Black Hole Hair: Quantum and Classical

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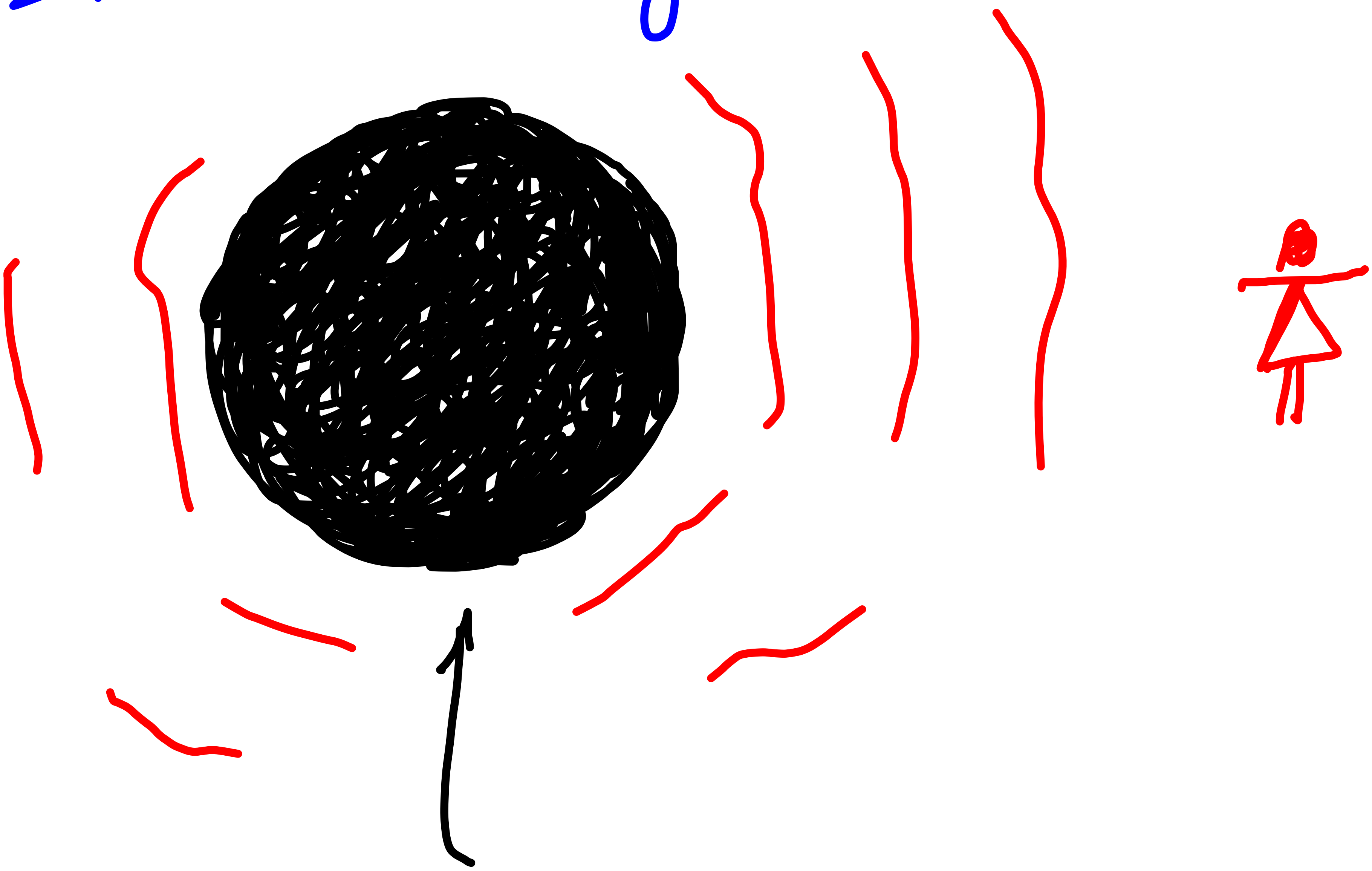
Skyrmion Black Hole Hair: Conservation of Baryon Number by Black Holes

with: Alexander Gußmann

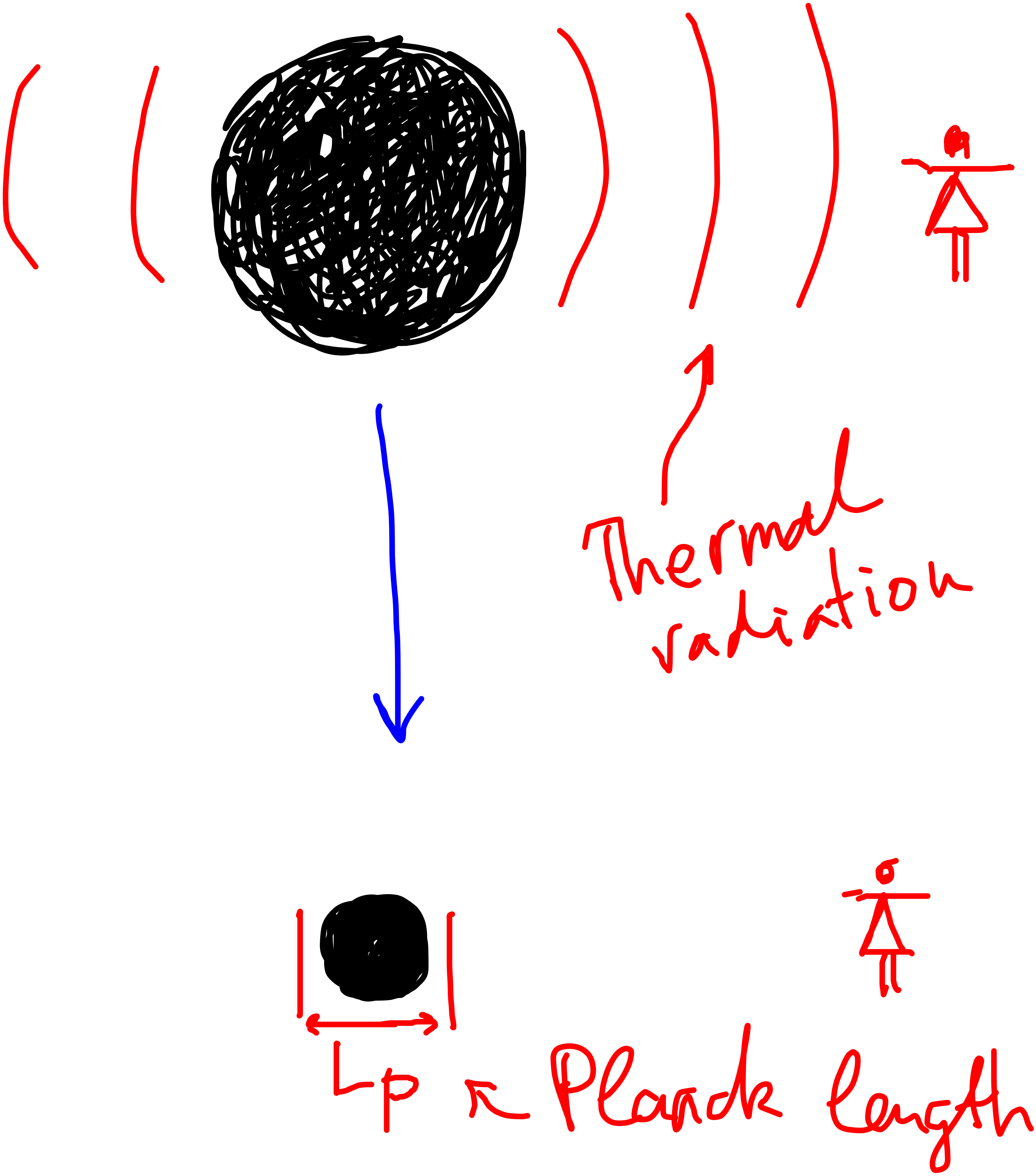
Folk theorem:
Semi-classical black hole physics is incompatible with conserved baryon number!

We shall find a loophole in the "proof".

Standard argument:



Black hole with baryons in it



Thermal radiation
L_p
Planck length

Assumptions:

- No baryonic hair;
- No deviation from thermality untill

$$x_h \sim L_P$$

Black hole horizon

Baryons as Skyrmions

QCD vacuum (take 2 massless quark flavors)

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \langle \bar{u} u \rangle = \langle \bar{d} d \rangle \neq 0$$

Spontaneous breaking of symmetry

$$SU(2)_L \otimes SU(2)_R \longrightarrow SU(2)_V$$

3 Nambu-Goldstone bosons: pions π^a $\quad a = 1, 2, 3$

Thus, the low energy theory below the QCD scale Λ_{QCD} is a Chiral theory of

$$U \equiv \exp\left(i \frac{\pi^a}{F_\pi} \sigma^a\right)$$

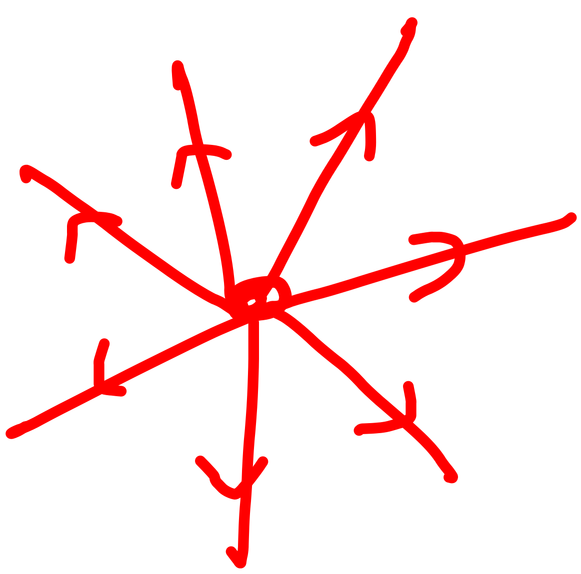
Pauli matrixes

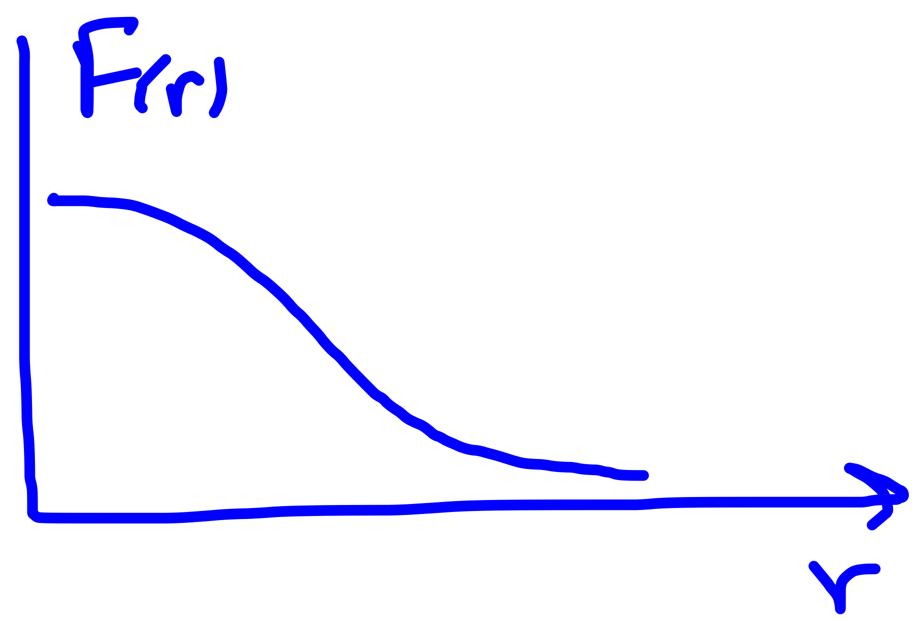
Where are the baryons?

The vacuum is S_3
and there are topological
defects (solitons) Skyrmions

$$\frac{1}{F_\pi}\pi^a = F(r)\,n^a$$

$$\vec{n} \equiv \frac{\vec{r}}{r}$$





Skyrmion hair.

$$\mathcal{L}_S = -F_\pi^2 \, \mathrm{Tr}\left(U^\dagger \partial_\mu U \, U^\dagger \partial^\mu U\right)$$

$$+ \frac{1}{e^2} \mathrm{Tr}\left([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2\right)$$

where

$$U = e^{\frac{i}{F_\pi} \pi^a \sigma^a} \leftarrow \underline{SU(2).}$$

$\pi^a \leftarrow$ Pions

Skyrmion size:

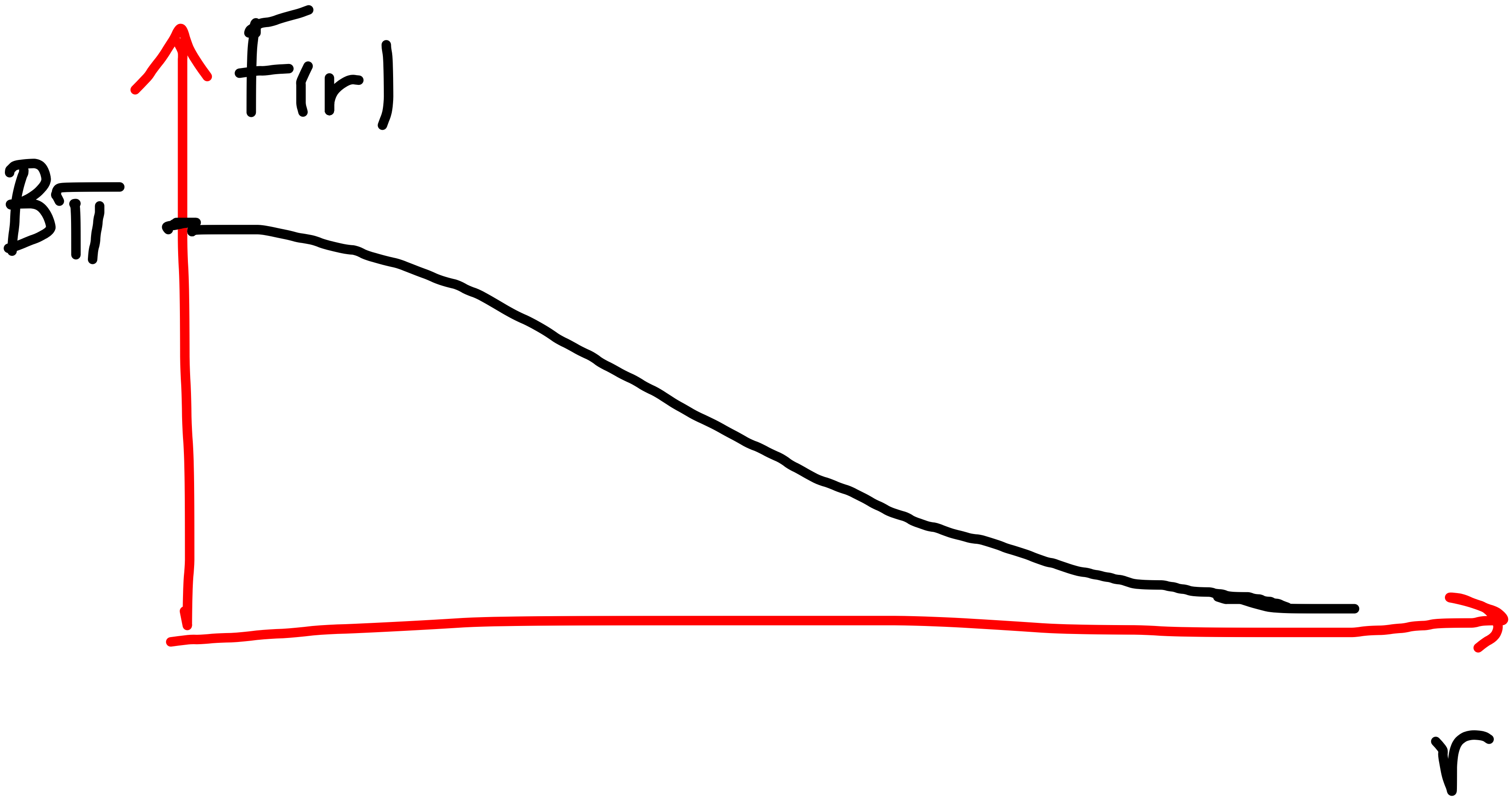
$$L = \frac{1}{e F_\pi}$$

Skyrmion Mass

$$M_S = \frac{F_\pi}{e}$$

Skyrmion:

$$\frac{\pi^a}{F_\pi} = F(r)\, n^a$$



$F(0) = B\pi, \quad F(\infty) = 0$

Skyrm topological Chern-Simons current

$$J_\mu \equiv -\varepsilon_{\mu\nu\alpha\beta} \mathrm{Tr}\left(U^{-1}\partial^\nu U\, U^{-1}\partial^\alpha U\, U^{-1}\partial^\beta U\right)$$

Topological charge

$$B = \int d\vec{x}\, J_0$$

Black holes with skyrmion hair have been found for

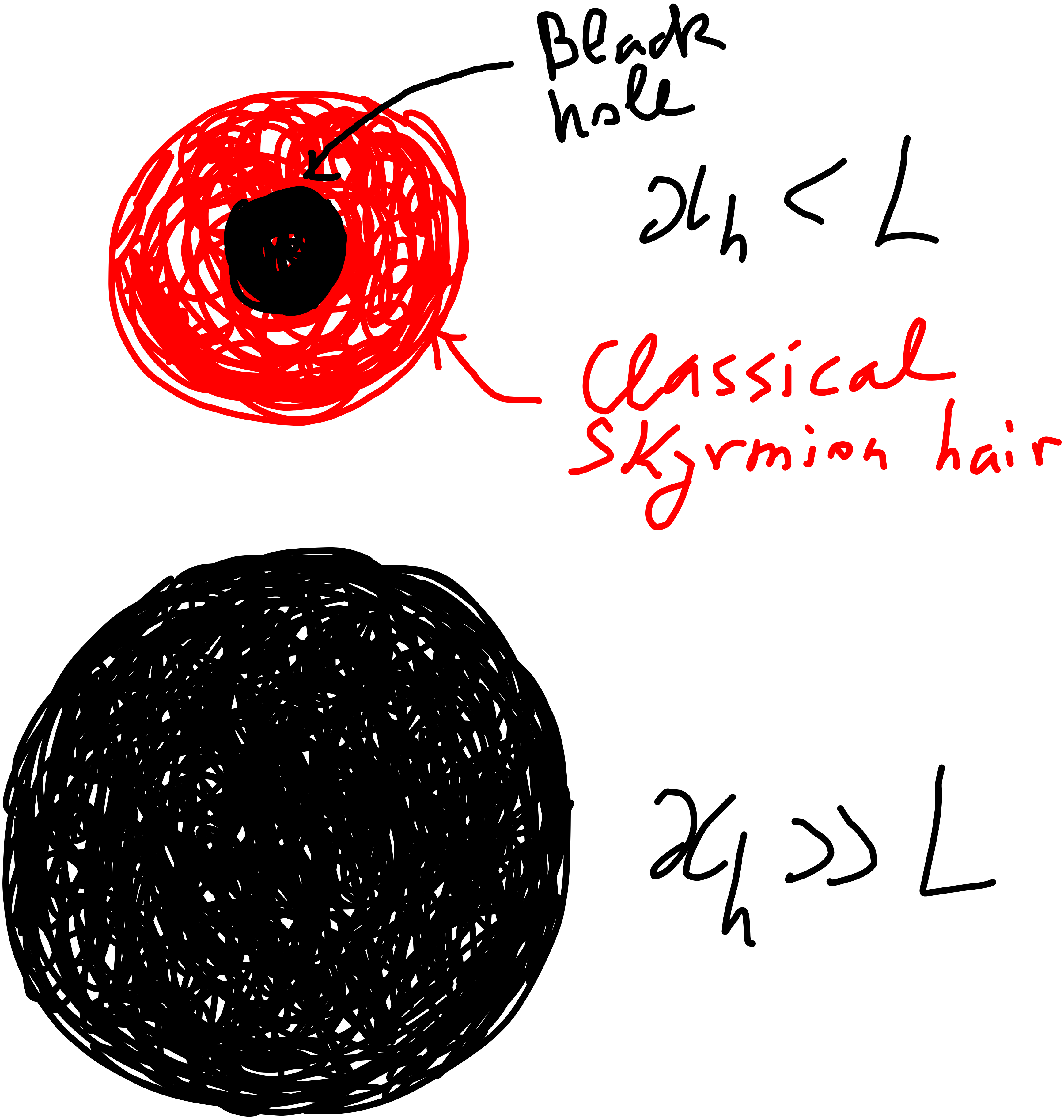
$$x_h \lesssim L$$

Black holes with Skyrmion hair (Luckock, Moss; Droz, Heusler, Straumann; Bizon, Chmaj; Shiiki, Nawado; ...)

$$ds^2 = N^2(r)\left(1 - \frac{2M(r)G_N}{r}\right)dt^2 - \left(1 - \frac{2M(r)G_N}{r}\right)^{-1}dr^2 - r^2 d\Omega^2$$

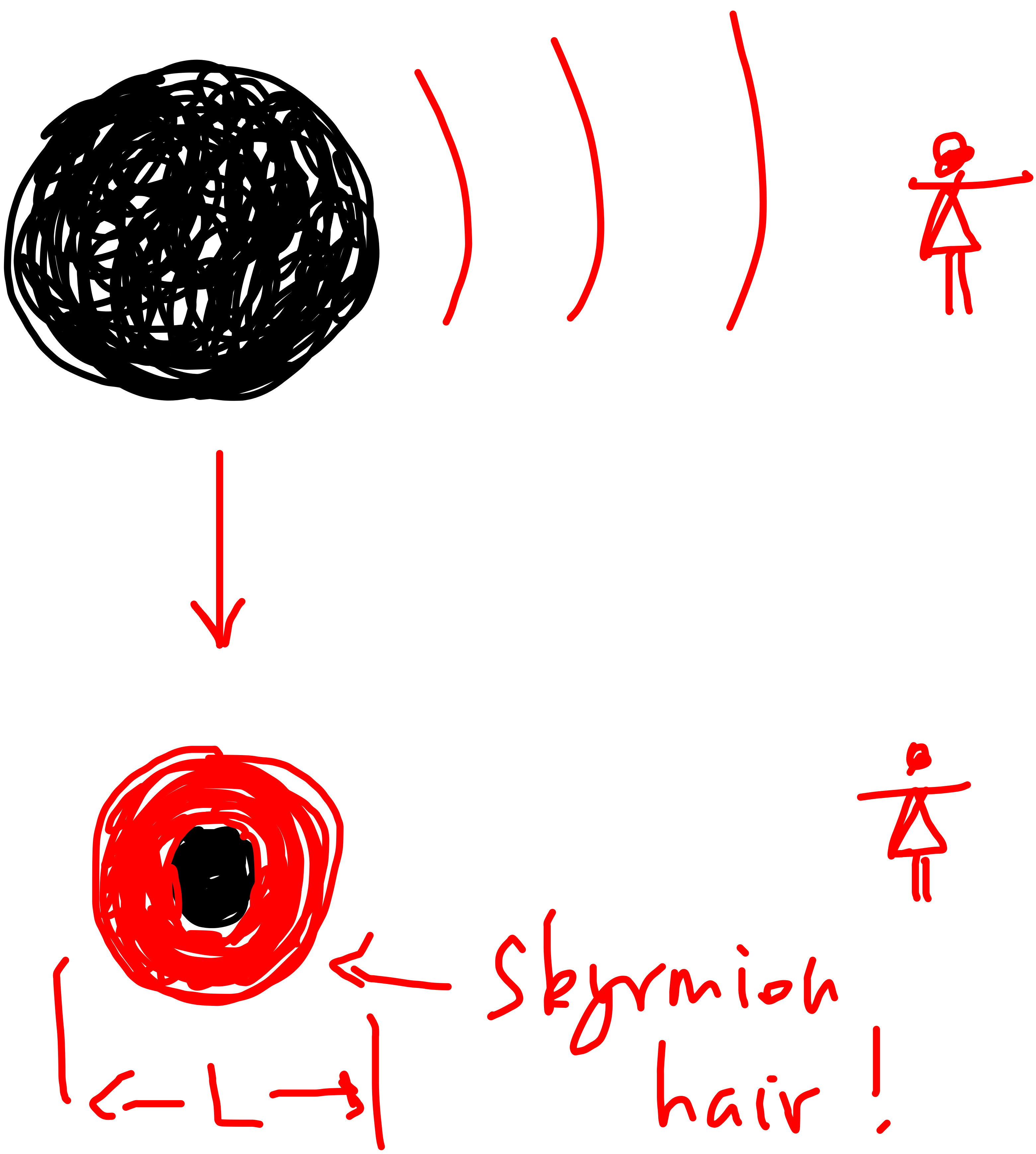
$$N(\infty) = 1 \qquad F(\infty) = 0$$

$$F(r) \propto \frac{1}{r^2} \qquad r \to \infty$$



Black hole
$x_h < L$
Classical Skyrmion hair
$x_h \gg L$

This opens up a logical possibility



Is this only a logical possibility or is a must?

What is the implication for baryon number?

Skyrmion-Baryon correspondence in QCD

$SU(N_c)$. **Witten**

$$U(1)_B \quad q \to e^{i\alpha} q$$

Baryon current $J_\mu = \frac{1}{N_c} \bar{q} \gamma_\mu q$

Skyrmion current

$$J_\mu = - \varepsilon_{\mu\nu\alpha\beta} \operatorname{Tr} \left(U^{-1} \partial^\nu U \, U^{-1} \partial^\alpha U \, U^{-1} \partial^\beta U \right)$$

Notice

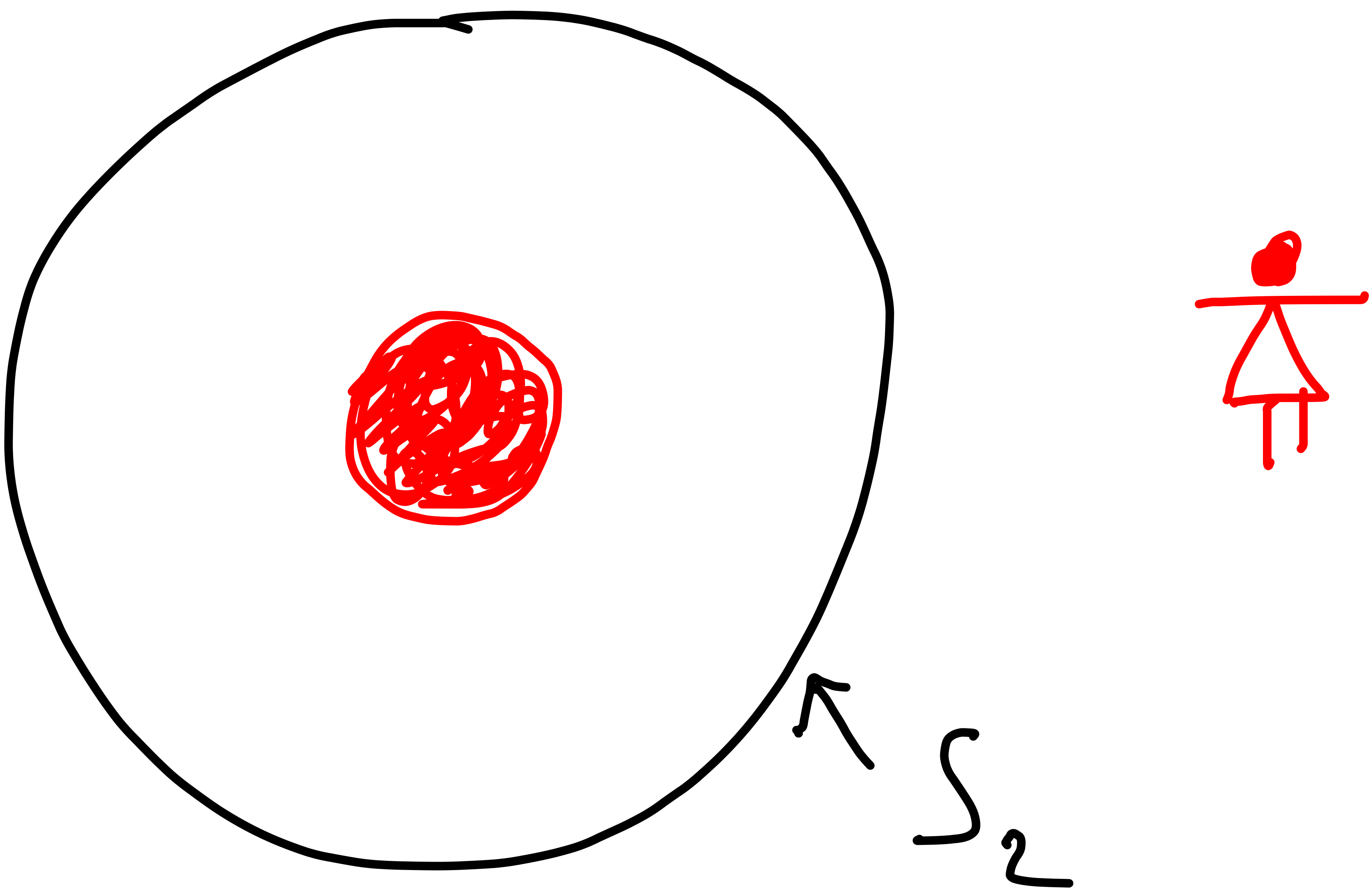
$$J_0 = {}^*d\,S$$

where

$$S_{\mu\nu} \equiv -\left(F(r) - \frac{1}{2}\sin(2F(r)) - \pi\right) \cdot \partial_{[\mu}\cos\theta\,\partial_{\nu]}\phi$$

Thus,

$$B = \int d^3x\, J_0 = \int_{S_2} dx^\mu \wedge dx^\nu S_{\mu\nu}$$



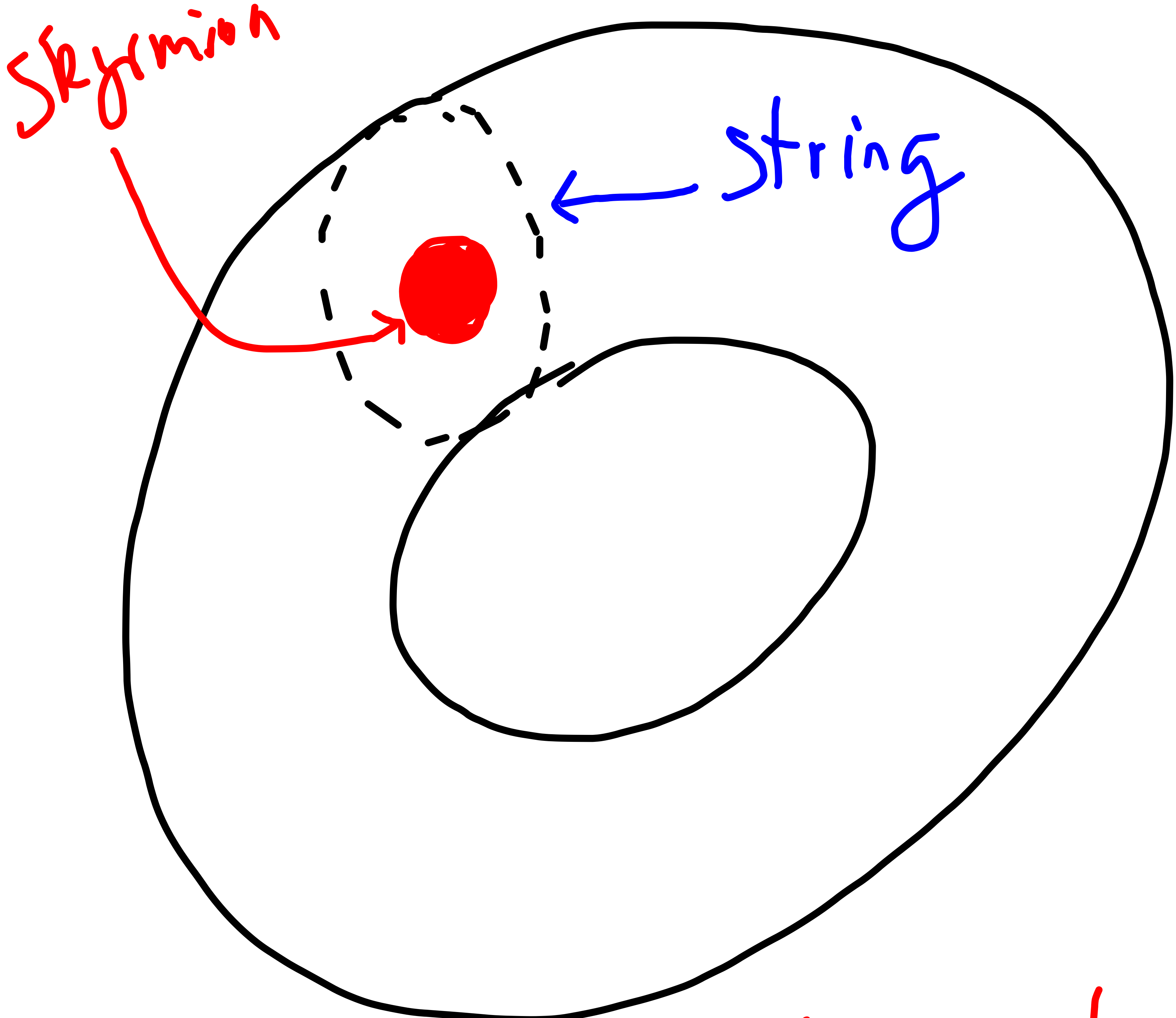
Thus, Skyrmion/baryon charge can be measured by a surface integral over S_2

$$B = \int_{S_2} dx^\mu \wedge dx^\nu \, S_{\mu\nu}$$

For this we couple $\tilde{S}_{\mu\nu}$ to a probe string

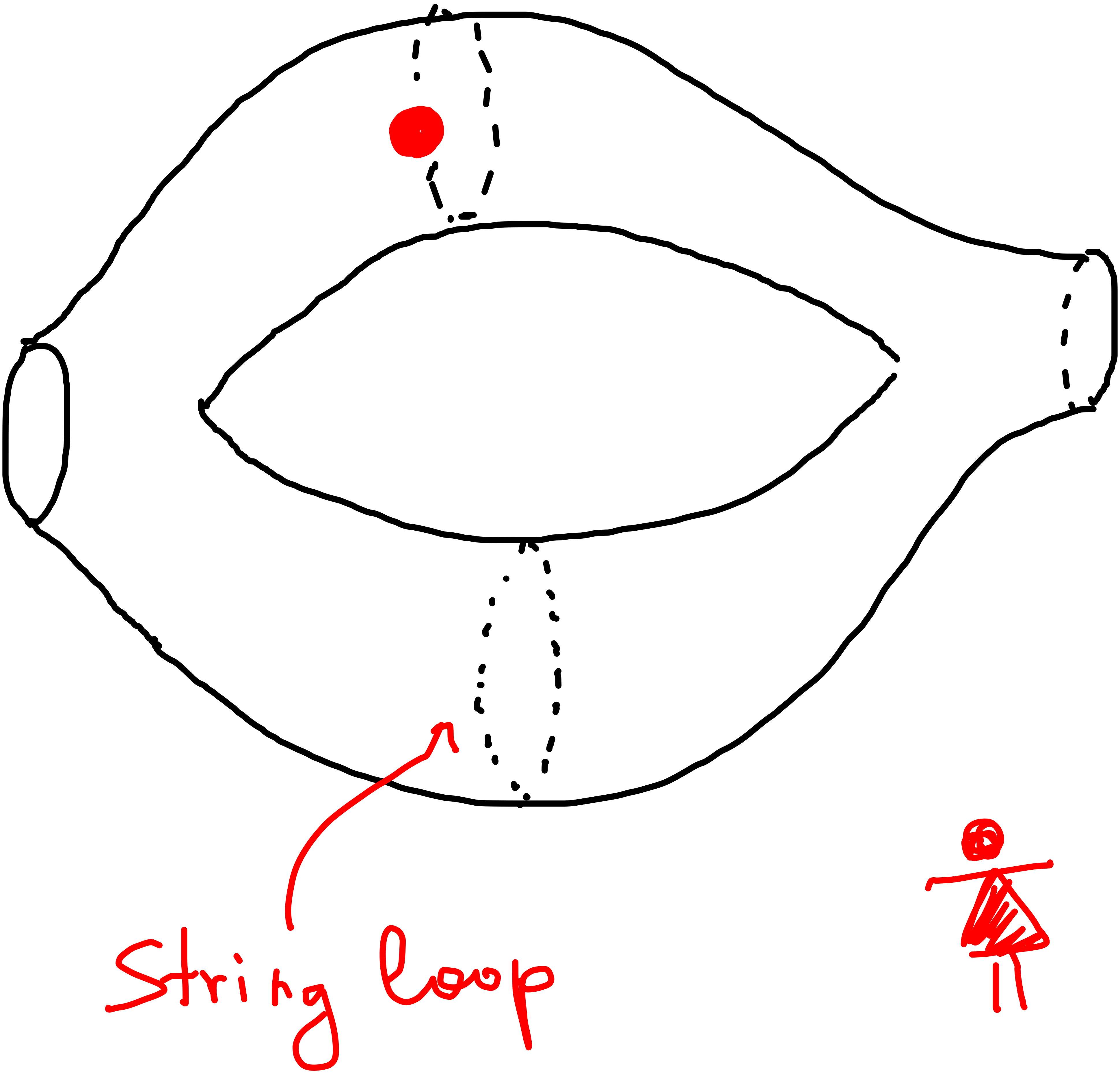
$$S = g \int dx^\mu \wedge dx^\nu \, \tilde{S}_{\mu\nu}$$

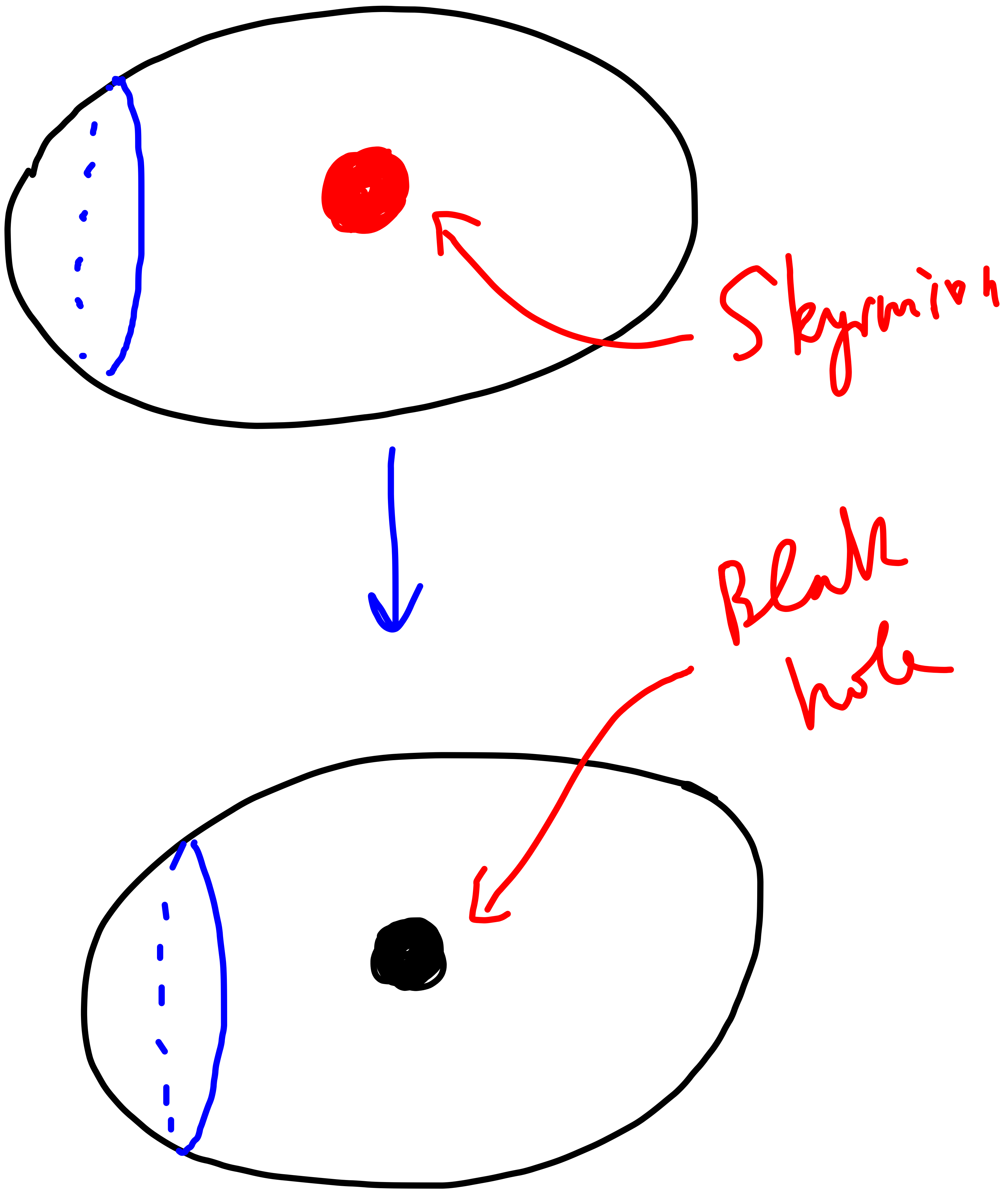
and perform Aharonov-Bohm type interference experiment.

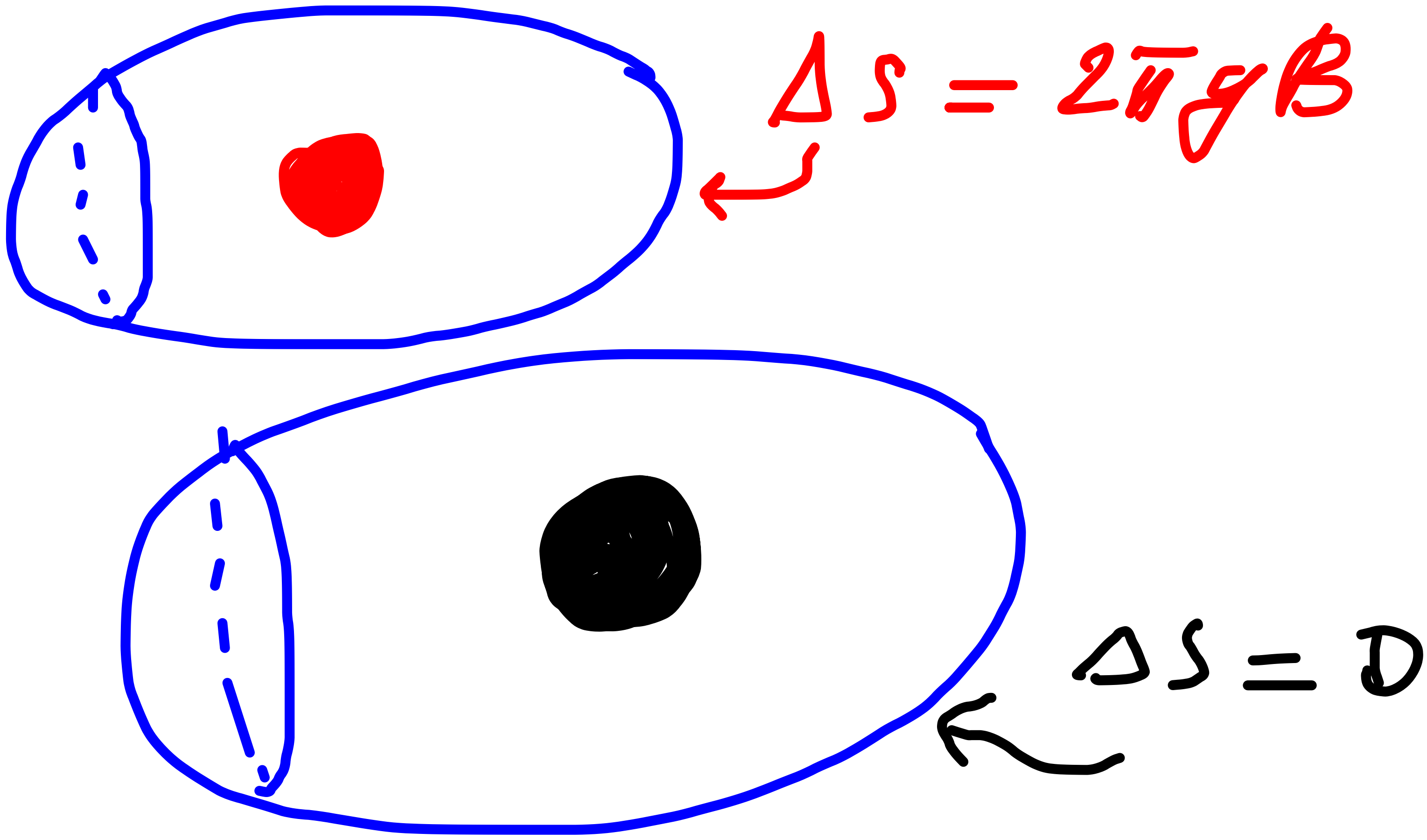


Aharonov-Bohm phase-shift

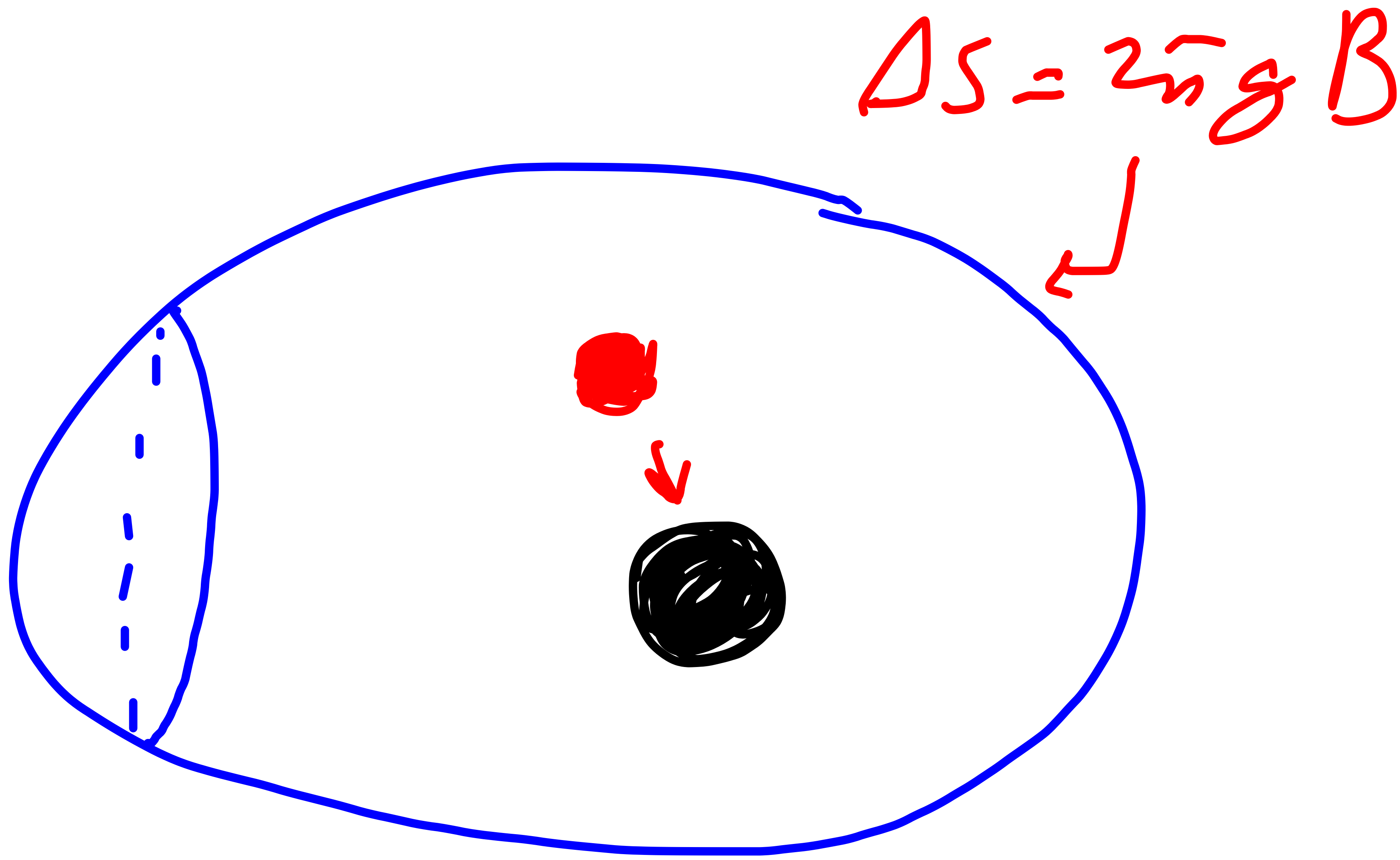
$$\Delta\Phi = 2\pi g B$$


String loop


Skyrmion
Black hole

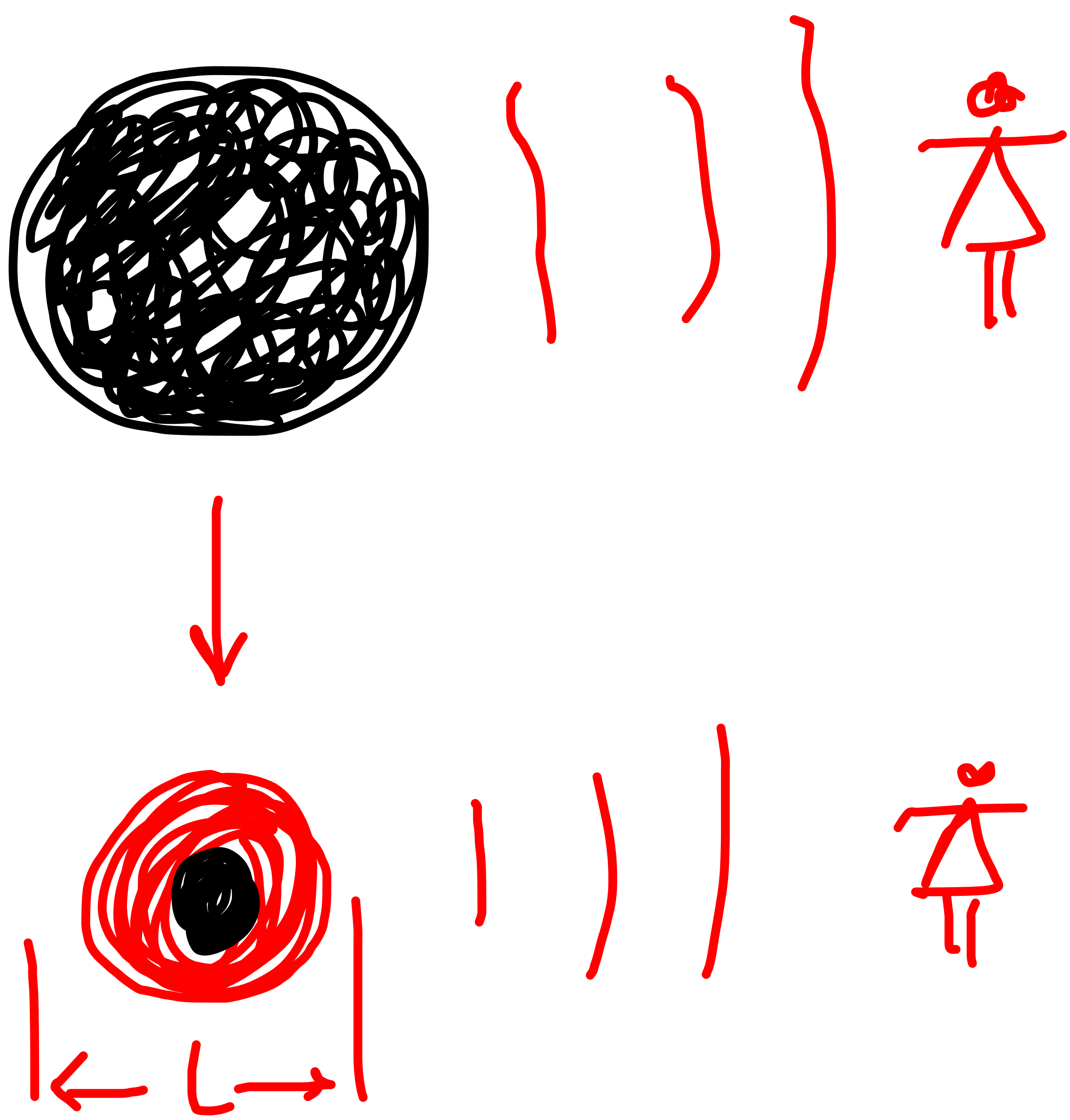


$\Delta S = 2\pi g B$
$\Delta S = 0$



$\Delta S = 2\pi g B$

Since Alice can measure AB-phse-shift she can monitor Skyrmion/baryon number at $r = \infty$.



Measurement of black hole <u>gauge</u> hair via AB phase shift was considered:

1. For $B_{\mu\nu}$ (Bowick, Giddings, Harvey, Horowitz and Strominger)
2. For gauged Z_N (Krauss & Wilczek; Preskill & Krauss)
3. For massive spin-2 $h_{\mu\nu}$ (Dvali)

In all these examples the hair is carried by gauge field!

It thus appears that Skyrmion 2-form $S_{\mu\nu}$ acts like a gauge potential and skyrmion-baryon charge acquires a secret topological protection.

Obviously $B = \int d^3x \, J_0$ is invariant under a gauge shift

$$S_{\mu\nu} \to S_{\mu\nu} + \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$$

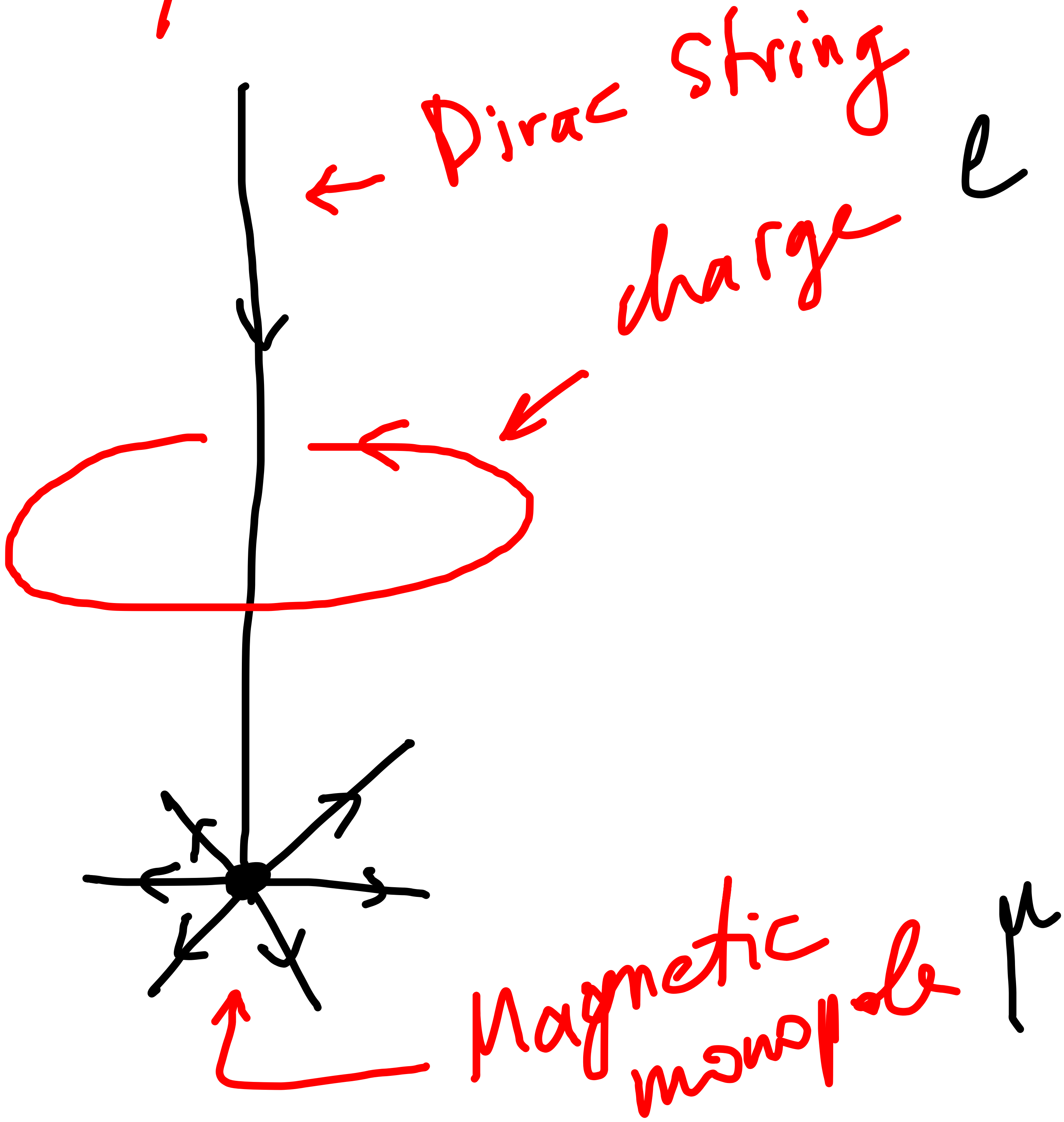
Physicallity of $S_{\mu\nu}$ depends on the charge of a probe string.

$$S_{string} = g \int dX^{\mu} \wedge dX^{\nu} S_{\mu\nu}$$

We must have

$$g \neq \frac{B}{n}$$

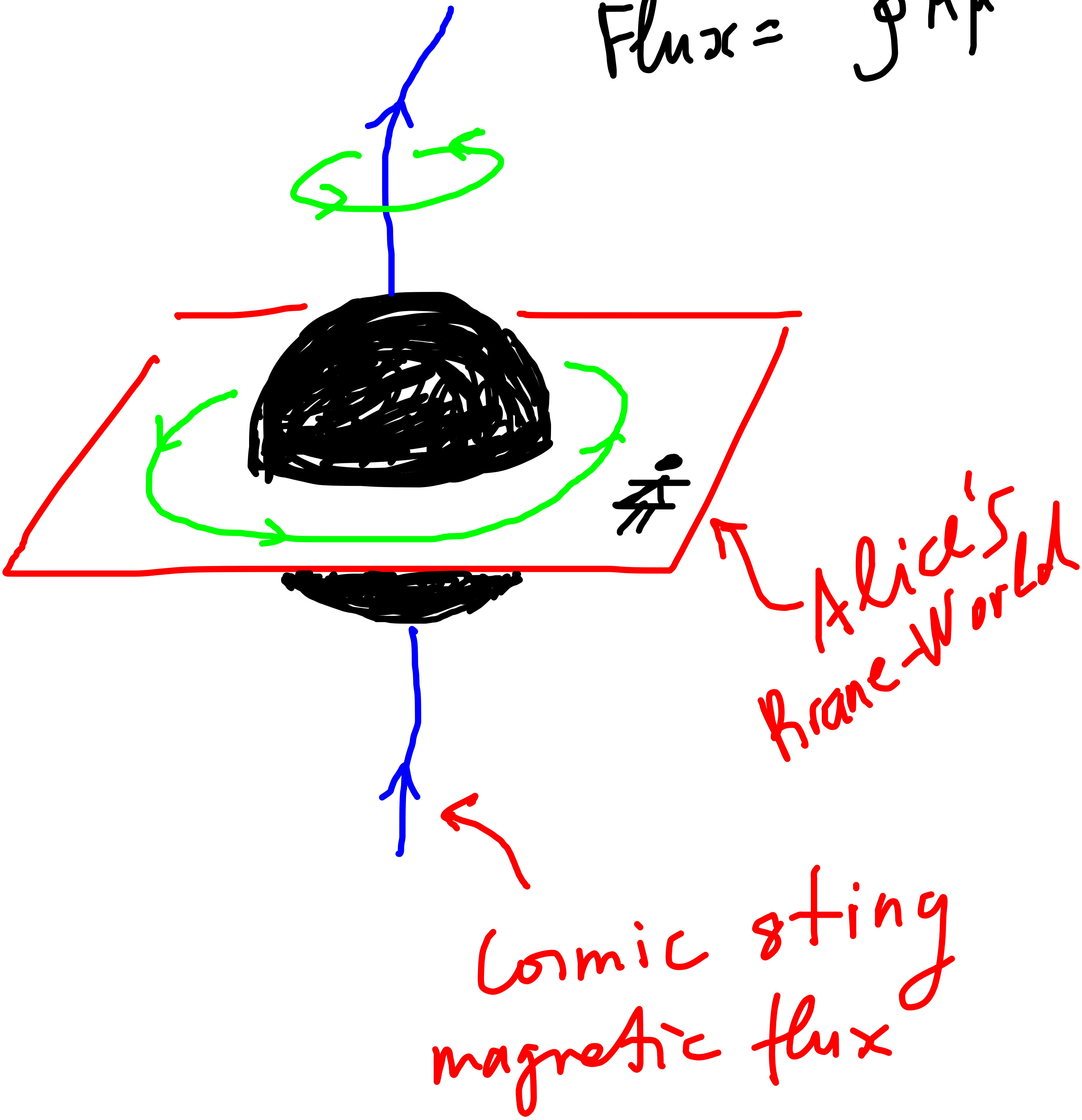
Dirac's quantization condition



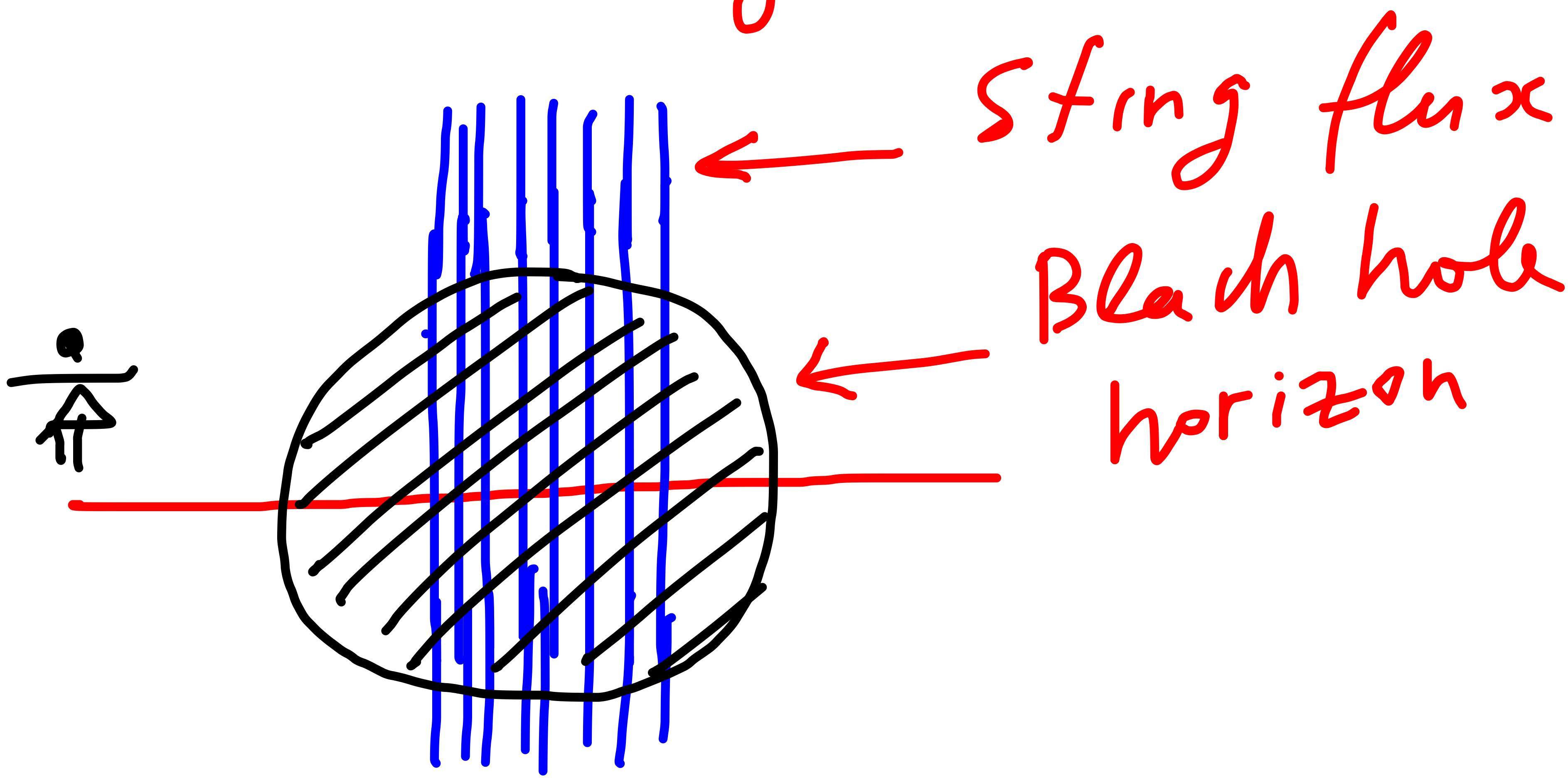
Phase-shift $\Delta S = 2\pi \mu g$

Cosmic string AB-hair

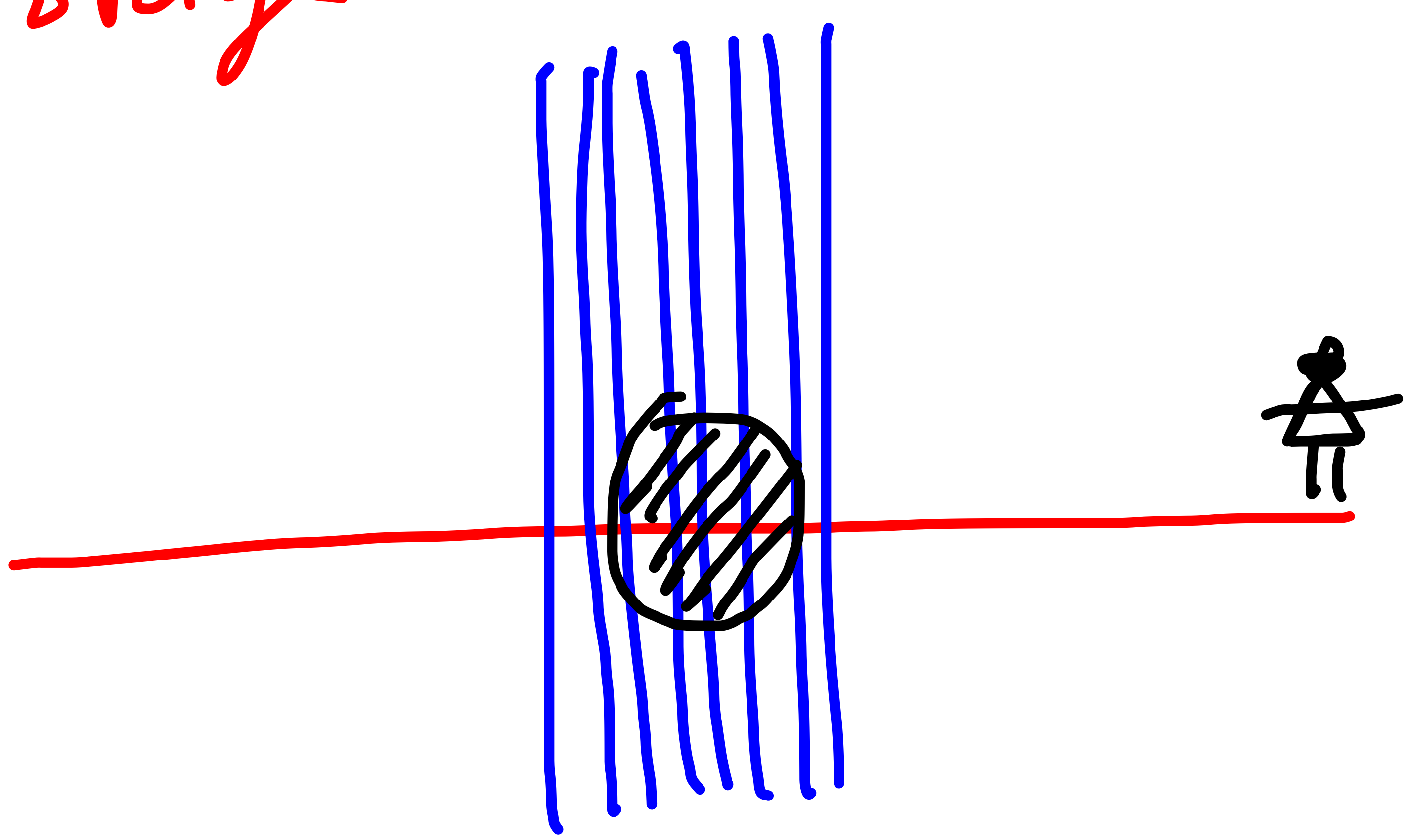
$$\text{Flux} = \oint A_\mu dx^\mu$$



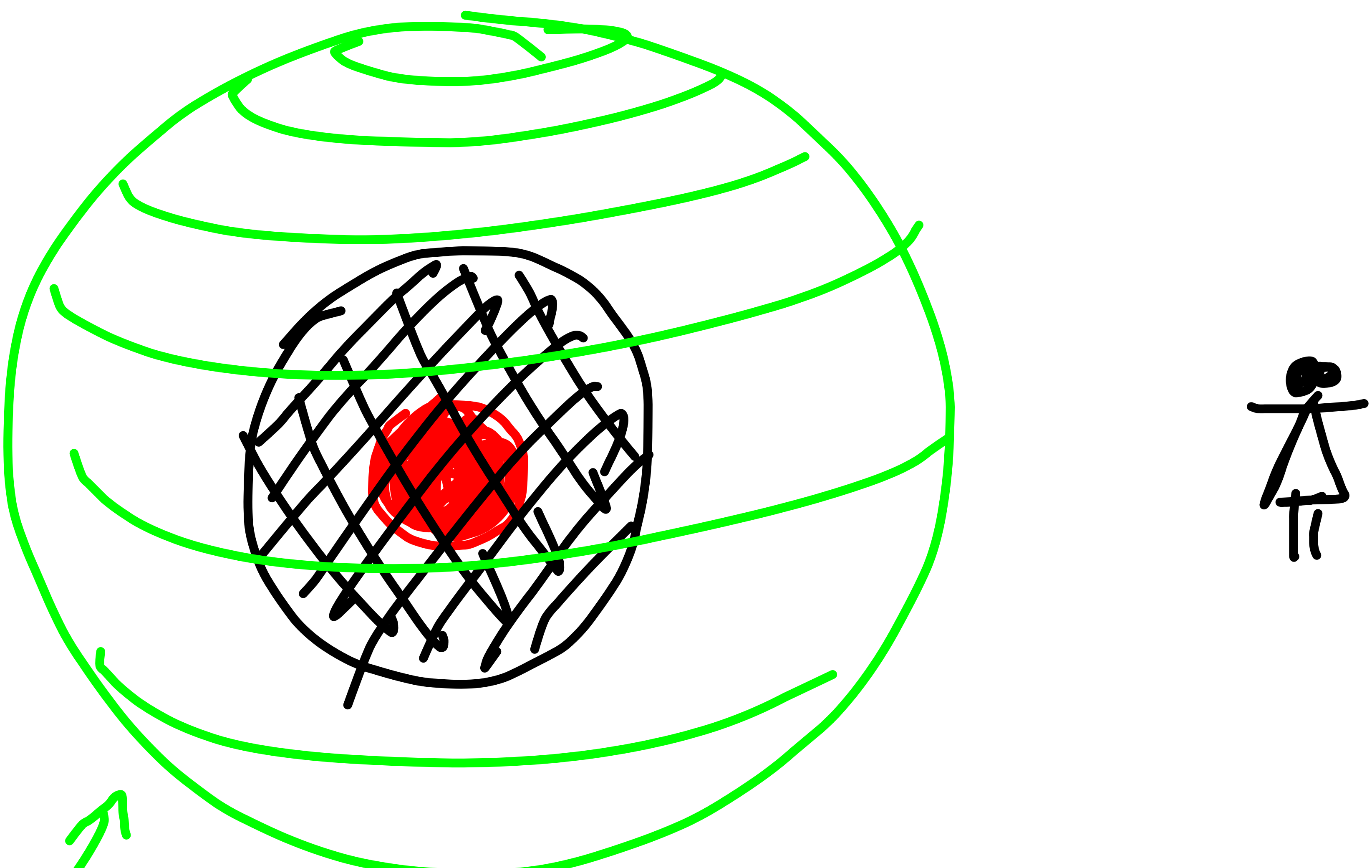
Initial stage



later stage



For Skyrmion we are in similar situation exept our "brane-world" is 3-dimensional



$$B = \int dx^\mu dx^\nu \, S_{\mu\nu}$$

So the baryon number cannot be lost!

Astrophysical implications?