZOHREH DAVOUDI
UNIVERSITY OF MARYLAND

THREE USQCD WHITEPAPERS FROM 2019 SET THE GUIDELINES IN FUNDAMENTAL SYMMETRY PROGRAM:

 Springer Link

Regular Article - Theoretical Physics | [Published: 14 November 2019](#)

Lattice QCD and neutrino-nucleus scattering

[Andreas S. Kronfeld](#), [David G. Richards](#) , [William Detmold](#), [Rajan Gupta](#), [Huey-Wen Lin](#), [Keh-Fei Liu](#),
[Aaron S. Meyer](#), [Raza Sufian](#) & [Sergey Syritsyn](#)

[The European Physical Journal A](#) **55**, Article number: 196 (2019) | [Cite this article](#)

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Review | [Published: 14 November 2019](#)

The role of Lattice QCD in searches for violations of fundamental symmetries and signals for new physics

[USQCD Collaboration](#), [Vincenzo Cirigliano](#), [Zohreh Davoudi](#) , [Tanmoy Bhattacharya](#), [Taku Izubuchi](#),
[Phiala E. Shanahan](#), [Sergey Syritsyn](#) & [Michael L. Wagman](#)

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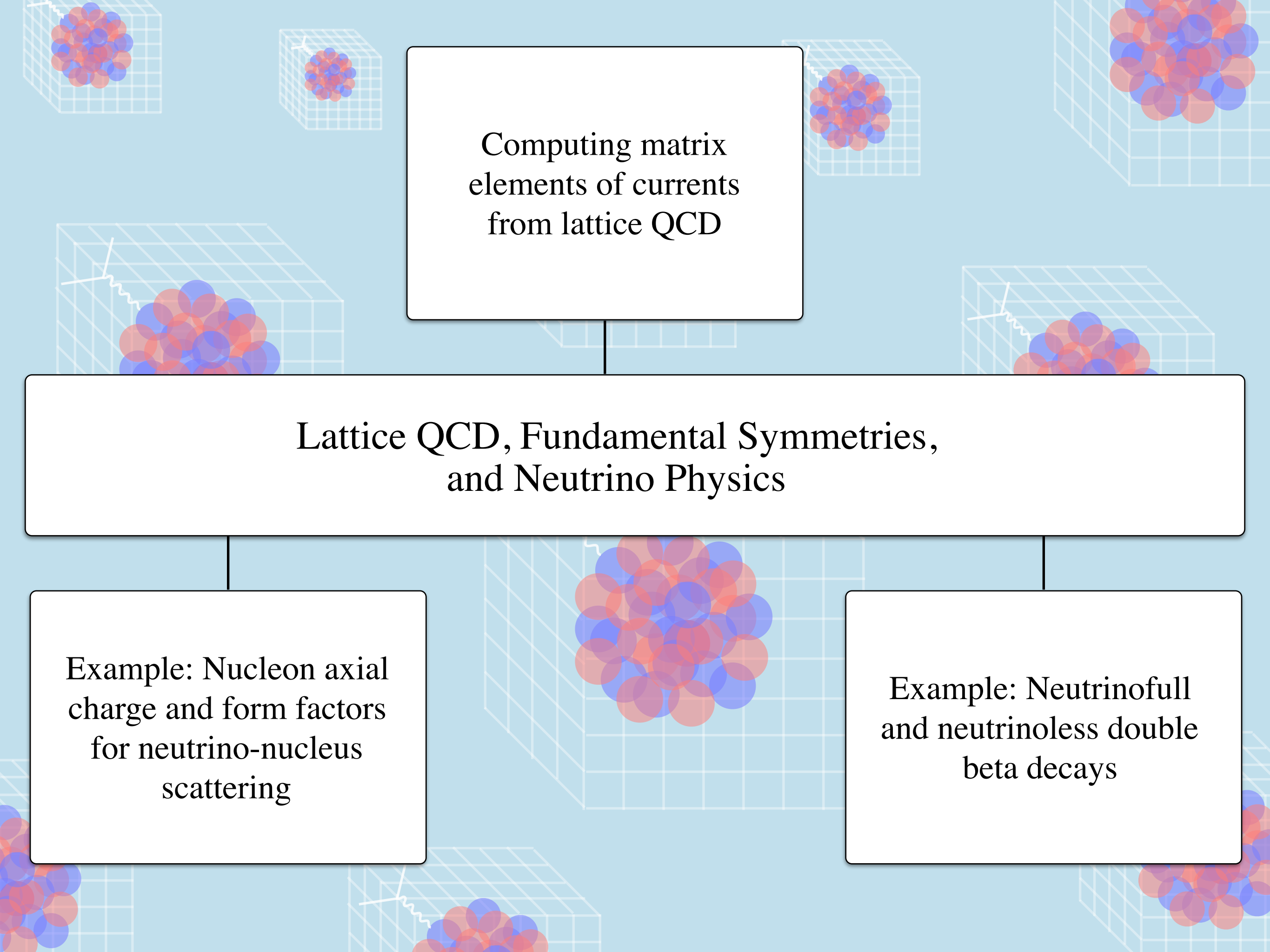
Review | [Published: 14 November 2019](#)

Opportunities for Lattice QCD in quark and lepton flavor physics

[USQCD Collaboration](#), [Christoph Lehner](#), [Stefan Meinel](#) , [Tom Blum](#), [Norman H. Christ](#), [Aida X. El-Khadra](#),
[Maxwell T. Hansen](#), [Andreas S. Kronfeld](#), [Jack Laiho](#), [Ethan T. Neil](#), [Stephen R. Sharpe](#) & [Ruth S. Van de Water](#)

[The European Physical Journal A](#) **55**, Article number: 195 (2019) | [Cite this article](#)

I WILL DISCUSS AT LEAST
TWO EXAMPLES...

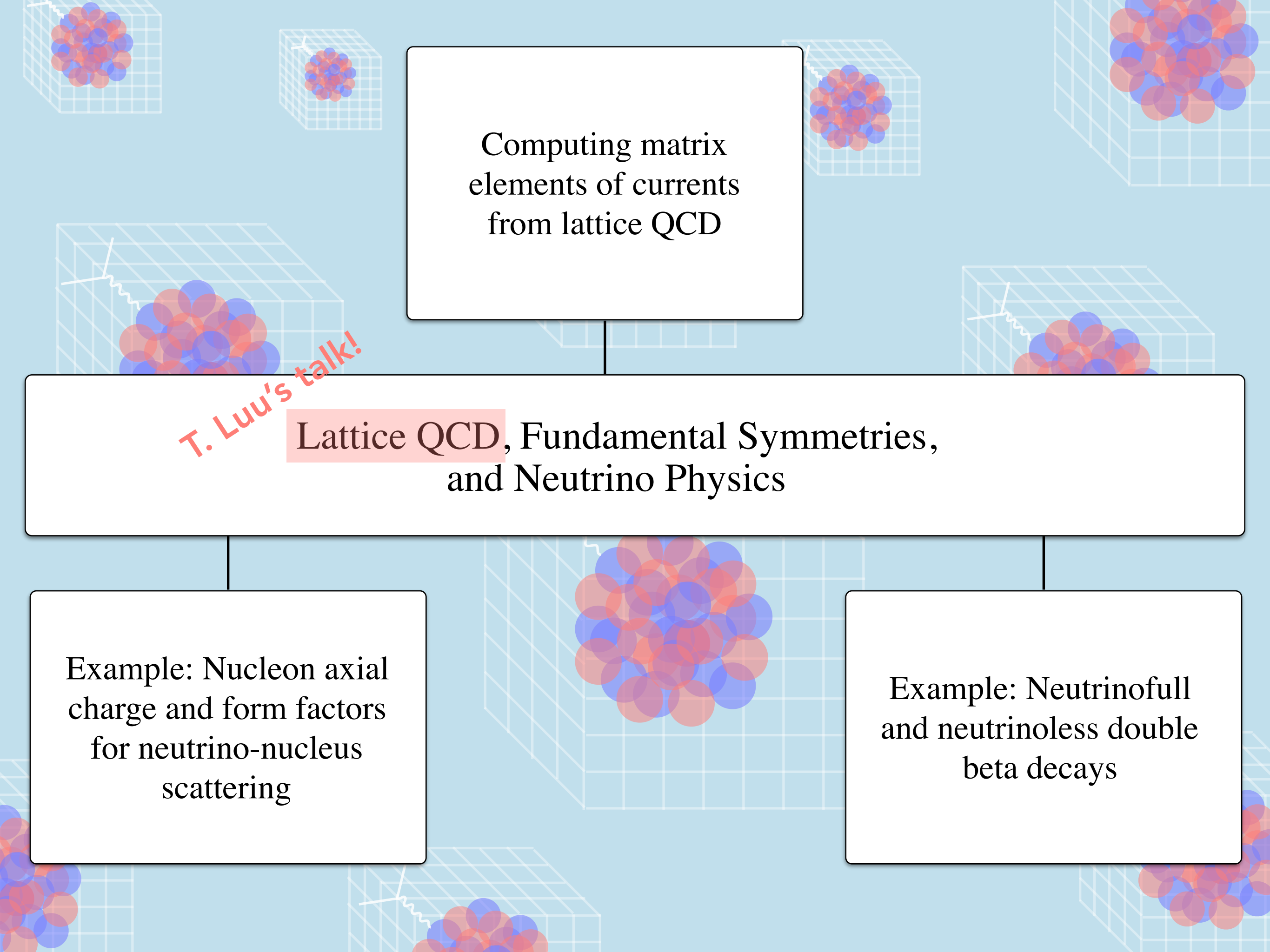


Computing matrix
elements of currents
from lattice QCD

Lattice QCD, Fundamental Symmetries, and Neutrino Physics

Example: Nucleon axial
charge and form factors
for neutrino-nucleus
scattering

Example: Neutrinoless
and neutrinoless double
beta decays



Computing matrix
elements of currents
from lattice QCD

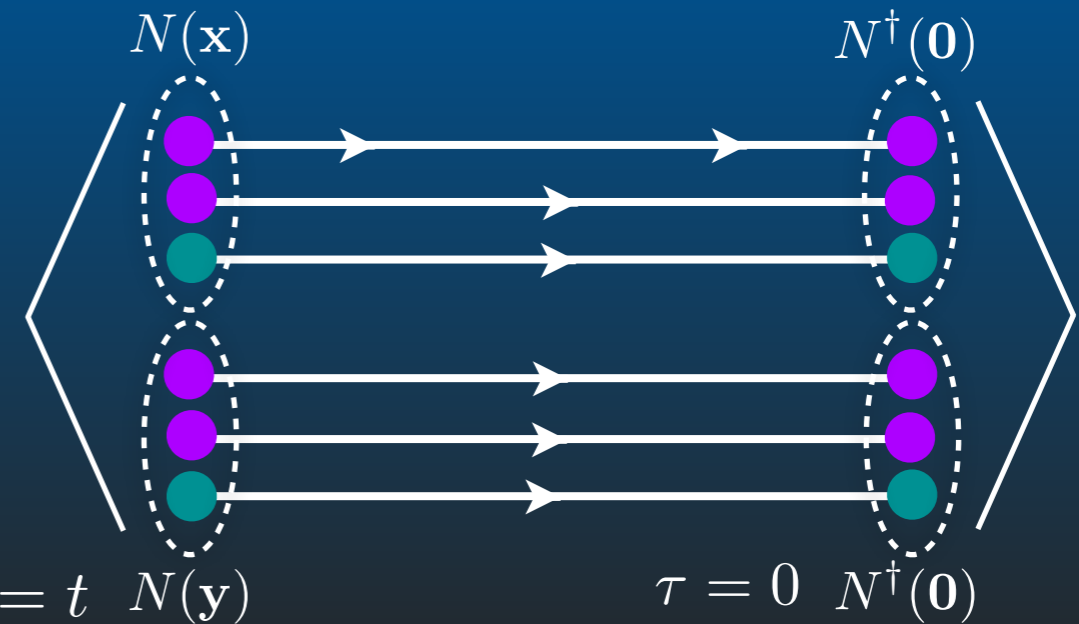
T. Luu's talk!

Lattice QCD, Fundamental Symmetries, and Neutrino Physics

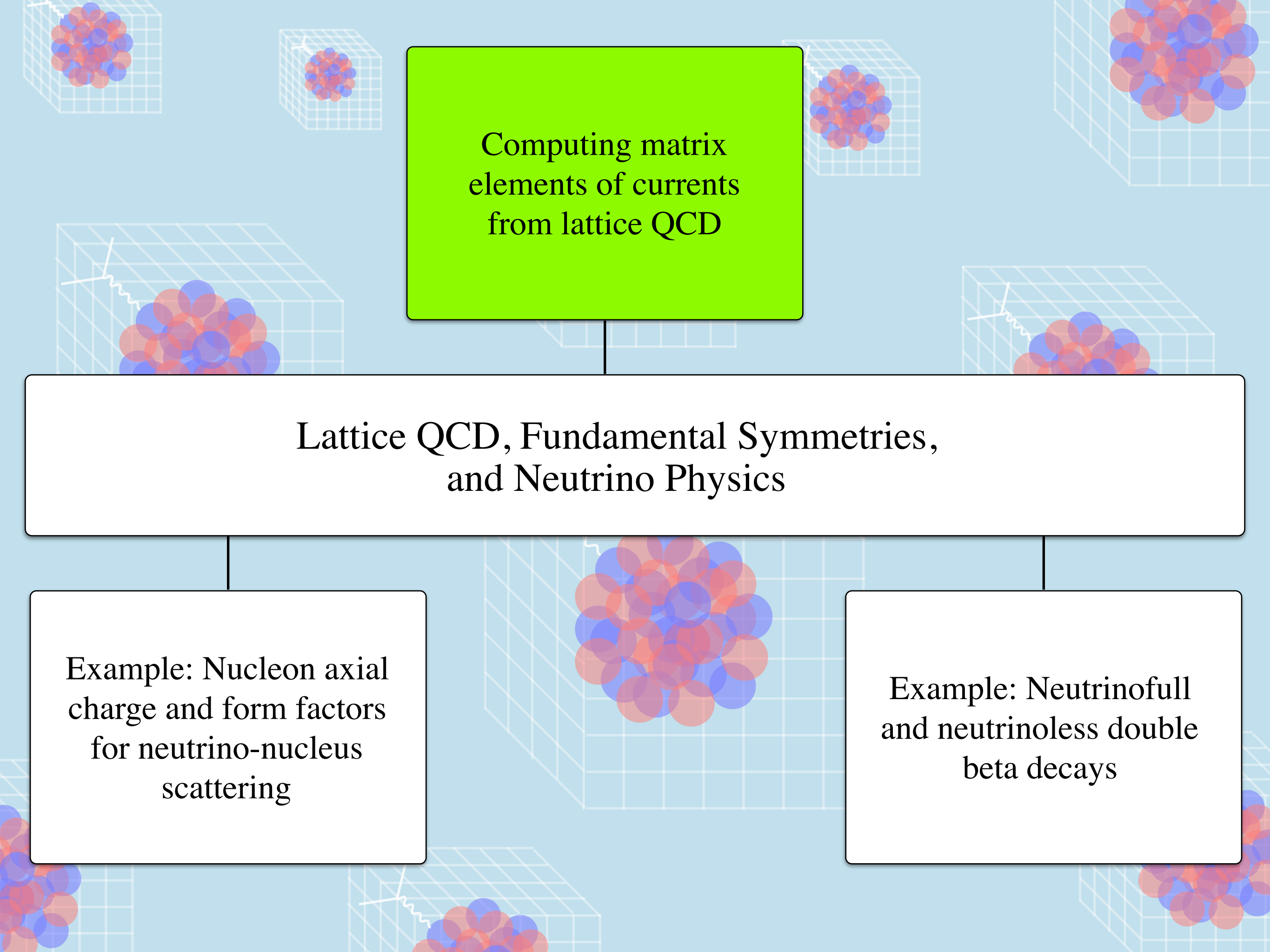
Example: Nucleon axial
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beta decays

The simplest objects to calculate using the lattice QCD method are two-point correlation functions...they give access to energy spectrum and more.

$$C(\mathbf{P}; t, t_0) = \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} \times$$


$$C_{\hat{O}, \hat{O}'}(\tau; \mathbf{d}) = \sum_{\mathbf{x}} e^{2\pi i \mathbf{d} \cdot \mathbf{x} / L} \langle 0 | \hat{O}'(\mathbf{x}, \tau) \hat{O}^\dagger(\mathbf{0}, 0) | 0 \rangle = \mathcal{Z}'_0 \mathcal{Z}_0^\dagger e^{-E^{(0)}\tau} + \mathcal{Z}'_1 \mathcal{Z}_1^\dagger e^{-E^{(1)}\tau} + \dots$$



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Computing matrix elements of currents from lattice QCD

Three(four)-point functions

For e.g., form factors, moments of structure functions, Compton amplitude, transition amplitudes

Background-field methods

For e.g., EM moments and polarizabilities, charge radius, form factors and transition amplitudes.

Feynman-Hellmann inspired methods

Similar to background fields. For e.g., axial charge, form factors, EM moments, transition amplitudes

Computing matrix elements of currents from lattice QCD

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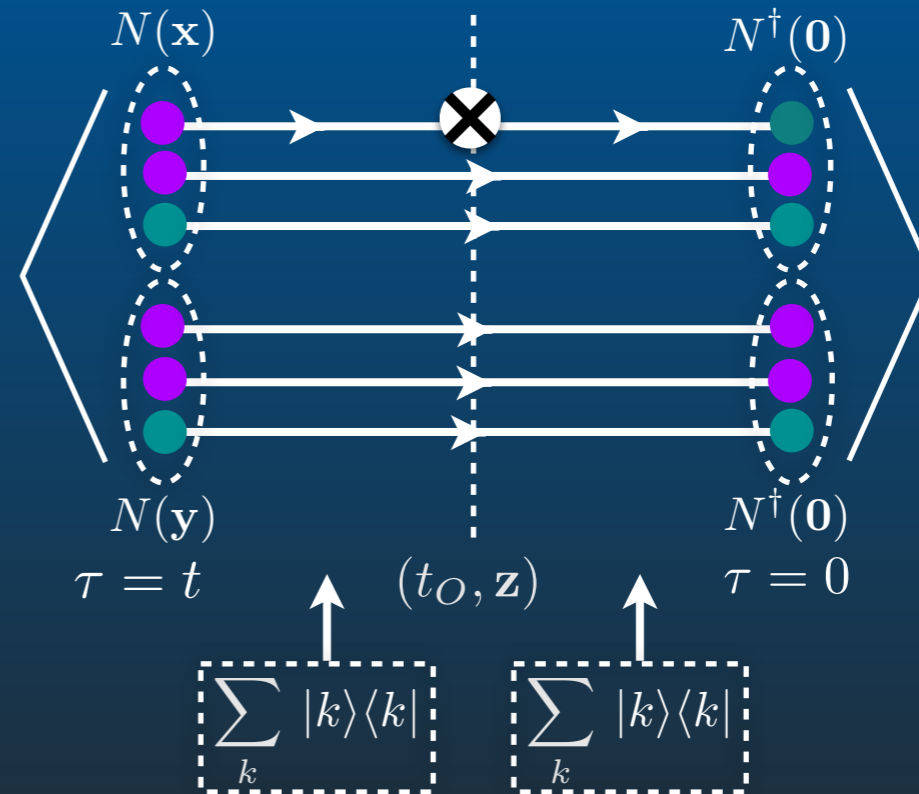
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Pictorially a three-point function looks like...

$$C(\mathbf{P}; t, t_0) = \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} \times$$



$$= Z_{0,pp}^{\text{src}} Z_{0,d}^{\text{snk}\dagger} e^{-E_{0,pp}t_0} e^{-E_{0,d}(t-t_0)} \langle pn | A | pp \rangle_L + \dots$$

Three points:

- i) Need to divide by appropriate two-point functions to cancel out unphysical overlap factors.
- ii) Need to renormalize the operator from lattice scheme to continuum scheme.
- iii) **Need to** turn the finite-volume matrix element to a physical transition amplitude.

Not essential for bound states!

Computing matrix elements of currents from lattice QCD

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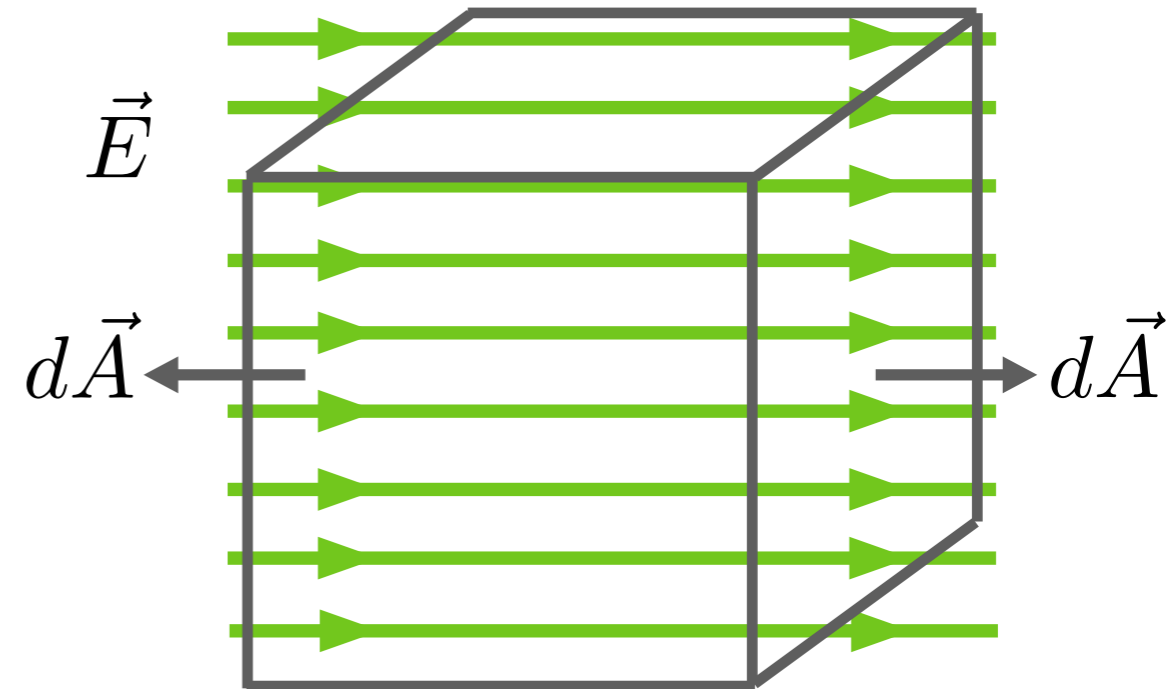
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Similar to background fields. For e.g., axial charge, form factors, EM moments, transition amplitudes

Background fields are non-dynamical, i.e., there will be no pair creation and annihilation in vacuum with a classical EM background field. This means the photon zero mode is no problem: it is absent from the calculation!

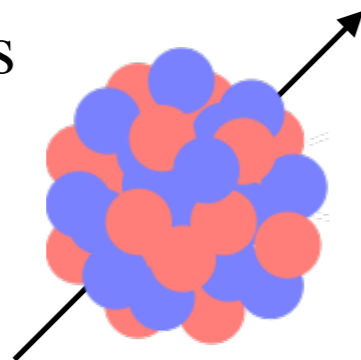


$$U^{(\text{QCD})} \rightarrow U^{(\text{QCD})} \times U^{(\text{QED})}$$

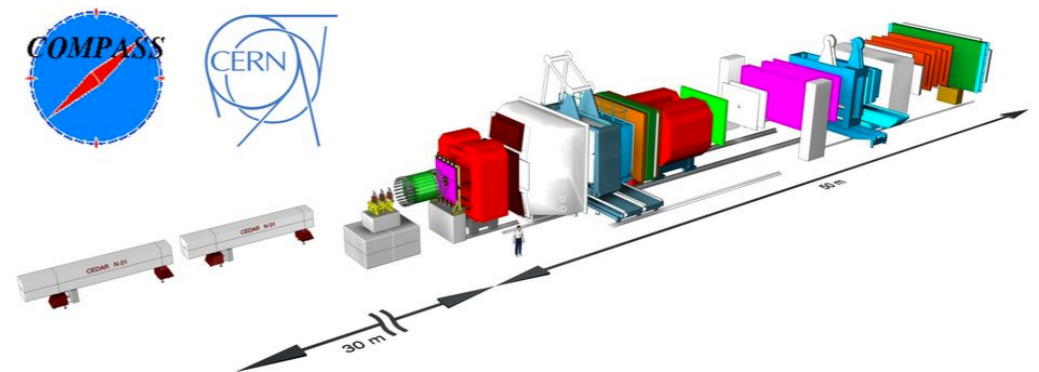
Modify the links when forming the quark propagators (quench approx).

Traditionally they are used for constraining the response of hadrons/nuclei to external probes:

Magnetic moments



Electric and magnetic polarizabilities



See e.g., BEANE et al (NPLQCD), Phys.Rev.Lett. 113 (2014) 25, 252001 and Phys.Rev. D92 (2015) 11, 114502. for nuclear-physics calculations.

Various other structure properties of hadrons and nuclei, as well as their transitions, can be studied using more complex background fields...just a two-point function calculation!

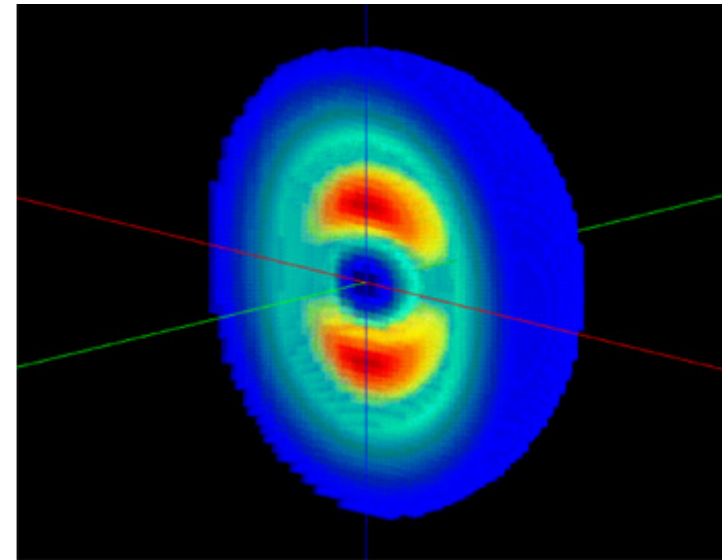
1) EM charge radius

ZD and Detmold,
Phys. Rev.
D 93,
014509
(2016).



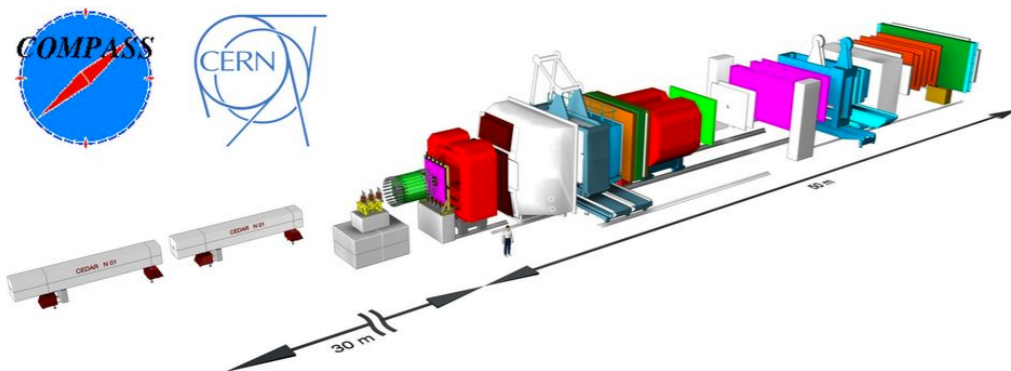
2) Electric quadrupole moment

ZD and Detmold, Phys. Rev. D 93, 014509 (2016).



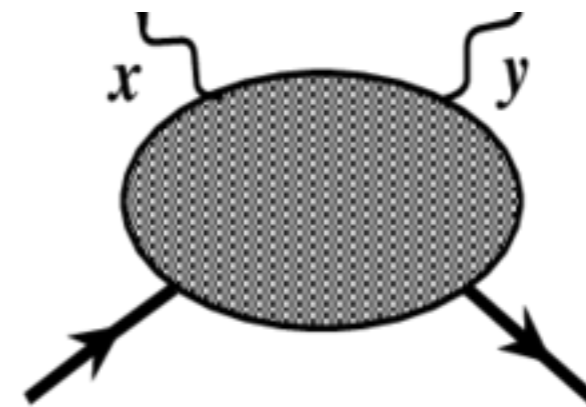
3) Form factors

Detmold, Phys. Rev. D 71, 054506 (2005).



4) Compton amplitude

Agadjanov, Meißner, Rusetsky,
Phys. Rev. D 95, 031502 (2017).



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Hamiltonian as a function
of a variable parameter

$$\hat{H}(\lambda) = \hat{H} + \lambda\hat{V}$$

Energy eigenvalue

$$\frac{dE_n}{d\lambda} = \frac{\langle \psi_n | \frac{d\hat{H}}{d\lambda} | \psi_n \rangle}{\langle \psi_n | \psi_n \rangle}$$

Energy eigenstate

Example: sigma term

$$m_q \left. \frac{\partial m_N}{\partial m_q} \right|_{m_q = m_q^{\text{phy}}} = \langle \mathcal{N} | m_q \bar{q} q | \mathcal{N} \rangle$$

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Generalization to correlation functions

$$C_\lambda(t) = \langle \lambda | \mathcal{O}(t) \mathcal{O}^\dagger(0) | \lambda \rangle = \frac{1}{\mathcal{Z}_\lambda} \int D\Phi e^{-S - S_\lambda} \mathcal{O}(t) \mathcal{O}^\dagger(0)$$

Just a 2pt function

$$S_\lambda = \lambda \int d^4x j(x)$$

Integrated matrix element

$$-\frac{\partial C_\lambda(t)}{\partial \lambda} \Big|_{\lambda=0} = -C(t) \int dt' \langle \Omega | \mathcal{J}(t') | \Omega \rangle + \int dt' \langle \Omega | T \{ \mathcal{O}(t) \mathcal{J}(t') \mathcal{O}^\dagger(0) \} | \Omega \rangle$$

$$\mathcal{J}(t) = \int d^3x j(t, \vec{x})$$

Buochard et al (CALLATT), Phys.Rev.D96,014504(2017).

Hamiltonian as a function of a variable parameter

$$\hat{H}(\lambda) = \hat{H} + \lambda \hat{V}$$

Energy eigenvalue

$$\frac{dE_n}{d\lambda} = \frac{\langle \psi_n | \frac{d\hat{H}}{d\lambda} | \psi_n \rangle}{\langle \psi_n | \psi_n \rangle}$$

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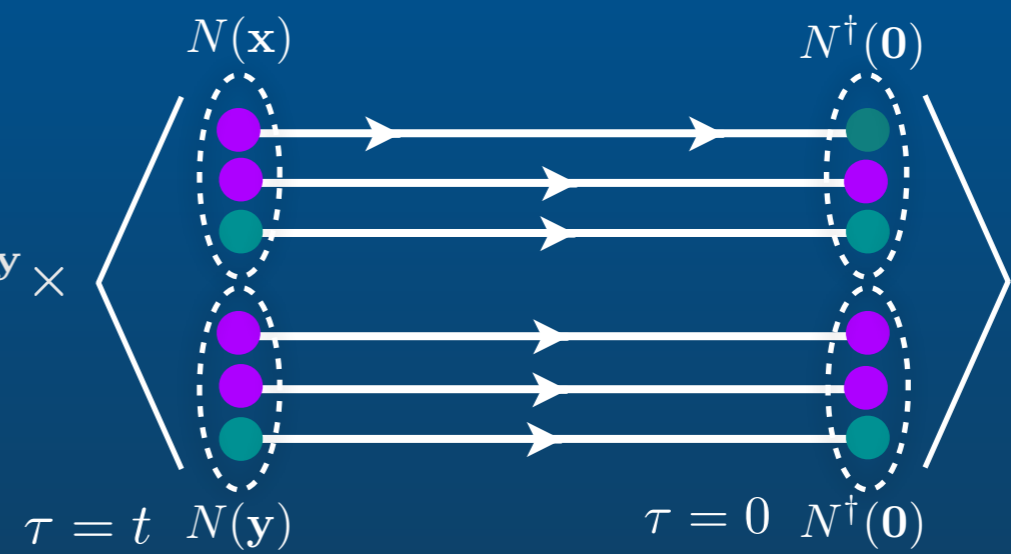
One way to implement this is to modify the usual quark propagators:

$$S_{\lambda_q; \Gamma}^{(q)}(x, y) = S^{(q)}(x, y) + \lambda \int dz S^{(q)}(x, z) \Gamma S^{(q)}(z, y)$$

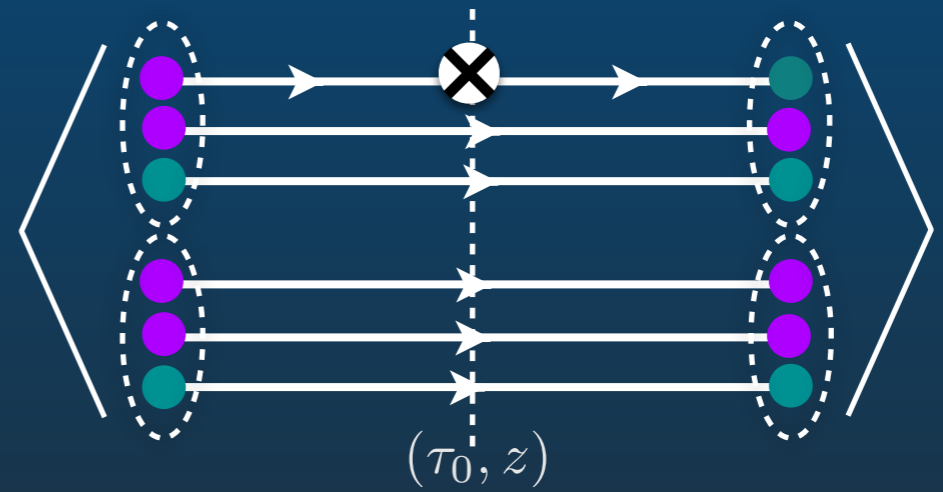
Savage et al (NPLQCD), Phys.Rev.Lett.119,062002(2017).

Let's look at an example:

$$C_{\lambda}(\mathbf{P}; t, t_0) = \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} \times$$

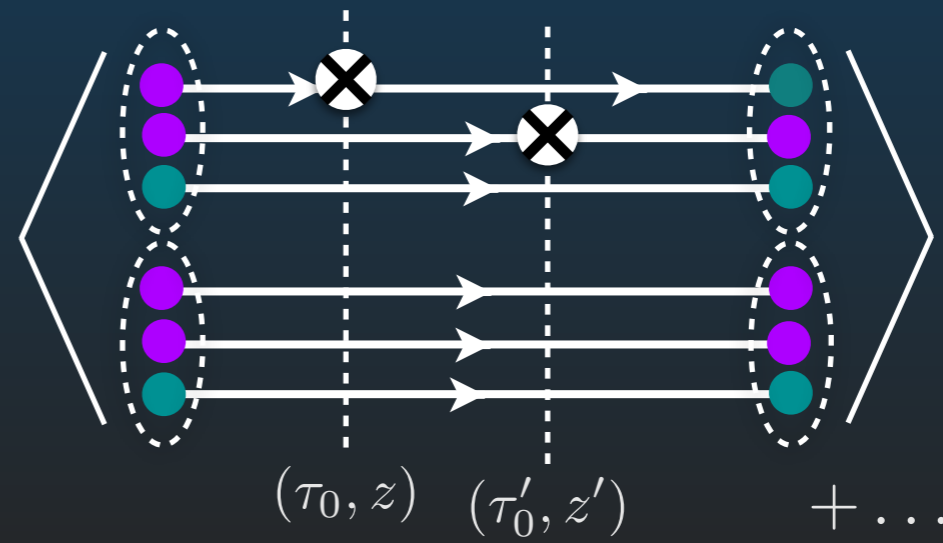


All possibilities $\longrightarrow + \lambda \sum_{\tau_0=0}^T \sum_z$

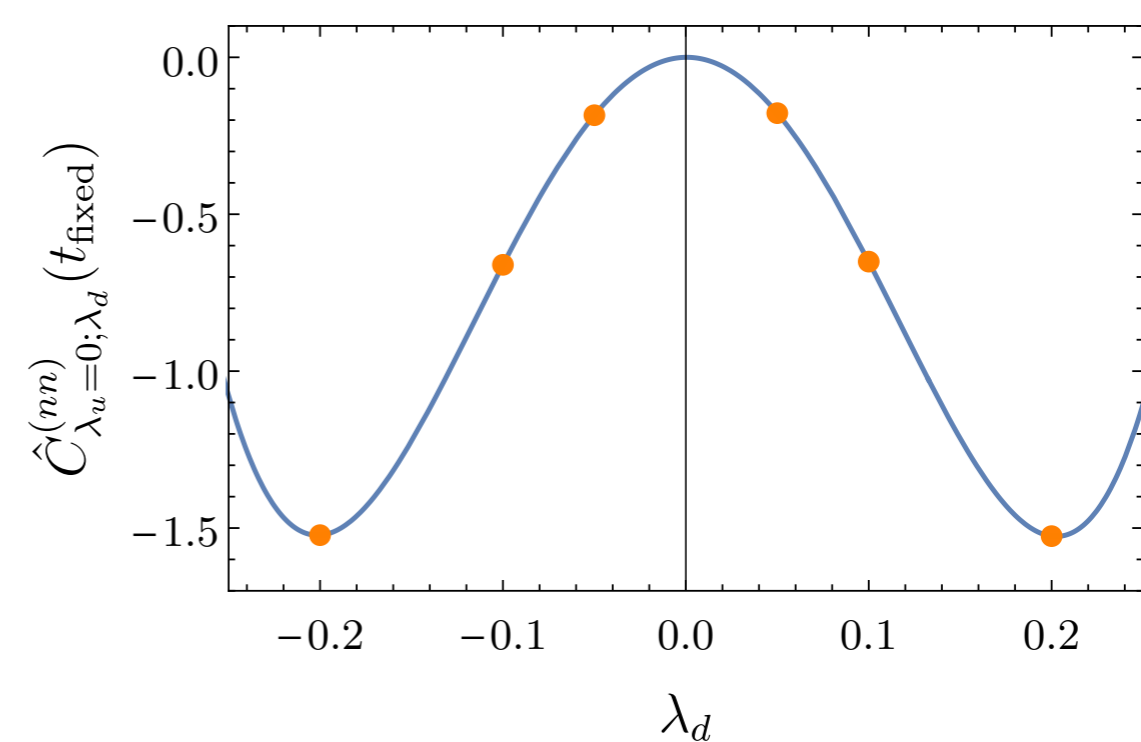
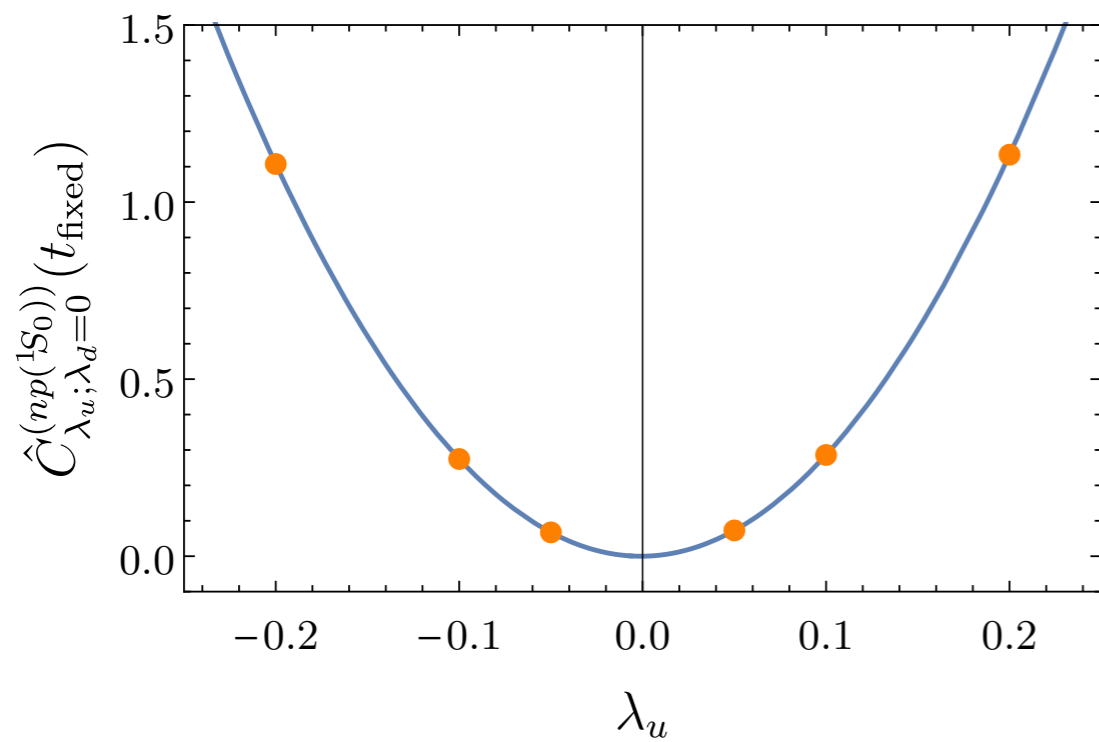
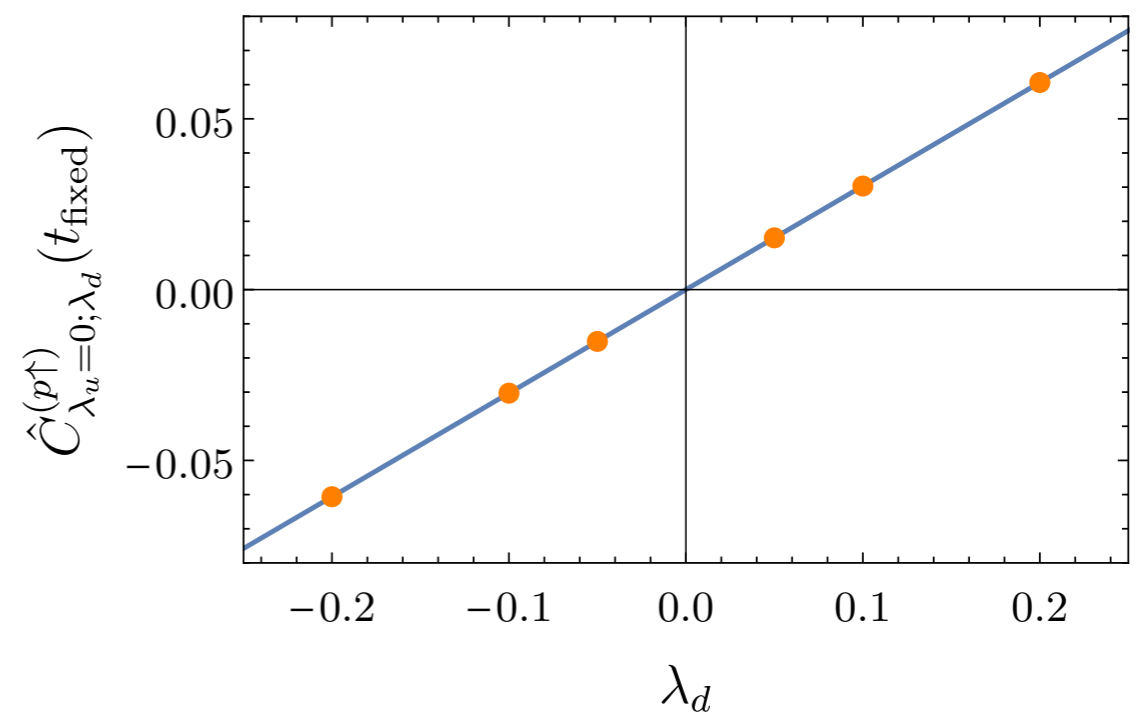
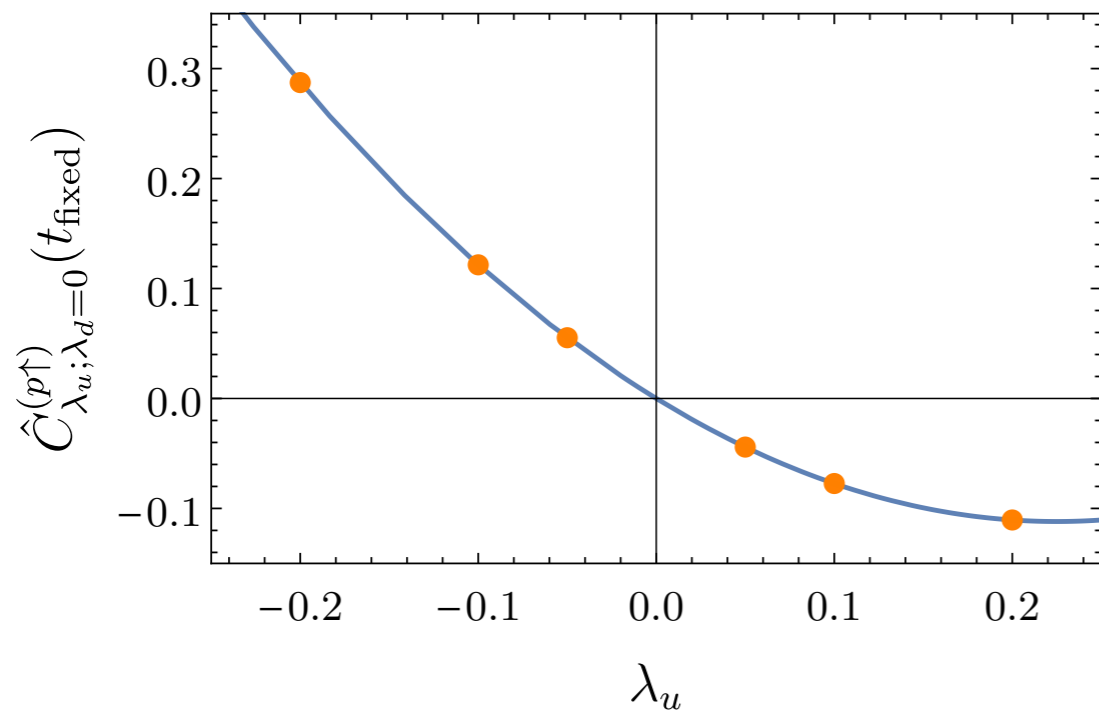


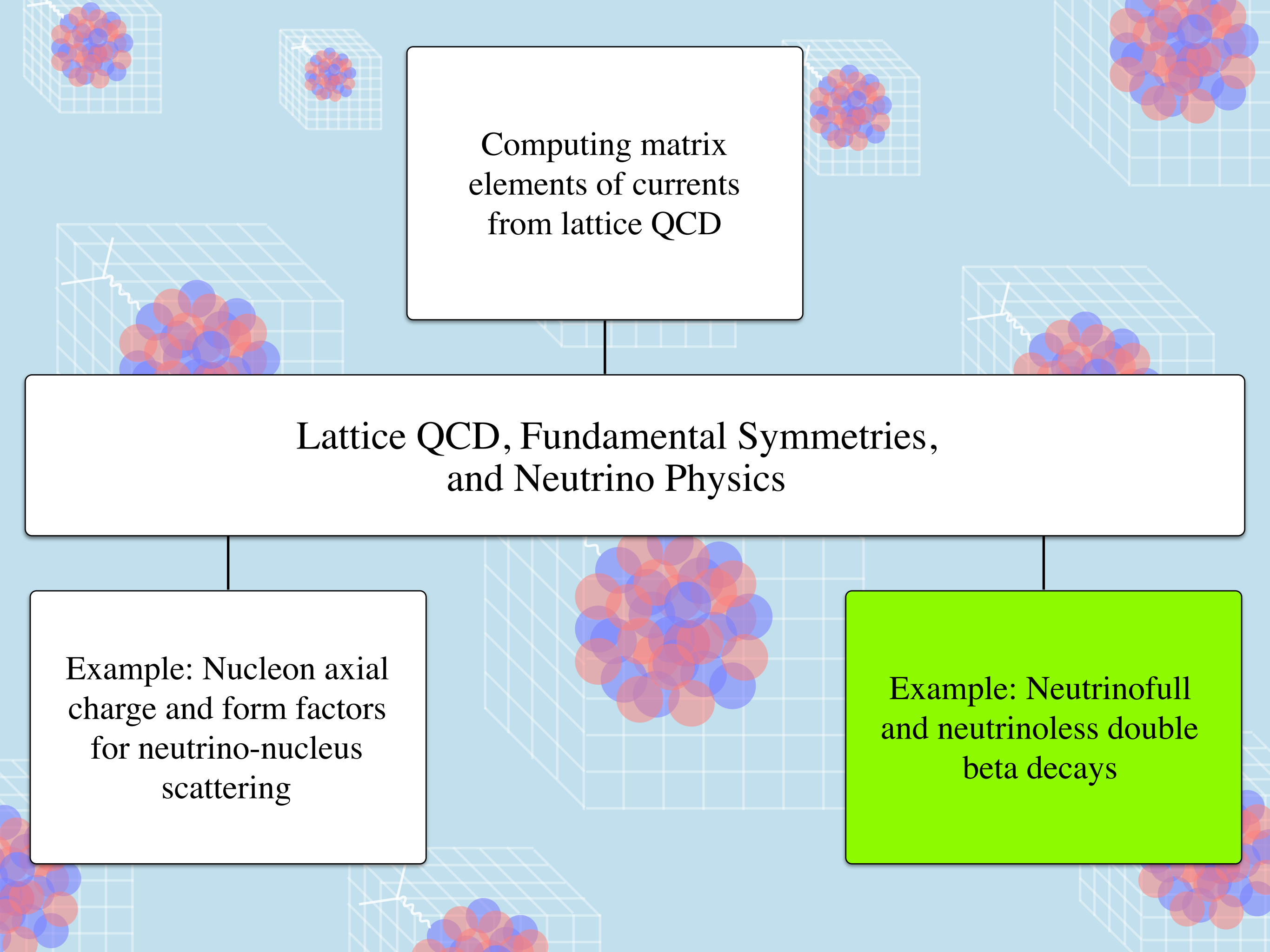
time-ordered product \downarrow

All possibilities $\longrightarrow + \lambda^2 \sum_{\tau_0=0}^T \sum_{\tau'_0=0}^T \sum_z \sum_{z'}$



Matrix elements from a compound propagator/background field



The background features several 3D wireframe cubes representing a lattice. Inside each cube is a cluster of overlapping red and blue spheres, representing a nucleon. Some cubes also have a white zigzag line extending from one corner, possibly representing a gluon field or a specific lattice action.

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Why neutrino-nucleus scattering?

Search for CP violation in neutrino oscillation experiment is entering a new era with the next generation experiments such as DUNE in the U.S. and HyperK in Japan.

Probability of muon neutrino to electron neutrino conversion, that holds information about CP violation, depends on the neutrino energy:

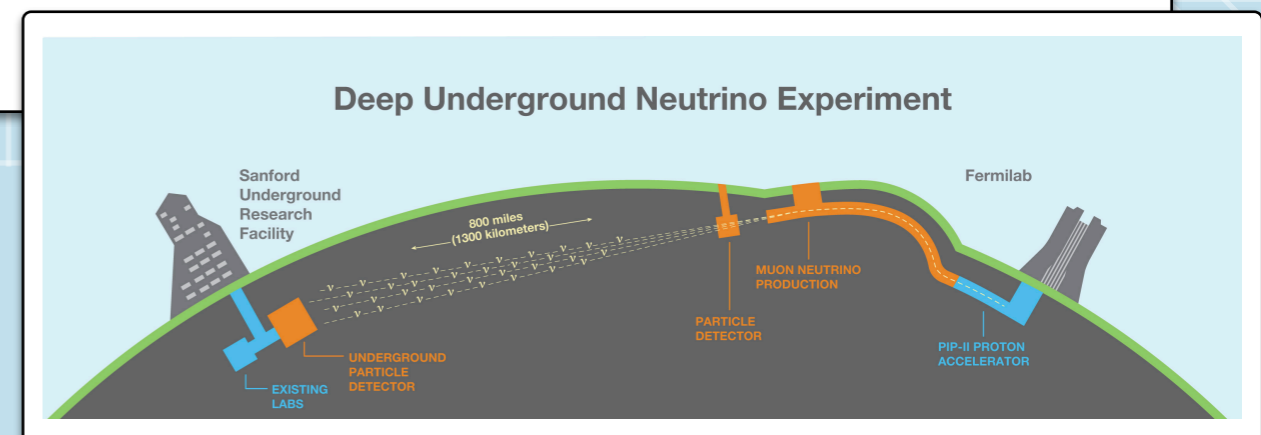
Desired CP-violating phase

$$P(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2[\Delta(A-1)]}{(A-1)^2} + \alpha J \sin \delta_{CP} \sin \Delta \frac{\sin(A\Delta) \sin[(1-A)\Delta]}{A(1-A)} + \alpha J \cos \delta_{CP} \cos \Delta \frac{\sin(A\Delta) \sin[(1-A)\Delta]}{A(1-A)} + \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(A\Delta)}{A^2}$$

Quantities that depend on neutrino energy

De Romeri et al, *JHEP* 09 (2016) 030.

Unfortunately the neutrino energy is undetermined a priori and must be reconstructed from its collision with target nuclear isotopes such as Argon: A very complex problem!



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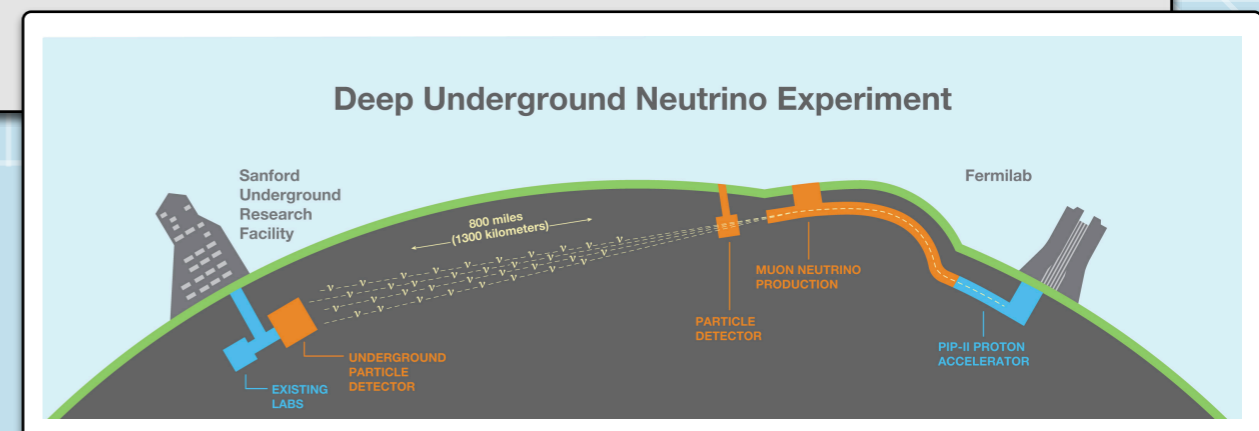
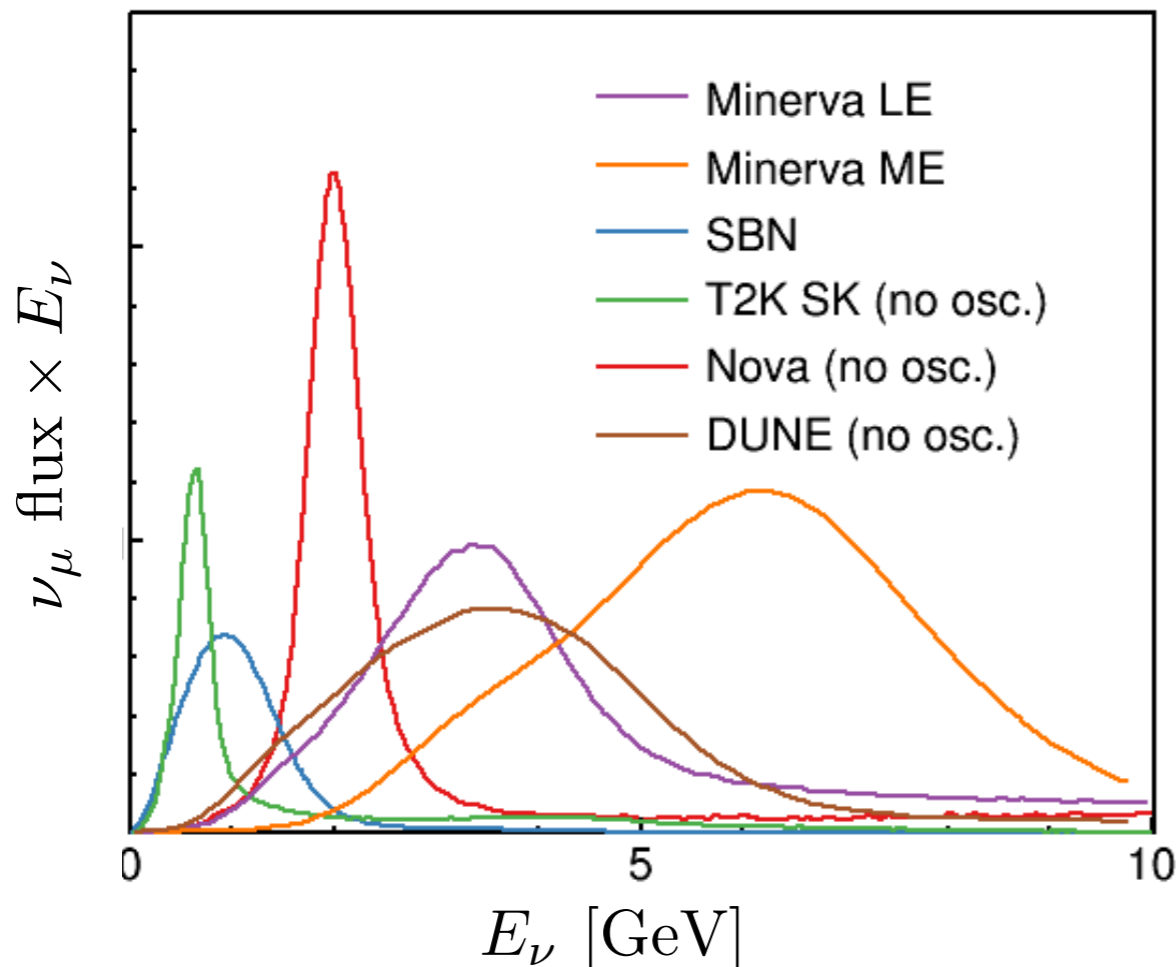
Desired CP-violating phase

$$\alpha J \sin \delta_{CP} \sin \Delta \frac{\sin(A\Delta) \sin[(1-A)\Delta]}{A(1-A)} + \frac{\sin(A\Delta)}{A} + \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(A\Delta)}{A^2}$$

De Romeri et al, *JHEP* 09 (2016) 030.

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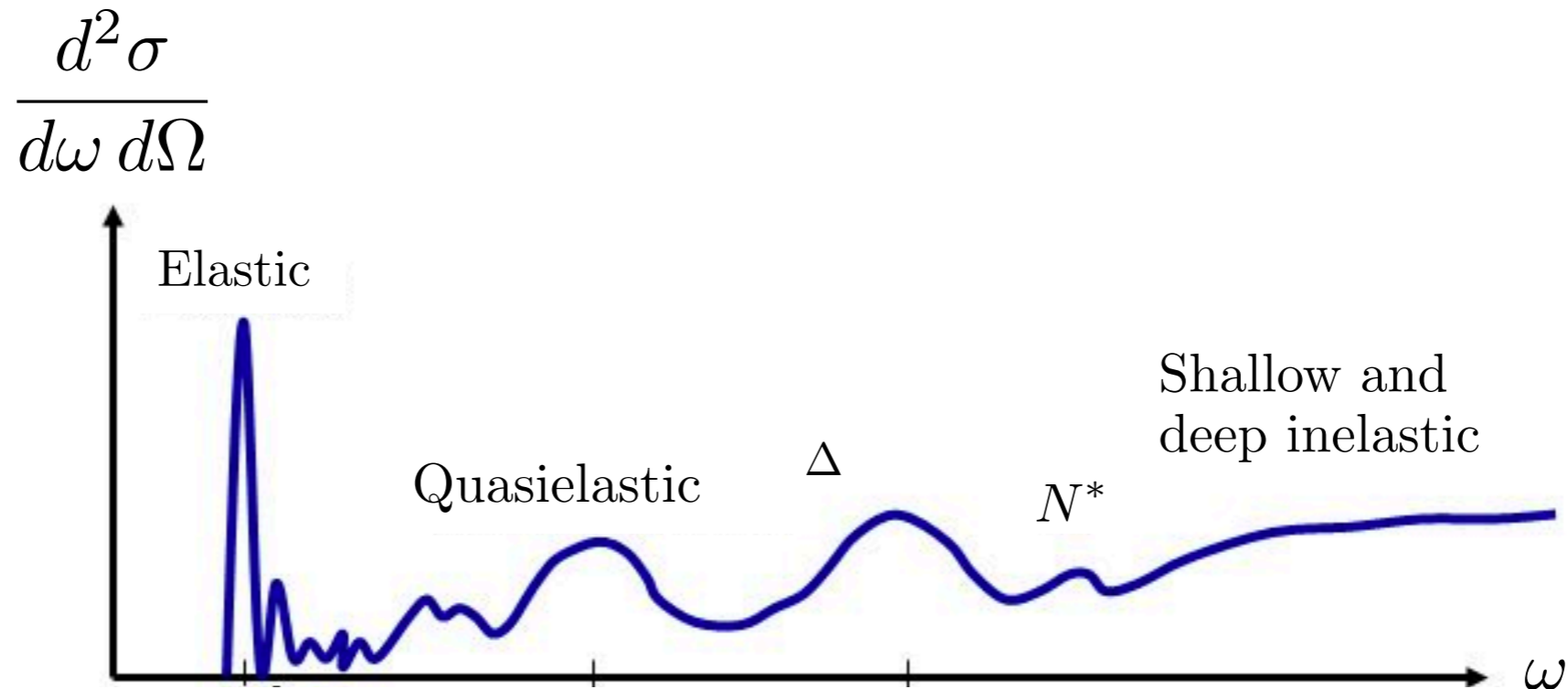
Kronfeld et al (USQCD collaboration), *Eur.Phys.J.A* 55 (2019)+Laura Fields.



One needs to constrain nuclear response to incoming neutrino of various energy. How can lattice QCD help?

ν -nucleus
scattering
at a fixed
momentum
transferred

$$\nu_l A \rightarrow l^- X$$



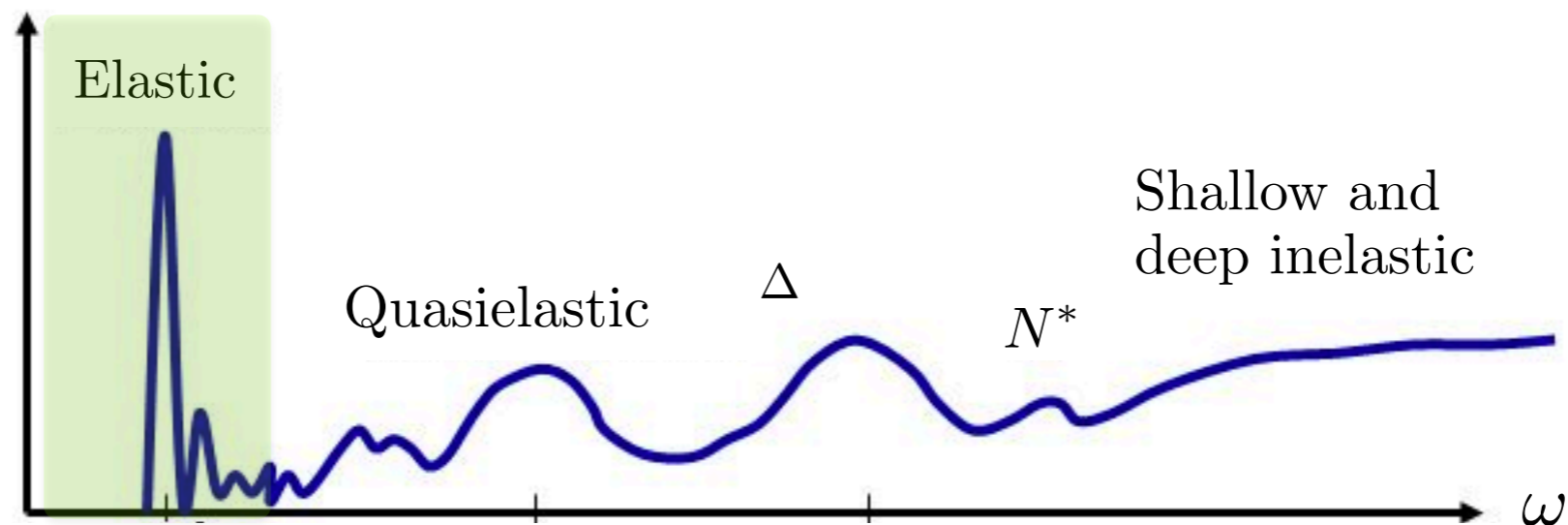
Kronfeld et al (USQCD
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$$\frac{d^2\sigma}{d\omega d\Omega}$$



Forward form
factors, radii

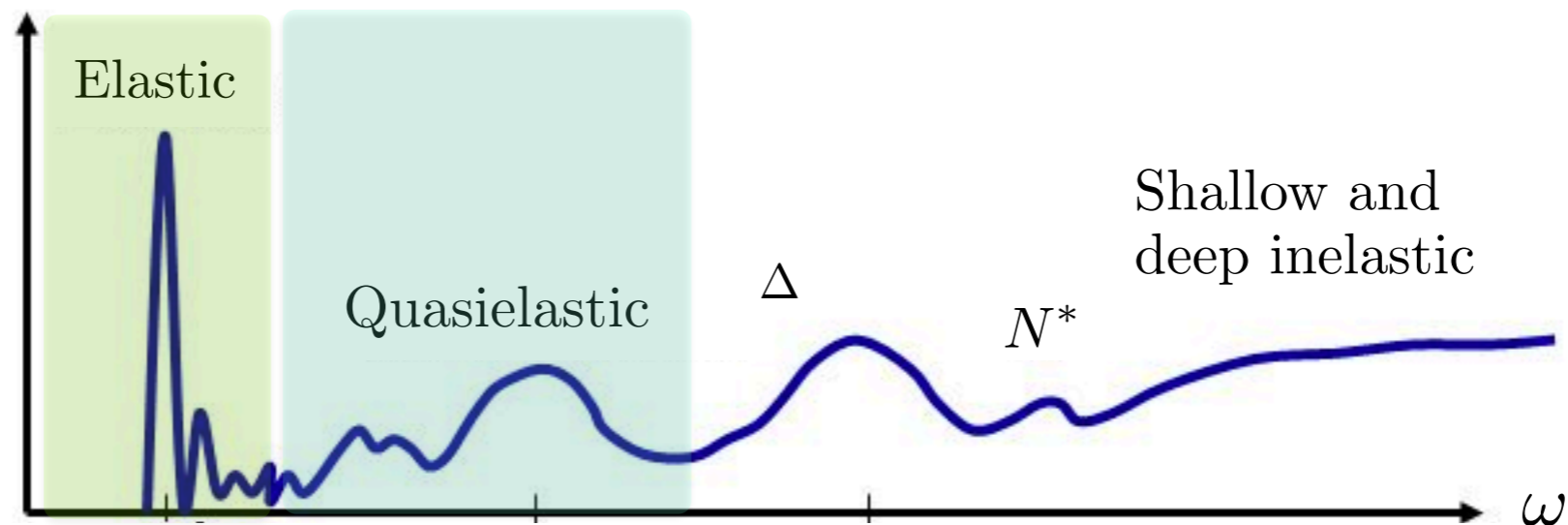
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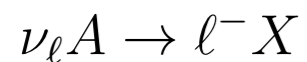
Forward form
factors, radii

Off-forward
form factors

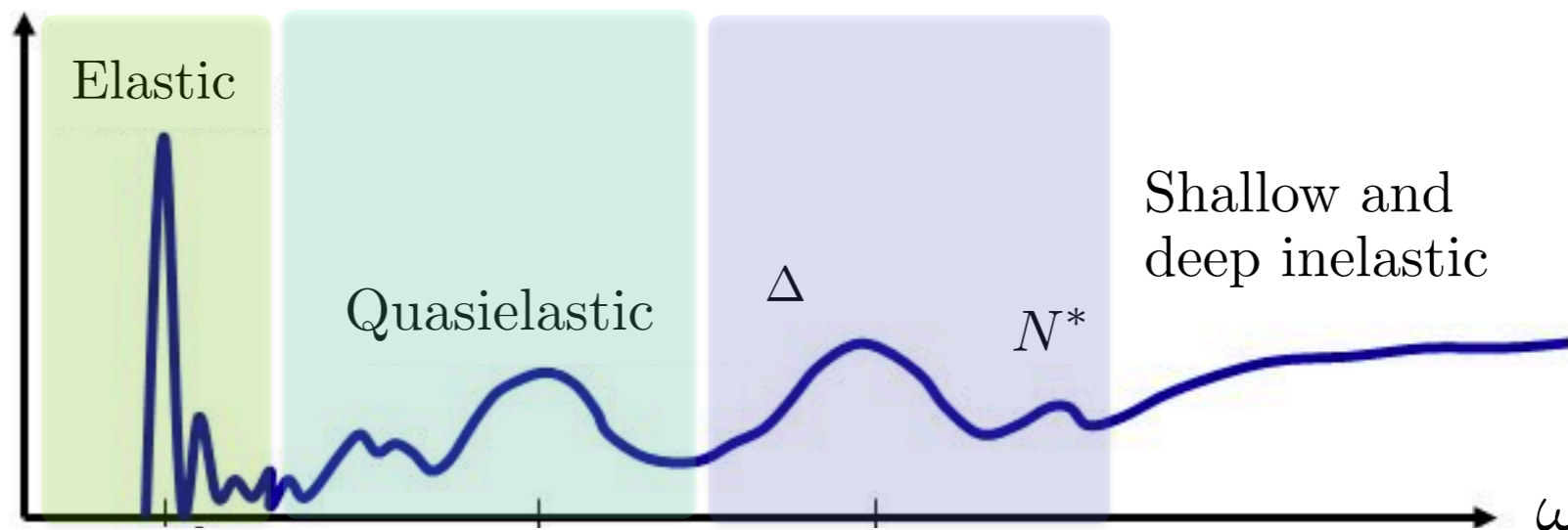
Kronfeld et al (USQCD
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ν -nucleus scattering at a fixed momentum transfer



$$\frac{d^2 \sigma}{d\omega d\Omega}$$



Forward form factors, radii

Off-forward form factors

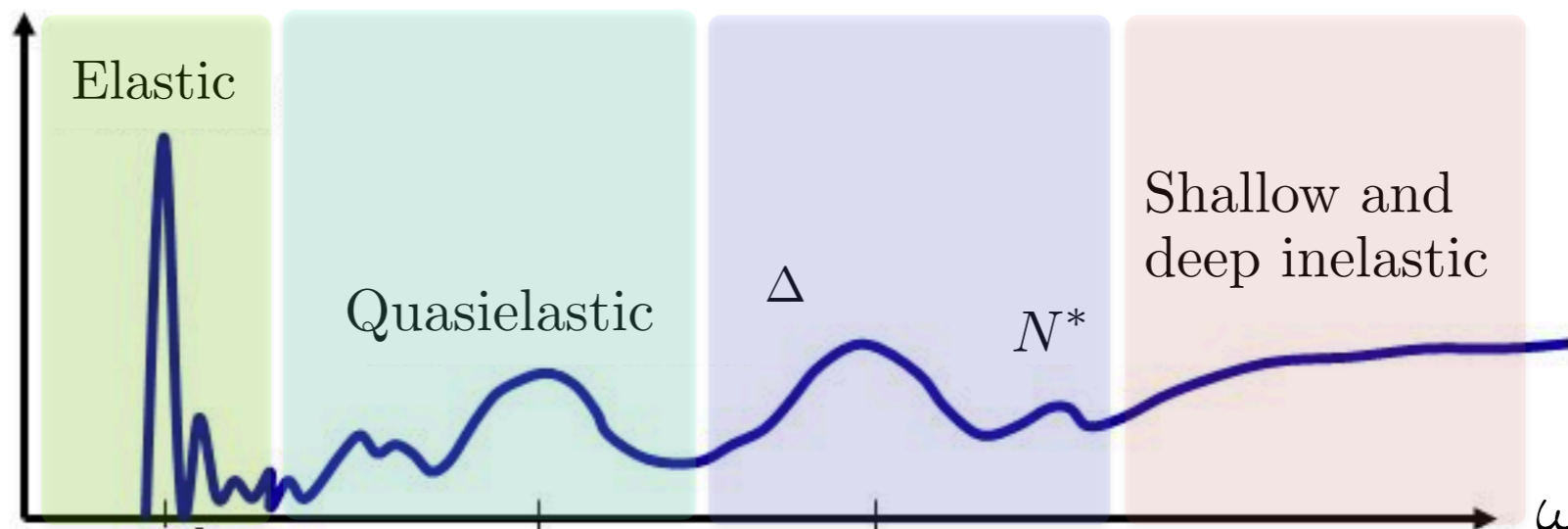
Kronfeld et al (USQCD collaboration),
Eur.Phys.J.A 55 (2019).

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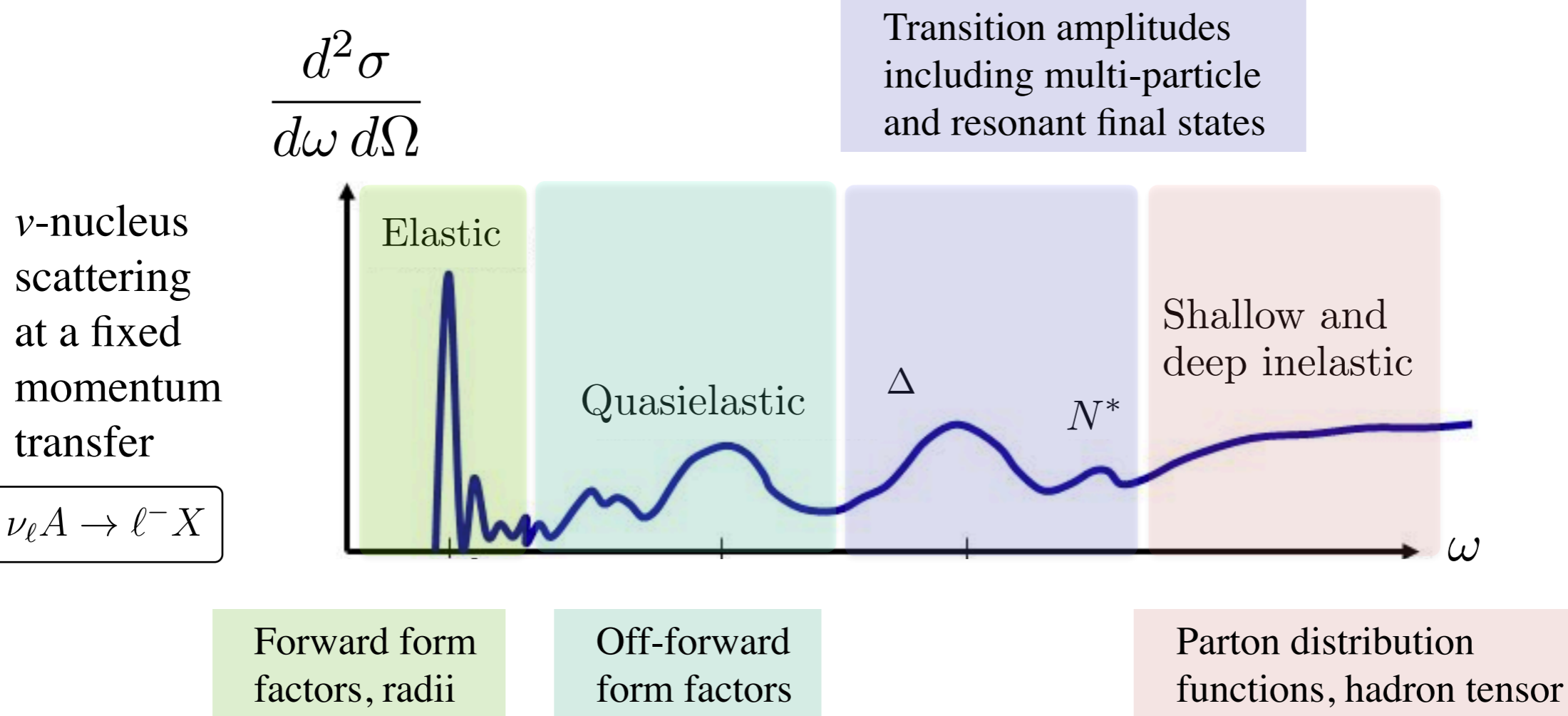
Forward form factors, radii

Off-forward form factors

Parton distribution functions, hadron tensor

Kronfeld et al (USQCD collaboration),
Eur.Phys.J.A 55 (2019).

One needs to constrain nuclear response to incoming neutrino of various energy. How can lattice QCD help?



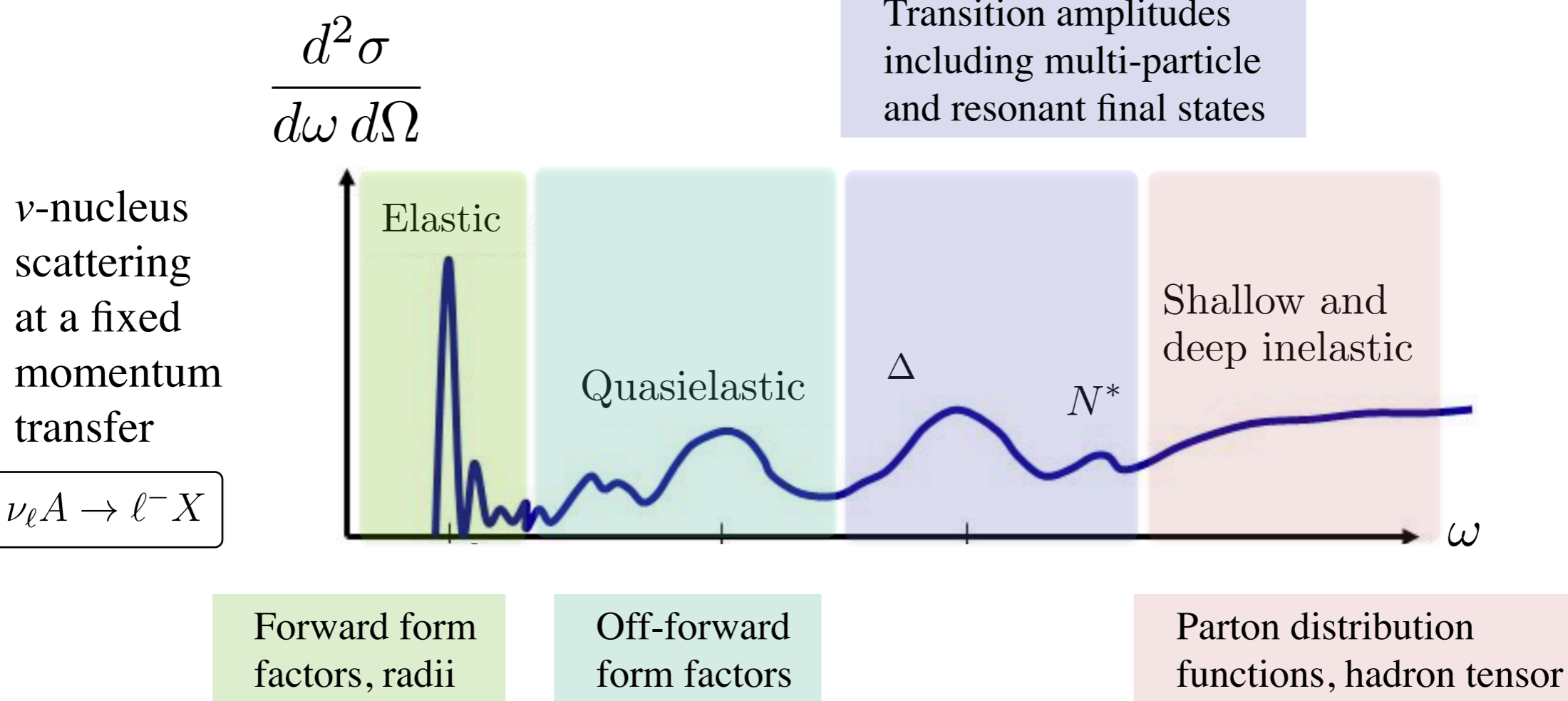
Need to compute various matrix elements in nucleon, multi-hadron states, and (light) nuclei:

$$\langle f | J_\nu | i \rangle, \quad \langle f | J_\mu^\dagger J_\nu | i \rangle, \quad \langle f | \mathcal{O} | i \rangle$$

and resort to EFTs to connect to large isotopes in experiments.

Kronfeld et al (USQCD collaboration),
Eur.Phys.J.A 55 (2019).

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I will not be able to tell you about the complete story, but I give you an example of a quantity lattice QCD can obtain well: the (nucleon) axial charge and form factors.

Kronfeld et al (USQCD collaboration),
Eur.Phys.J.A 55 (2019).

Example: The application of 3pt function method to obtain the axial charge/form factors of the nucleon

Constantinou, arXiv:1411.0078 [hep-lat].

$$\langle N(p', s') | \bar{\psi}(x) \gamma_\mu \gamma_5 \psi(x) | N(p, s) \rangle = i \left(\frac{m_N^2}{E_N(\mathbf{p}') E_N(\mathbf{p})} \right)^{1/2} \bar{u}_N(p', s') \left[G_A(q^2) \gamma_\mu \gamma_5 + \frac{q_\mu \gamma_5}{2m_N} G_P(q^2) \right] u_N(p, s)$$

Axial-vector current

Nucleon spinor

Axial and pseudo scalar form factors

$$G_A(0) = g_A$$

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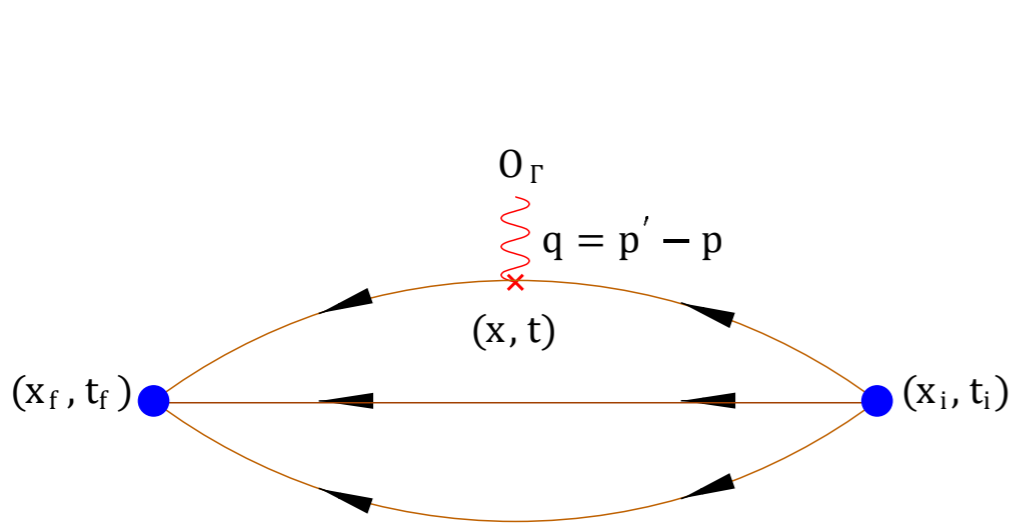
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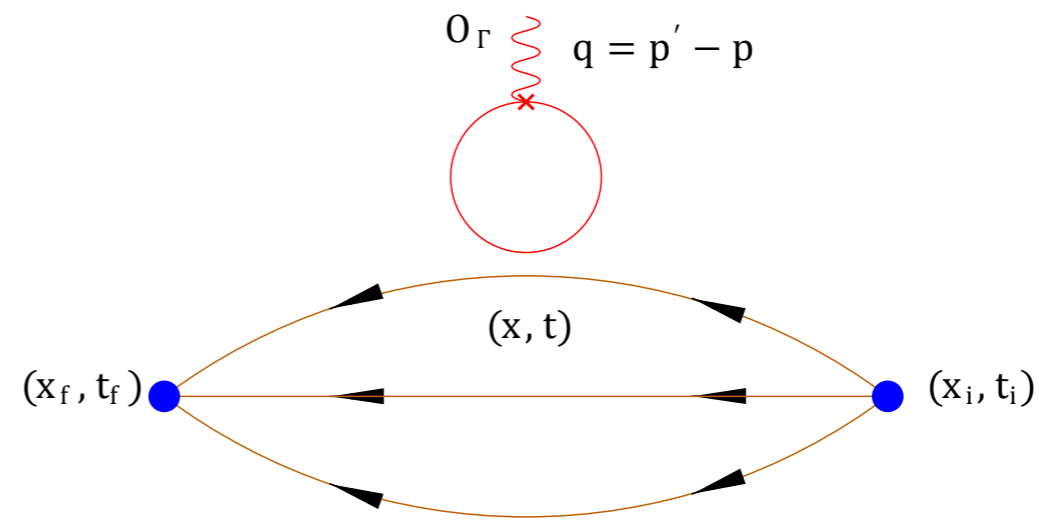
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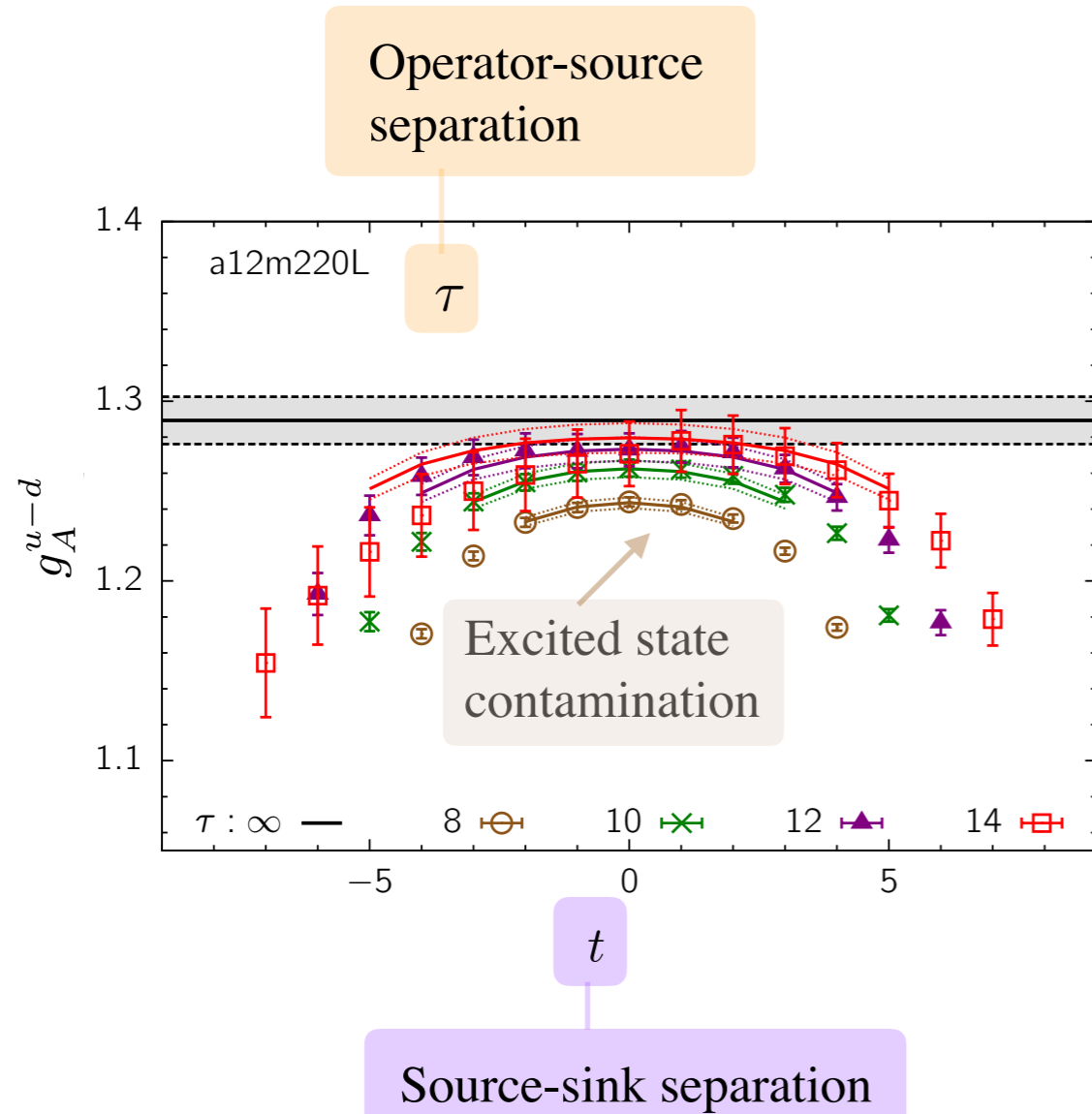
Connected contribution



Disconnected contribution
 (vanishes at isospin limit for isovector quantities)

Example: The application of 3pt function method to obtain the axial charge/form factors of the nucleon

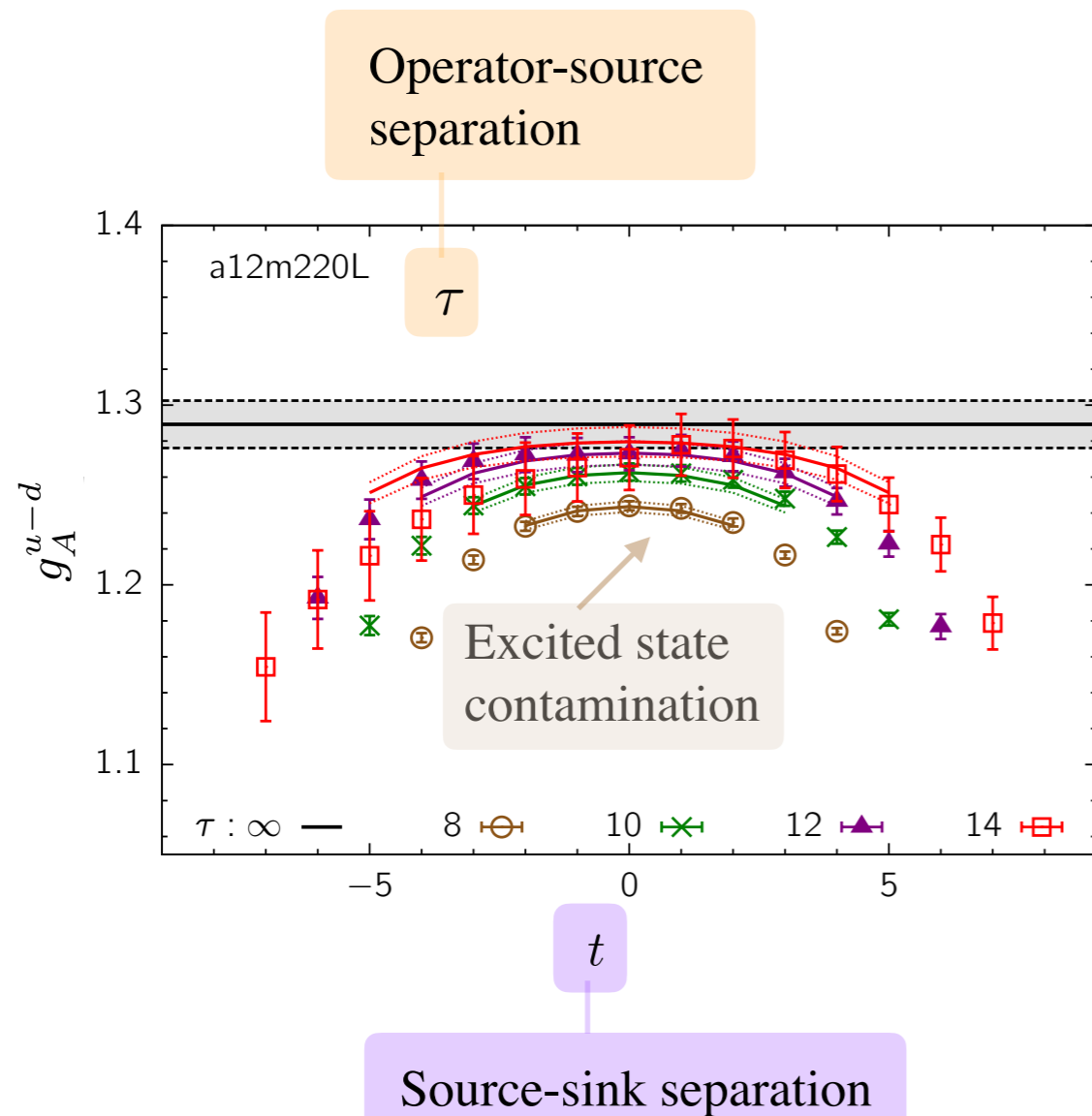
Gupta et al (PNDME), Phys. Rev. D 98, 034503 (2018)



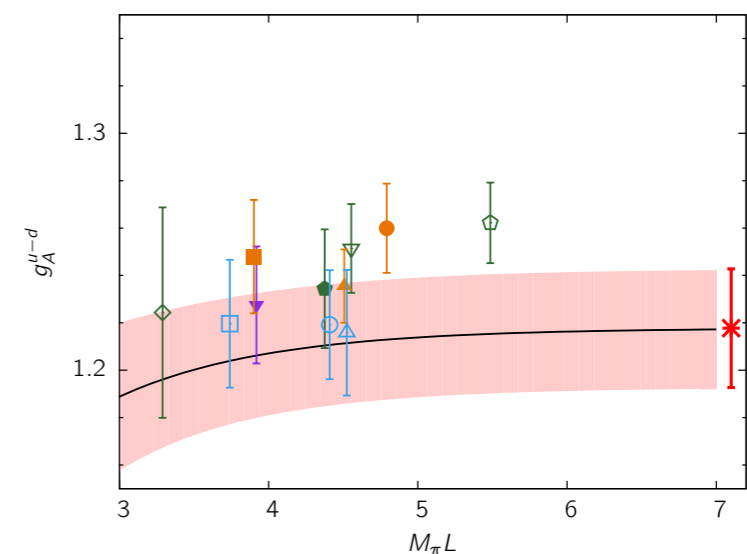
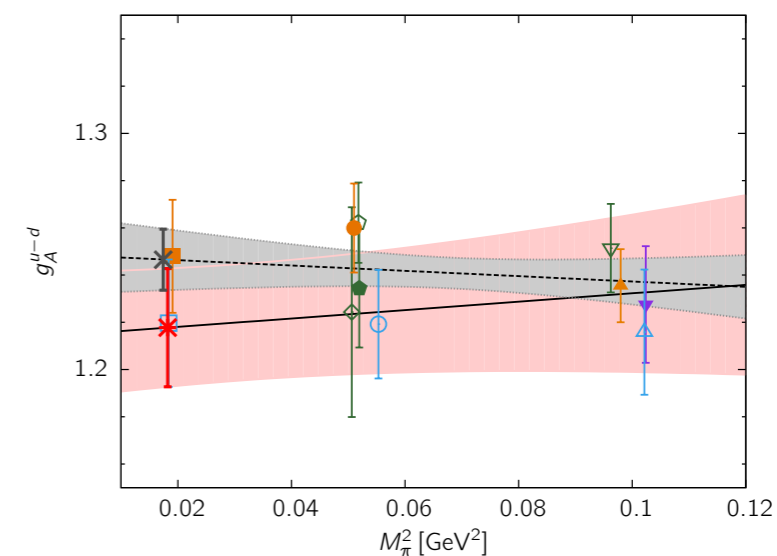
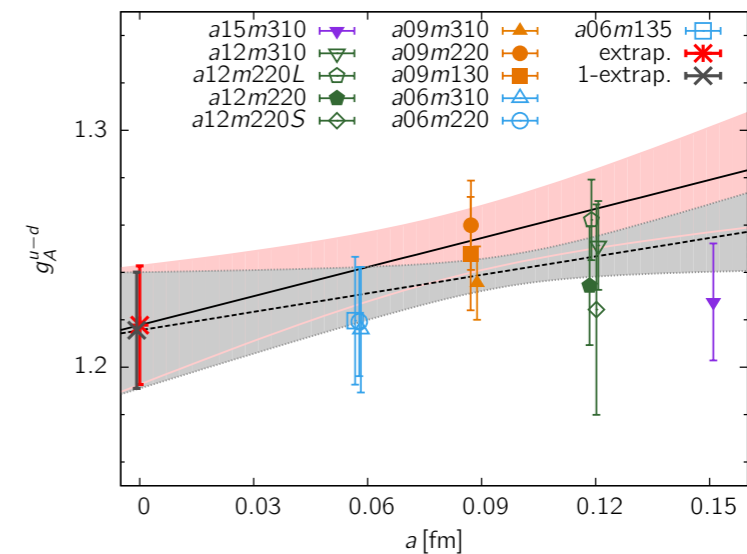
3pt function for a single lattice spacing, volume and quark masses

Example: The application of 3pt function method to obtain the axial charge/form factors of the nucleon

Gupta et al (PNDME), Phys. Rev. D 98, 034503 (2018)



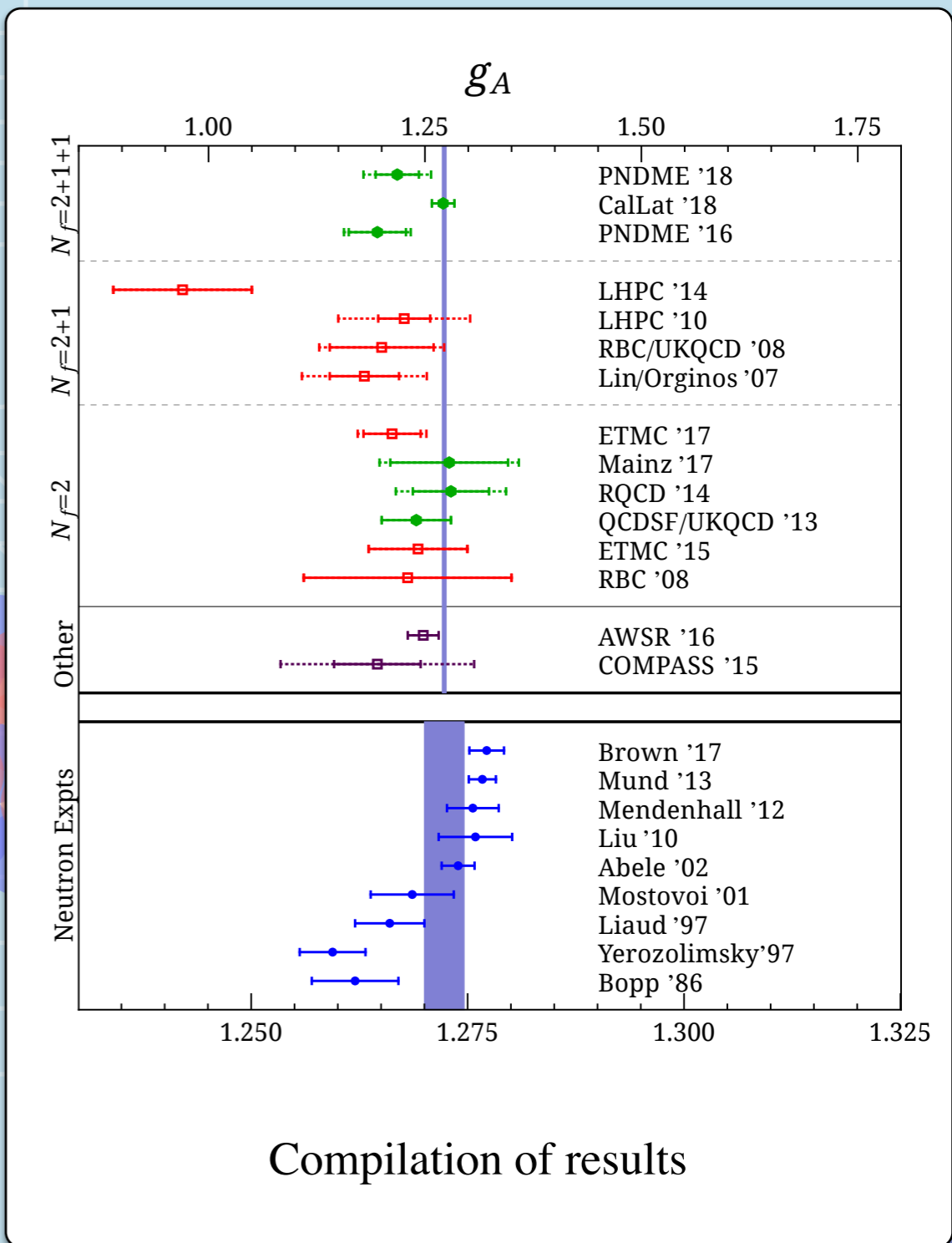
3pt function for a single lattice spacing, volume and quark masses



Extrapolation to continuum, infinite volume, and physical quark masses

Example: The application of 3pt function method to obtain the axial charge/form factors of the nucleon

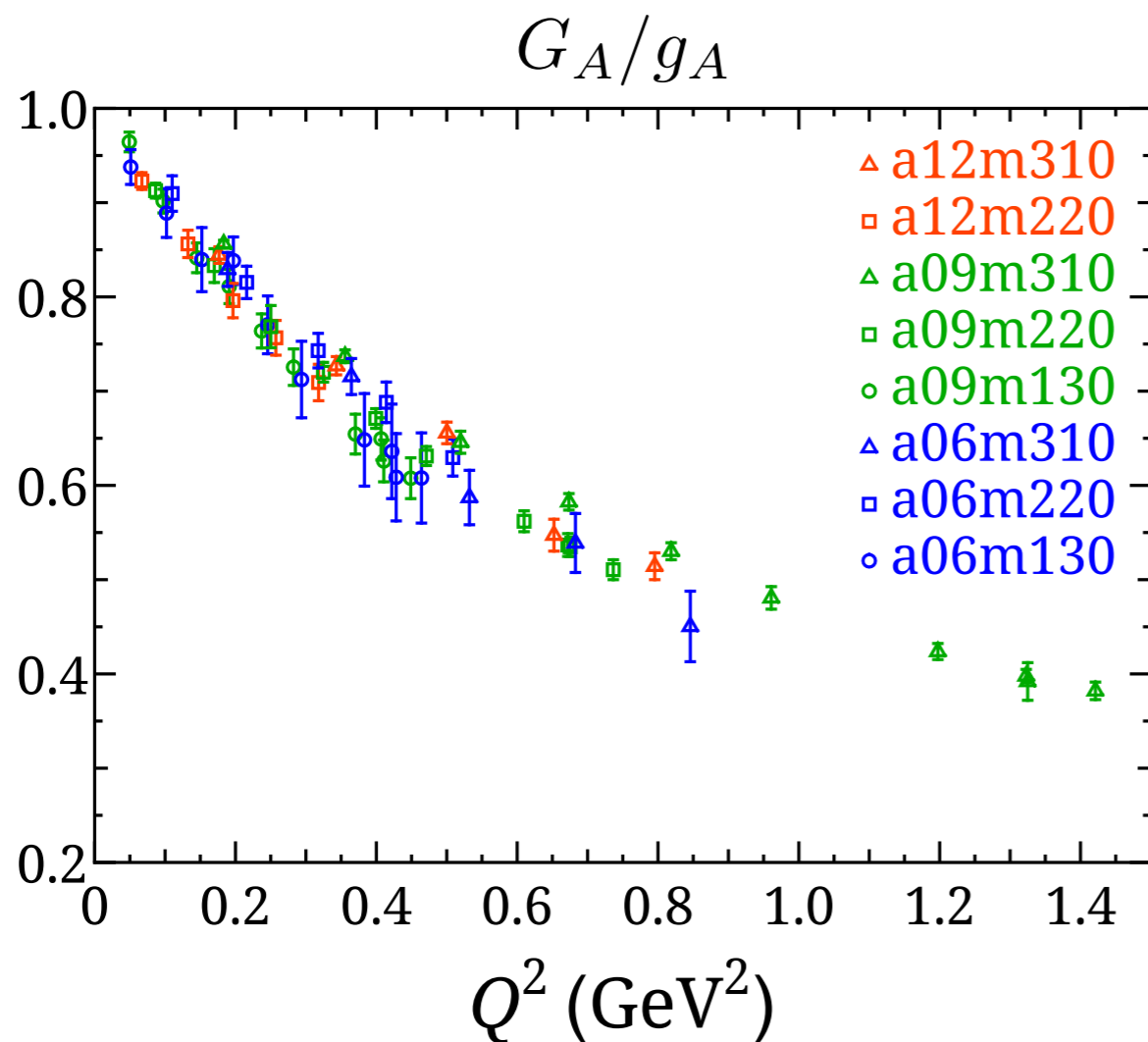
FLAG Review (2019), EPJC 80, 113 (2020).



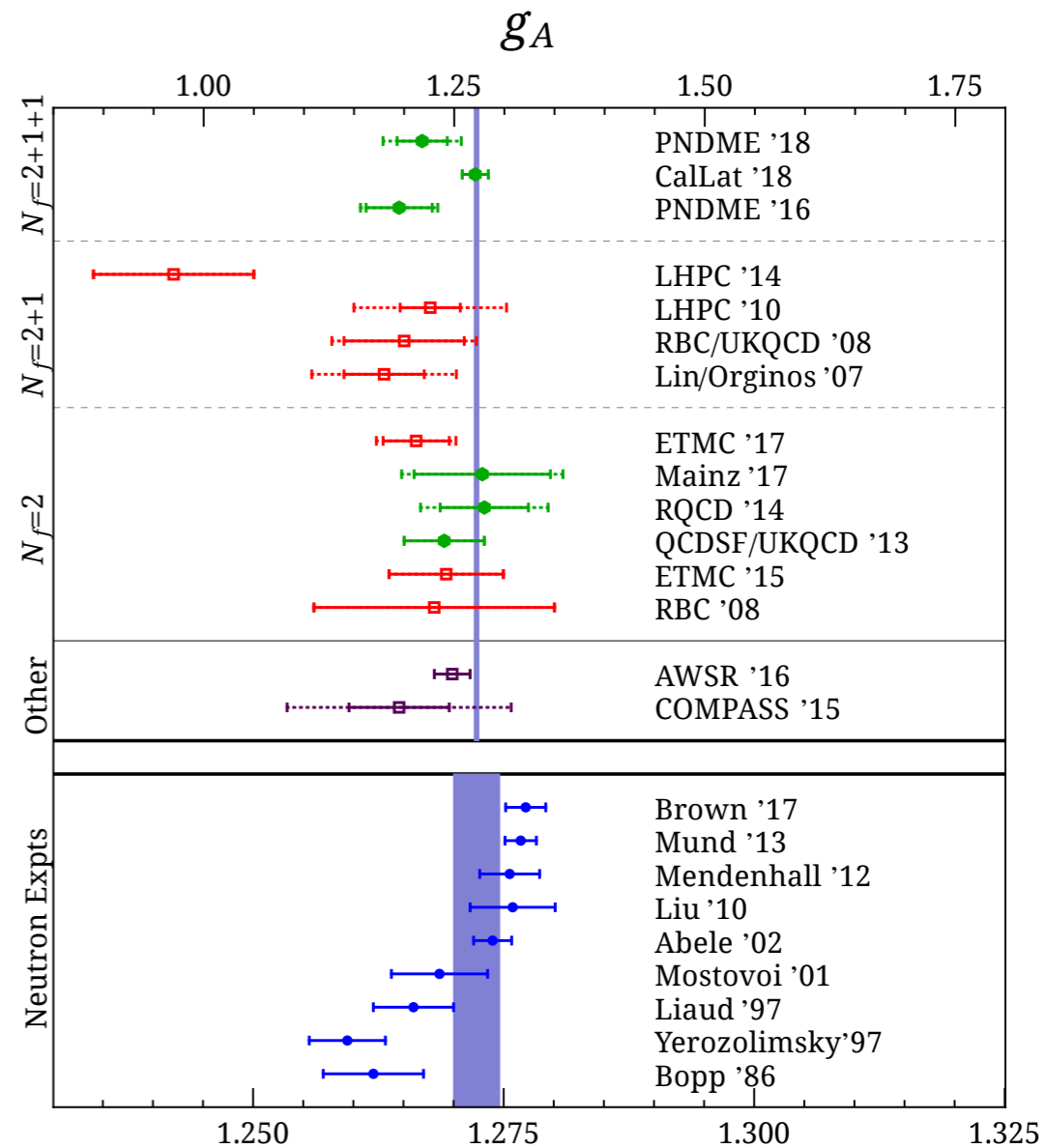
Example: The application of 3pt function method to obtain the axial charge/form factors of the nucleon

FLAG Review (2019), EPJC 80, 113 (2020).

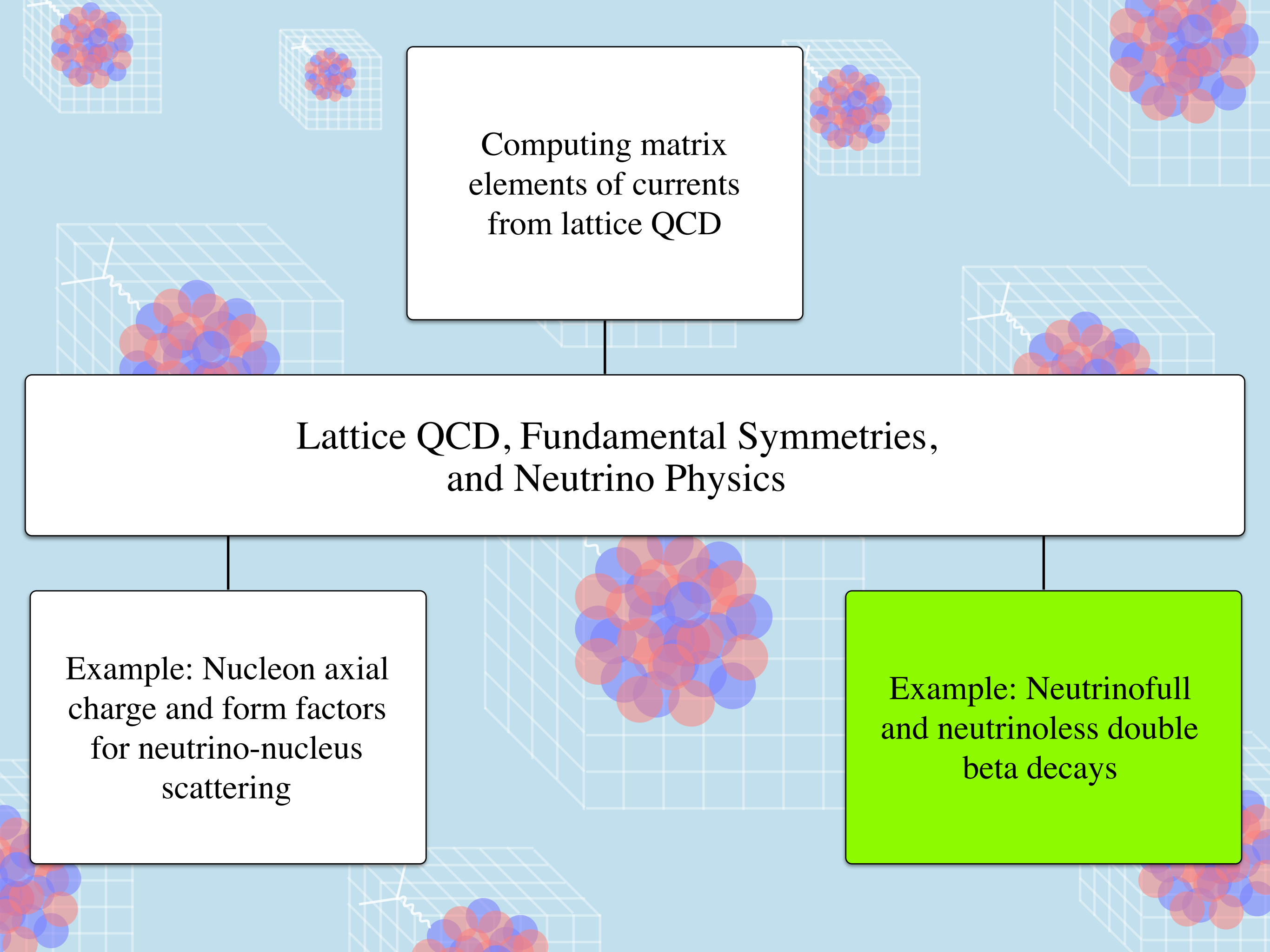
Jang et al, EPJ Web Conf. 175, 06033 (2018)



Axial form factor results



Compilation of results

The background features several 3D wireframe cubes representing a lattice. Inside each cube is a cluster of overlapping red and blue spheres, representing a nucleon. Some cubes also have a white zigzag line extending from a vertex, representing a gluon field.

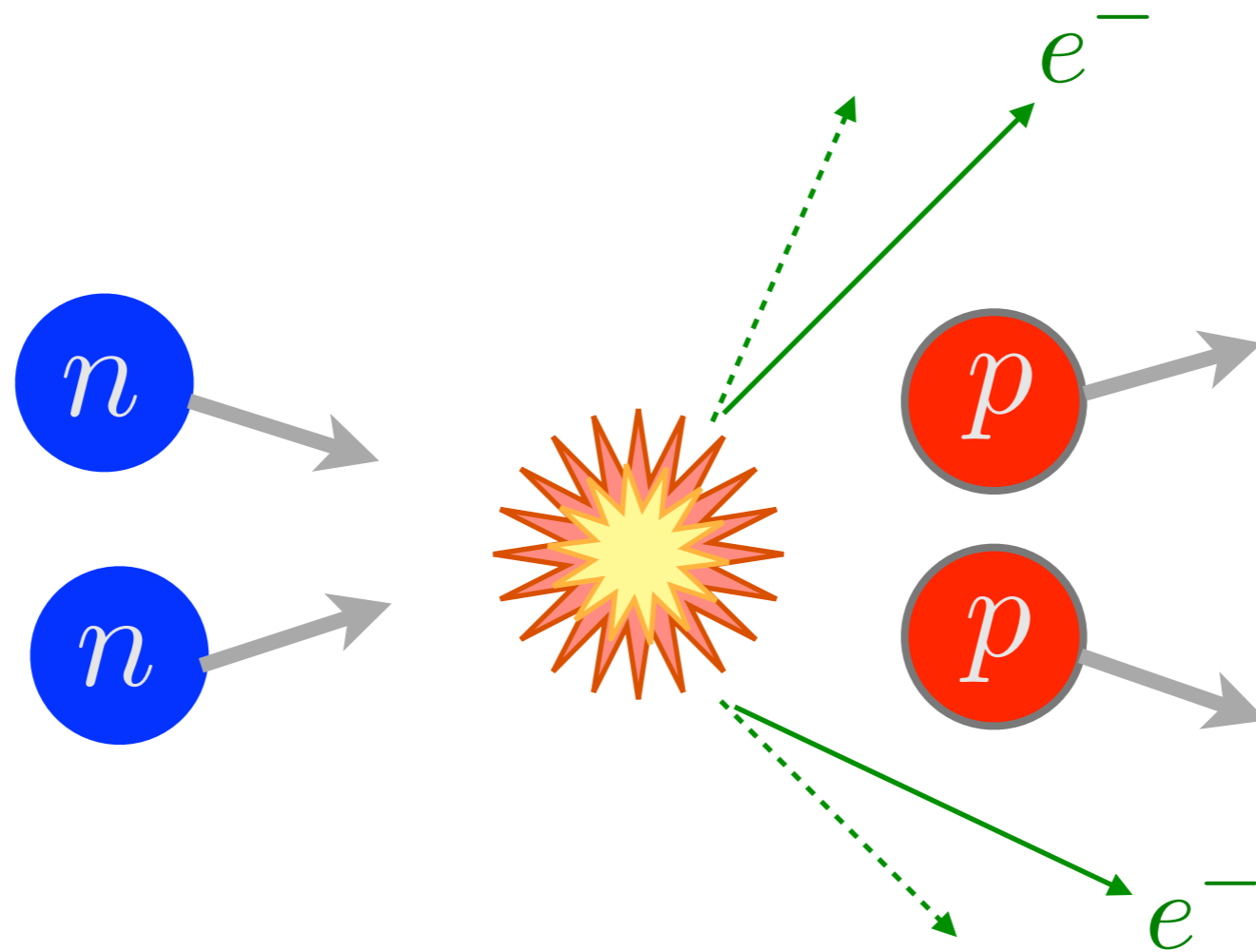
Computing matrix
elements of currents
from lattice QCD

Lattice QCD, Fundamental Symmetries,
and Neutrino Physics

Example: Nucleon axial
charge and form factors
for neutrino-nucleus
scattering

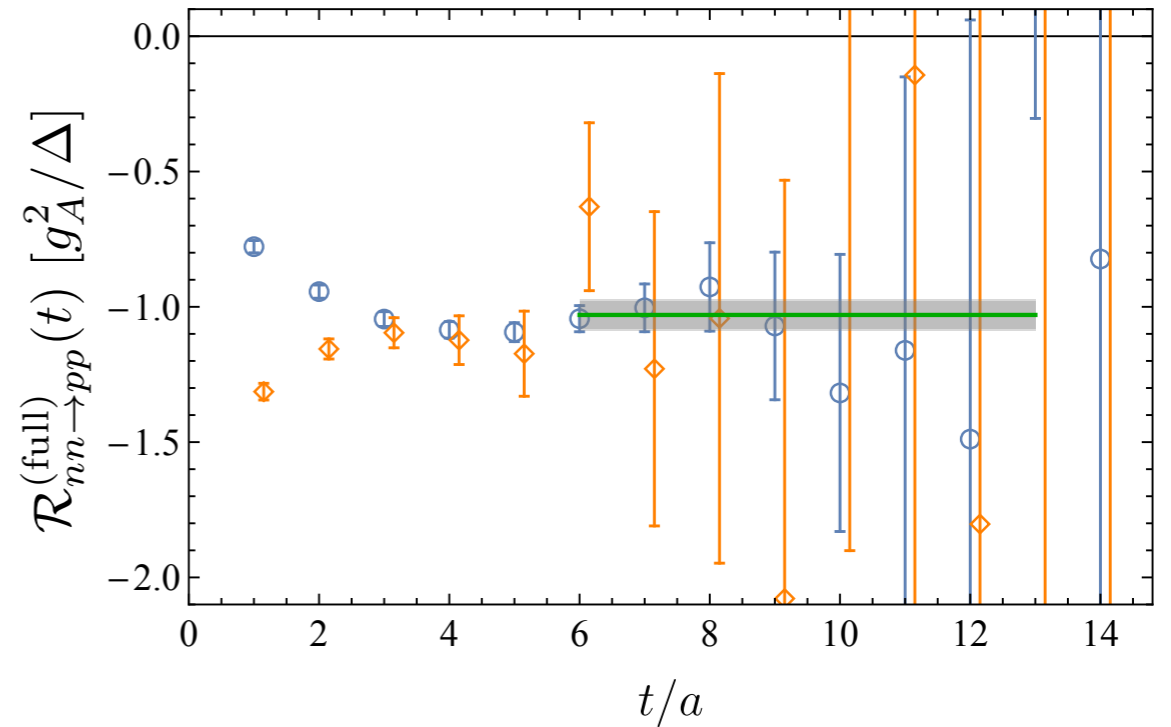
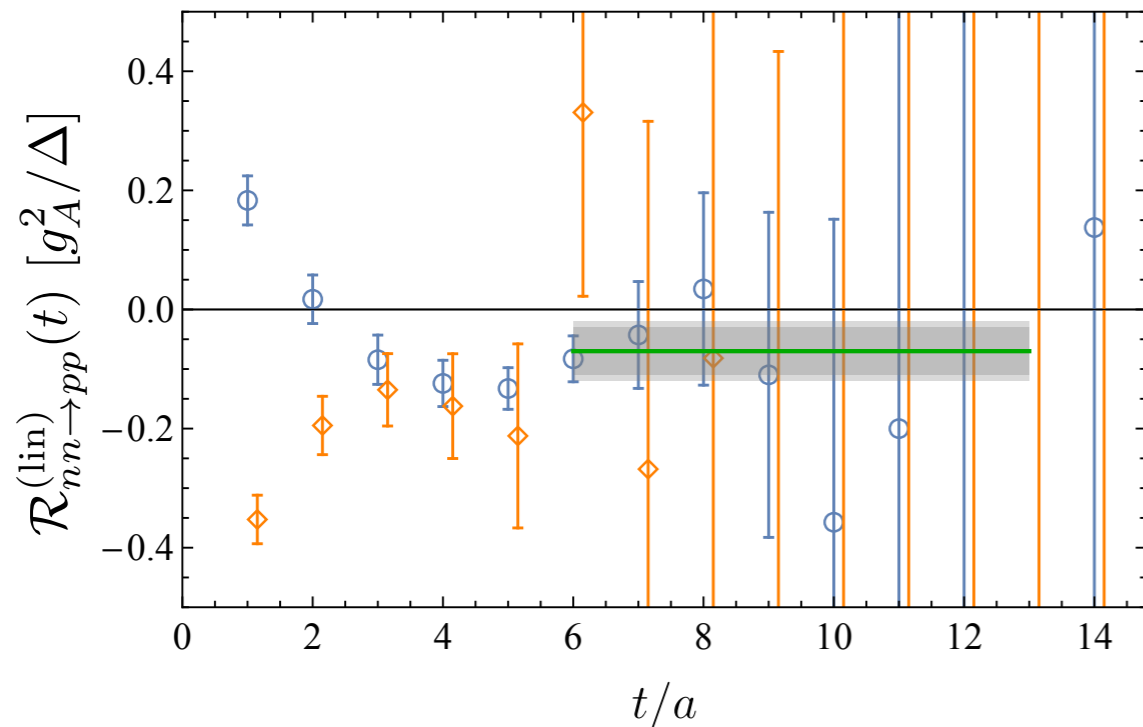
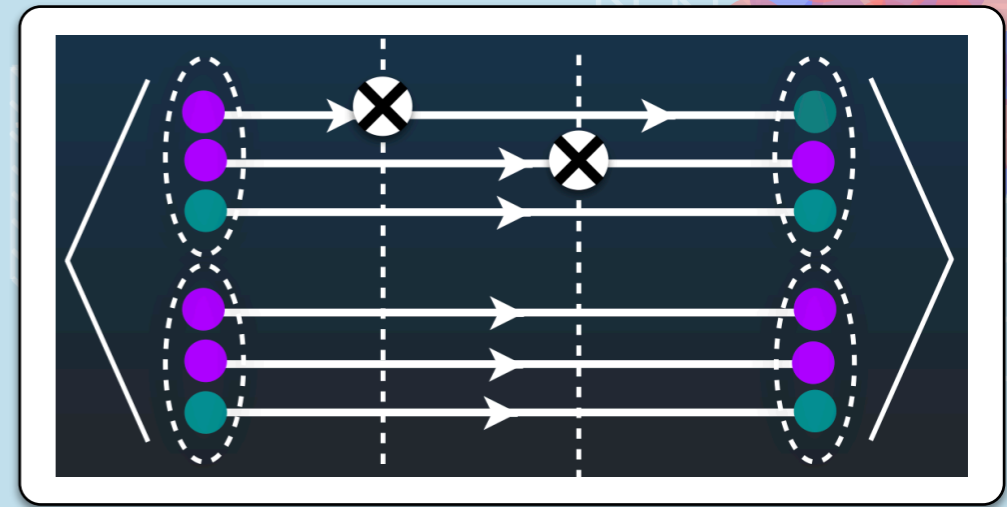
Example: Neutrinoless
and neutrinoless double
beta decays

NEUTRINOFUL DOUBLE-BETA DECAY



Matrix element from QCD using the Feynman-Hellmann (modified propagator) method (hiding all technicalities):

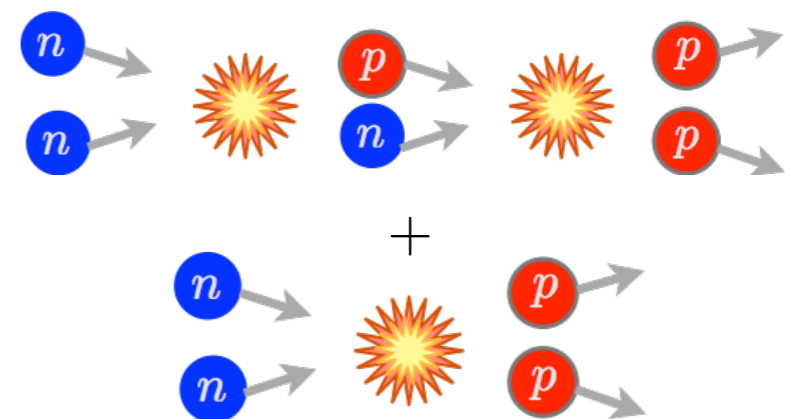
$$N_f = 3, \quad m_\pi = 0.806 \text{ GeV}, \quad a = 0.145(2) \text{ fm}$$



SHORT-DISTANCE CONTRIBUTION

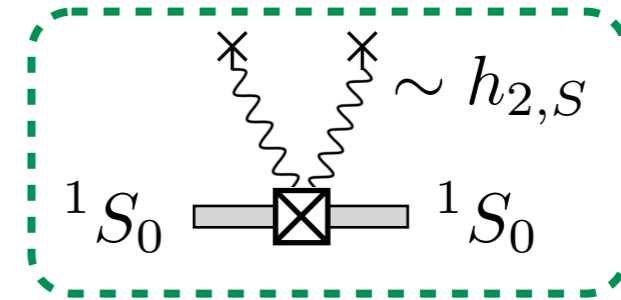
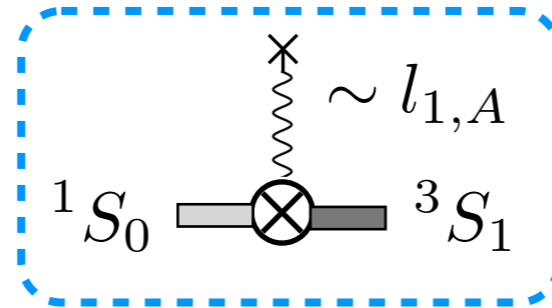
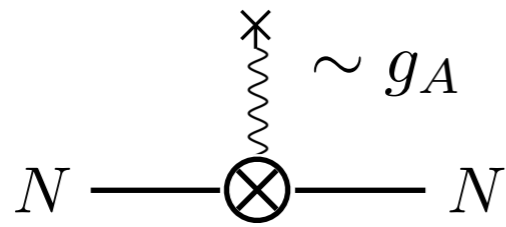


FULL CONTRIBUTION

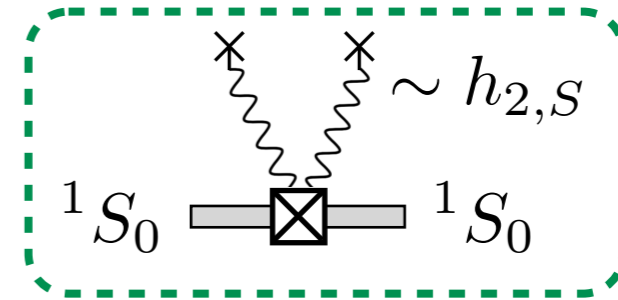
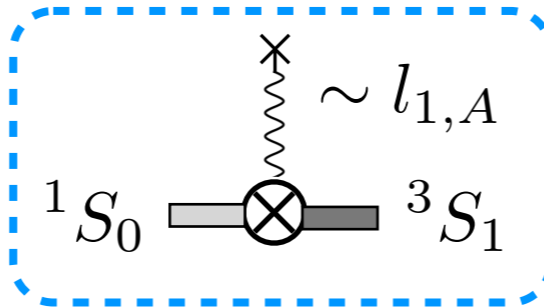
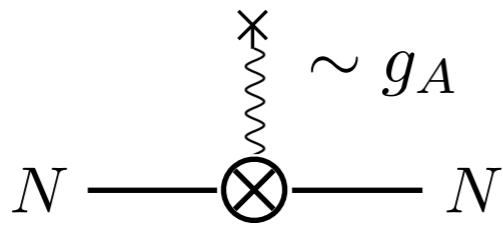


Let's see how we can match this result to a low-energy EFT and constrain unknown LECs

* Note "dibaryon" fields are used.



Let's see how we can match this result to a low-energy EFT and constrain unknown LECs



$$i\mathcal{C}_{nn \rightarrow pp} = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]} + \dots$$

The equation shows four diagrams representing different pion exchange topologies between two nucleons. The first diagram is a box diagram with 1S_0 and 3S_1 states. The second is a triangle diagram with 1S_0 and 3S_1 states. The third is a triangle diagram with 1S_0 and 1S_0 states. The fourth is a contact diagram with two 1S_0 states.

$$\text{[diagram 5]} + \text{[diagram 6]} + \mathcal{O}(\lambda^4)$$

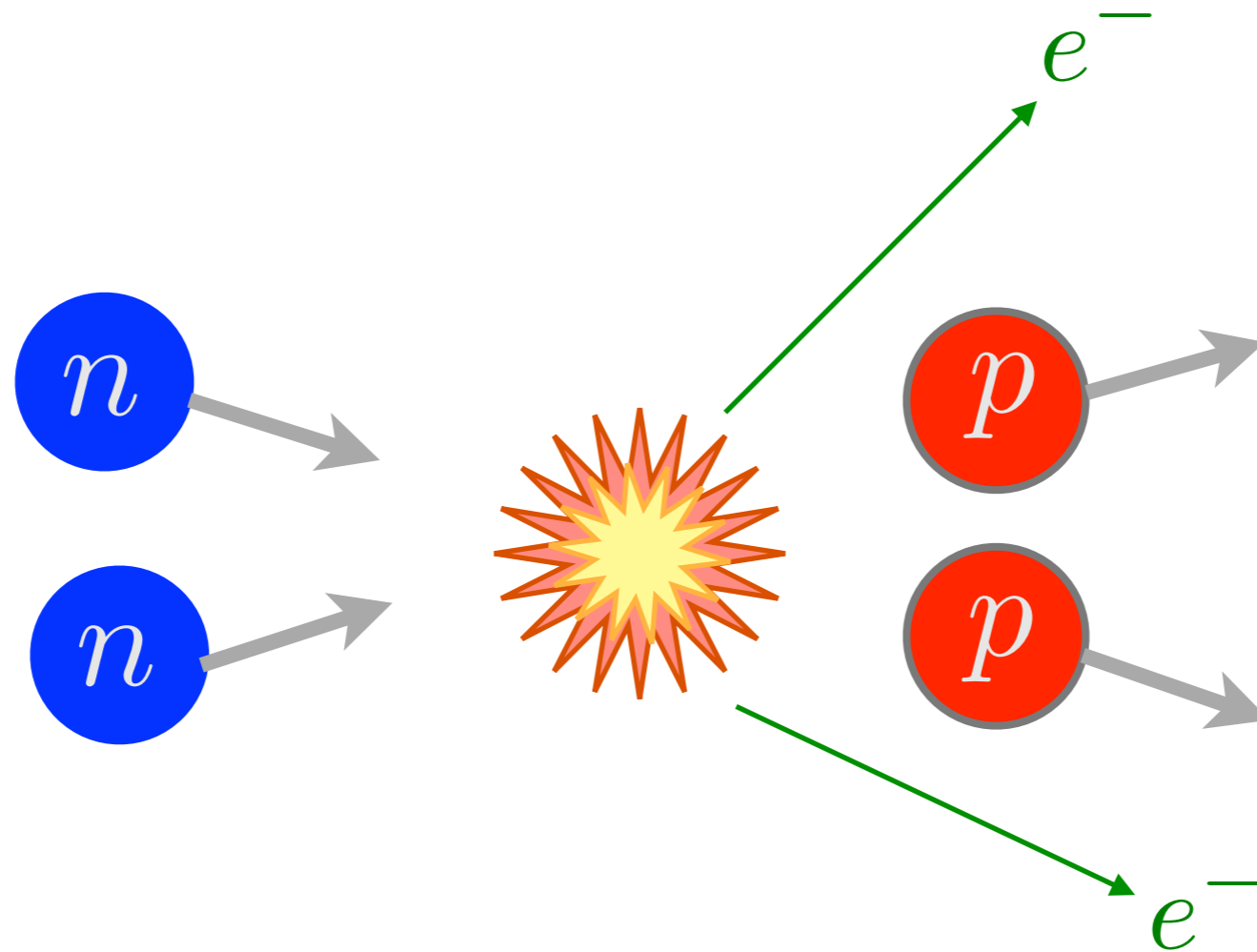
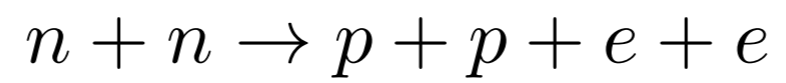
Diagram 5 is a box diagram with four pion lines. Diagram 6 is a contact diagram with two pion lines, enclosed in a green dashed box.

A coupling related to $\sim h_{2,S}$ above.

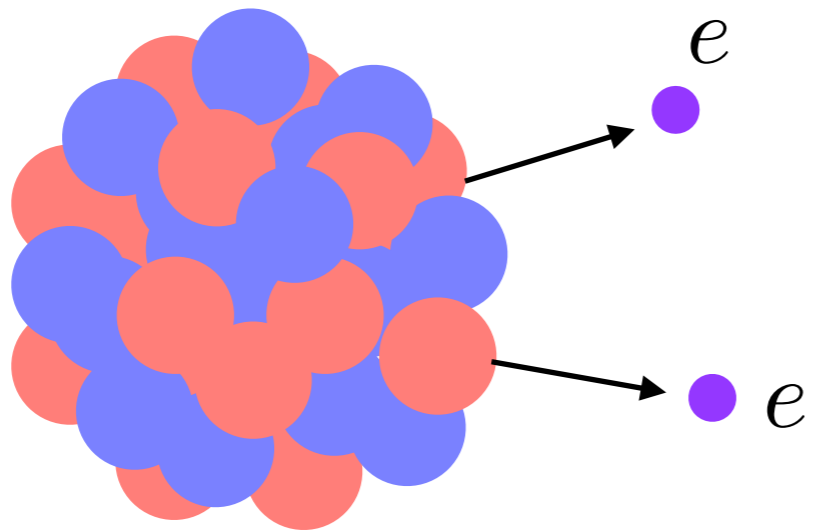
$$\mathbb{H}_{2,S} = 4.7(1.3)(1.8) \text{ fm}$$

$$@ m_\pi \approx 800 \text{ MeV}$$

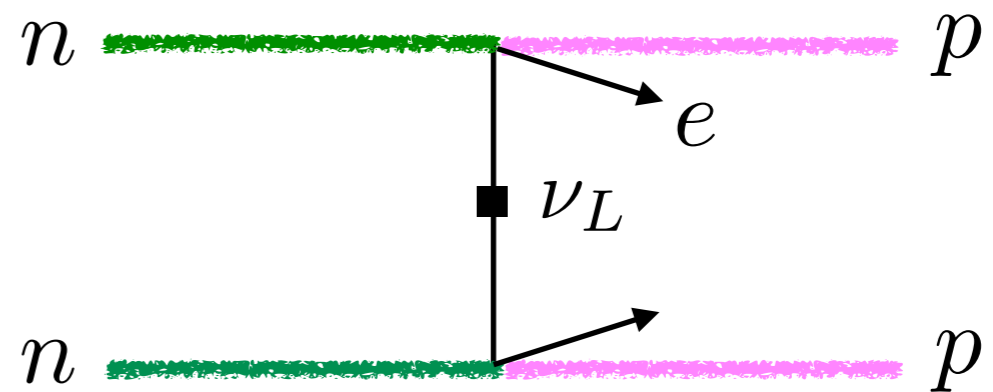
NEUTRINOLESS DOUBLE-BETA DECAY



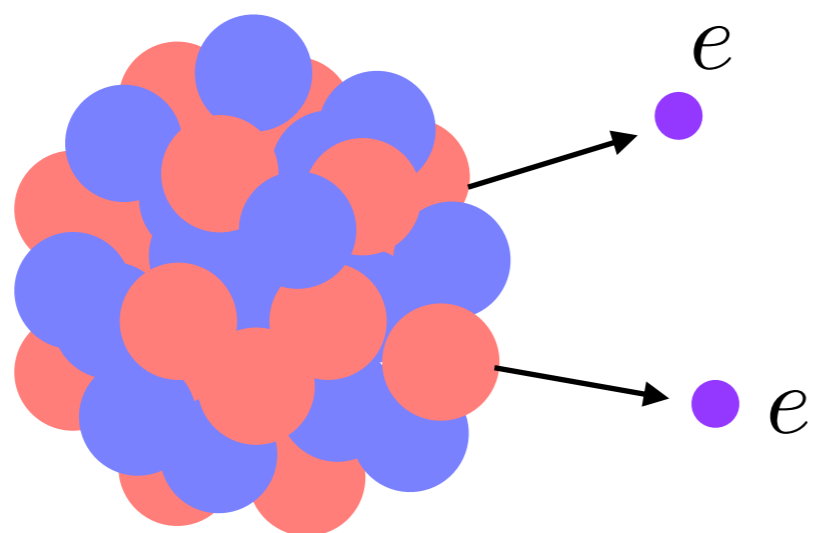
We cannot study the matrix elements of Germanium or other experimentally relevant isotopes directly, but lattice QCD combined with EFT can help improve nuclear structure predictions of the rates.



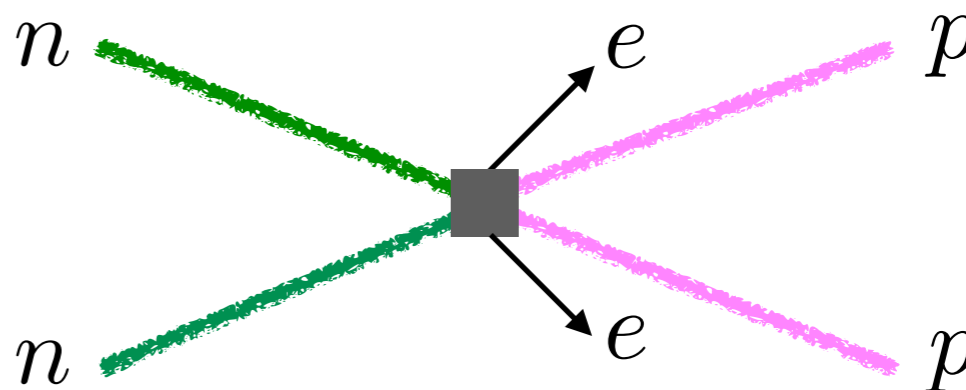
Momentum exchanged: $q \sim 100$ MeV
Three and multi-nucleon effects?



We cannot study the matrix elements of Germanium or other experimentally relevant isotopes directly, but lattice QCD combined with EFT can help improve nuclear structure predictions of the rates.



Momentum exchanged: $q \sim 100$ MeV
Three and multi-nucleon effects?

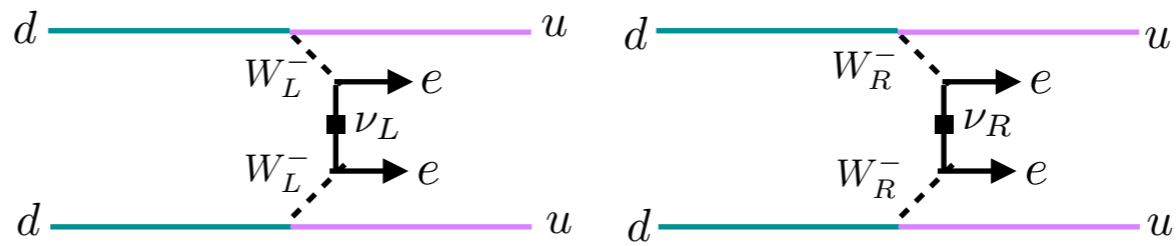


Ask Jordy all about it!

Matching high scale to low scale for $0\nu BB$ decay

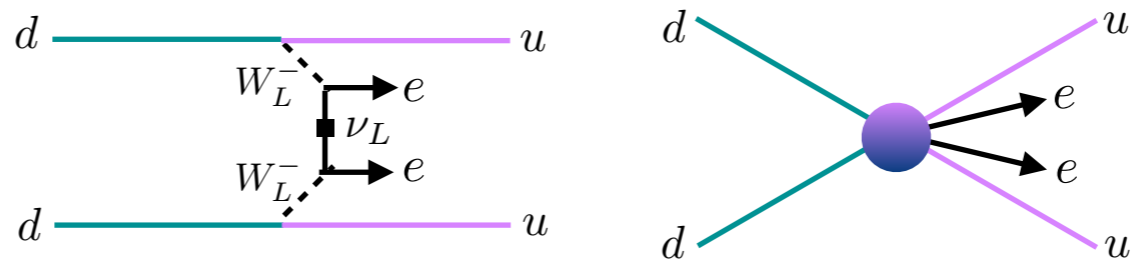
$\Lambda > \text{TeV}$

START WITH YOUR FAVORITE HIGH-SCALE MODEL, E.G.:



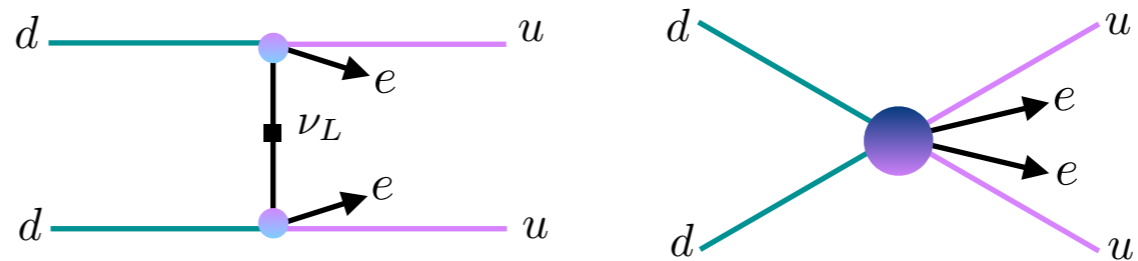
$\Lambda \sim 10^2 \text{ GeV}$

RUN IT DOWN TO THE SCALE WHERE THE HIGH-SCALE PHYSICS CAN BE INTEGRATED OUT:



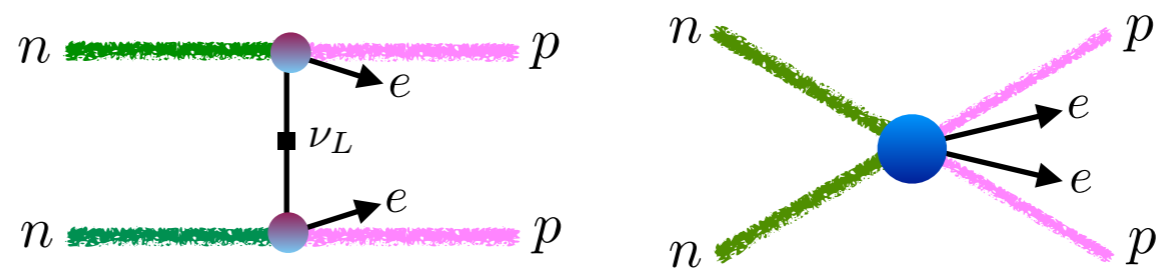
$\Lambda \sim 2 \text{ GeV}$

RUN IT DOWN TO PERTURBATIVE QUARK-LEVEL MATRIX ELEMENTS:



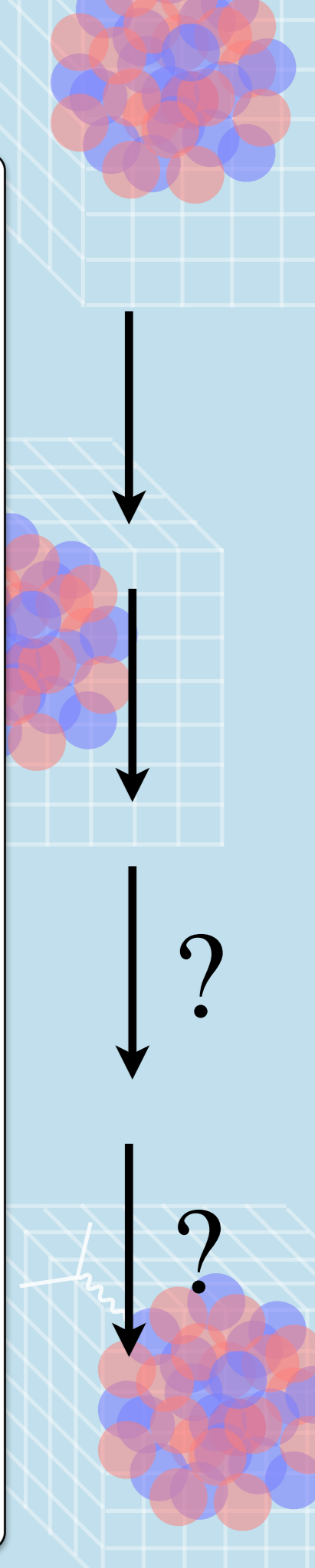
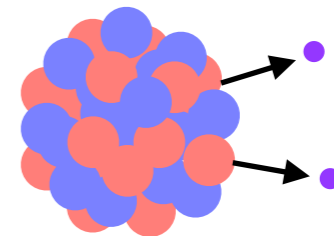
$\Lambda < \text{GeV}$

RUN IT DOWN TO THE HADRNIC SCALE:



$\Lambda < \text{MeV}$

USE NUCLEAR MANY-BODY CALCULATION TO MATCH IT TO NUCLEAR MATRIX ELEMENTS:





Two scenarios

NON-LOCAL MATRIX ELEMENTS OF TWO DIMENSION-6 FOUR- FERMION STANDARD MODEL WEAK CURRENTS

Constrains more reliably the limits on the effective Majorana neutrino mass (a combination of masses and mixing angles) in the minimal extension of SM.

LOCAL MATRIX ELEMENTS OF DIMENSION-9 SIX-FERMION OPERATORS

Helps to find out if predictions of high-scale models are within the reach of current and future experimental limits. Will eventually constrain such models more reliably.



Two scenarios

NON-LOCAL MATRIX ELEMENTS OF TWO DIMENSION-6 FOUR-FERMION STANDARD MODEL WEAK CURRENTS

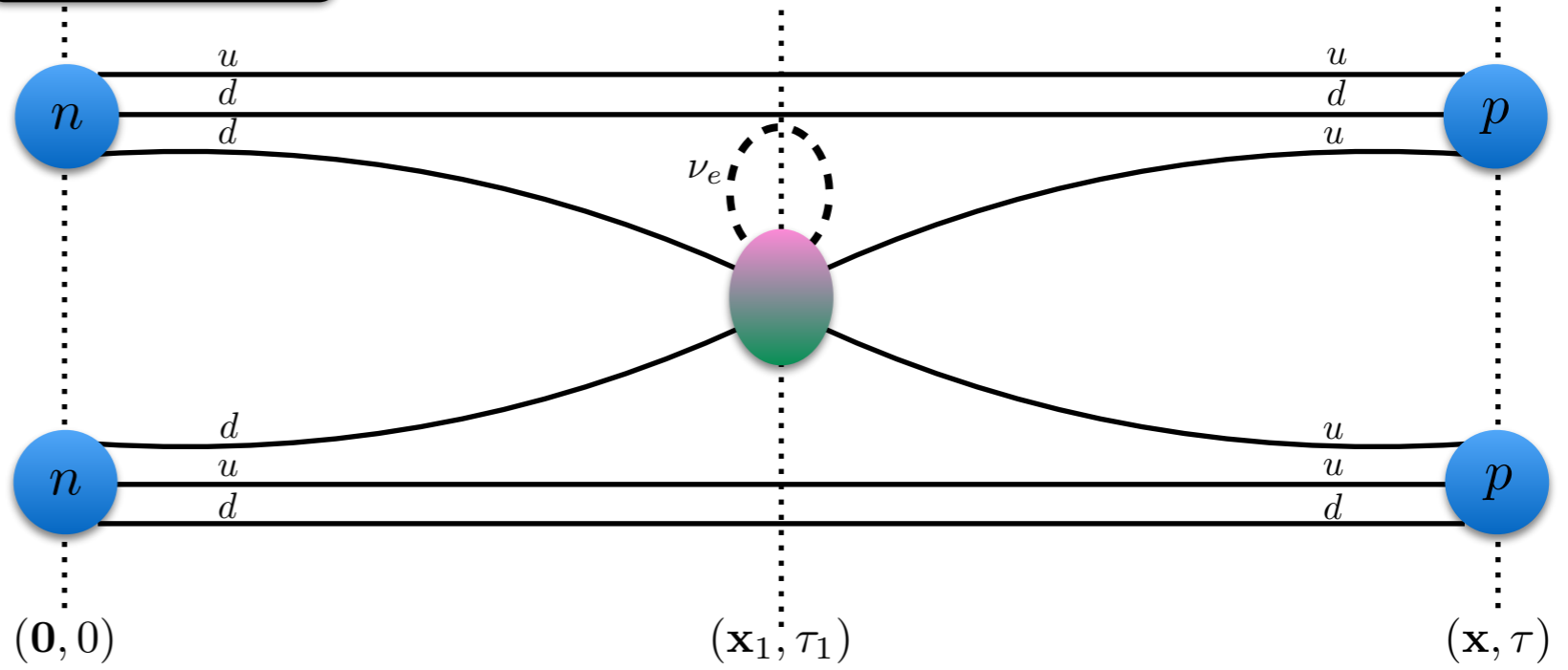
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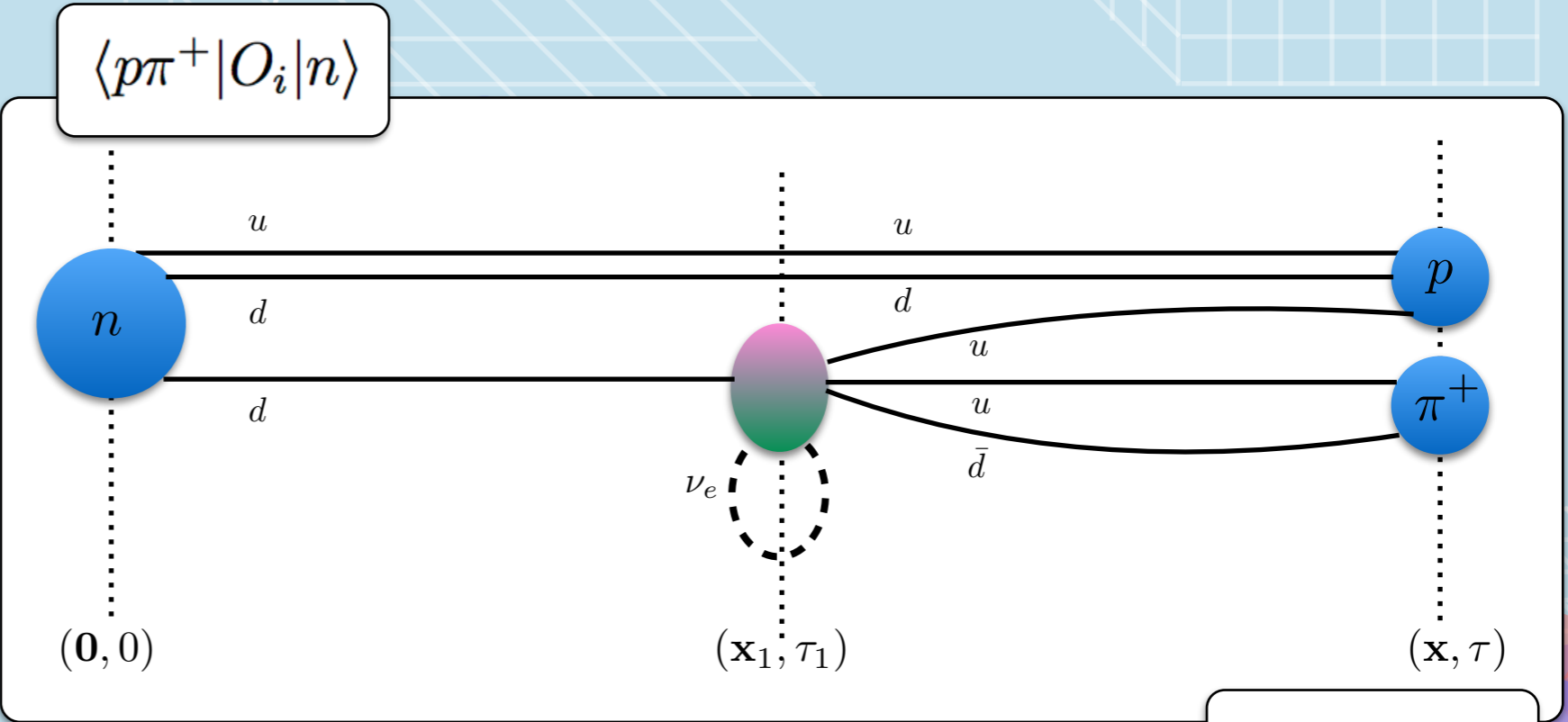
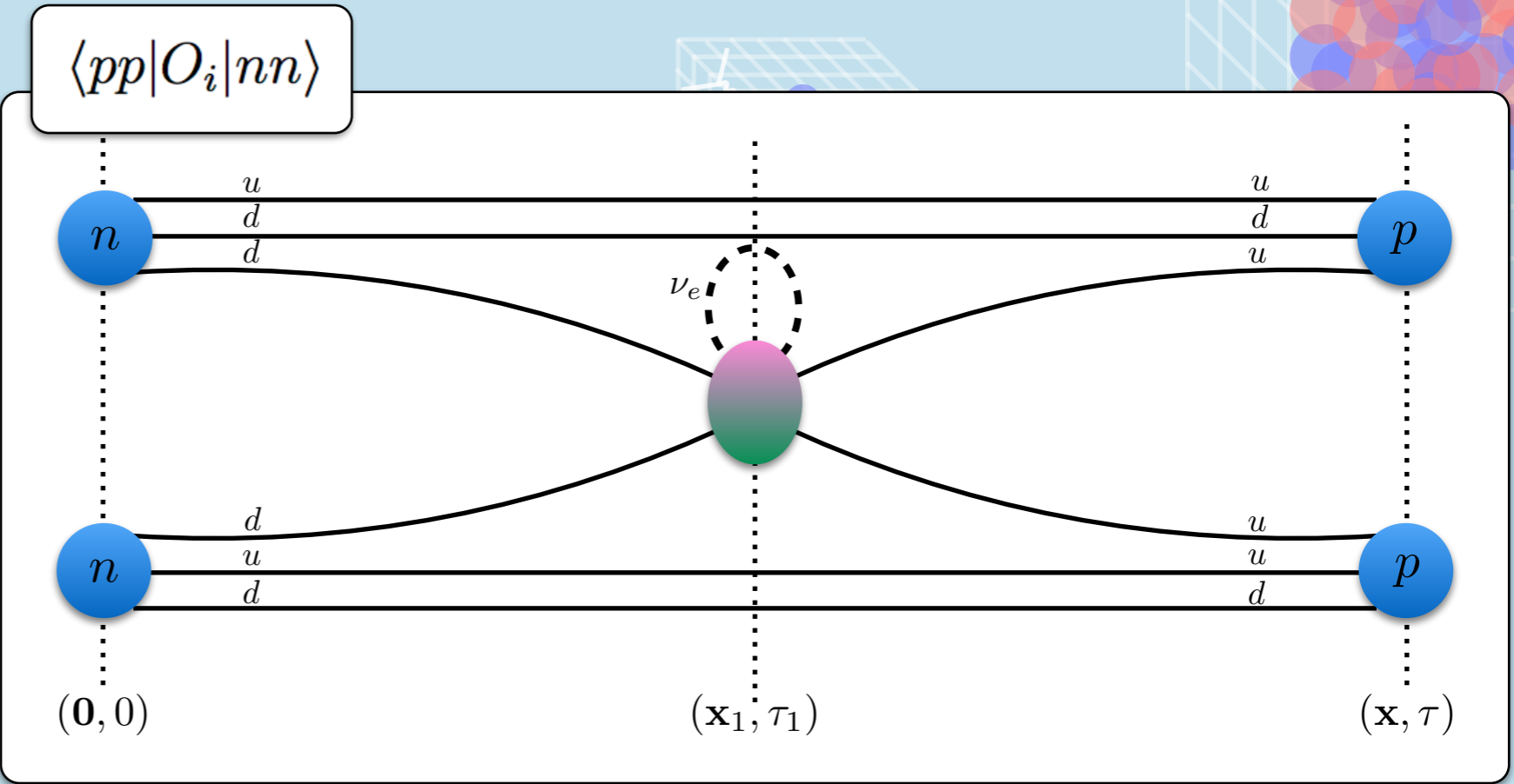
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MATRIX ELEMENTS OF LOCAL
FOUR-QUARK OPERATORS

$$\langle pp|O_i|nn\rangle$$



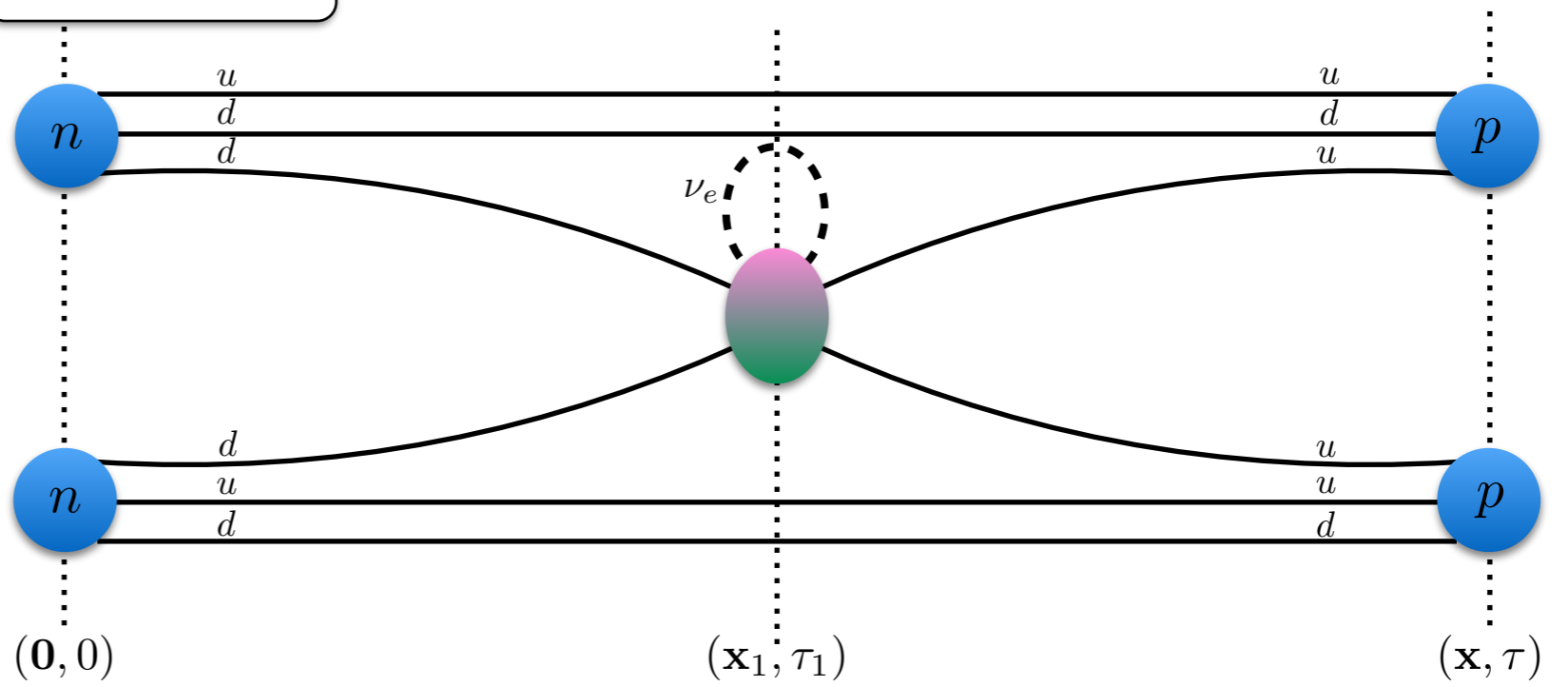
MATRIX ELEMENTS OF LOCAL
FOUR-QUARK OPERATORS



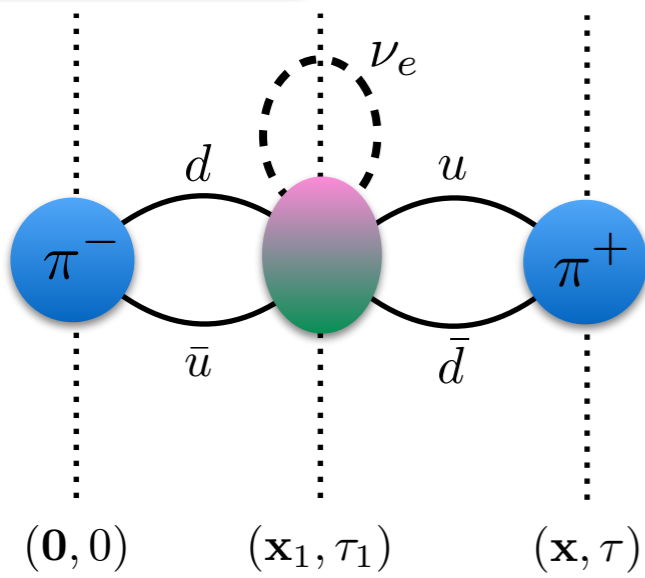
$\langle p|O_i|n\pi^- \rangle$

MATRIX ELEMENTS OF LOCAL
FOUR-QUARK OPERATORS

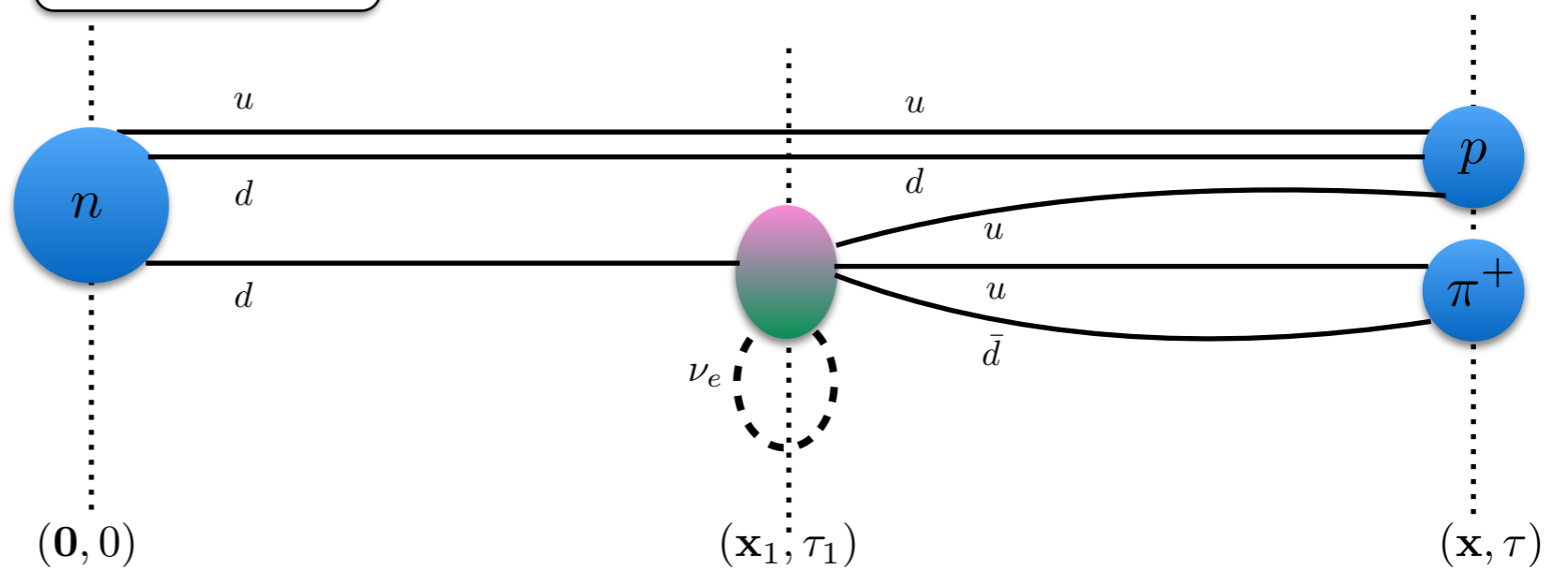
$$\langle pp|O_i|nn\rangle$$



$$\langle \pi^+|O_i|\pi^- \rangle$$



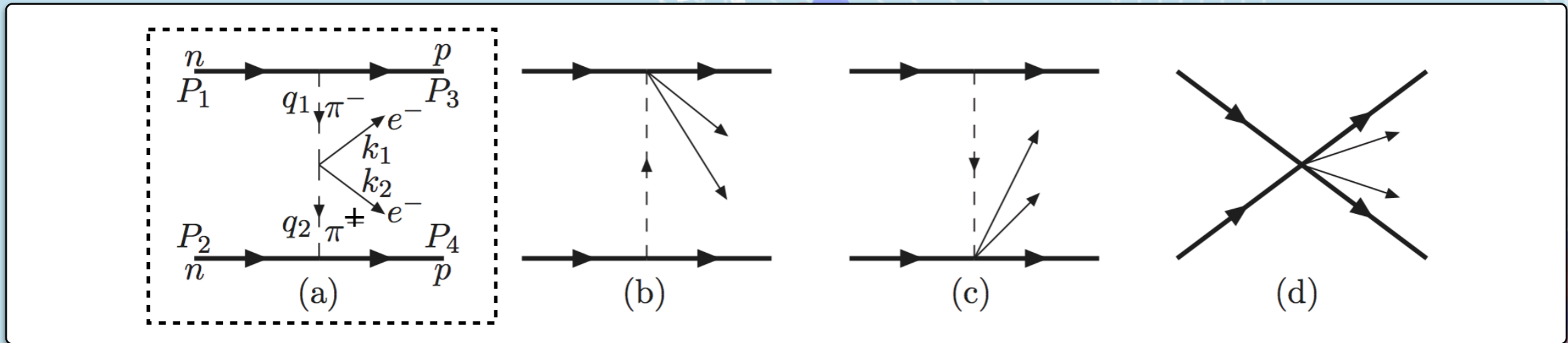
$$\langle p\pi^+|O_i|n\rangle$$



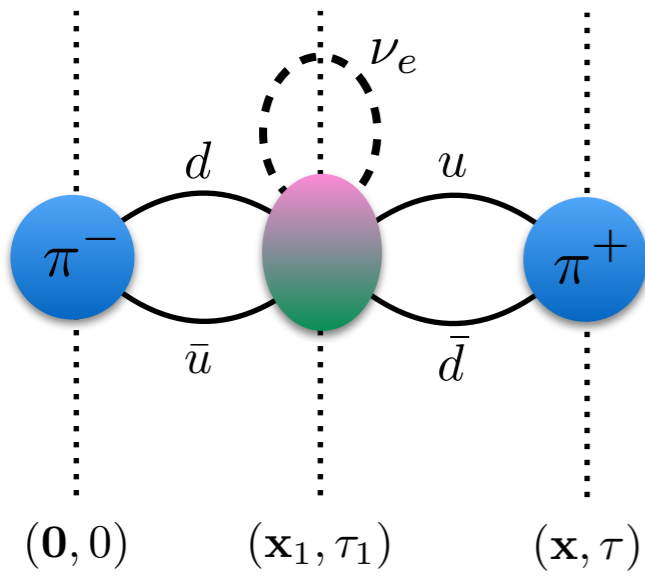
$$\langle p|O_i|n\pi^- \rangle$$

MATRIX ELEMENTS OF LOCAL FOUR-QUARK OPERATORS

Prezeau, Ramsey-Musolf, Vogel Phys.Rev. D68 03401 (2003).



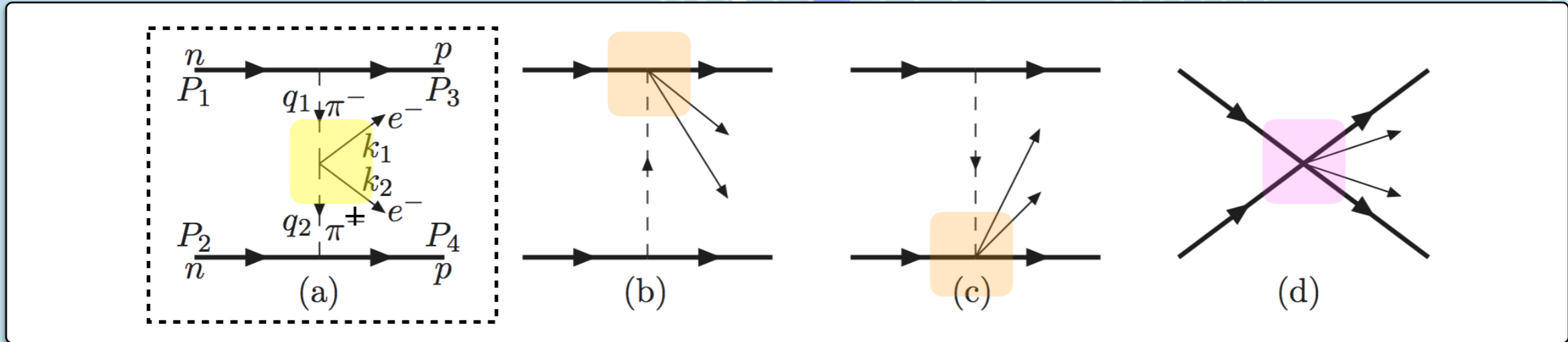
$$\langle \pi^+ | O_i | \pi^- \rangle$$



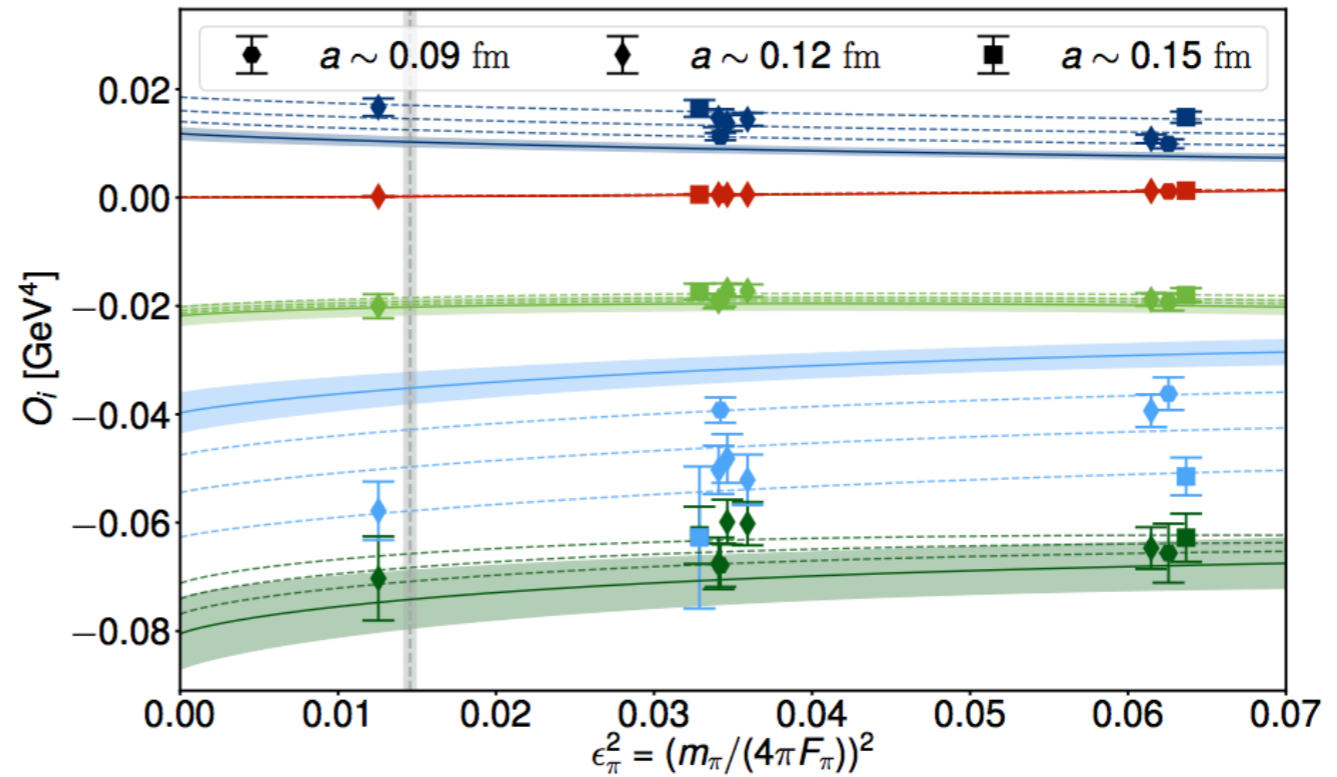
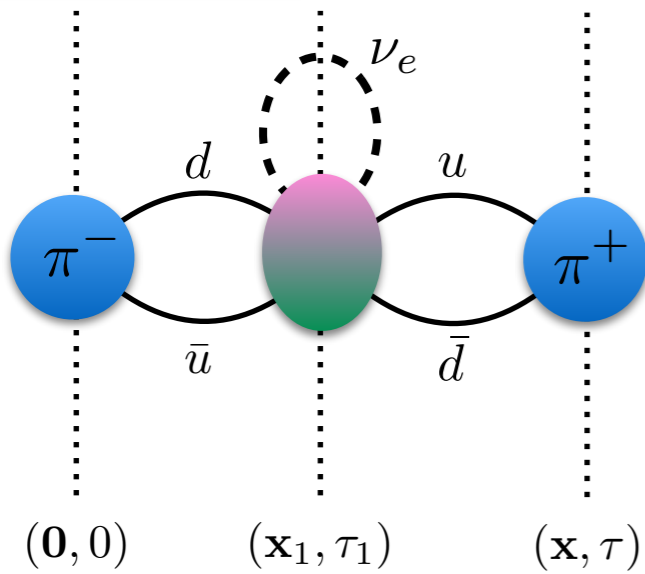
MATRIX ELEMENTS OF LOCAL FOUR-QUARK OPERATORS

Indirect methods to constrain the same LECs:
 Savage, Phys. Rev. C59, 2293 (1999).
 Cirigliano, Dekens, Graesser and Mereghetti,
 PLB Volume 769, 2017, 460-464.

Prezeau, Ramsey-Musolf, Vogel Phys.Rev. D68 03401 (2003).



$$\langle \pi^+ | O_i | \pi^- \rangle$$



A lattice QCD determination:
 Nicholson et al (CALLATT collaboration),
 Phys. Rev. Lett. 121, 172501 (2018).



Two scenarios

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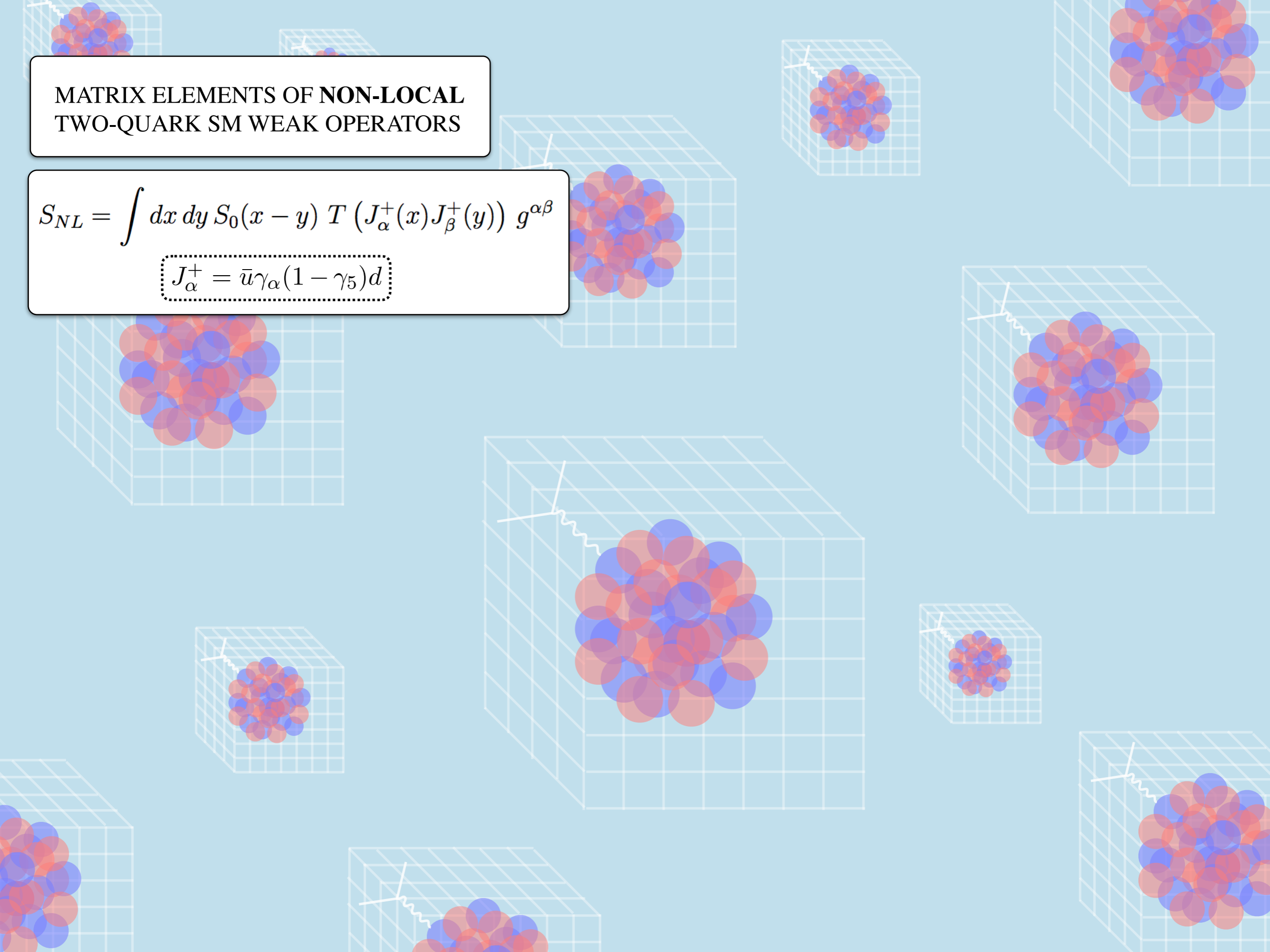
LOCAL MATRIX ELEMENTS OF DIMENSION-9 SIX-FERMION OPERATORS

Helps to find out if predictions of high-scale models are within the reach of current and future experimental limits. Will eventually constrain such models more reliably.

MATRIX ELEMENTS OF **NON-LOCAL**
TWO-QUARK SM WEAK OPERATORS

$$S_{NL} = \int dx dy S_0(x - y) T (J_\alpha^+(x) J_\beta^+(y)) g^{\alpha\beta}$$

$$J_\alpha^+ = \bar{u} \gamma_\alpha (1 - \gamma_5) d$$



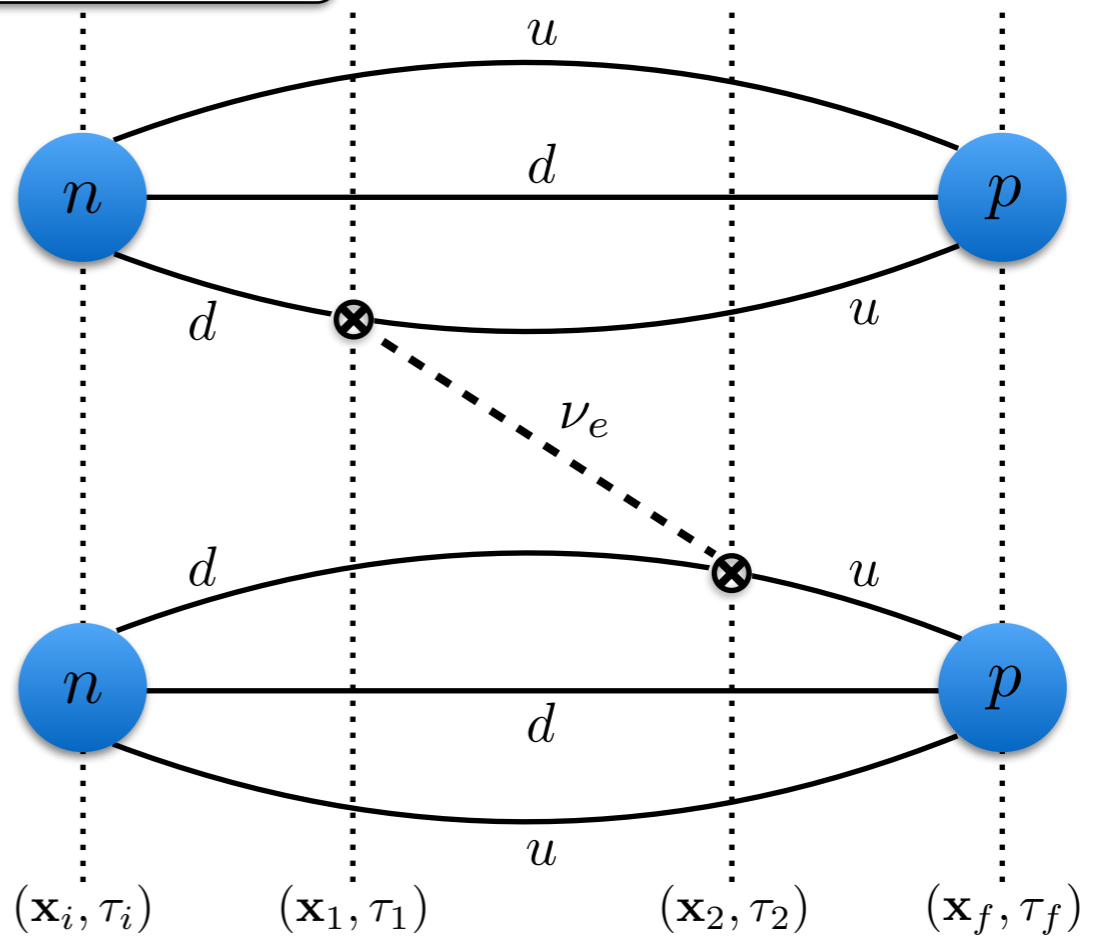
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EFT approach makes the case for LQCD even stronger, see e.g., Cirigliano, Dekens, De Vries, Graesser, Mereghetti, Pastore, and Van Kolck, Phys. Rev. Lett. 120, 202001 (2018), arXiv:1802.10097 [hep-ph].

$$\langle pp | S_{NL} | nn \rangle$$



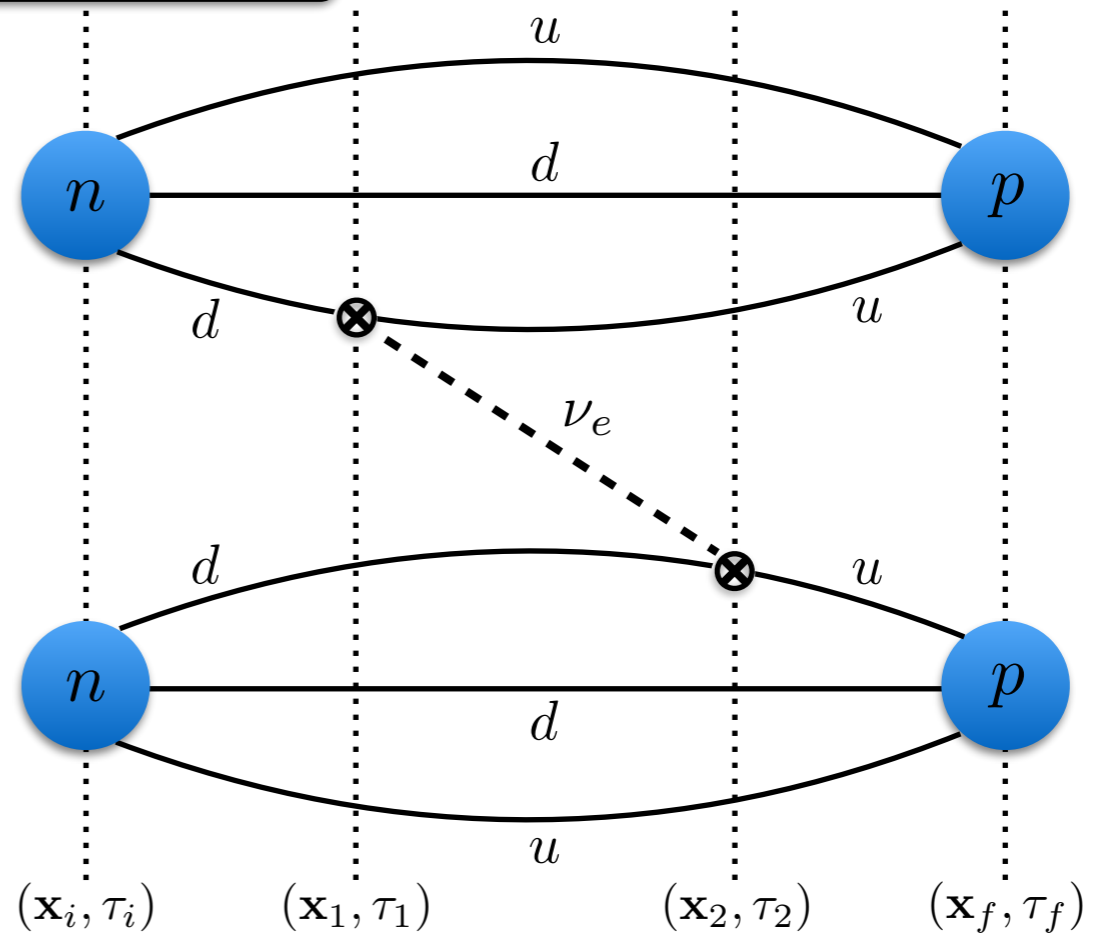
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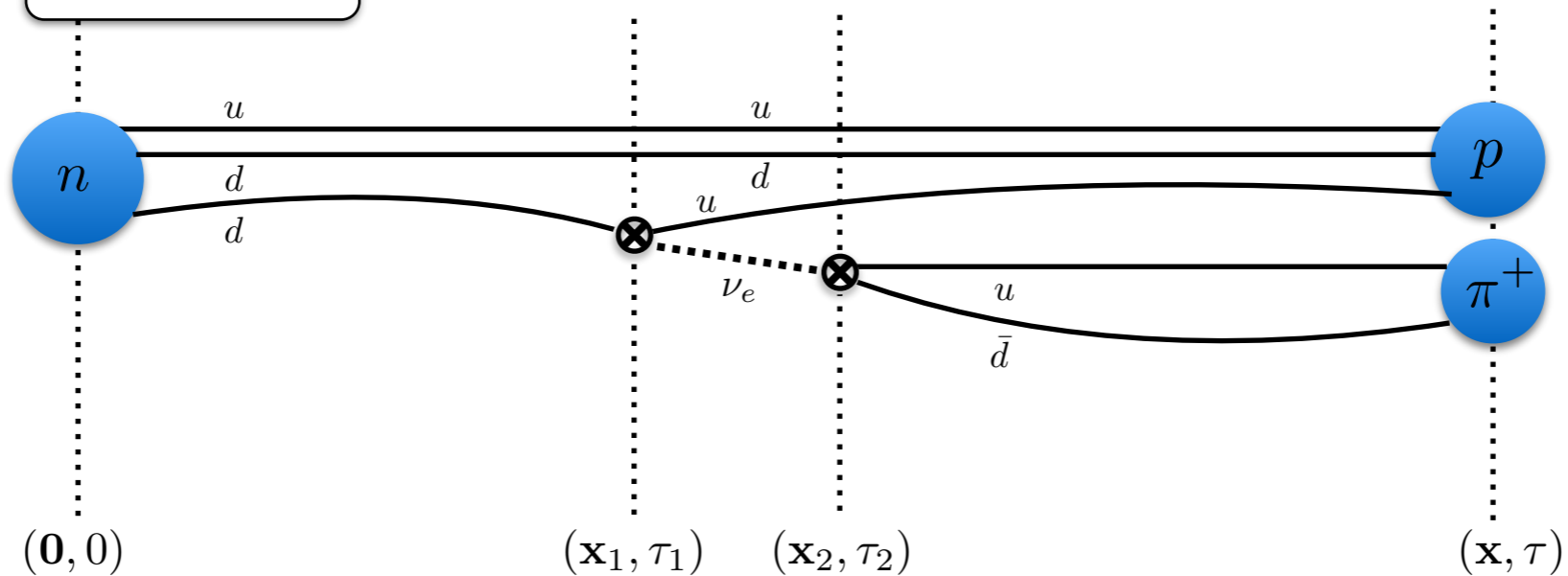
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$$\langle pp | S_{NL} | nn \rangle$$



$$\langle p\pi^+ | S_{NL} | n \rangle$$



$$\langle p | S_{NL} | n\pi^- \rangle$$

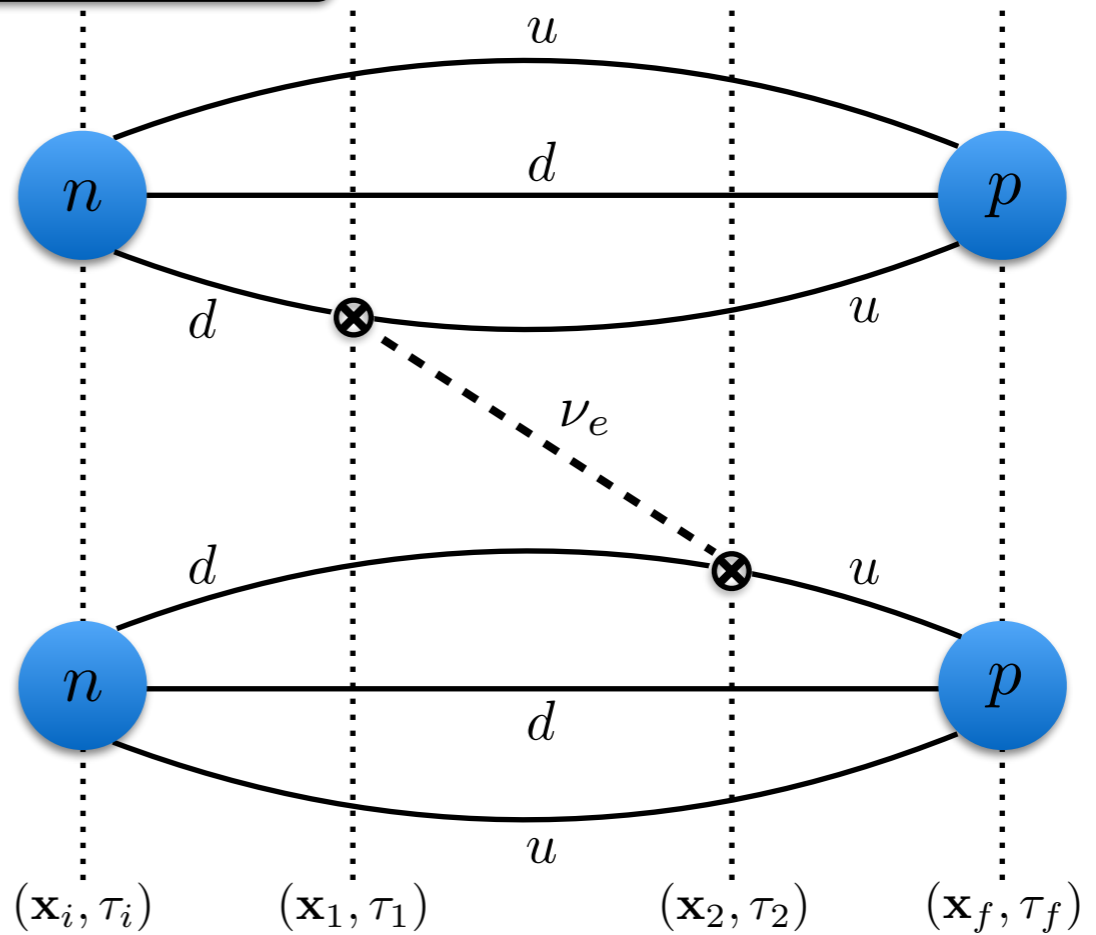
MATRIX ELEMENTS OF NON-LOCAL TWO-QUARK SM WEAK OPERATORS

$$S_{NL} = \int dx dy S_0(x - y) T (J_\alpha^+(x) J_\beta^+(y)) g^{\alpha\beta}$$

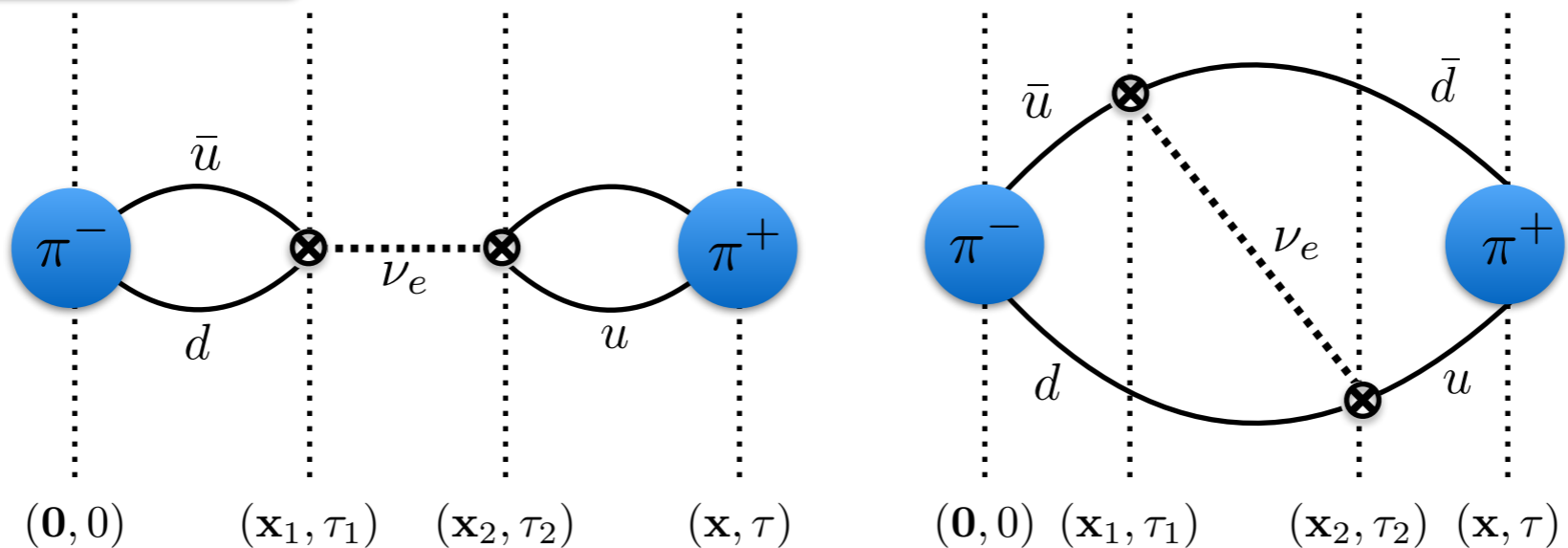
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$$\langle pp | S_{NL} | nn \rangle$$



$$\langle \pi^+ | S_{NL} | \pi^- \rangle$$



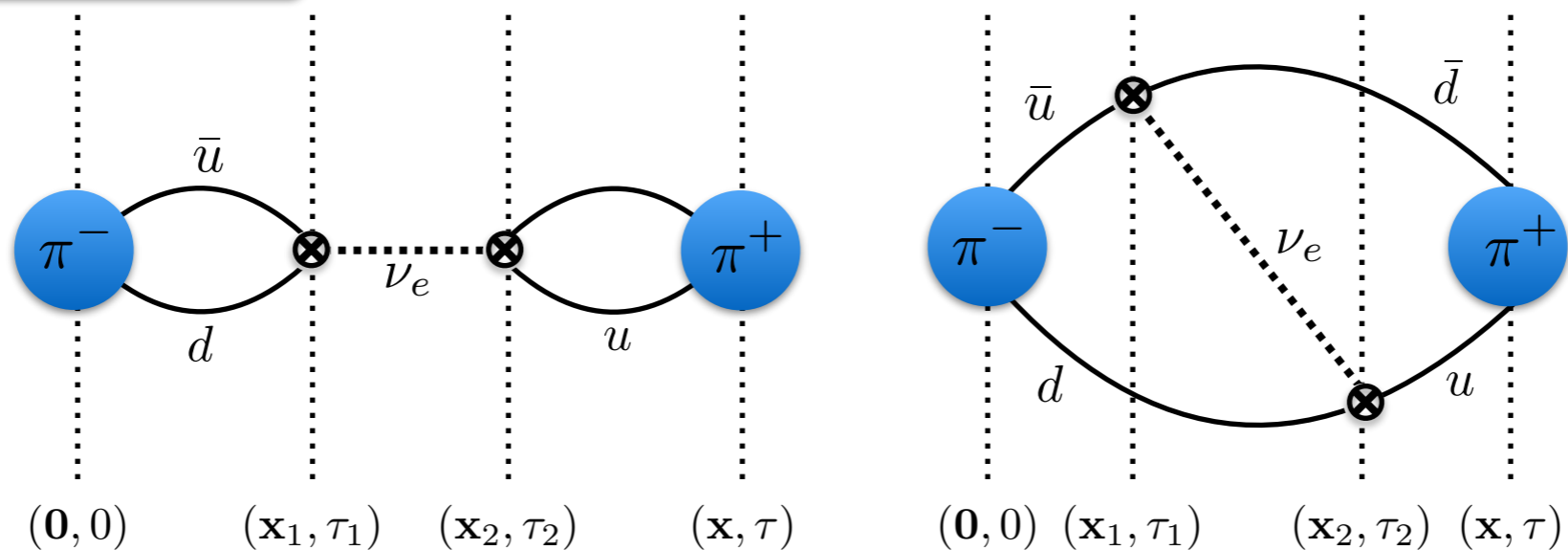
MATRIX ELEMENTS OF **NON-LOCAL**
TWO-QUARK SM WEAK OPERATORS

Ratio of next-to-leading to leading order amplitude
in chiral perturbation theory:

$$S_{\pi\pi} = 1 + \frac{m_\pi^2}{8\pi^2 f_\pi^2} \left(3 \log \left(\frac{\mu^2}{m_\pi^2} \right) + 6 + \frac{5}{6} g_\nu^{\pi\pi}(\mu) \right)$$

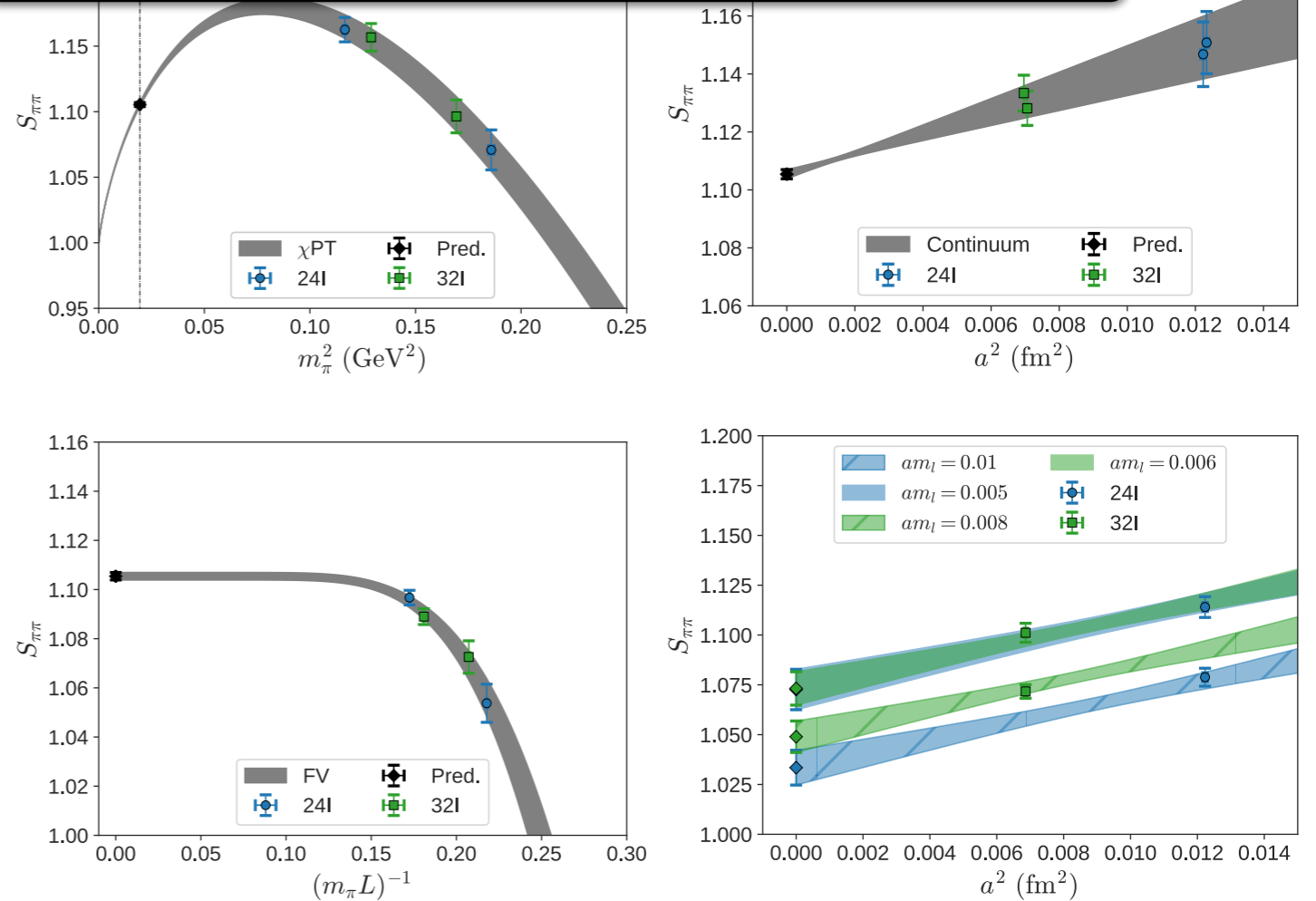
The unknown ChiPT LEC to be
determined with lattice QCD.

$\langle \pi^+ | S_{NL} | \pi^- \rangle$



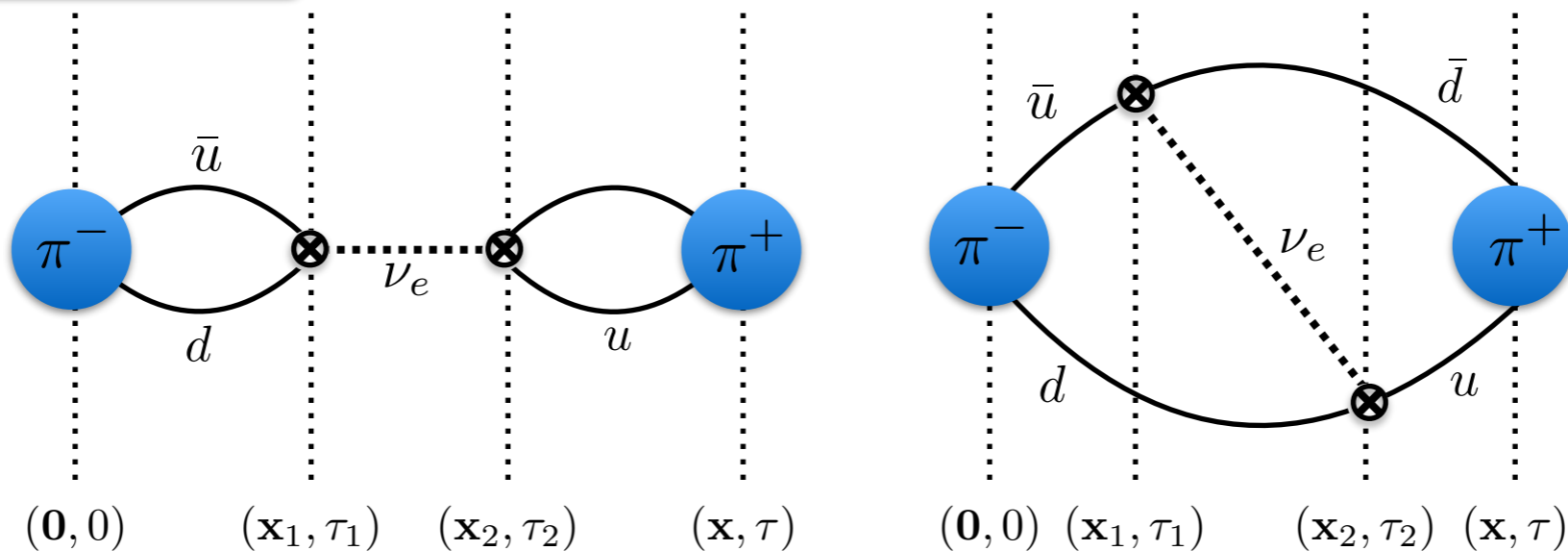
MATRIX ELEMENTS OF NON-LOCAL TWO-QUARK SM WEAK OPERATORS

$$g_\nu^{\pi\pi}(770 \text{ MeV}) = -10.78(12)_{\text{stat}}(4)_{\text{fit}}(50)_{\text{FV}}(9)_{\chi\text{PT}}$$

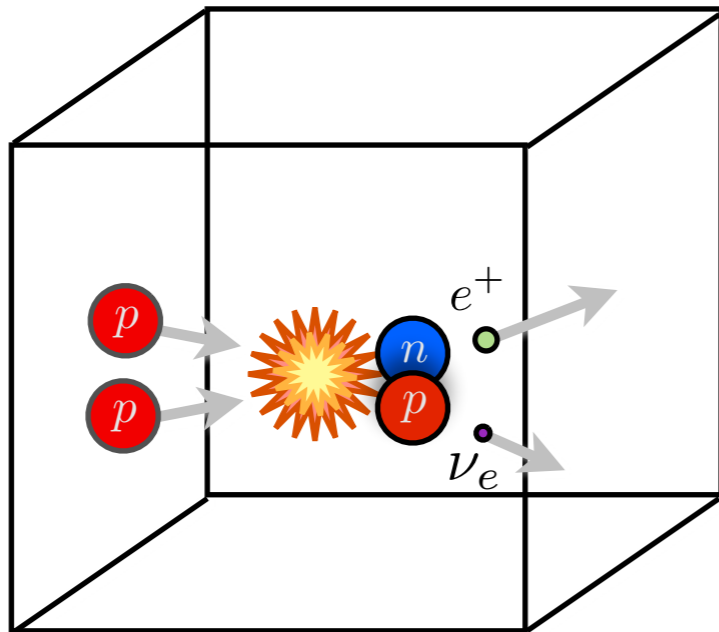


See also: Tuo, Feng, Jin, *phys.Rev.D* 100 (2019) 9, 094511.

$$\langle \pi^+ | S_{NL} | \pi^- \rangle$$

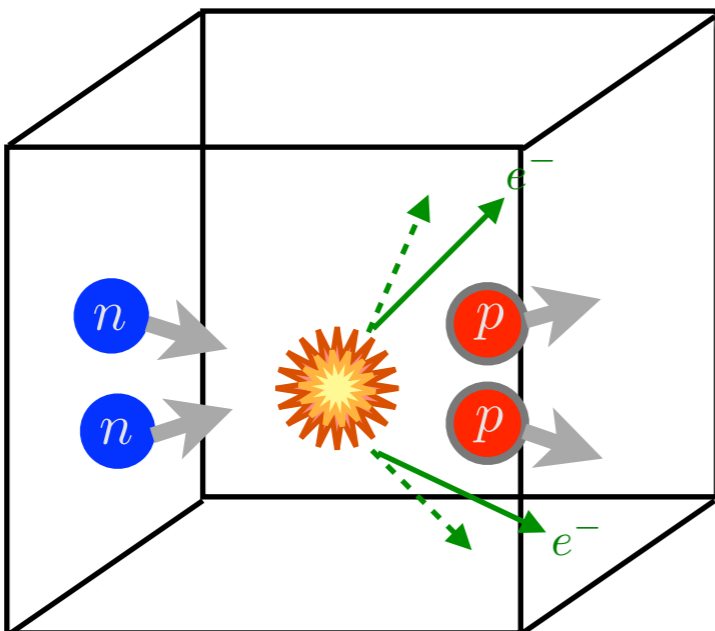
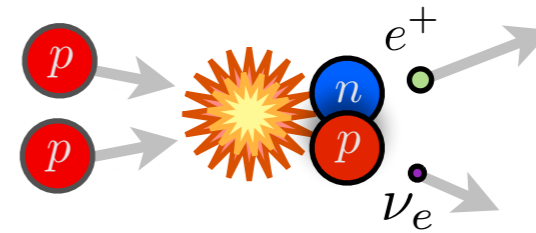


EUCLIDEAN

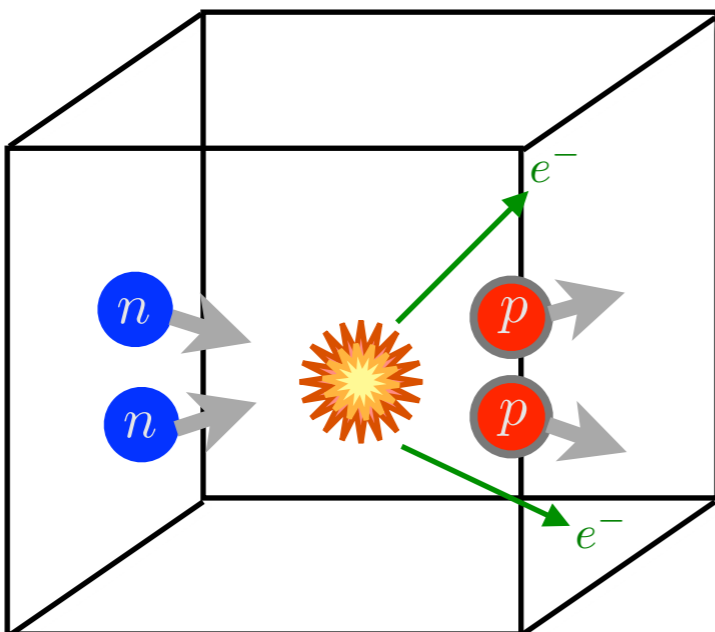
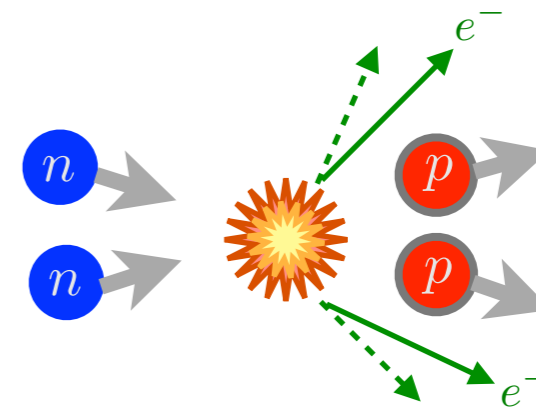


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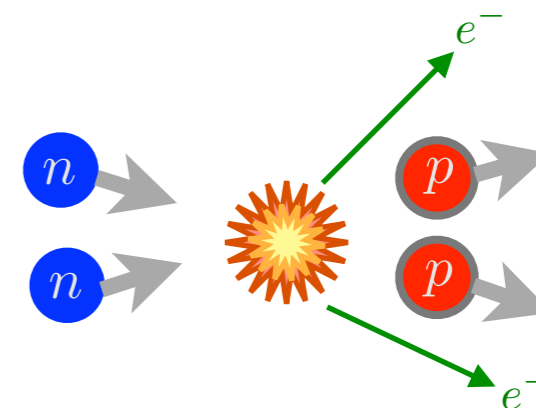
MINKOWSKI



?



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The **finite-volume technology** for electroweak matrix elements is crucial for the success of the program and builds upon many valuable developments of the past, to mention a few...

Weak transition matrix elements from finite volume correlation functions

Lellouch and Luescher, Commun. Math. Phys. 219, 31–44 (2001).

Finite-volume effects for two-hadron states in moving frames

Kim, Sachrajda, and Sharpe, Nucl. Phys. B 727, 218–243 (2005).

Electroweak matrix elements in the two-nucleon sector from lattice QCD

Detmold and Savage, Nucl. Phys. A743 170–193 (2004).

Matrix elements of unstable states

Bernard, Hoja, Meißner, Rusetsky JHEP, Vol 2012, 23 (2012) .

Multichannel one-to-two transition form factors in a finite volume

Briceno, Hansen, and Walker-Loud, Phys. Rev. D 91, 034501 (2015).

Moving Multi-Channel Systems in a Finite Volume with Application to Proton-Proton Fusion

Briceno and Davoudi, Phys. Rev. D 88, 094507 (2013).

Relativistic, model-independent, multichannel $2 \rightarrow 2$ transition amplitudes in a finite volume

Briceno and Hansen, Phys. Rev. D 94, 013008 (2016).

Effects of finite volume on the KL-KS mass difference

Christ, Feng, Martinelli, and Sachrajda, Phys. Rev. D 91, 114510 (2015).

Long-range electroweak amplitudes of single hadrons from Euclidean finite-volume correlation

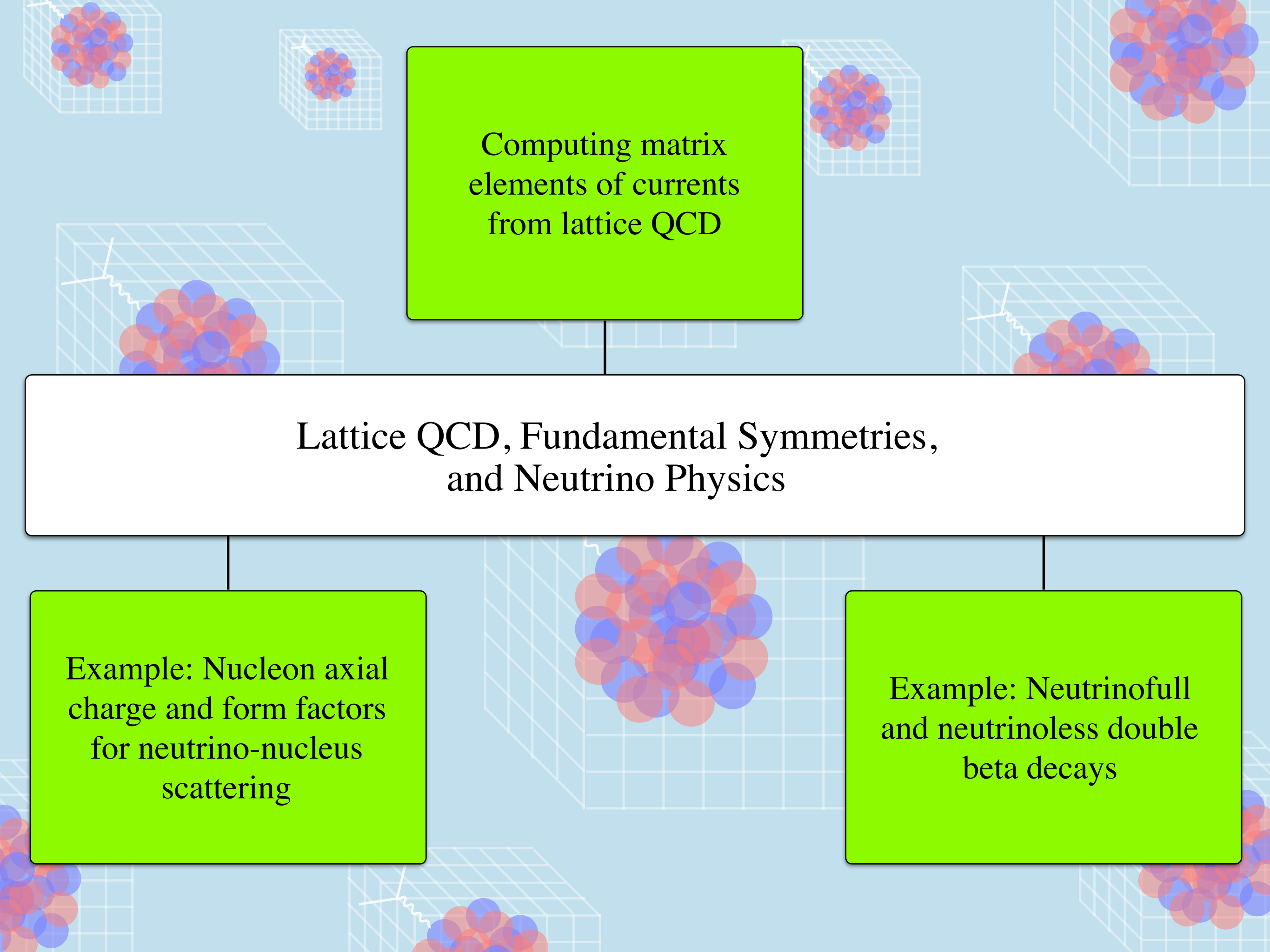
Briceno, Davoudi, Hansen, Schindler, and Baroni, Phys. Rev. D101, 14509 (2020).

Two-neutrino double-beta decay in pionless effective field theory from a Euclidean finite-volume correlation function

Davoudi and Kadam et al, Phys. Rev. D 102, 114521 (2020)

The path from lattice QCD to the short distance contribution to $0\nu\beta\beta$ decay with a light Majorana neutrino

Davoudi and Kadam et al, Phys. Rev. Lett. 126, 152003 (2021).

The background features several 3D wireframe cubes representing a lattice. Inside each cube is a cluster of overlapping red and blue spheres, representing a nucleon. Some cubes also have a white wavy line extending from one corner, possibly representing a gluon field.

Computing matrix
elements of currents
from lattice QCD

Lattice QCD, Fundamental Symmetries,
and Neutrino Physics

Example: Nucleon axial
charge and form factors
for neutrino-nucleus
scattering

Example: Neutrinoless
and neutrinoless double
beta decays

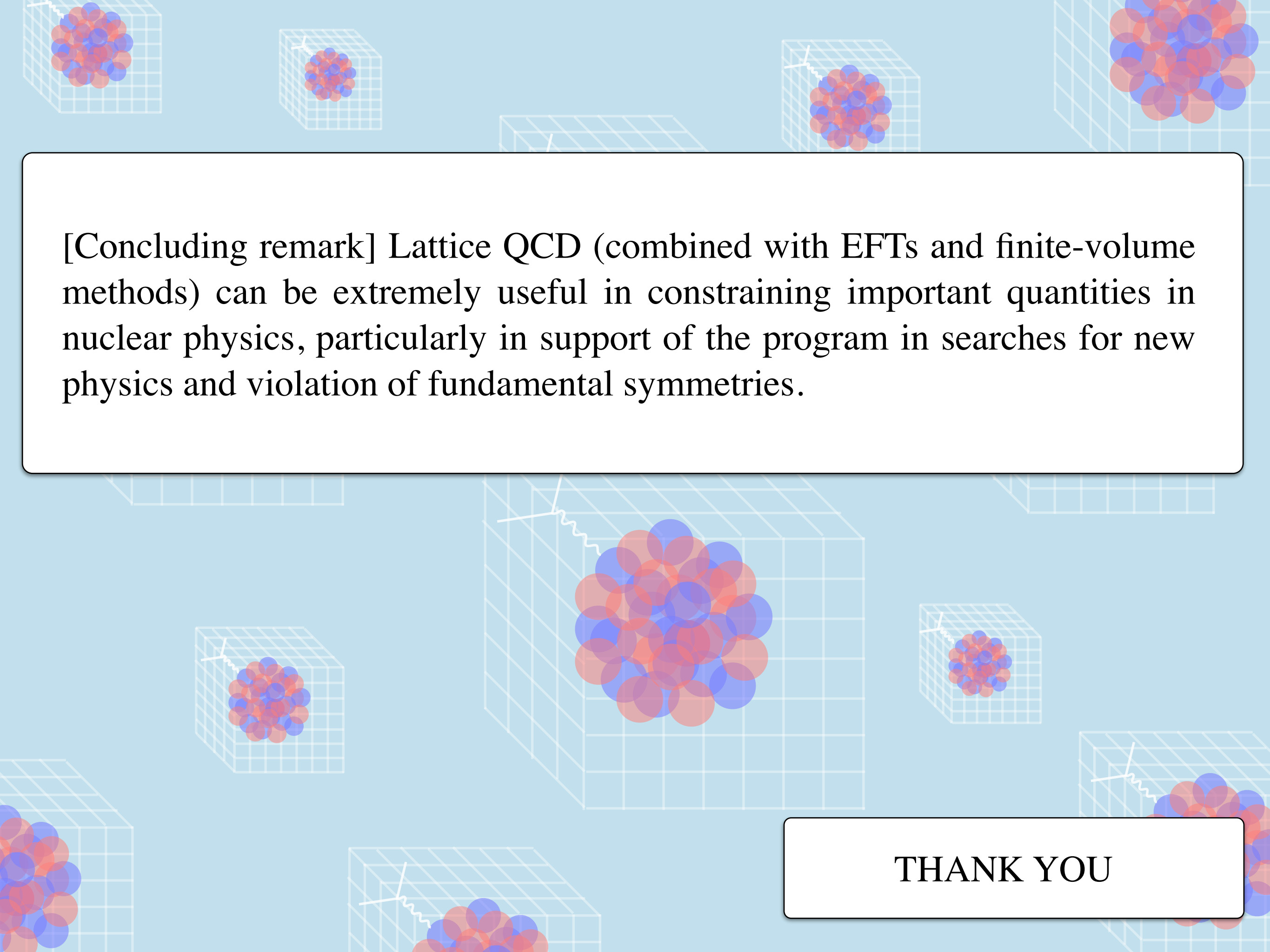
A few more examples where lattice QCD can have an impact...

Physics	Target Quantity	Experiments
Baryon Number Violation and Grand Unified Theories	Proton Decay Matrix Elements	DUNE, Hyper-Kamiokande
Baryon Number minus Lepton Number Violation	Neutron-antineutron Matrix Elements	ILL, ESS Super-K, DUNE and other reactors
Lepton Flavor Violation	Nucleon and Nuclei Form Factors	Mu2e, COMET
Lepton Number Violation	$0\nu\beta\beta$ Matrix Elements	EXO, Tonne-scale $0\nu\beta\beta$
CP Violation and Baryon Asymmetry in Universe	Electric Dipole Moment	Hg, Ra, n EDM at SNS and LANL
Dark Matter and New Physics Searches	Nucleon and Nuclei Form Factors	Dark Matter Experiments, Precision Measurements

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T. Luu's talk!



[Concluding remark] Lattice QCD (combined with EFTs and finite-volume methods) can be extremely useful in constraining important quantities in nuclear physics, particularly in support of the program in searches for new physics and violation of fundamental symmetries.

THANK YOU