Technische Universität München

Non-local effects in exclusive $b \to s\ell\ell$ decays

Danny van Dyk

Technische Universität München

Fifth Autumn School & Workshop: Physics of the Standard Model and Beyond
Regional Training Network in Theoretical Physics – Tbilisi, Georgia

funded by

Emmy Noether-Programm

DFG Deutsche Forschungsgemeinschaft

Motivation

Experimental measurements on $b \to s\ell\ell$

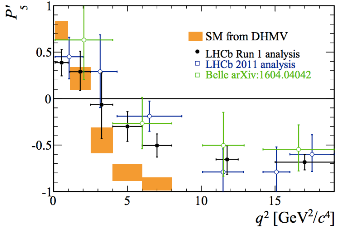
- LHCb measurements $B \to K\mu\mu$, $B \to K^*\mu\mu$, $B_s \to \phi\mu\mu$
- Analogous measurements by Belle, ATLAS and CMS
- Lepton-Flavor Non-Universality

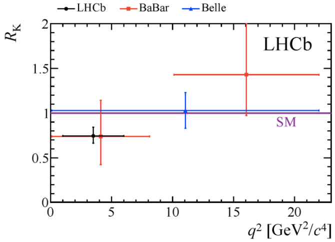
Raised a lot of interest, lot of work from theory + experiment

- Mostly: Interest in ”Anomalies” and New Physics
- Here: strive to get a better handle on hadronic matrix elements of non-local operators

Motivation

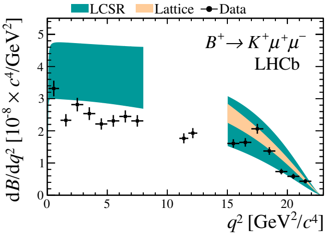
Intriguing "anomalies" in some observables



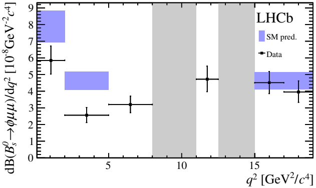


[Phys. Rev. Lett. 113, 151601 (2014)]

Less significant yet intriguing deviations in branching ratios



[LHCb JHEP06(2014)133]

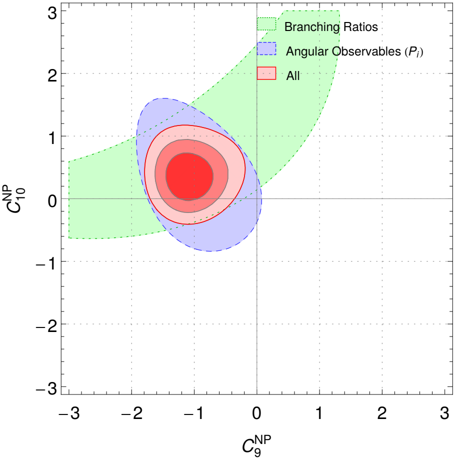


[LHCb JHEP09(2015)179]

Motivation

Significant SM pulls in global fits

[Descotes-Genon, Hofer, Matias, Virto 2015 + others]



Significance already at the level of $\sim 5\sigma$ *****

Effective Theory ($b \to s$, but analogous $b \to d$)

For $\Lambda_{\rm EW}, \Lambda_{\rm NP} \gg M_B$: Flavour and CP mediated by $D = 6$ ops :

$$\mathcal{L}_W = \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm QED} + \frac{4G_F}{\sqrt{2}} \left[\lambda_t \sum_i \mathcal{C}_i \mathcal{O}_i + \lambda_c \sum_i \mathcal{C}_i^{\,c} \mathcal{O}_i^{\,c} + \lambda_u \sum_i \mathcal{C}_i^{\,u} \mathcal{O}_i^{\,u} \right]$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu} \qquad \mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R T^A b) G^A_{\mu\nu}$$

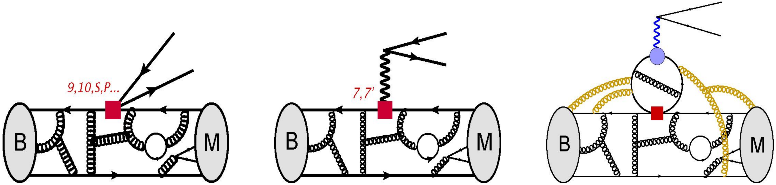
$$\mathcal{O}_{9\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell) \qquad \mathcal{O}_{10\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

$$\mathcal{O}_1^{\,c} = (\bar{c}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L c) \qquad \mathcal{O}_2^{\,c} = (\bar{c}\gamma_\mu P_L T^a b)(\bar{s}\gamma^\mu P_L T^a c)$$

$$\mathcal{O}_1^{\,u} = (\bar{u}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L u) \qquad \mathcal{O}_2^{\,u} = (\bar{u}\gamma_\mu P_L T^a b)(\bar{s}\gamma^\mu P_L T^a u)$$

$$\mathcal{O}_i = (\bar{s}\gamma_\mu P_X b) \sum_q (\bar{q}\gamma^\mu q)$$

SM contributions to $\mathcal{C}_i(\mu_b)$ known to NNLL [Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06]



$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- Local (Form Factors) : $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \, \bar{s} \, \Gamma_\lambda^{(T)} \, b \, | \bar{B}(k+q) \rangle$
- Non-Local : $\mathcal{H}_\lambda(q^2) = i \, \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq\cdot x} \, \langle \bar{M}_\lambda(k) | T\{ \mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$
- CKM structure : $\mathcal{H}_\lambda = -\frac{\lambda_u}{\lambda_t} \mathcal{H}_\lambda^{(u)} - \frac{\lambda_c}{\lambda_t} \mathcal{H}_\lambda^{(c)}$

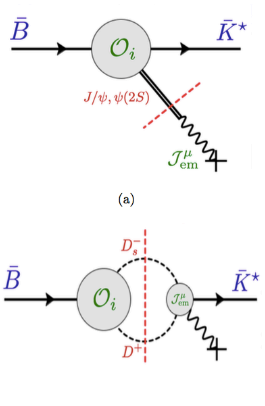
Lorentz Decomposition

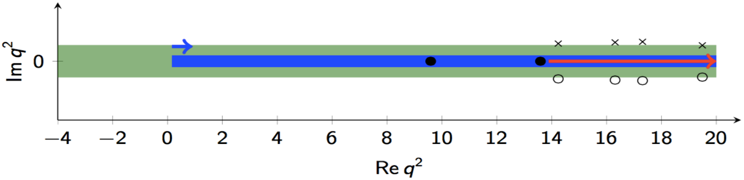
$$\mathcal{H}^\mu(q,k) \equiv i \int \mathrm{d}^4x\, e^{iq\cdot x}\, \langle \bar{K}^*(k,\eta)|T\{\mathcal{J}^\mu_{\mathrm{em}}(x), \mathcal{C}_i\mathcal{O}_i(0)\}|\bar{B}(k+q)\rangle$$

$$\equiv M_B^2\, \eta^*_\alpha\, \left[S_\perp^{\alpha\mu}\, \mathcal{H}_\perp(q^2) - S_\parallel^{\alpha\mu}\, \mathcal{H}_\parallel(q^2) - S_0^{\alpha\mu}\, \mathcal{H}_0(q^2)\right]$$

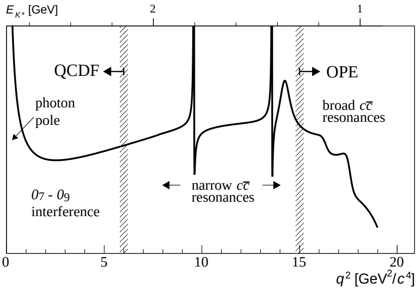
- $S_\lambda^{\alpha\mu}$ – basis of Lorentz structures (carefully chosen)
- $\mathcal{H}_\lambda$ – Lorentz invariant correlation functions
- λ – polarization states ($\perp$, $\parallel$, 0) [for vector meson]

Analytic structure





Strategy



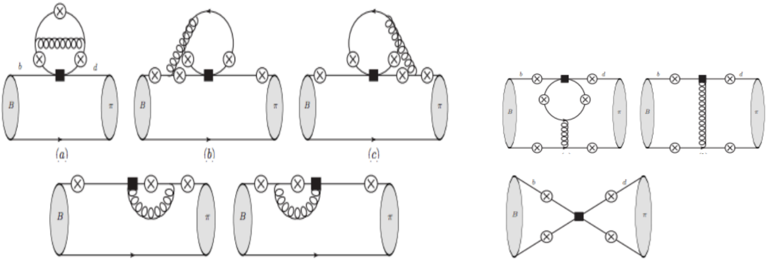
[sketch from Blake, Gershon, Hiller 2015]

- Calculate non-local ME at negative q^2
- Extrapolate to $q^2 > 0$ via some type of analytic continuation
- Constrain two narrow resonances at $q^2 > 0$ from data on $B \to \psi_n K^*$

Calculations at negative q^2

- **QCD Factorization** [Beneke, Feldmann, Seidel 2001 & 2004]

$$\mathcal{H}_\lambda = C_\lambda\, \mathcal{F}_\lambda + \sum_{\pm} \int \frac{d\omega}{\omega} \Phi^B_\pm(\omega) \int_0^1 du\, T^\pm_\lambda(u,\omega)\, \phi^\pm_M(u) + \mathcal{O}(\Lambda/m_B, \Lambda/E)$$

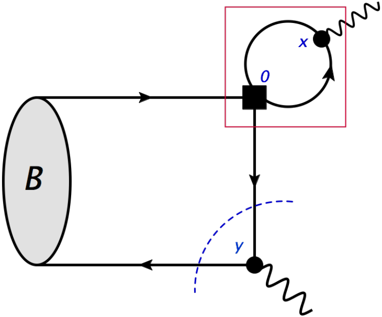


$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_{\lambda;\mathrm{fact,LO}}(q^2) + \mathcal{H}_{\lambda;\mathrm{fact,NLO}}(q^2) + \mathcal{H}_{\lambda;\mathrm{spect}}(q^2) + \mathcal{H}_{\lambda;\mathrm{WA}}(q^2) + \cdots$$

Calculations at negative q^2

- LCSRs with B-meson DAs

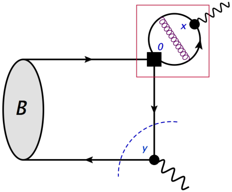
[Khodjamirian, Mannel, Pivovarov, Wang 2010]

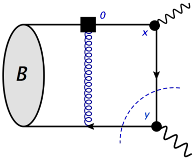


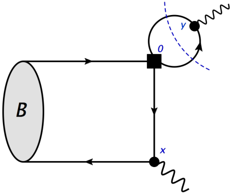
LC exp. of charm prop. [Balitsky, Braun 1989]

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{matching coeff}} [\bar{s}\,\Gamma\, b] + \cdots$$

$$\Rightarrow \mathcal{H}_\lambda = (\text{matching coeff}) \times \mathcal{F}_\lambda^{LCSR}$$



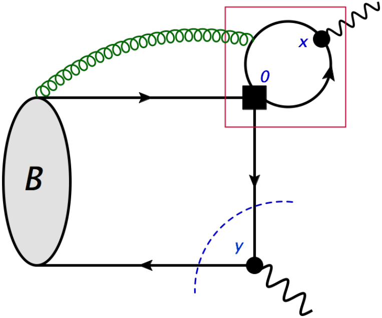




Calculations at negative q^2

▶ LCSRs with B-meson DAs

[Khodjamirian, Mannel, Pivovarov, Wang 2010]

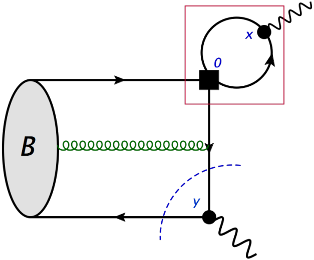


LC exp. of charm prop. [Balitsky, Braun 1989]

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{matching coeff}} [\bar{s}\,\Gamma\, b] +$$

$$+ \,(\text{coeff}) \times [\,\bar{s}_L \gamma^\alpha (i n_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L\,] \; + \cdots$$

3-particle correction to $\mathcal{F}_\lambda \longrightarrow$



Calculations at negative q^2

- At the end of the day

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_{\lambda;\text{fact,LO}}(q^2) + \mathcal{H}_{\lambda;\text{fact,NLO}}(q^2) + \mathcal{H}_{\lambda;\text{spect}}(q^2) + \mathcal{H}_{\lambda;\text{WA}}(q^2) + \\ + \mathcal{H}_{\lambda;\text{soft}}(q^2) + \mathcal{H}_{\lambda;\text{soft},O_8}(q^2) + \cdots$$

  - $\mathcal{H}_{\lambda;\text{soft}}$ and $\mathcal{H}_{\lambda;\text{fact,LO}}$ cancel to large extent
  - $\mathcal{H}_{\lambda;\text{soft},,O_8}$ contributions negligible

Accessing $q^2 > 0$: dispersion relations

Dispersion relation relating $\mathcal{H}(q_0^2 < 0)$ to $\mathcal{H}(q^2 > 0)$

[Khodjamirian, Mannel, Pivovarov, Wang 2010] [Hambrock, Khodjamirian, Rusov 2015]

$$\mathcal{H}^{(p)}(q^2) - \mathcal{H}^{(p)}(q_0^2) = (q^2 - q_0^2)\Bigg[\sum_V \frac{f_V\, \mathcal{A}^p(B \to VM)}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{\rm tot})} + \int_{s_h}^{\infty} ds \frac{\rho_h^{(p)}(s)}{(s^2 - q_0^2)(s - q^2 - i\epsilon)}\Bigg] \qquad (p = u, c)$$

- $V = \rho, \omega, \phi, J/\psi, \psi(2S)$
- For $b \to s \Rightarrow$ Neglect λ_u and OZI suppressed contributions

  $\Rightarrow \mathcal{A}^c(B \to VM_s) \sim \mathcal{A}(B \to \psi_n M_s)$ can be determined from data.
- For $b \to d$ both $\mathcal{A}^{u,c}(B \to VM)$ important $\Rightarrow$ Need extra theory input (QCDF)
- Light-hadron spectral density $\Rightarrow$ QH-Duality
- Open-charm spectral density $\simeq a_p + b_p \dfrac{q^2}{4m_D^2}$ (expansion for $q^2 < m_{J/\psi}^2$)

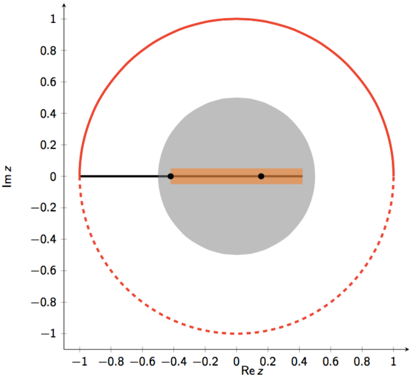
Accessing $q^2 > 0$: z expansion $[B \to K^* \ell\ell]$

Ansatz in z valid below the $D\bar{D}$ threshold

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

Motivated by "z-parametrization" of form factors. [Boyd et al '94, Bourelly et al '08]

1. Extract the poles : $\hat{\mathcal{H}}_\lambda(q^2) = (q^2 - M^2_{J/\psi})(q^2 - M^2_{\psi(2S)})\,\mathcal{H}_\lambda(q^2)$



2. $\hat{\mathcal{H}}_\lambda(q^2)$ is analytic except for $D\bar{D}$ cut.
3. Perform conformal mapping $q^2 \mapsto z(q^2)$.
4. $\hat{\mathcal{H}}_\lambda(z)$ analytic within unit circle.
5. Taylor expand $\hat{\mathcal{H}}_\lambda(z)$ around $z = 0$.
6. Good convergence expected since $|z| < 0.52$ for $-7\,\text{GeV}^2 \leq q^2 \leq 14\text{GeV}^2$

Accessing $q^2 > 0$: z expansion $[B \to K^* \ell\ell]$

Some details for actual parametrisation

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

- Try to capture most features of the expansion (better convergence)
- Parametrize the ratios $\mathcal{H}_\lambda(q^2)/\mathcal{F}_\lambda(q^2)$ instead
- The poles should not modify the asymptotic behaviour at $|q^2| \to \infty$

$$\mathcal{H}_\lambda(z) = \frac{1 - z z^*_{J/\psi}}{z - z_{J/\psi}} \frac{1 - z z^*_{\psi(2S)}}{z - z_{\psi(2S)}} \hat{\mathcal{H}}_\lambda(z)$$

$$\hat{\mathcal{H}}_\lambda(z) = \left[\sum_{k=0}^{K} \alpha_k^{(\lambda)} z^k \right] \mathcal{F}_\lambda(z)$$

where $\alpha_k^{(\lambda)}$ are complex coefficients, and the expansion is truncated after the term z^K. We will take $K = 2$ (16 real parameters).

- The modified EOS source code is available upon request (public repo and web page should be updated soon!)

Experimental constraints

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

- The residues of the poles are given by $B \to K^* \psi_n$:

$$\mathcal{H}_\lambda(q^2 \to M_{\psi_n}^2) \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2 (q^2 - M_{\psi_n}^2)} + \cdots$$

- Angular analyses determine [Belle, Babar, LHCb]

$$|r_\perp^{\psi_n}|,\ |r_\parallel^{\psi_n}|,\ |r_0^{\psi_n}|,\ \arg\{r_\perp^{\psi_n} r_0^{\psi_n *}\},\ \arg\{r_\parallel^{\psi_n} r_0^{\psi_n *}\},$$

where $\quad r_\lambda^{\psi_n} \equiv \underset{q^2 \to M_{\psi_n}^2}{\mathrm{Res}} \frac{\mathcal{H}_\lambda(q^2)}{\mathcal{F}_\lambda(q^2)} \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2\, \mathcal{F}_\lambda(M_{\psi_n}^2)}$

- We produce correlated pseudo-observables from a fit (5+5).

Prior Fit to z parametrisation [$B \to K^*\ell\ell$]

(Prior) Fit to Experimental and theoretical pseudo-observables

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

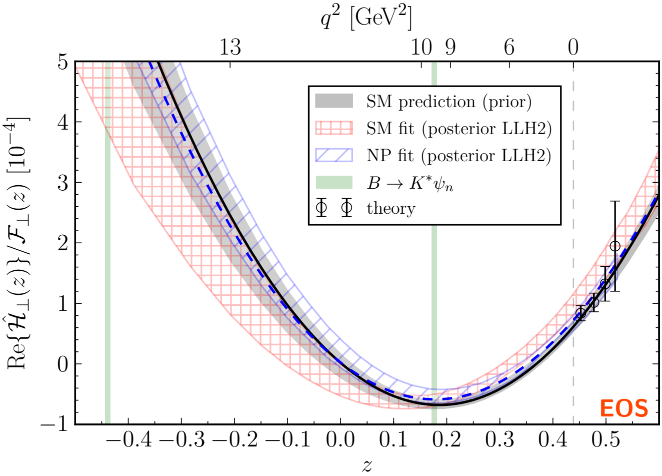
k	0	1	2
$\mathrm{Re}[\alpha_k^{(\perp)}]$	-0.06 ± 0.21	-6.77 ± 0.27	18.96 ± 0.59
$\mathrm{Re}[\alpha_k^{(\parallel)}]$	-0.35 ± 0.62	-3.13 ± 0.41	12.20 ± 1.34
$\mathrm{Re}[\alpha_k^{(0)}]$	0.05 ± 1.52	17.26 ± 1.64	–
$\mathrm{Im}[\alpha_k^{(\perp)}]$	-0.21 ± 2.25	1.17 ± 3.58	-0.08 ± 2.24
$\mathrm{Im}[\alpha_k^{(\parallel)}]$	-0.04 ± 3.67	-2.14 ± 2.46	6.03 ± 2.50
$\mathrm{Im}[\alpha_k^{(0)}]$	-0.05 ± 4.99	4.29 ± 3.14	–

Table 1: Mean values and standard deviations (in units of 10^{-4}) of the prior PDF for the parameters $\alpha_k^{(\lambda)}$.

Prior Fit to z parametrisation [$B \to K^*\ell\ell$]

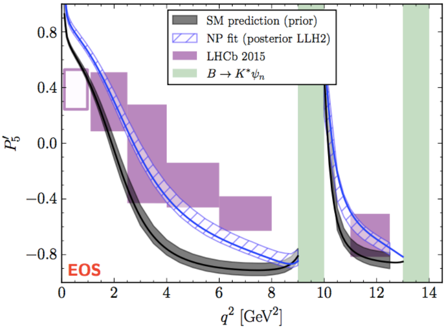
(Prior) Fit to Experimental and theoretical pseudo-observables

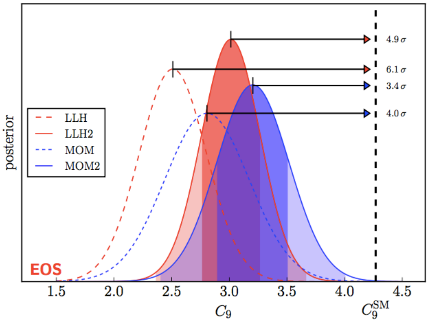
[Bobeth, Chrzaszcz, van Dyk, Virto 2017]



SM predictions and Fit including $B \to K^* \mu^+ \mu^-$ data and $\mathcal{C}_9^{\text{NP}}$

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]





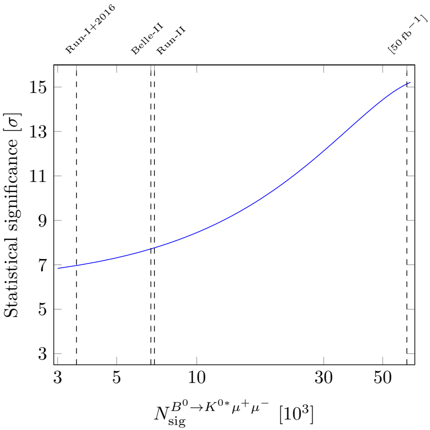
The NP hypothesis with $\mathcal{C}_9^{\text{NP}} \sim -1$ is strongly favoured by the fit

- pulls $> 3.4\sigma$ in 1D posterior of the parameter
- posterior odds (for some fits strongly) in favour of NP interpretation

Sensitivity to New Physics in $\mathcal{C}_9$ in $B \to K^* \mu^+ \mu^-$ from an unbinned fit

[Chrzaszcz, Mauri, Serra, Silva Coutinho, van Dyk w.i.p]

Preliminary



- Use $\mathcal{C}_9^{\text{NP}} = -1$ as benchmark point
- Use theory inputs exactly as in pheno analysis [Bobeth, Chrzaszcz, van Dyk, Virto 2017]
- Sensitivity to higher order in z
- Sensitivity to z coefficients in absence of theory priors!!

Summary

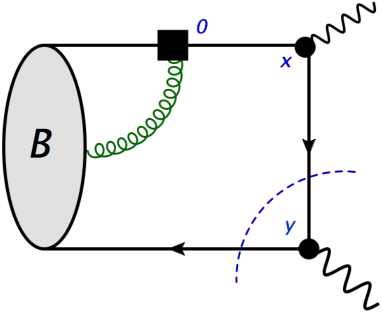
- Non-local effects are half of the amplitude. Must include them!
- Technically challenging, but good advances since 2001
- Theory calculations most reliable at spacelike q^2
  - QCD Factorization [Beneke, Feldmann, Seidel 2001 & 2004]
  - LCSRs [Khodjamirian, Mannel, Pivovarov, Wang 2010]
- Access timelike q^2 via dispersion relations or z-parametrizations
  - Global analyses to exploit parametrical correlations
  - Experimental colleagues have begun work to incorporate z-parametrization in their analyses
- Did not discuss large q^2!!

Backup slides

Calculations at negative q^2

► LCSRs with B-meson DAs [Khodjamirian, Mannel, Wang 2012]

Soft gluon correction to O_8 contribution



Simpler calculation than charm-loop

Numerically very small

Calculations at negative q^2

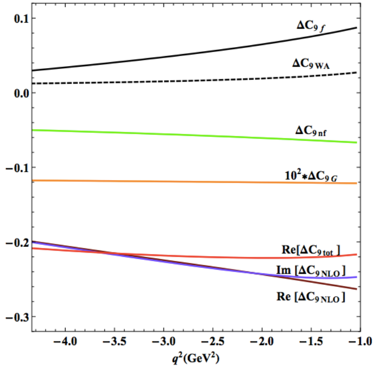
▶ Results for $\mathcal{H}^{(c)}$

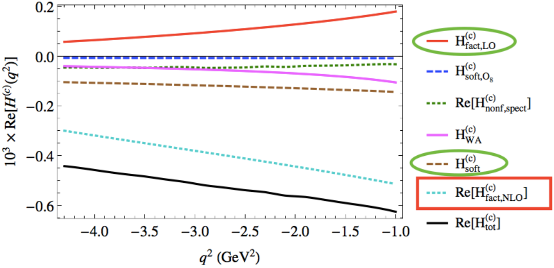
[Khodjamirian, Mannel, Wang 2012], [Hambrock, Khodjamirian, Rusov 2015]

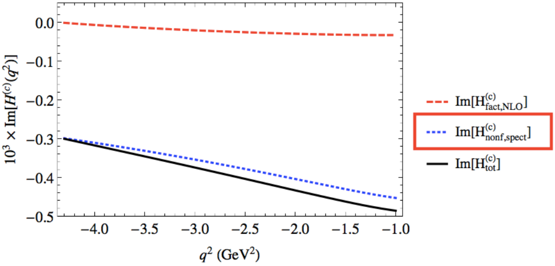
$B^- \to \pi^- \ell\ell \longrightarrow$

$B^- \to K^- \ell\ell$

$\downarrow$





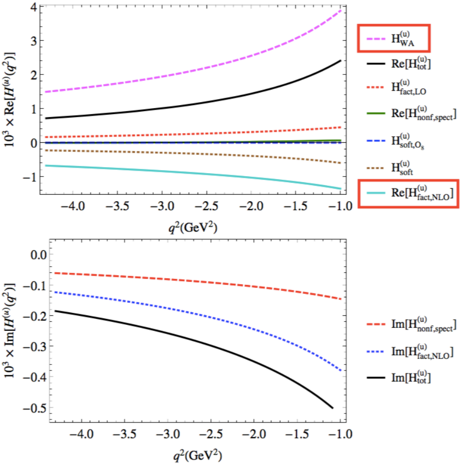


Calculations at negative q^2

- Results for $\mathcal{H}^{(u)}$

[Hambrock, Khodjamirian, Rusov 2015]

$B^- \to \pi^- \ell\ell$



$B^0 \to \pi^0 \ell\ell$