Towards NLL precision for the inclusive decay $B \to X_s\gamma\gamma$

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Introduction

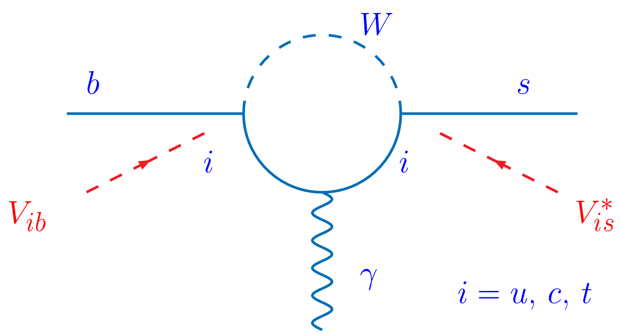
The decay $b \to s\gamma\gamma$ is a so-called rare decay.

The decay $b \to s\gamma$ is another example of such a decay. Let me make a few statements on $b \to s\gamma$ which also hold for $b \to s\gamma\gamma$:

$b \to s\gamma$ does not exist at tree-level in the SM

However, it is induced at the one-loop level:

typical diagram (e.m. penguin)



-tests SM at the QT level

-sensitive to certain CKM matrix elements

The loop-induction naturally suppresses the BR.

Structure of the decay amplitude:

$$A(b \to s\gamma) = \sum_{i=u,c,t} V_{ib} V_{is}^* \; f[m_i^2/m_b^2]$$

If the up-type masses were degenerate $[m_i = m]$, then

$$A(b \to s\gamma) = f(m^2/m_b^2) \sum_{i=u,c,t} V_{ib} V_{is}^* = 0$$

due to unitarity of the CKM-matrix!

In reality, we have a strong splitting of the up-type masses: $m_t \gg m_c \gg m_u$. As a consequence, in part. because of the large m_t, a BR of to order of 10^{-4} results!

$b \to s\gamma$, $b \to s\gamma\gamma$ etc. sensitive to the heaviest particles in the SM.

Therefore a high sensitivity to extensions of the SM is expected!

E.g. in the 2HDM of type II, the most stringent bound on the charged Higgs mass comes from $b \to s\gamma$.

Theoretical framework to calculate these decays

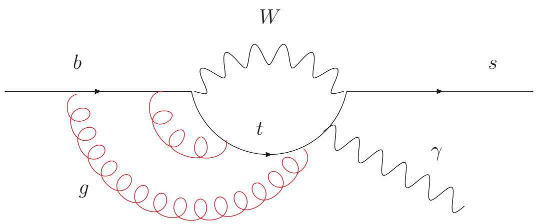
HQE: $\Gamma[B \to X_s \gamma\gamma] = \Gamma[b \to s\gamma\gamma(g)] +$ corr. in Λ_{QCD}/m_b.

- no linear corrections in Λ_{QCD}/m_b
- Corr. start at $\mathcal{O}(\Lambda_{QCD}^2/m_b^2)$; they are related to the motion of the b-quark inside the meson

Today, we only discuss the main contributions: the free b-quark decay $b \to X_s\gamma\gamma$.

Well-known: This partonic decay rate is significantly enhanced by QCD-effects.

There are large logs of the form (n gluons exchanged)



$$\left(\frac{\alpha_s}{\pi}\right)^n \log^n \frac{m_b^2}{M^2} \qquad M = m_t, m_W: \text{ leading logs (LL)}$$

$$\left(\frac{\alpha_s}{\pi}\right)^n \log^{n-1} \frac{m_b^2}{M^2} \qquad \text{next-to-leading logs (NLL)}$$

To get a reasonable result, one has to resum at least the LL and NLL terms.

Useful machinery to achieve resummation: construct effective Hamiltonian and resum logs using RGE techniques.

Keeping only operators up to dim. 6, the effective Hamiltonian for $b \to s\gamma\gamma$ is the same as for $b \to s\gamma$:

$$\mathcal{H} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i(\mu) O_i(\mu) \quad .$$

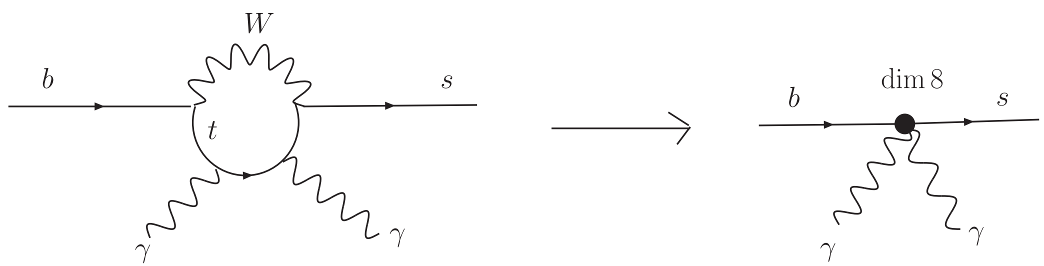
The operators relevant in the following are:

$O_1 = (\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{s}_{L\alpha}\gamma_\mu c_{L\beta}) \quad O_2 = (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{s}_{L\beta}\gamma_\mu c_{L\beta})$

$O_7 = \frac{e}{16\pi^2} m_b(\mu)\,(\bar{s}\sigma_{\mu\nu} R b)\, F^{\mu\nu}$ phot. dipole

$O_8 = \frac{g_s}{16\pi^2} m_b(\mu)\,(\bar{s}_\alpha \sigma_{\mu\nu} T^A_{\alpha\beta} R b_\beta)\, G^{\mu\nu,A}$ gluonic dipole

Note, there is a local $bs\gamma\gamma$ operator, but it is of dim. 8, i.e. suppressed by $(m_b/m_W)^2$ and therefore neglected.



Let's look at the structure of the eff. Hamiltonian:

$$\mathcal{H}_{eff} \quad \sim \quad \sum_i C_i(\mu) O_i(\mu)$$

$\mathcal{H}_{eff}$ independent of μ, while C_i and O_i depend on μ:

$\rightarrow$ RGE for $C_i(\mu)$:

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ij}^T C_j(\mu) ; \quad \gamma_{ij} \text{ : anomalous dim. matrix}$$

Matching usually done at high scale μ_W, i.e. $\mu_W \sim O(m_W)$:

μ_W: full theory and mat. el. of op. have same large log's:

Corr. to $C_i(\mu_W)$ rel. small.

$\downarrow$ RGE

$\mu_b = O(m_b)$: mat. el. of op. don't have large log's: They are contained in the $C_i(\mu_b)$.

Calculation of the branching ratio consists of three steps:

	LL	NLL	NNLL
-matching at $\mu = \mu_W$: $\to C_i(\mu_W)$	α_s^0	α_s^1	α_s^2
-RGE: $\to C_i(\mu_b)$ [with $\mu_b = O(m_b)$]	α_s^1	α_s^2	α_s^3
-calc. of matrix element for specific decay	α_s^0	α_s^1	α_s^2

The Wilson coeff. are all available even for NNLL precision.

For the matrix elements the situation is different:

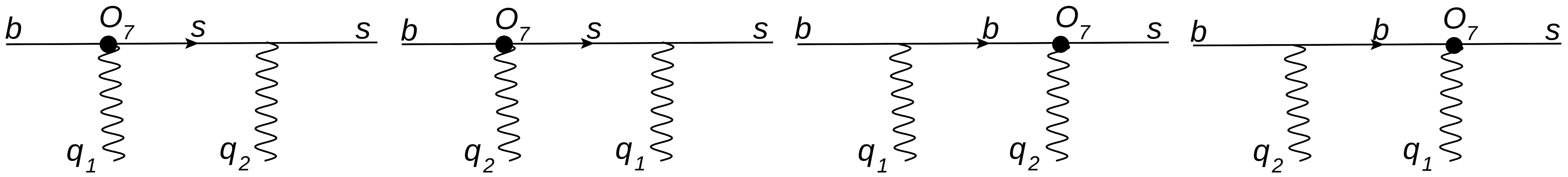
For $B \to X_s\gamma$ the matrix elements are known at NNLL precision.

For $B \to X_s\gamma\gamma$ they are known only at LL precision. We therefore started the NLL program by working out the QCD corrections the dominant O_7 contribution on which I report now.

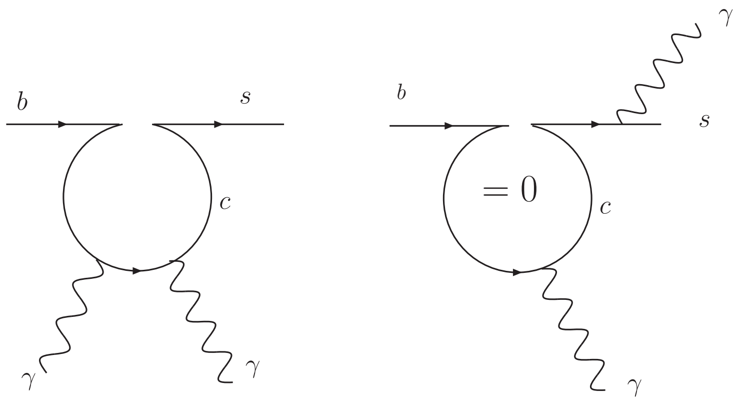
$B \to X_s\gamma\gamma$: Situation without QCD

We take into account the effects of the dominant operators O_1, O_2 and O_7.

The matrix elements associated with O_7:



The matrix elements associated with $O_{1,2}$:



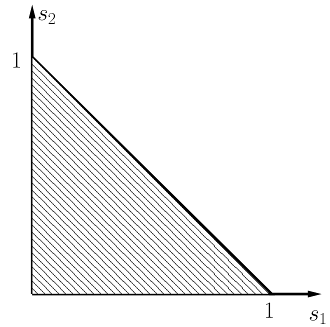
First look at the kinematics of the process $b \to s\gamma\gamma$.

The corresponding fully differential decay width has two independent kinematical variables. We choose them to be s_1 and s_2:

$$s_1 = (p_b - q_1)^2/m_b^2 \qquad s_2 = (p_b - q_2)^2/m_b^2\,; \qquad (q_1, q_2 \text{ photons; } p_b \text{ } b\text{-quark})$$

In the b-rest frame they are related to the photon energies (E_1 and E_2):

$$s_1 = 1 - 2E_1/m_b$$
$$s_2 = 1 - 2E_2/m_b$$



E_1 and E_2 must be away from zero to be observed $\leftrightarrow$ $s_1 < 1$ and $s_2 < 1$.

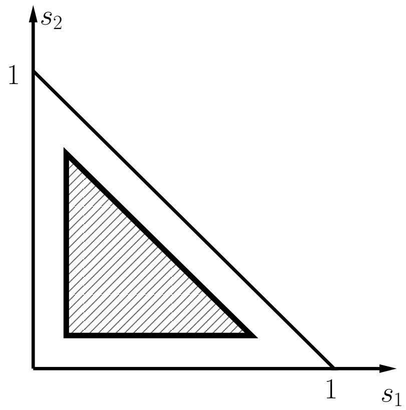
Require additionally that $s_1 > 0$ and $s_2 > 0$: By this condition we exclude collinear photon emission from the s-quark, because

$$(p_s + q_1)^2 = (p_b - q_2)^2 = s_2\, m_b^2 \qquad \text{and} \qquad (p_s + q_2)^2 = (p_b - q_1)^2 = s_1\, m_b^2\,.$$

The invariant mass squared s of the two photons also has to be away from 0

$$s = (q_1 + q_2)^2/m_b^2 = 1 - s_1 - s_2 \qquad \text{it is zero on the diagonal line}$$

We work out the double double diff. decay width in the window below, parametrized by c (as suggested by Reina, Ricciardi, Soni 1997):



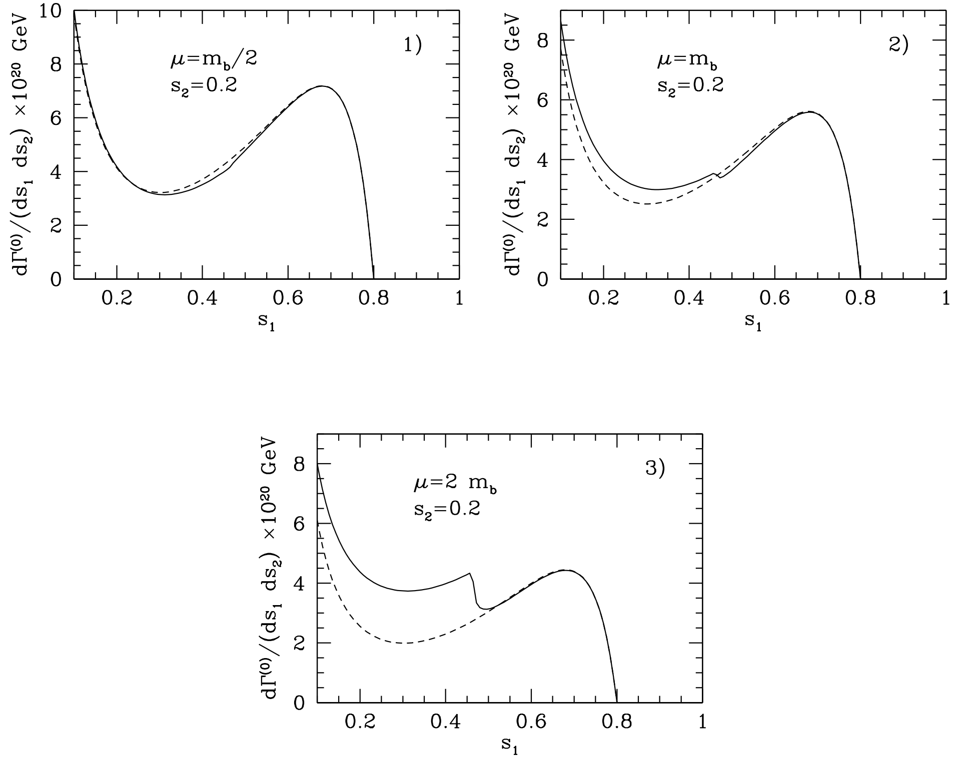
$$s_1 \geq c\,; \qquad s_2 \geq c\,;$$

$$1 - s_1 - s_2 \geq c\,.$$

Our aim is to give the double differential decay width is this restricted area.

Note: When later taking into account bremsstrahlung gluons, the kinematical (s_1, s_2)-range becomes larger. But, nevertheless we restrict ourselves to the shaded area also in this case.

We illustrate the result for the double differential decay width by fixing s_2 at 0.2 and vary s_1 (kinematical endpoint at $s_1 = 0.8$).



dashed-line: (O_7, O_7)-contr. only.

Solid: all contributions

The contributions from O_1 and O_2 involve the combination $C_2(\mu) + \frac{4}{3}C_1(\mu)$. For $\mu = m_b/2$ this combination is (accidentally) almost vanishing. This does not hold at other scales.

For the (O_7, O_7)-interference we get [r_0 is a (symmetric) polynomial in s_1 and s_2]:

$$\frac{d\Gamma_{77}^{(0)}}{ds_1 ds_2} = \frac{G_F^2 m_b^5 \alpha |C_7(\mu)|^2 |V_{tb} V_{ts}|^2 Q_d^2}{1024\pi^5} \frac{1 - s_1 - s_2}{(1 - s_1)^2 s_1 (1 - s_2)^2 s_2} \, r_0 \, .$$

The remaining interferences $(O_{1,2}, O_7)$, $(O_{1,2}, O_{1,2})$ etc. lead to (Reina et al. 1997; Hiller et al. 1997; Cao et al. 2001; Asatrian 2011):

$$\frac{d\Gamma^{(0)}}{ds_1 ds_2} = \frac{G_F^2 m_b^5 \alpha |V_{tb} V_{ts}|^2}{1024\pi^5} \times$$
$$\Bigg\{ 4\, Q_u^4 \left(C_2(\mu) + \frac{4}{3} C_1(\mu) \right)^2 \frac{(s_1 + s_2)}{(1 - s_1 - s_2)^2} \left| 1 - s_1 - s_2 - 4\, \hat{m}_c^2 \, \arcsin^2(z) \right|^2$$
$$+ 16\, Q_d\, Q_u^2 \left(C_2(\mu) + \frac{4}{3} C_1(\mu) \right) C_7(\mu) \, \left(1 - s_1 - s_2 - 4\, \hat{m}_c^2 \, \mathrm{Re} \left(\arcsin^2(z) \right) \right) \Bigg\} \, ,$$

with $z = \sqrt{(1 - s_1 - s_2)/(4\hat{m}_c^2)}$. $\hat{m}_c^2$ is understood to have a small negative imaginary part.

Some numbers to get a rough idea for the branching ratio:

Using $c = 1/100$ for the kinematical cut-parameter, we get at LL precision:

For $\mu = m_b/2$: 4.0×10^{-7};

For $\mu = m_b$: 3.1×10^{-7};

For $\mu = 2m_b$: 2.5×10^{-7};

Or, when using $c = 1/50$ for the kinematical cut-parameter:

For $\mu = m_b/2$: 2.4×10^{-7};

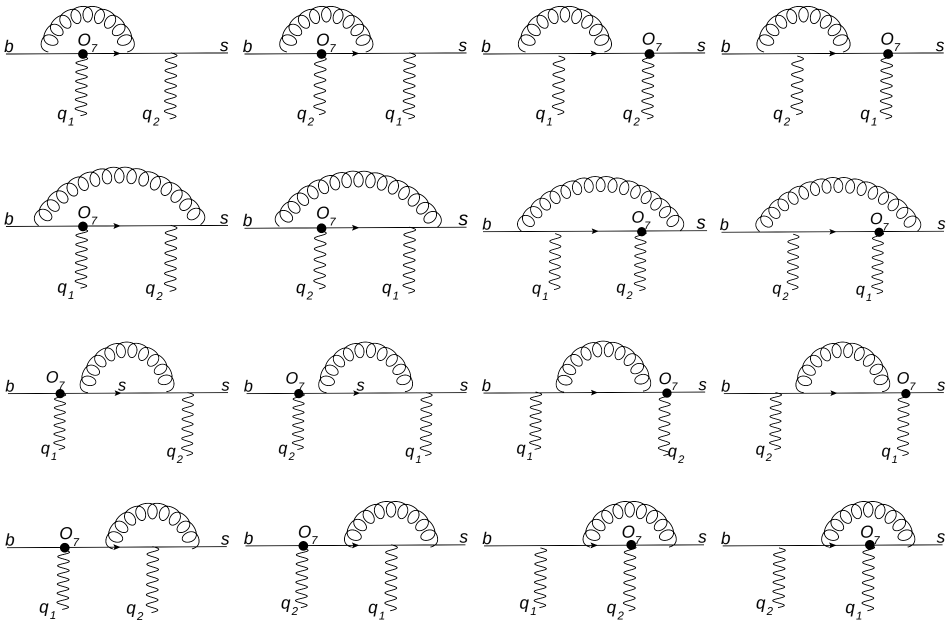
For $\mu = m_b$: 1.9×10^{-7};

For $\mu = 2m_b$: 1.6×10^{-7};

$\rightarrow$ The LL results strongly dependent on the renormalization scale μ. Complete NLL corrections should reduce it!

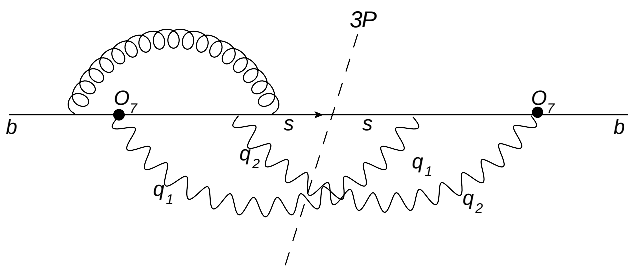
$B \to X_s\gamma\gamma$: Virtual gluon corrections to the O_7 contribution

The diagrams defining the (unrenormalized) virtual corrections are:



not shown: diags. with self-energy insertions on the external fermion legs: Taken into account through renormalization.

Technically, we directly calculated the interference with the lowest order diagrams, e.g



These objects contain loop- and phase space integrals!

We converted the phase-space integrals to loop-integrals, used systematically the IBP relations (with the AIR and FIRE implementations) and back-converted the obtained MI's to mixed loop/phase-space integrals. Then diff. eqs.

Taking into account all counterterms, the structure of the renormalized result is ($r = m_s^2/m_b^2$):

$$\frac{d\Gamma_{77}^{(1),\mathrm{virt}}}{ds_1\,ds_2} = \frac{\alpha_s}{4\pi} C_F \left[\frac{4\log(s_1+s_2) - 4 - 2\log(r)}{\epsilon} + \log^2(r) - \log(r) \right] \left(\frac{\mu}{m_b} \right)^{2\epsilon} \frac{d\Gamma_{77}^{(0,d)}}{ds_1\,ds_2} + \text{“fin. terms”}\,.$$

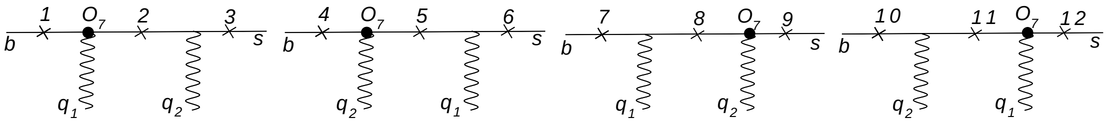
A few remarks:

1. The infrared singularites associated with soft gluons were regulated dimensionally. Note: The photons do not become soft, we want to observe them.

2. Collinear singularites: were regulated with a non-zero strange quark mass m_s. All these singularities are due to collinear gluons in our restricted phase space (when considering virtual corrections).

$B \to X_s \gamma\gamma$: Gluon bremsstrahlung corrections to the O_7 contribution

The diagrams at the amplitude level are:



x : gluon emission

The kinematical range of s_1 and s_2 is larger in this case ($0 < s_1 < 1$, $0 < s_2 < 1$), but we restrict to the region discussed above, which is also accessible to the lowest order.

The four-particle final state is described by 5 independent kin. variables, s_1 and s_2 are just two of them.

We integrated over the three remaining variables, i.e. we only keep s_1 and s_2 differential, leading to

$$\frac{d\Gamma_{77}^{(1),\mathrm{brems}}}{ds_1 ds_2} \,.$$

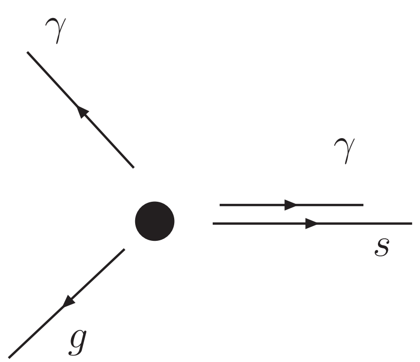
We found: When combining virtual- and bremsstrahlung corrections, there is

no cancellation of the $\log(m_s)$ terms (the $1/\epsilon$ and $\log^2(m_s)$ terms however, do cancel).

First guess: Somewhere is simply an error, but this was not the case.

Solution to the problem:

In the bremsstrahlung process there are configurations where one of the photons can become collinear with the s-quark even within our restricted phase space region, leading to uncancelled $\log(m_s)$ terms.



When combining virtual- and bremsstrahlung corrections in this setup, we have to following situation:

1. The sing. induced by soft/coll. gluons cancel

2. The sing. induced by coll. photons do no cancel

The point is that our observable is inclusive concerning gluons, but not w.r.t. photons. [The combination $b \to s\gamma$ (including QED corrections) plus $b \to s\gamma\gamma$ would be inclusive].

In principle, the configuration with coll. photon emission could be treated using fragmentation functions.

We recently considered the fragmentation function stuff in connection with specific contributions the process $B \to X_s\gamma$. There we saw, that simply treating m_s as a consituent mass gives similar results.

For $B \to X_s\gamma\gamma$ we proceed in the latter way and use $m_s = 400, 500, 600$ MeV in the numerics.

Virtual- and bremsstrahlung combined

The final result for the $O(\alpha_s)$ corrections reads

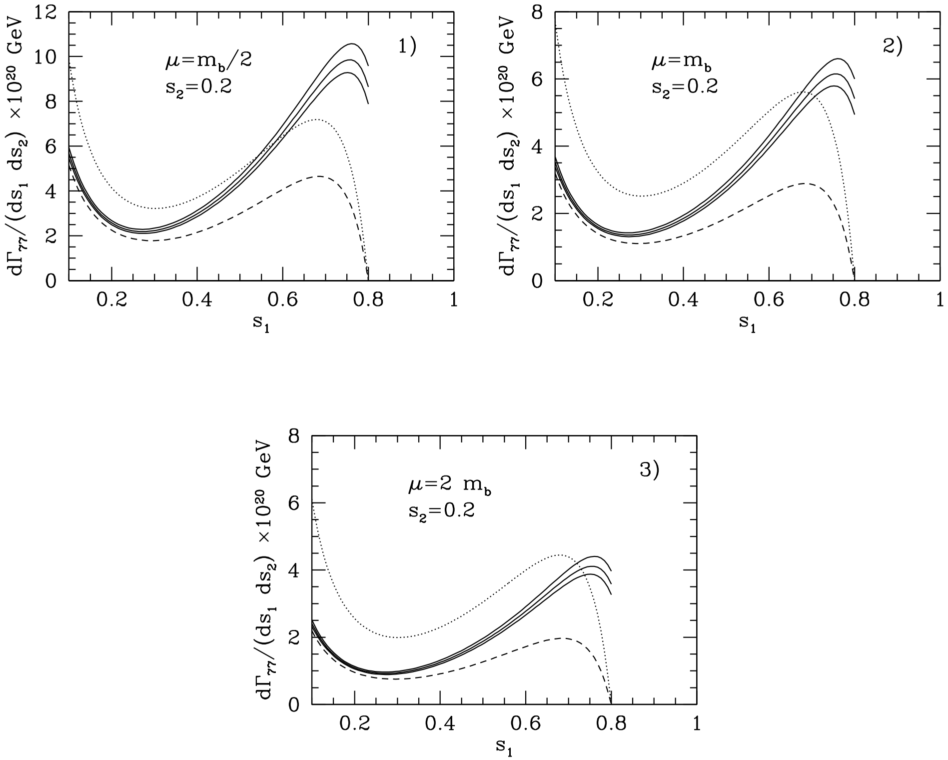
$$\frac{d\Gamma_{77}^{(1)}}{ds_1\, ds_2} = \frac{\alpha^2\, \bar{m}_b^2(\mu)\, m_b^3\, |C_{7,eff}(\mu)|^2\, G_F^2\, |V_{tb}V_{ts}^*|^2\, Q_d^2}{1024\, \pi^5} \times$$

$$\frac{\alpha_s}{4\pi}\, C_F\, \left[\frac{-4\, r_0\, (1 - s_1 - s_2)}{(1 - s_1)^2\, s_1\, (1 - s_2)^2\, s_2}\, \log\frac{\mu}{m_b} + f + g\log((m_s/m_b)^2) + h \right] .$$

The functions f and g in [] were given in analytic form, while for h we made a fit to a set of simple “basis-functions”, which is very accurate.

→ Main result of arXiv:1403.4502

$B \to X_s\gamma\gamma$: NLL Numerical results, O_7 contr. (arXiv:1403.4502)



Again: s_2 fixed at 0.2.

dotted: LL result;

dashed: forget it;

solid: NLL for $m_s = 400, 500, 600$ MeV.

When gluon bremsstrahlung is absent, the kin. endpoint in s_1 is at 0.8. $\to$ LL curve goes to zero at $s_1 = 0.8$.

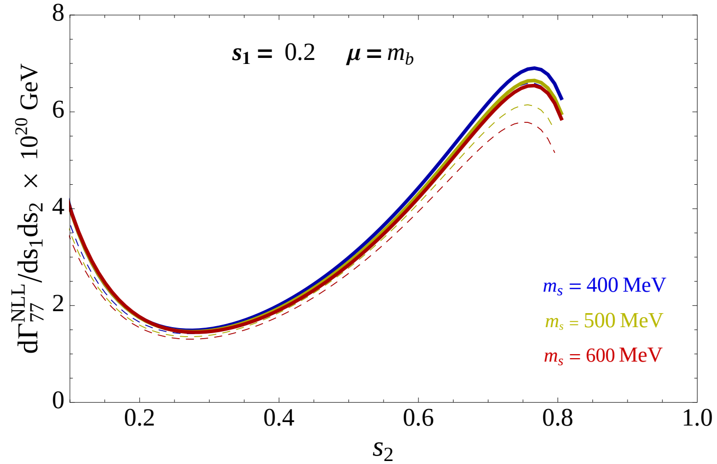
At NLL we also have gluon bremsstrahlung, $s_1^{\max} = 1 \to$ NLL is not zero at $s_1 = 0.8$.

Comparing LL and NLL: QCD corrections are important: they modify shape of spectra, not only normalization.

Non-logarithmic m_s effects in the (O_7, O_7) contribution to $B \to X_s\gamma\gamma$

In the results just discussed, only the logarithmic and constant terms in m_s were kept, while the power terms $m_s^1, m_s^2, \ldots$ were discarded. As we finally work with rather large m_s, we recently did a computation where the full m_s-dependence is kept (Asatrian, Greub, Kokulu, arXiv:1611.08449)

Plot: [solid $\leftrightarrow$ full; dashed $\leftrightarrow$ power terms in m_s discarded]



Comment on the (O_8, O_8) contribution to $B \to X_s\gamma\gamma$

In addition, we have also worked out the (O_8, O_8) contribution. It is very small in the full phase space (Asatrian, Greub, Kokulu, 1511.00153 [hep-ph]) .

The (O_8, O_8) is naively suppressed by a factor of $|C_8 Q_d / C_7|^2 \sim 1/36$ relative to the QCD corrections to the (O_7, O_7) interference contribution.

Potentially this naive suppression could be mildered:

In the O_7 contribution only one photon can be emitted from the strange quark, while in the O_8 contribution both photons can be emitted from the strange, leading potentially to a certain enhancement of the O_8 due to propagator effects.

A detailed analysis shows, however, that the (O_8, O_8) contribution influences the branching ratio by about +0.1% (Asatrian, Greub, Kokulu, 1511.00153 [hep-ph])

Missing NLL contributions; alternative observables

QCD corrections to $(O_{1,2}, O_7)$ and $(O_{1,2}, O_{1,2})$ are important, but difficult to calculate. Nevertheless, we started ($\rightarrow$ future talk!).

Reduction of scale dependence (μ) only will happen if these contributions are included.

We also plan to work out observables where the photons are isolated (isolation cuts à la Frixione, hep-ph/9801442).

$B \to X_s\gamma\gamma$: Kinematical branching ratios

We use $c = 1/100$ (upper part of table) and $c = 1/50$ (lower part). All numbers in units of 10^{-7}.

O_7-columns: only contributions from O_7;
"all"-columns: O_7 + $O_{1,2}$ contrib. at lowest order

	O_7	all	O_7	all	O_7	all
	$\mu = m_b/2$	$\mu = m_b/2$	$\mu = m_b$	$\mu = m_b$	$\mu = 2m_b$	$\mu = 2m_b$
LL	3.96	3.96	3.10	3.11	2.45	2.53
NLL_1	3.81	3.81	2.37	2.39	1.60	1.68
NLL_2	3.35	3.34	2.08	2.10	1.41	1.49
NLL_3	2.97	2.97	1.85	1.87	1.25	1.33
LL	2.40	2.40	1.87	1.89	1.48	1.55
NLL_1	2.39	2.39	1.49	1.51	1.01	1.08
NLL_2	2.17	2.17	1.35	1.37	0.91	0.99
NLL_3	1.99	1.99	1.24	1.26	0.84	0.91

$$\text{NLL}_1 \leftrightarrow m_s = 400 \text{ MeV} \; ; \text{NLL}_2 \leftrightarrow m_s = 500 \text{ MeV} \; ; \text{NLL}_3 \leftrightarrow m_s = 600 \text{ MeV} \; .$$

Summary

The branching ratio for $B \to X_s\gamma\gamma$ is systematically known only at LL precision.

We did a first step towards NLL precision by calculating QCD corrections to the matrix element associated with O_7.

The corrections are large. They modify the spectra, not only the normalization.

Note: Calculations of the matrix elements in the process $B \to X_s\gamma\gamma$ at NLL precison are of similar complexity as those for $B \to X_s\gamma$ at NNLL. .... and this was a long enterprise!

So it will take some time to work out the important QCD corrections involving the operators O_1 and O_2. The Armenian-Swiss collaboration will go on!