Theory of nucleon and nuclear EDMs

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CONTENTS

Introduction

ELECTRIC DIPOLE MOMENT: FUNDAMENTALS

- consider here nucleons and light nuclei
- a (permanent) electric dipole moment (EDM) violates time-reversal symmetry
  $\to$ if $\mathcal{CPT}$ holds, this is equivalent to $\mathcal{CP}$-violation
- In the Standard Model (SM) small $\mathcal{CP}$-violation
  $\to$ tiny nEDM, pEDM, ...

$$|d_n| < 10^{-30}\, e\, cm$$

- current limits: $|d_n| < 2.6 \times 10^{-26}\, e\,\text{cm}$ [dir.] $\to$ fig.

$$|d_p| < 2.0 \times 10^{-25}\, e\,\text{cm} \text{ [indir.]}$$

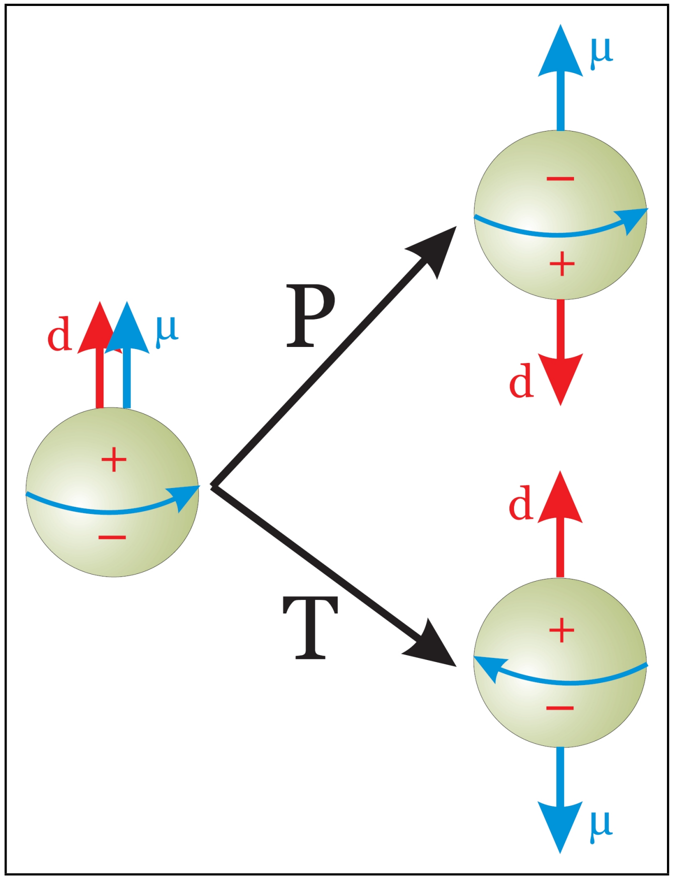


fig. from Wikipedia

$\Rightarrow$ very sensitive to physics beyond the SM (also $\mu \to e\gamma, (g-2)_\mu, \ldots$)
$\to$ precision low-energy hadron physics might well outdeliver the LHC

ELECTRIC DIPOLE MOMENT: DETAILS

- Permanent EDM of a subatomic particle:

$$\boxed{\vec{d} = d\,\vec{\sigma}}$$

- Energy of a particle with magnetic moment $\vec{\mu}$ and electric dipole moment $\vec{d}$:

$$H = -\mu\vec{\sigma}\cdot\vec{B} - d\vec{\sigma}\cdot\vec{E}$$

$\mathcal{T}$: $$H = -\mu\vec{\sigma}\cdot\vec{B} + d\vec{\sigma}\cdot\vec{E}$$

$\mathcal{P}$: $$H = -\mu\vec{\sigma}\cdot\vec{B} + d\vec{\sigma}\cdot\vec{E}$$

$\Rightarrow$ a non-vanishing permanent EDM of a subatomic particle is thus $\mathcal{T}$- and $\mathcal{P}$-violating

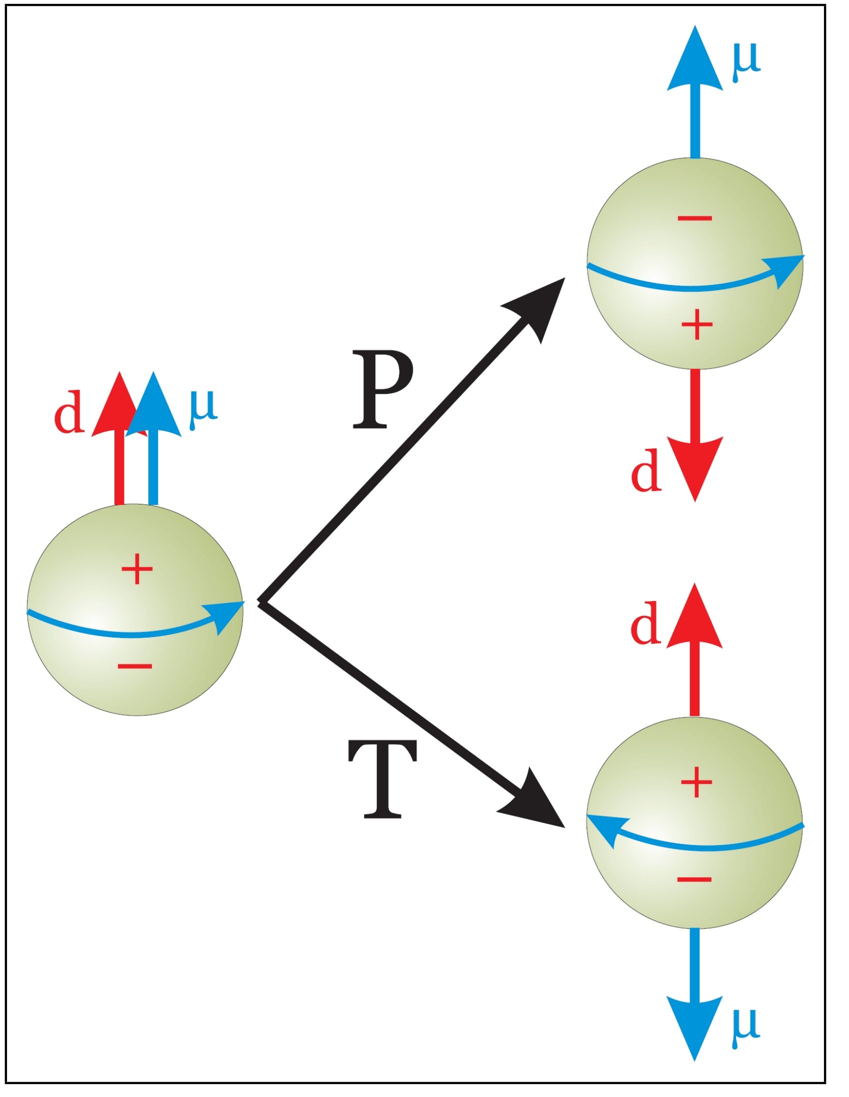
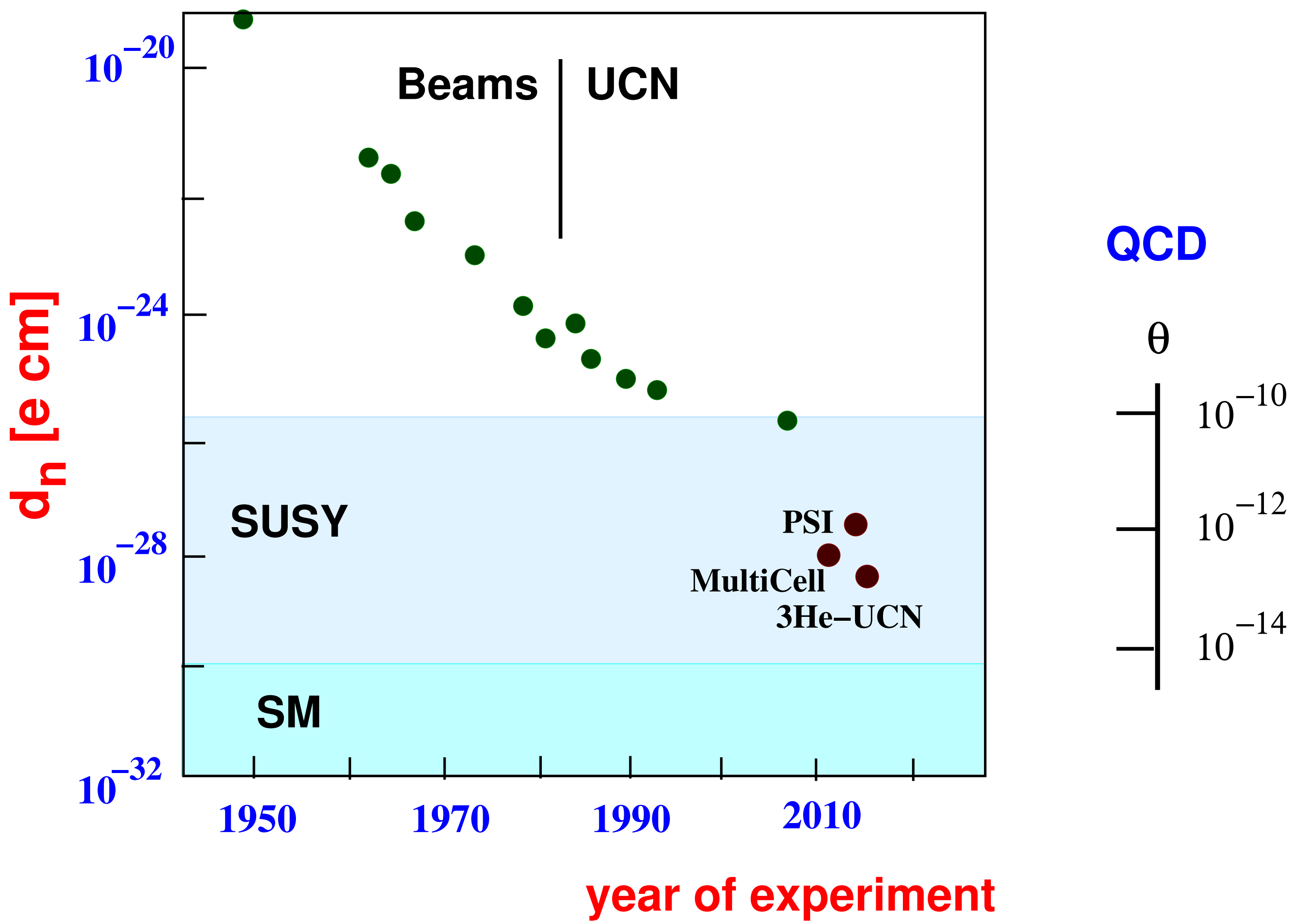


fig. from Wikipedia

- Be aware: non-degenerate ground state, otherwise level mixing

EXPERIMENTS on the NEUTRON EDM

- performed and projected experiments (UCN):



A SIMPLE ESTIMATE of the NUCLEON EDM

Khriplovich & Lamoreaux (1997), Nikolaev (2012)

- CP & P conserving magnetic moment $\sim$ nuclear magneton μ_N

$$\mu_N = \frac{e}{2m_P} \sim 10^{-14}\, e\text{ cm}$$

- A nonzero EDM requires

parity (P) violation: the price to pay is 10^{-7} $[G_F F_\pi^2 \sim 10^{-7}]$

and additionally CP violation: the price to pay is 10^{-3} $[|\eta_{+-}| \sim 10^{-3}]$

- In summary: $|d_N| \sim 10^{-7} \times 10^{-3} \times \mu_N \sim 10^{-24}\, e$ cm

- In the SM (without θ-term): extra $G_F F_\pi^2$ factor to *undo* flavor change

$$\hookrightarrow \; |d_N^{\rm SM}| \sim 10^{-7} \times 10^{-24}\, e\text{ cm} \sim 10^{-31}\, e\text{ cm}$$

$\hookrightarrow$ empirical window for BSM searches ($\theta = 0$) is:

$$10^{-24}\, e\text{ cm} > |d_N| > 10^{-30}\, e\text{ cm}$$

STRONG $\mathcal{CP}$–VIOLATION

- Many SM and BSM sources of EDMs $\rightarrow$ concentrate on one: QCD θ-term
- QCD has non-trivial topological vacua: $|\theta\rangle = \sum_n \mathrm{e}^{i\,n\,\theta}|n\rangle$
- Consider strong $\mathcal{CP}$-violation induced by the θ-vacuum
- QCD in the presence of strong $\mathcal{CP}$-violation

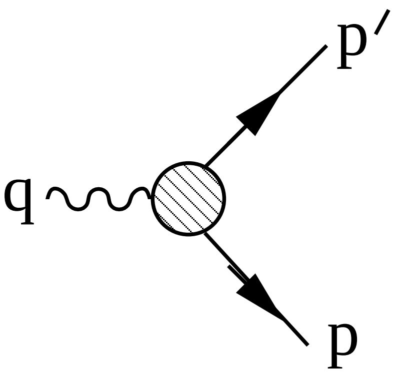
$$\mathcal{L}_{QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} + \sum_{\text{flavors}} \bar{q}\,(i\not{D} - \mathcal{M})\,q + \theta_0\,\frac{g^2}{32\pi^2}\,\underbrace{G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}}_{\sim\,\vec{E}^a\cdot\vec{B}^a}$$

- A non-vanishing vacuum angle θ_0 entails $d_n \neq 0$ (also $d_p \neq 0$)

$$d_N \approx |\theta_0|\,e\,\frac{M_\pi^2}{m_N^3} \approx 10^{-16}|\theta_0|\,e\,\mathrm{cm} \rightarrow \;\theta_0 = \mathcal{O}(10^{-10})$$

BARYON EDMs - BASIC DEFINITIONS

- Baryon electromagnetic form factors in the presence of strong $\mathcal{CP}$-violation [or other sources of it]



$$\langle p'|\, J^{\nu}_{em}\, |p\rangle = \bar{u}\,(p')\,\Gamma^{\nu}\,\left(q^2\right)u\,(p)$$

$$\Gamma^{\nu} = \gamma^{\nu} F_1\left(q^2\right) - \frac{i}{2m_B}\sigma^{\mu\nu} q_\mu F_2\left(q^2\right) - \frac{1}{2m_B}\sigma^{\mu\nu} q_\mu \gamma_5 F_3\left(q^2\right) + \ldots$$

$$q^2 = (p' - p)^2$$

- Dipole moments and radii:

$$d_B = \frac{F_{3,B}(0)}{2m_B}\,, \qquad \left\langle r^2_{ed}\right\rangle = 6\frac{dF_3(q^2)}{dq^2}\bigg|_{q^2=0}$$

- Similar formulae for light nuclei [see later]
- Calculation requires non-perturbative technique: EFT and/or LQCD

The baryon EDM in chiral perturbation theory

Ottnad, Kubis, UGM, Guo, Phys. Lett. B **687** (2008) 42
Guo, UGM, JHEP **1212** (2012) 097

BARYON EDMs - CALCULATIONS

- non-perturbative methods: baryon chiral perturbation theory and/or lattice QCD
- study the nucleon/baryon electric dipole form factors in baryon CHPT
  - ⋆ already quite a number of investigations (mostly the neutron)
    Crewther, Veneziano, Witten, Pich, de Rafael, . . .,
    Borasoy, Narison, Hockings, van Kolck, de Vries, . . .
  - ⋆ complete one-loop calculation   Ottnad, Kubis, M., Guo, Phys. Lett. B **687** (2008) 42
  - ⋆ based on $\mathcal{L}_{\rm eff}[\phi_a, \phi_0, B]$ supplemented by power counting
- LQCD calculations for θ and CP-violating form factors become available

  ⇒ must address quark mass and finite volume corrections

  ⇒ fruitful interplay between LQCD and CHPT/EFT practitioners

EFFECTIVE LAGRANGIAN

- Effective Lagrangian based on $U(3)_{\rm L} \times U(3)_{\rm R}$ symmetry:

Herrera-Siklódy et al., Borasoy

$$\mathcal{L}_{\rm eff} = \mathcal{L}[U, B]\ , \quad U = \exp\Bigg(\underbrace{\sqrt{\frac{2}{3}}\frac{i}{F_0}\eta_0}_{U(1)} + \underbrace{\frac{2i}{F_\phi}\phi}_{SU(3)}\Bigg)$$

- Power counting [δ = small parameter]:

Kaiser, Leutwyler, . . .

$$p = \mathcal{O}(\delta)\ , \qquad m_q = \mathcal{O}\left(\delta^2\right)\ , \qquad 1/N_c = \mathcal{O}\left(\delta^2\right)$$

- Determination of the low-energy constants

  – meson sector: meson masses, η-η' mixing, . . .

  – meson-baryon sector: em form factors, large N_c + naturalness

EFFECTIVE LAGRANGIAN

- $U(3) \times U(3)$ effective meson-baryon Lagragian Borasoy (2000)

$$\mathcal{L}_{\phi B} = i\,\mathrm{Tr}\big[\bar{B}\gamma^\mu[D_\mu, B]\big] - \mathring{m}\,\mathrm{Tr}[\bar{B}B] - \frac{D/F}{2}\mathrm{Tr}\big[\bar{B}\gamma^\mu\gamma_5[u_\mu, B]_\pm\big]$$
$$+ b_{D/F}\,\mathrm{Tr}\big[\bar{B}\big[\chi_+ - i\mathcal{A}(U - U^\dagger), B\big]_\pm\big] + b_0\,\mathrm{Tr}[\bar{B}B]\,\mathrm{Tr}\big[\chi_+ - i\mathcal{A}(U - U^\dagger)\big]$$
$$+ 4\mathcal{A}\,w'_{10}\frac{\sqrt{6}}{F_0}\eta_0\mathrm{Tr}[\bar{B}B] + i\Big(w'_{13/14}\,\bar{\theta}_0 + w_{13/14}\frac{\sqrt{6}}{F_0}\eta_0\Big)\mathrm{Tr}\big[\bar{B}\sigma^{\mu\nu}\gamma_5[F^+_{\mu\nu}, B]_\pm\big]$$
$$+ w_{16/17}\,\mathrm{Tr}\big[\bar{B}\sigma^{\mu\nu}[F^+_{\mu\nu}, B]_\pm\big] + \frac{\lambda}{2}\,\mathrm{Tr}\big[\bar{B}\gamma^\mu\gamma_5 B\big]\mathrm{Tr}[u_\mu]$$

- tree-level LECs: $w_{13}, w_{14}, w'_{13}, w'_{14}$ [some get renormalized] } more later!
- loop LECs: w'_{10}, λ
- further LECs are absorbed in masses, magnetic moments, etc
- chiral invariant: $\bar{\theta}_0(x) = \theta_0(x) - i\ln\det U(x)$

BARYON EDMS at ONE LOOP

Guo, UGM, JHEP**12** (2012) 097

- Consider the ground state baryon octet $(N, \Lambda, \Sigma, , \Xi)$

  $\hookrightarrow$ not only interesting in itself, but also provides sufficient data to fix LECs

- tree-level contributions to baryon EDMs $\left[\alpha = 144 V_0^{(2)} V_3^{(1)} / (F_0 F_\pi M_{\eta_0})^2\right]$:

$$d(n) = d(\Lambda)/2 = -d(\Sigma^0)/2 = d(\Xi^0) = 8e\bar{\theta}_0 \left(\alpha w_{13} + w'_{13}\right)/3$$

$$d(p) = d(\Sigma^+) = -4e\bar{\theta}_0 \left[\alpha \left(w_{13} + 3w_{14}\right) + w'_{13} + 3w'_{14}\right]/3$$

$$d(\Sigma^-) = d(\Xi^-) = -4e\bar{\theta}_0 \left[\alpha \left(w_{13} - 3w_{14}\right) + w'_{13} - 3w'_{14}\right]/3$$

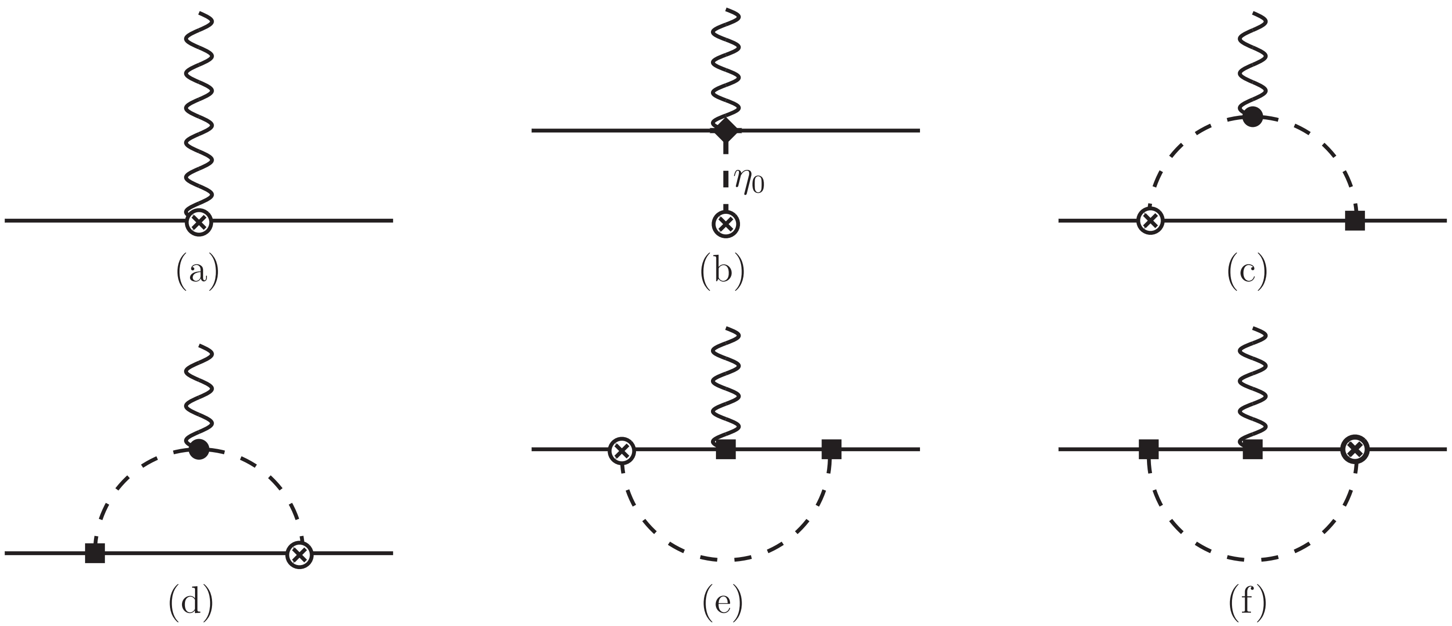
- Charged particles feature more loop contributions than neutral ones

  $\hookrightarrow$ Neutron case: $\{\pi^-, p\}$ and $\{K^+, \Sigma^-\}$

  $\hookrightarrow$ Proton case: $\{\pi^+, n\}$, $\{\pi^0(\eta_8, \eta_0), p\}$, $\{K^+, \Sigma^0(\Lambda)\}$ and $\{K^0, \Sigma^+\}$

FEYNMAN GRAPHS for $F_3^B(q^2)$

- tree (a,b) and one-loop (c,d,e,f) graphs (complete one-loop)



- other loop graphs (see Ottnad et al.) mutually cancel

$\otimes$ $\mathcal{CP}$-odd

■ dim. 2 $\mathcal{CP}$-even

NEUTRON LOOP CONTRIBUTIONS

- one-loop contributions to the neutron EDM (other neutral baryons similar):

$$
\begin{aligned}
\frac{F_{3,n}^{\rm loop}(q^2)}{2m_N} = \frac{V_0^{(2)} e\bar\theta_0}{\pi^2 F_\pi^4} \Bigg\{ & (D+F)\,(b_D+b_F) \\
& \times \left[1 - \ln\frac{M_\pi^2}{\mu^2} + \sigma_\pi \ln\frac{\sigma_\pi - 1}{\sigma_\pi + 1} + \frac{\pi\left(2M_\pi^2 - q^2\right)}{2m_N\sqrt{-q^2}} \arctan\frac{\sqrt{-q^2}}{2M_\pi} \right] \\
& -(D-F)\,(b_D-b_F) \left[1 - \ln\frac{M_K^2}{\mu^2} + \sigma_K \ln\frac{\sigma_K - 1}{\sigma_K + 1} \right. \\
& + \frac{\pi}{\sqrt{-q^2}} \left(\frac{2M_K^2 - q^2}{2m_N} - 8\,(b_D - b_F)\left(M_K^2 - M_\pi^2\right) \right) \\
& \left. \times \arctan\frac{\sqrt{-q^2}}{2M_K} \right] \Bigg\} \qquad \left[\sigma_{\pi(K)} = \sqrt{1 - 4M_{\pi(K)}^2/q^2} \right]
\end{aligned}
$$

- only known masses and couplings !

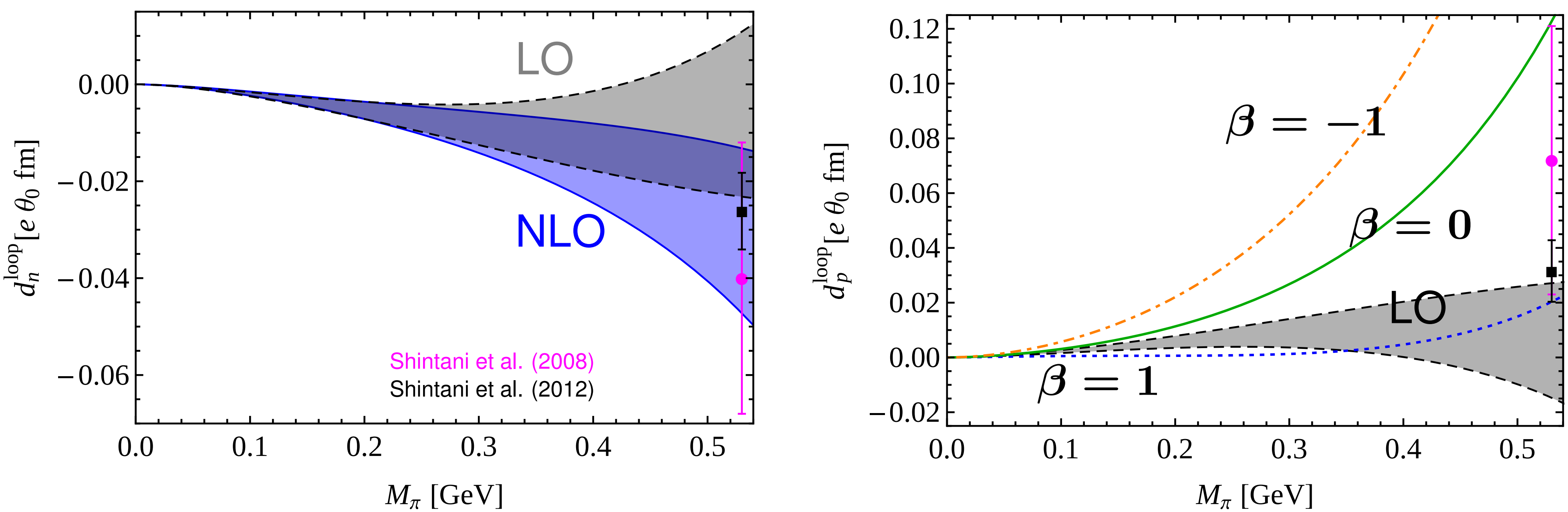
PROTON LOOP CONTRIBUTIONS

- one-loop contributions to the proton EDM (other charged baryons similar):

$$
\begin{aligned}
\frac{F_{3,p}^{\rm loop}(q^2)}{2m_N} = -\frac{V_0^{(2)} e\bar{\theta}_0}{6\pi^2 F_\pi^4} \Bigg\{ & 6(D+F)\,(b_D+b_F) \\
& \times \left[1 - \ln\frac{M_\pi^2}{\mu^2} + \sigma_\pi \ln\frac{\sigma_\pi - 1}{\sigma_\pi + 1} + \frac{3\pi M_\pi}{2m_N} + \frac{\pi\left(2M_\pi^2 - q^2\right)}{2m_N\sqrt{-q^2}} \arctan\frac{\sqrt{-q^2}}{2M_\pi} \right] \\
& + 4\,(Db_D + 3Fb_F)\left(1 - \ln\frac{M_K^2}{\mu^2} + \sigma_K \ln\frac{\sigma_K - 1}{\sigma_K + 1} + \frac{\pi M_K}{m_N} \right) \\
& + \frac{4\pi}{\sqrt{-q^2}} \arctan\frac{\sqrt{-q^2}}{2M_K} \left[\frac{(Db_D + 3Fb_F)}{2m_N}\left(2M_K^2 - q^2\right) \right. \\
& \left. + 8\left(M_K^2 - M_\pi^2\right)\left(F\left(b_D^2 + 3b_F^2\right) - \tfrac{2}{3} Db_D\,(b_D - 3b_F)\right) \right] \\
& + \frac{\pi}{m_N} \left[6(D-F)\,(b_D - b_F)\,M_K + (D-3F)\,(b_D - 3b_F)\,M_{\eta_8} \right. \\
& \left. + \frac{2F_\pi^2}{F_0^2} \underbrace{(2D - 3\lambda)\,(2b_D + 3b_0 + 6w'_{10})}_{\equiv\,\beta} M_{\eta_0} \right] \Bigg\}
\end{aligned}
$$

NEUTRON & PROTON EDMS

- Pion mass dependence of the neutron and proton loop contributions:



- β in 1/GeV for the proton
- preliminary data from Shintani et al just for illustration
- similar for the other neutral and charged baryons

MAKING PREDICTIONS

- Close inspection of the tree and one-loop expressions reveals

$\Rightarrow$ only **two** combinations of LECs appear at NLO in **all** baryon EDMs:

$$w_a(\mu) \equiv \alpha w_{13} + w'^{\,r}_{13}(\mu)$$

$$w_b(\mu) \equiv 3[\alpha w_{14} + w'^{\,r}_{14}(\mu)] + \frac{V_0^{(2)}\beta}{4\pi F_0^2 F_\pi^2 m_{\rm ave}} M_{\eta_0}$$

- use this formalism to analyze lattice data $\longrightarrow$ next section

- there are relations free of LECs such as

$$d_{\Sigma^0} + d_\Lambda \sim \left(M_K^2 - M_\pi^2\right)\left(Fb_D^2 + 2Db_Db_F + 3Fb_F^2\right) + \mathcal{O}(\delta^4)$$

$$d_n - d_{\Xi^0} \sim (Db_D + Fb_F)\left(2\ln\frac{M_K^2}{M_\pi^2} + \pi\frac{M_\pi - M_K}{m_{\rm ave}}\right) + \frac{8\pi}{M_K}\left(M_K^2 - M_\pi^2\right)\left(Db_D^2 + 2Fb_Db_F + Db_F^2\right) + \mathcal{O}(\delta^4)$$

$\Rightarrow$ now lets apply this formalism!

Finite volume effects for baryon EDMs

Akan, Guo, UGM, Phys.Lett. B **736** (2014) 163

CALCULATIONAL SCHEME

- Lattice QCD operates in a finite volume

  $\Rightarrow$ must consider finite volume corrections

  $\Rightarrow$ LO nEDM corrections known — O'Connell, Savage (2006)

- work in the limit of infinite time and large volumes L^3

- momenta quantized $p_n = 2\pi\vec{n}/L$

  $\rightarrow$ mode sums: $i\int\frac{d^4p}{(2\pi)^4} \rightarrow \frac{i}{L^3}\sum_{\vec{n}}\int\frac{dp^0}{2\pi}$

- finite volume corrections: $\delta_L[\mathcal{Q}] = \mathcal{Q}(L) - \mathcal{Q}(\infty)$

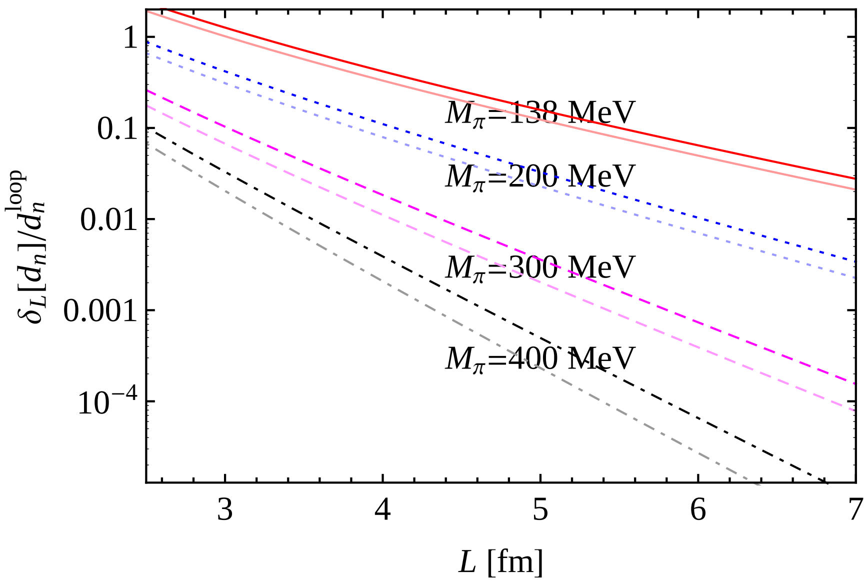
  $\hookrightarrow$ these are entirely generated by loops

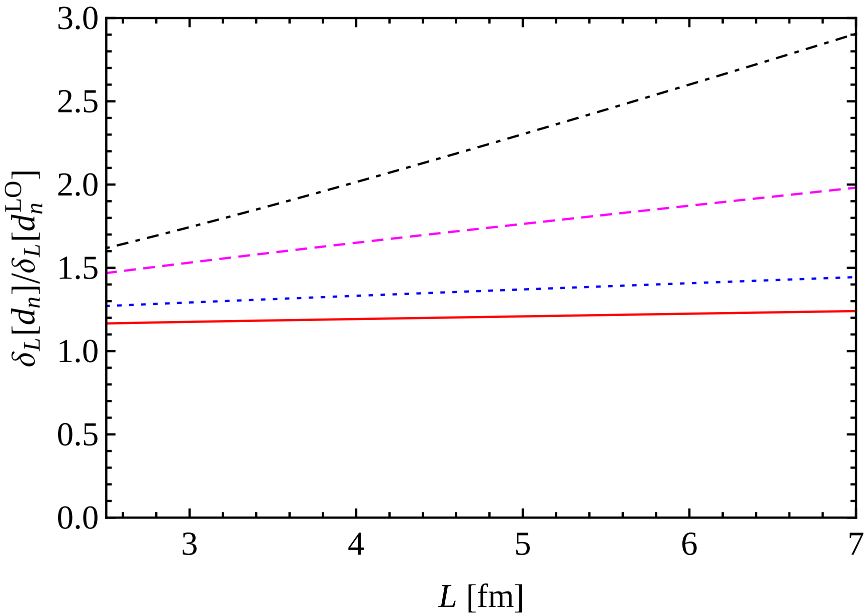
- here: perform NLO calculations for all baryon EDMs — Guo, UGM (2012)

RESULTS for the NEUTRON EDM

- At LO, recover the results of O'Connell and Savage
- Find sizeable NLO corrections $\longrightarrow$ must be included

$$\delta_L[d_n]/d_n^{\rm loop} \qquad\qquad \delta_L^{\rm NLO}[d_n]/\delta_L^{\rm LO}[d_n]$$





- available also for the proton and the hyperons

The neutron EDM from 2+1 flavor lattice QCD

Guo, Horsley, UGM, Nakamura, Rakow, Schierholz, Zanotti, Phys. Rev. Lett. **115** (2015) 062001 [arXiv:1502.02295]

LATTICE FORMULATION: GENERALITIES

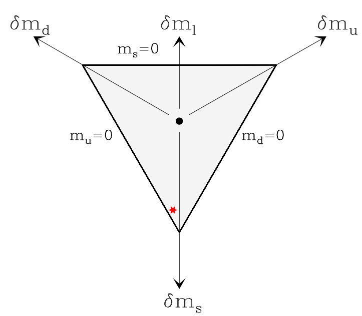
- work around the SU(3) symmetry point, keeping the singlet mass $\bar{m}$ fixed:

$$\bar{m} = (m_u + m_d + m_s)$$

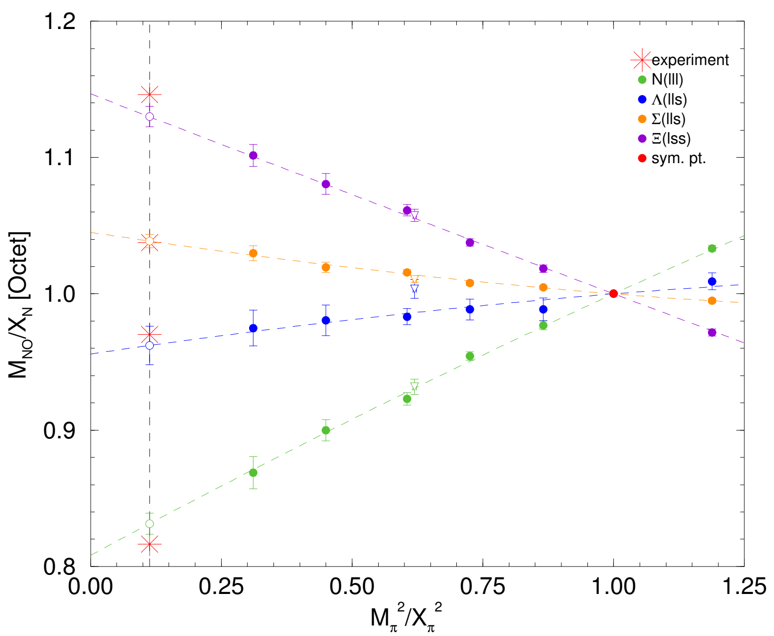
Bietenholz et al., Phys. Rev. D84 (2010) 054059

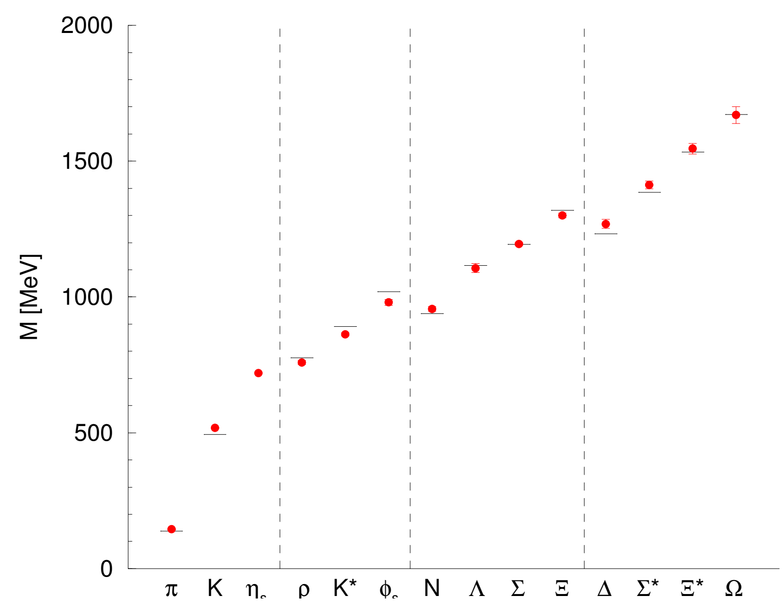
$\Rightarrow$ keeps the kaon mass low for varying strange quark masses

constrained polynomials fits



$\Rightarrow$ works fine for $N_f = 2+1$ $[m_u = m_d = m_\ell]$





LATTICE FORMULATION at FIXED TOPOLOGY

- The θ-term on the lattice [at fixed topological charge Q]:

$$S = S_0 + S_\theta, \quad S_\theta = i\,\theta\,Q,$$
$$Q = -\frac{1}{64\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\,a^4 \sum_x G^b_{\mu\nu} G^b_{\rho\sigma} \in \mathbb{Z}$$

- rotate the θ-term into the fermionic part of the action

Baluni (1979)

$$S_\theta = -\frac{i}{3}\theta \hat{m} a^4 \sum_x \left(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s\right)$$
$$\hat{m}^{-1} = \frac{1}{3}\left(m_u^{-1} + m_d^{-1} + m_s^{-1}\right) = \frac{1}{3}\left(2m_\ell^{-1} + m_s^{-1}\right)$$

- take the vacuum angle imaginary: $\theta = i\,\bar{\theta}$

[th'y analytic at $\theta = 0$]

$$\Rightarrow \quad S_\theta = \bar{\theta}\,\frac{m_\ell\, m_s}{2m_s + m_\ell}\, a^4 \sum_x \left(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s\right)$$

$\Rightarrow$ real action, vanishes at $m_l = 0$ and $m_s = 0$

LATTICE SET-UP

- Wilson fermions with a clover term & Symanzik improved gluon action (SLiNC)

$$S^q = S_0^q + S_\theta^q = a^4 \sum_x \bar{q} \left(D - \frac{1}{4} c_{SW}\, \sigma_{\mu\nu}\, G_{\mu\nu} + m_q + \frac{\lambda}{2a}\, \gamma_5 \right) q$$

- lattice set-ups ($\beta = 5.50$)

#	κ_ℓ	κ_s	V	M_π [MeV]	M_K [MeV]	λ
1	0.1209	0.1209	$24^3 \times 48$	465	465	0.003
2	0.1209	0.1209	$24^3 \times 48$	465	465	0.005
3	0.1210	0.1206	$24^3 \times 48$	360	505	0.003
4	0.1210	0.1206	$24^3 \times 48$	360	505	0.005
5	0.1210	0.1206	$32^3 \times 64$	360	505	0.003
6	0.1211	0.1205	$32^3 \times 64$	310	520	0.003

- $a = 0.074(2)$m from the average baryon octet mass
- only ensembles 1 to 4 in the PRL publication

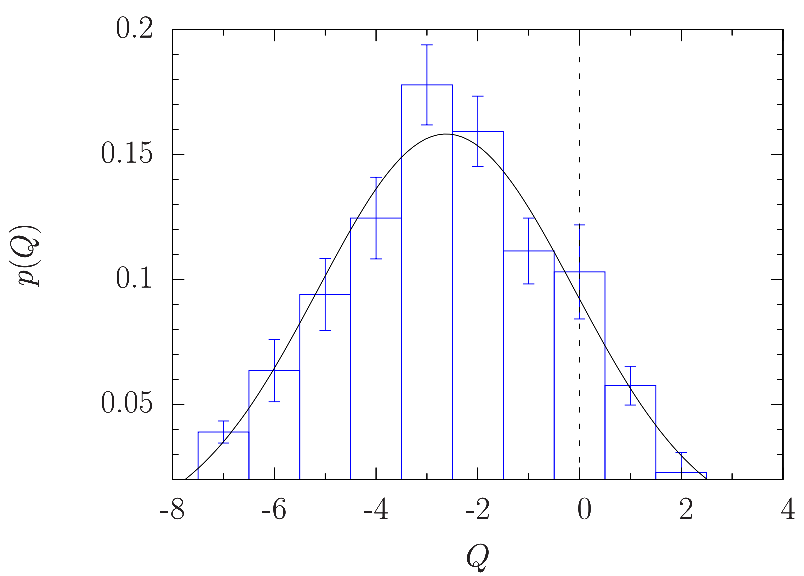
LATTICE SET-UP: TOPOLOGICAL CHARGE

- keep the singlet mass $\bar{m}$ fixed at its physical value
  and vary $\delta m_q = m_q - \bar{m}$

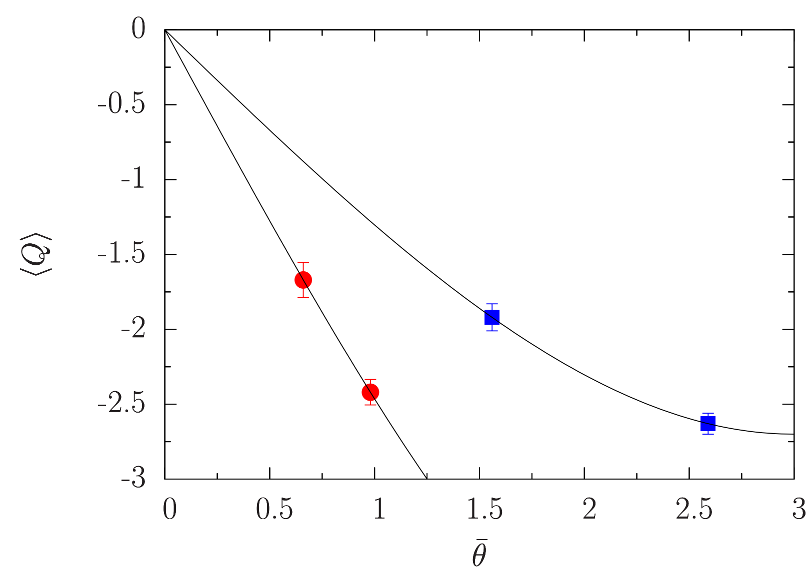
$$\lambda = \bar{\theta}\, 2a\, \frac{m_\ell\, m_s}{2m_s + m_\ell}, \qquad am_q = \frac{1}{2\kappa_q} - \frac{1}{2\kappa_{0,c}}, \quad \kappa_{0,c} = 0.1211$$

- Ensembles carry topological charge, $\langle Q \rangle \sim \bar{m}$

topological charge distribution #4



average topological charge

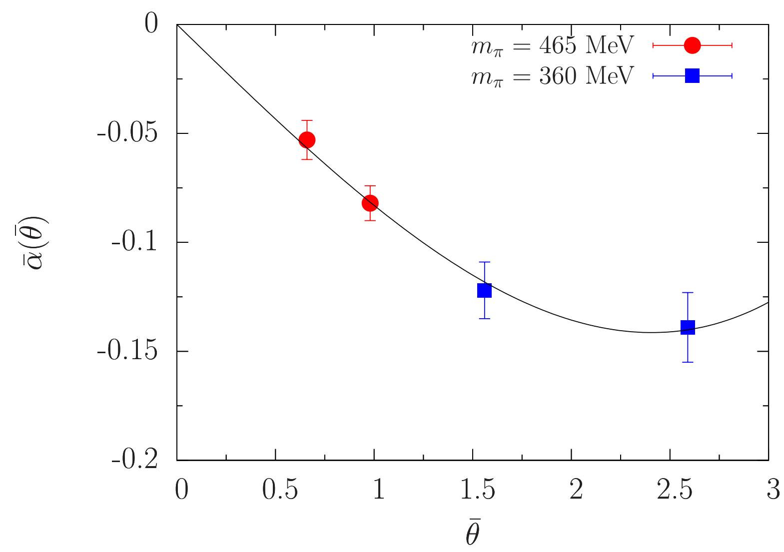


EVALUATION

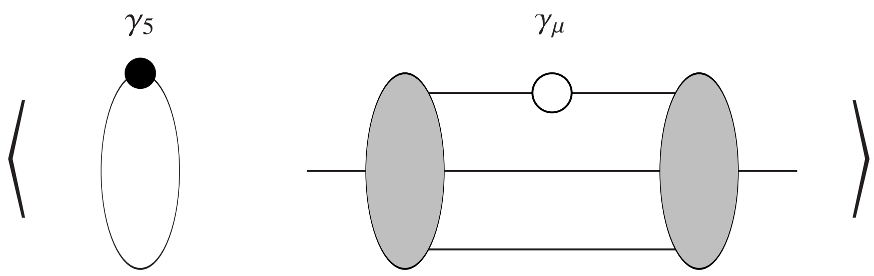
- at non-vanishing θ, Dirac spinors pick up a phase $\sim \alpha(\theta)$

$\rightarrow$ can be obtained from a ratio of two-point functions

$$\frac{\mathrm{Tr}[G^{\theta}_{NN}(t;0)\Gamma_4\gamma_5]}{\mathrm{Tr}[G^{\theta}_{NN}(t;0)\Gamma_4]} = i\frac{\sin 2\alpha(\theta)}{1+\cos 2\alpha(\theta)}$$



- flavor-singlet ps density interacts with the nucleon through quark-line disconnected diagrams only



- Form factor $F_3(q^2)$ extracted from a ratio of 3-point and 2-point functions generalizing the method of Capitani et al., Nucl. Phys. Proc. Suppl. 73 (1999) 294

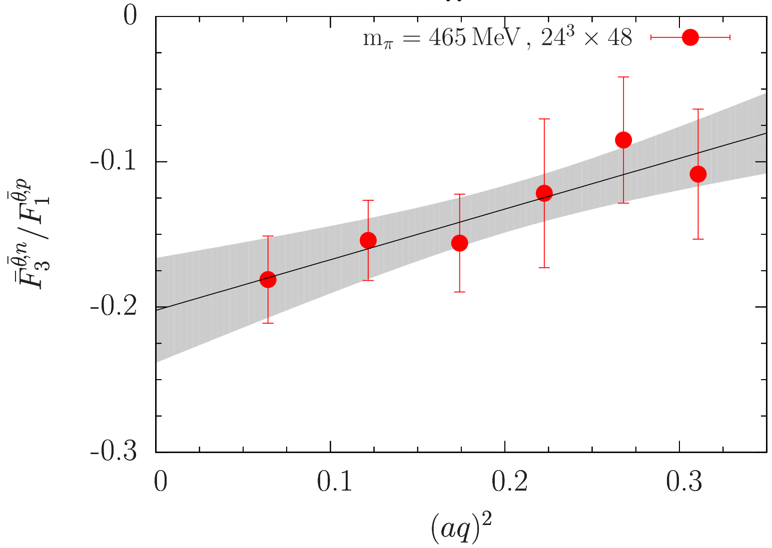
FORM FACTOR RATIOS

- Extracting $F_3^{\bar{\theta},n}(q^2)/F_1^{\bar{\theta},p}(q^2)$

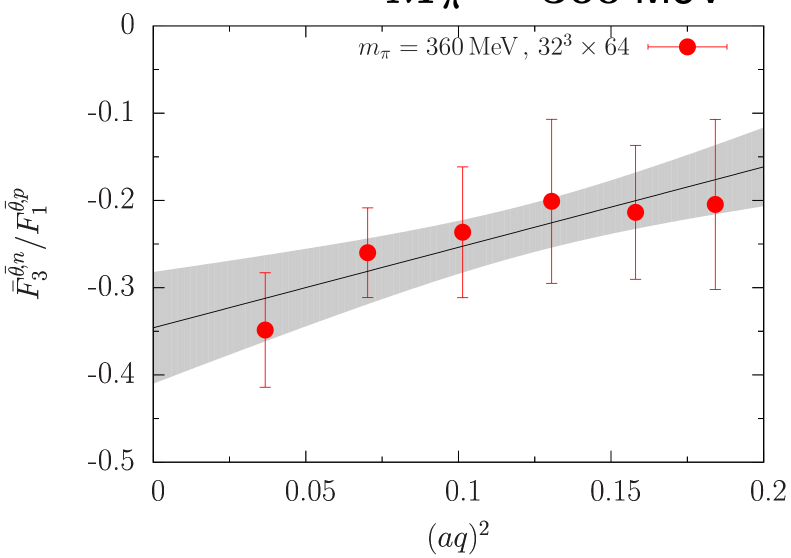
  at finite θ:

  easier extrapolation to $q^2 = 0$

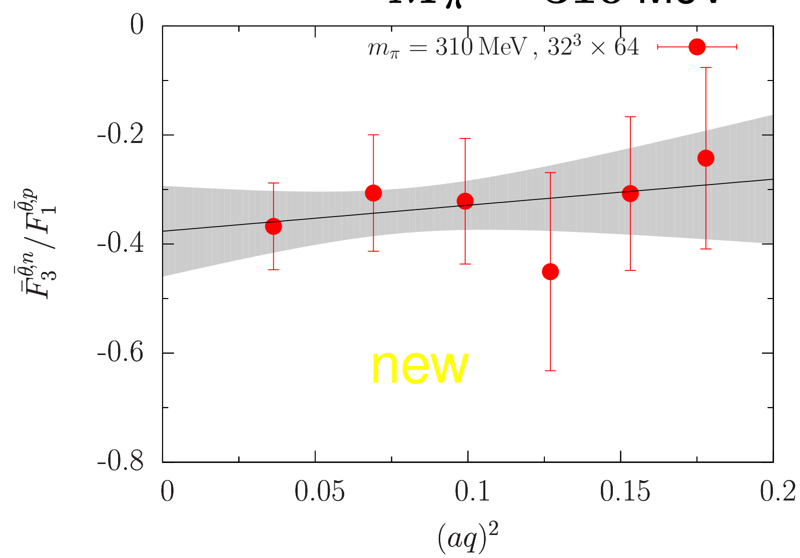
$M_\pi = 465$ MeV



$M_\pi = 360$ MeV



$M_\pi = 310$ MeV



RESULTS

- Form factor as a fct of $\bar{\theta}$
- continue θ and $F_3^{\theta}(0)$ to real values
- expand around $\theta = 0 : F_3^{\theta}(0) = F_3^{(1)}(0)\theta + \ldots$

$\rightarrow d_n = eF_3^{(1)}(0)\theta/2m_N$

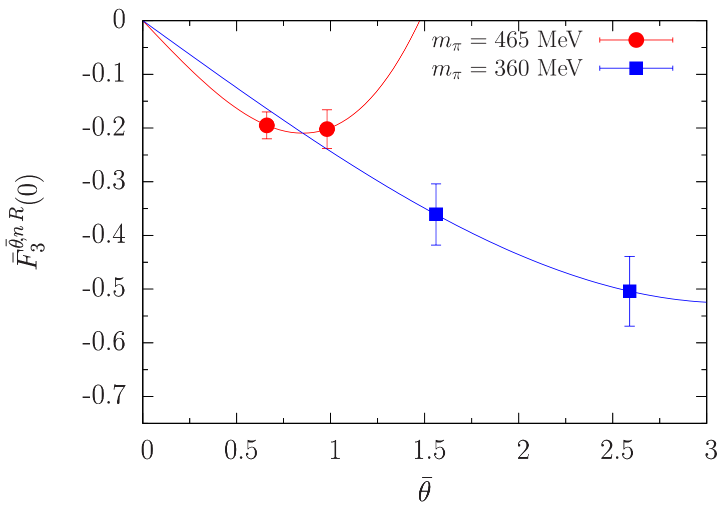
- chiral extrapolation with $2M_K^2 + M_\pi^2$ =const.

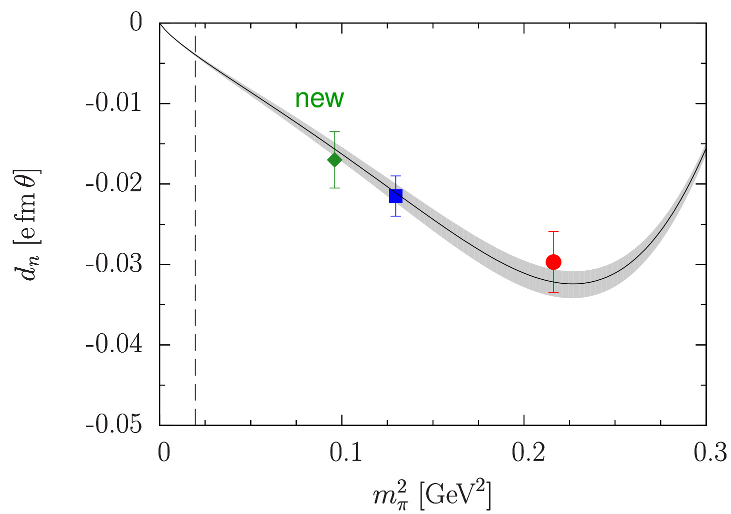
$\rightarrow w_a(\mu = 1\,\text{GeV}) = 0.04(1)\,\text{GeV}^{-1}$

$\rightarrow$ $d_n = -0.0039(2)(9)\ [e\,\text{fm}\,\theta]$

$\rightarrow |\theta| \lesssim 7.4 \times 10^{-11}$

- proton under analysis





OTHER RESULTS

- Neutron EDM using $N_f = 2+1+1$ twisted mass fermions

$$|d_n| = 0.045(6)(1)|\theta|\, e\,\text{fm}, \text{ one ensemble @ } M_\pi = 373\,\text{MeV}$$

Alexandrou et al., Phys. Rev. D **93** (2016) 074503

- Neutron EDM and tensor charges from LQCD

$$d_n < 4 \times 10^{-28}\, e\,\text{cm in split SUSY w/ gaugino mass unification}$$

Bhattacharya et al., Phys. Rev. Lett. **115** (2015) 212002

- other groups are close to announce numbers

  domain wall fermions $N_f = 2+1$, low pion masses, various volumes

  Shintani et al., Phys. Rev. D **93** (2016) 094503

  nucleon EDM from gradient flow, tested in quenched QCD

  Shindler et al., Phys. Rev. D **92** (2015) 094518

EDMs of light nuclei and models of CP violation

Bsaisou, Hanhart, Liebig, UGM, Nogga, Wirzba, Eur. Phys. J. A **49**: 31 (2013)
Bsaisou, de Vries, Hanhart, Liebig, UGM, Minosi, Nogga, Wirzba, JHEP **03** (2015) 104
Bsaisou, UGM, Nogga, Wirzba, Annals Phys. **359** (2015) 317
Wirzba, Bsaisou, Nogga, Int. J. Mod. Phys. E **26** (2017) 1740031

see also: de Vries, Hockings, Mereghetti, van Kolck, Timmermans (2005 - 2015)
Yamanaka, Hiyama (2015-2017)

MOTIVATION

- Why nuclei? $\Rightarrow$ non-trivial test of the θ-scenario and other models of $\mathcal{CP}$V

  $\Rightarrow$ allow access to other $\mathcal{CP}$-violating couplings

  $\Rightarrow$ allow to disentangle various model of $\mathcal{CP}$V

- **two-flavor** effective Lagrangian in standard heavy baryon formulation

  Bernard, UGM, Kaiser, Int.J.Mod.Phys. E4 (1995) 193

$$
\begin{aligned}
\mathcal{L}^{\pi N}_{\not{CP}} = & -d_n N^\dagger(1-\tau^3)S^\mu v^\nu N F_{\mu\nu} - d_p N^\dagger(1+\tau^3)S^\mu v^\nu N F_{\mu\nu} \\
& + (m_N \Delta)\,\pi_3\pi^2 + g_0 N^\dagger \vec{\pi}\cdot\vec{\tau}\, N + g_1 N^\dagger \pi_3 N \\
& + C_1 N^\dagger N\, \mathcal{D}_\mu(N^\dagger S^\mu N) + C_2 N^\dagger \vec{\tau}\, N \cdot \mathcal{D}_\mu(N^\dagger \vec{\tau}\, S^\mu N)
\end{aligned}
$$

- various contributions at next-to-leading order:

  * single nucleon EDMs – d_n, d_p
  * CP-violating 3-pion and pion-nucleon couplings – Δ, g_0, g_1
  * CP-violating nucleon-nucleon contact interactions – C_1, C_2

CP-VIOLATING NUCLEAR OPERATORS

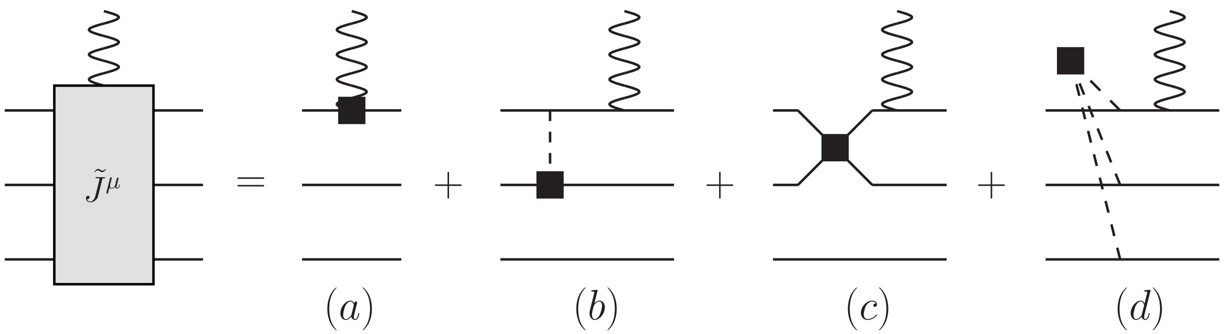
- EDM matrix element ($A = {}^2$H, ^{3}He, ^{3}H) [Breit frame]:

$$iq\frac{F_3^A(q^2)}{2m_A} = \left\langle A; M_J = J \,\middle|\, \tilde{J}^0(q) \,\middle|\, A; M_J = J \right\rangle \;, \quad d_A = \lim_{q^2 \to 0} \frac{F_3^A(q^2)}{2m_A}$$

- CP-violating transition current (only linear terms):

$$\tilde{J}^\mu = J^\mu_{\not{CP}} + V_{\not{CP}}\, G\, J^\mu + J^\mu\, G\, V_{\not{CP}} + \cdots$$

[G = 2N, 3N Greens function]

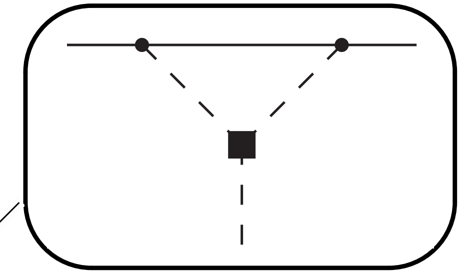


- single nucleon current (a):

$$J_i^\mu = \frac{e}{2}\left(1 + \tau^3_{(i)}\right) v^\mu \;, J^\mu_{\not{CP},i} = \frac{1}{2}\left[d_n\left(1 - \tau^3_{(i)}\right) + d_p\left(1 + \tau^3_{(i)}\right)\right] i\vec{q}\cdot\vec{\sigma}_{(i)}\, v^\mu$$

CP-VIOLATING NUCLEAR OPERATORS continued

irreducible 2N potential (b) + (c):



$$V^{NN}_{\not{CP},ij}(\vec{k}_i) = i\frac{g_A}{2F_\pi}\frac{\vec{k}_i}{\vec{k}_i^{\,2}+M_\pi^2}\, g_0\, \vec{\sigma}^{-}_{(ij)}\vec{\tau}_{(i)}\cdot\vec{\tau}_{(j)}$$
$$+\, i\frac{g_A}{4F_\pi}\frac{\vec{k}_i}{\vec{k}_i^{\,2}+M_\pi^2}\left[g_1 + \Delta\, f_{g_1}(|\vec{k}_i|)\right]\left(\vec{\sigma}^{+}_{(ij)}\tau^{-}_{(ij)} + \vec{\sigma}^{-}_{(ij)}\tau^{+}_{(ij)}\right)$$
$$+\,\frac{i}{2}\frac{\beta^2 M_\pi^2 \vec{k}_i}{\vec{k}_i^{\,2}+\beta^2 M_\pi^2}\left[C_1\vec{\sigma}^{-}_{(ij)} + C_2\,\vec{\sigma}^{-}_{(ij)}\vec{\tau}_{(i)}\cdot\vec{\tau}_{(j)}\right]$$

$$f_{g_1}(k) = -\frac{15}{32}\frac{g_A^2 M_\pi m_N}{\pi F_\pi^2}\left[1+\left(\frac{1+2\vec{k}^{\,2}/(4M_\pi^2)}{3|\vec{k}\,|/(2M_\pi)}\arctan\left(\frac{|\vec{k}\,|}{2M_\pi}\right)-\frac{1}{3}\right)\right]$$

- $\Delta\ f_{g_1}$ induced from 3-pion vertex
- $\beta\to\infty$ in calcs, used as diagnostic tool

CP-VIOLATING NUCLEAR OPERATORS continued

- irreducible 3N potential (d):

$$V^{3N}_{\not{CP}}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = -i\Delta \frac{m_N g_A^3}{4F_\pi^3} \left(\delta^{ab}\delta^{c3} + \delta^{ac}\delta^{b3} + \delta^{bc}\delta^{a3}\right) \tau^a_{(1)}\tau^b_{(2)}\tau^c_{(3)}$$
$$\times \frac{(\vec{\sigma}_{(1)} \cdot \vec{k}_1)(\vec{\sigma}_{(2)} \cdot \vec{k}_2)(\vec{\sigma}_{(3)} \cdot \vec{k}_3)}{\left[\vec{k}_1^2 + M_\pi^2\right]\left[\vec{k}_2^2 + M_\pi^2\right]\left[\vec{k}_3^2 + M_\pi^2\right]}$$

- evaluate 2N and 3N bound states using Faddeev equations

$$\langle\psi_A|\tilde{J}^\mu|\psi_A\rangle = \langle\psi_A|J^\mu_{\not{CP}} + V_{\not{CP}}\, G\, J^\mu + J^\mu\, G\, V_{\not{CP}} + \cdots|\psi_A\rangle$$

- employ wave functions from chiral EFT at N^{2}LO (precise enough)

- uncertainty from varying the cut-off in the chiral EFT, better for ^{2}H

RESULTS for LIGHT NUCLEI

- evaluating the nuclear matrix elements gives:

$$d_{^2\mathrm{H}} = (0.936\pm0.008)(d_n+d_p)+\left[(0.183\pm0.002)\,g_1-(0.646\pm0.023)\,\Delta\; f_{g_1}\right] e\,\mathrm{fm}$$

$$\begin{aligned} d_{^3\mathrm{He}} = {} & (0.90\pm0.01)\,d_n-(0.03\pm0.01)\,d_p-(0.017\pm0.006)\,\Delta \\ & -\left\{(0.61\pm0.14)\,\Delta\; f_{g_1}-(0.11\pm0.01)\,g_0-(0.14\pm0.02)\,g_1\right. \\ & \left.-\left[(0.04\pm0.02)C_1-(0.09\pm0.02)C_2\right]\times\mathrm{fm}^{-3}\right\} e\,\mathrm{fm} \end{aligned}$$

$$\begin{aligned} d_{^3\mathrm{H}} = {} & -(0.03\pm0.01)\,d_n+(0.92\pm0.01)\,d_p-(0.017\pm0.006)\,\Delta \\ & -\left\{(0.61\pm0.14)\,\Delta\; f_{g_1}-(0.11\pm0.01)\,g_0-(0.14\pm0.02)\,g_1\right. \\ & \left.+\left[(0.04\pm0.02)C_1-(0.09\pm0.02)C_2\right]\times\mathrm{fm}^{-3}\right\} e\,\mathrm{fm} \end{aligned}$$

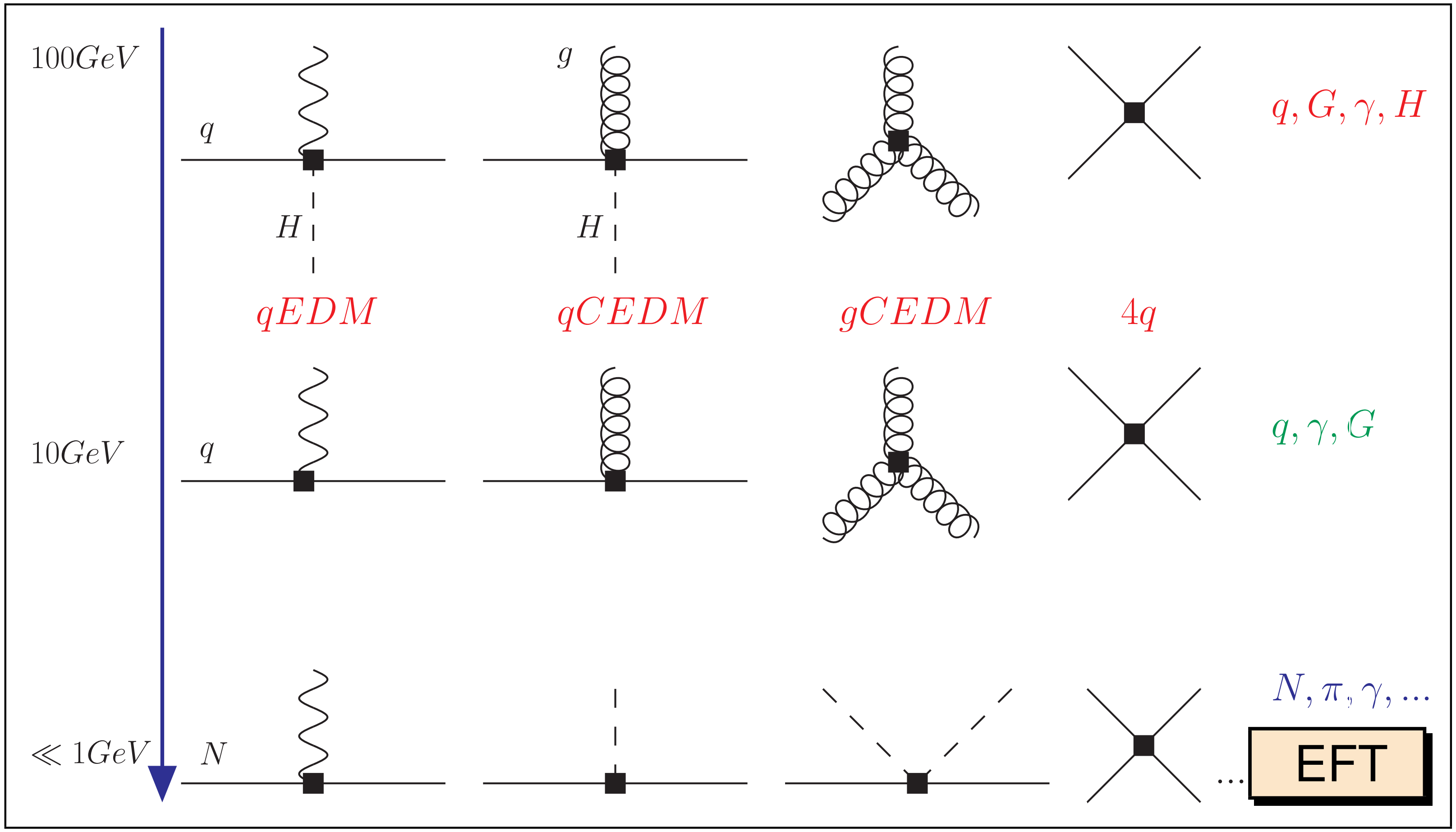
$\Rightarrow$ various models of CP violation give different predictions for the various coupling constants

[Note: terms $\sim C_{3,4}$ only relevant for L-R symm. models not given]

BSM OPERATORS at LOW ENERGIES

- translate (B)SM sources of P- and T-odd interactions into tailored EFTs

Fig. courtesy of A. Wirzba, J. de Vries



GLOSSARY OF CPV HADRONIC VERTICES

- Leading dimension-6 terms plus dimension-4 θ-term [beyond CKM]:

$$\mathcal{L}^{\not{CP}} = \underbrace{-\bar{\theta}\frac{g_s^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}}_{\theta\text{-term}}$$

$$\underbrace{-\frac{i}{2}\sum_{q=u,d} d_q\bar{q}\gamma_5\sigma_{\mu\nu}F^{\mu\nu}q}_{\text{qEDM}} \underbrace{-\frac{i}{2}\sum_{q=u,d}\tilde{d}_q\bar{q}\gamma_5\frac{1}{2}\lambda^a\sigma^{\mu\nu}G^a_{\mu\nu}q}_{\text{qCEDM}}$$

$$\underbrace{+\frac{d_{\rm W}}{6}f_{abc}\epsilon^{\mu\nu\alpha\beta}G^a_{\alpha\beta}G^b_{\mu\rho}G_\nu^{c\,\rho}}_{\text{gCEDM}} + \underbrace{\sum_{i,j,k,l=u,d} C^{\rm 4q}_{ijkl}\bar{q}_i\Gamma q_j\bar{q}_k\Gamma' q_l}_{\text{4qEDM}}$$

- Note that the 4q-terms from L-R symmetric models are treated separately

- Scaling: $\{d_q, \tilde{d}_q\} \sim v_{\rm EW}/\Lambda^2_{\not{CP}} \sim m_{u,d}/\Lambda^2_{\not{CP}}$, $\{d_{\rm W}, C^{\rm 4q}_{ijkl}\} \sim 1/\Lambda^2_{\not{CP}}$

SCALING OF CPV HADRONIC VERTICES

- from the θ term to BSM sources

coupling	g_0	g_1	d_0, d_1	$(m_N \Delta)$	$C_{1,2} (C_{3,4})$
CP, isospin	$\not{CP}$, IC	$\not{CP}$, IV	$\not{CP}$, IC+IV	$\not{CP}$, IV	$\not{CP}$, IC (IV)
θ-term	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi/m_N)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(M_\pi^2/m_N^2)$
qEDM	$\mathcal{O}(\alpha_{\rm EM}/(4\pi))$	$\mathcal{O}(\alpha_{\rm EM}/(4\pi))$	$\mathcal{O}(1)$	$\mathcal{O}(\alpha_{\rm EM}/(4\pi))$	$\mathcal{O}(\alpha_{\rm EM}/(4\pi))$
qCEDM	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(M_\pi^2/m_N^2)$
gCEDM	$\mathcal{O}(M_\pi^2/m_N^2)^\star$	$\mathcal{O}(M_\pi^2/m_N^2)^\star$	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(1)$
4qLR	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi^3/m_N^3)$	$\mathcal{O}(M_\pi/m_N)$	$\mathcal{O}(M_\pi^2/m_N^2)$
4q	$\mathcal{O}(M_\pi^2/m_N^2)^\star$	$\mathcal{O}(M_\pi^2/m_N^2)^\star$	$\mathcal{O}(1)$	$\mathcal{O}(M_\pi^2/m_N^2)$	$\mathcal{O}(1)$

*) Goldstone theorem $\to$ relative $\mathcal{O}(M_\pi^2/m_N^2)$ suppression of πN interactions

SPECIFIC CALCULATIONS

- Nuclear contribution from the QCD θ-term:

$$
\begin{aligned}
d^{\theta}_{^2\mathrm{H}} - 0.94(d^{\theta}_{p} + d^{\theta}_{n}) &= \bar{\theta}\cdot(0.89 \pm 0.30)\cdot 10^{-16}\, e\,\mathrm{cm} \\
d^{\theta}_{^3\mathrm{He}} - 0.90\, d^{\theta}_{n} + 0.03\, d^{\theta}_{p} &= -\bar{\theta}\cdot(1.01 \pm 0.42)\cdot 10^{-16}\, e\,\mathrm{cm} \\
d^{\theta}_{^3\mathrm{H}} - 0.92\, d^{\theta}_{p} + 0.03\, d^{\theta}_{n} &= \bar{\theta}\cdot(2.37 \pm 0.42)\cdot 10^{-16}\, e\,\mathrm{cm}.
\end{aligned}
$$

- Nuclear contribution from the FQLR-term:

$$
\begin{aligned}
d^{LR}_{^2\mathrm{H}} - 0.94(d^{LR}_{p} + d^{LR}_{n}) &= -\Delta\cdot(2.1 \pm 0.5)\, e\,\mathrm{fm} \\
d^{LR}_{^3\mathrm{He}} - 0.90\, d^{LR}_{n} + 0.03\, d^{LR}_{p} &= -\Delta\cdot(1.7 \pm 0.5)\, e\,\mathrm{fm} \\
d^{LR}_{^3\mathrm{H}} - 0.92\, d^{LR}_{p} + 0.03\, d^{LR}_{n} &= -\Delta\cdot(1.7 \pm 0.5)\, e\,\mathrm{fm}
\end{aligned}
$$

TESTING STRATEGIES

- Deuteron EDM might distinguish between $\bar{\theta}$ and other scenarios
  allows extraction of the g_1 coupling through $d_D - 0.94(d_p + d_n)$
- ^{3}He (or ^{3}H) EDM necessary for a proper test of $\bar{\theta}$ and LR scenarios
- a2HDM scenario: both helion & triton EDMs would be needed
- Deuteron & helion work as complementary **isospin filters** of EDMs
- gCEDM, 4q chiral singlet: disentanglement difficult, may be lattice calcs?
- ultimate progress may come from combining LQCD and experiments
- of course, these various models also predict EDMs for leptons etc.

$\Rightarrow$ precision calcs in hadronic physics are an absolute must!

SUMMARY & OUTLOOK

- Baryon EDMs evaluated in $U(3) \times U(3)$ CHPT at NLO
  $\hookrightarrow$ Chiral extrapolation formulae worked out
  $\hookrightarrow$ Only two LECs at complete one-loop order
  $\hookrightarrow$ Finite volume corrections available

- LQCD calculation of the neutron EDM for 2+1 flavors
  $\hookrightarrow$ simulation at various pion masses & lattice volumes
  $\hookrightarrow$ working with an imaginary θ [th'y assumed to be analytic at $\theta = 0$]
  $\hookrightarrow$ using CHPT, a precise value of d_n at the physical point emerges:

$$d_n = -0.0039(2)(9)\;[e\,\text{fm}\,\theta]$$

SUMMARY & OUTLOOK

- Theory of nuclear EDMs
  $\hookrightarrow$ general formulas as functions of all NLO CP-violating operators
  $\hookrightarrow$ explicit calculations for ^{2}H, ^{3}He, ^{3}H
  $\hookrightarrow$ testing strategies for various specific models in and beyond the SM

- UCN experiments on-going
- protons and charged light nuclei: storage ring measurements
- JEDI collaboration performs proof-of-principle exp. for the proton at COSY
  $\hookrightarrow$ for details, see `http://collaborations.fz-juelich.de/ikp/jedi/`

$\rightarrow$ interesting times ahead . . .

SPARES

CALCULATIONAL SCHEMES

- Chiral perturbation theory (CHPT) and extensions thereof allow to study the quark mass dependence and finite volume representation of hadron and nuclear properties

- Lattice QCD allows for ab initio calculations at (un)physical quark masses

  $\hookrightarrow$ Horsley et al., Shintani et al., Bhattacharya et al., **Guo et al.**

- Consider one “exotic” property of elementary particles and nuclei

  here: the baryon & light nuclei electric dipole moment

  $\hookrightarrow$ θ-vacua of QCD (topology)

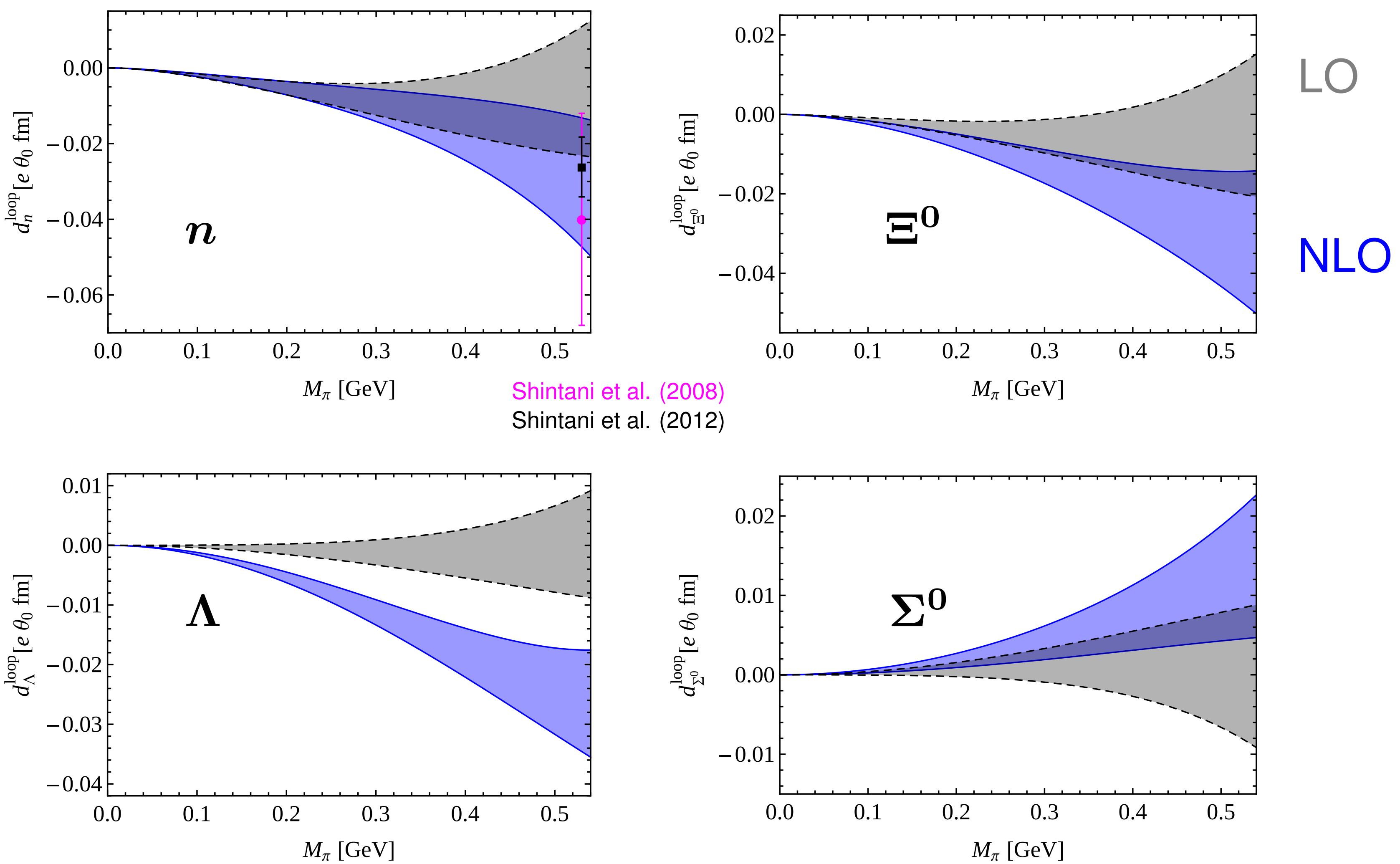
  $\hookrightarrow$ strong $\mathcal{CP}$-violation

  $\hookrightarrow$ also sensitive to physics beyond the SM

  $\hookrightarrow$ challenge for precision calculations

EDMS of NEUTRAL BARYONS

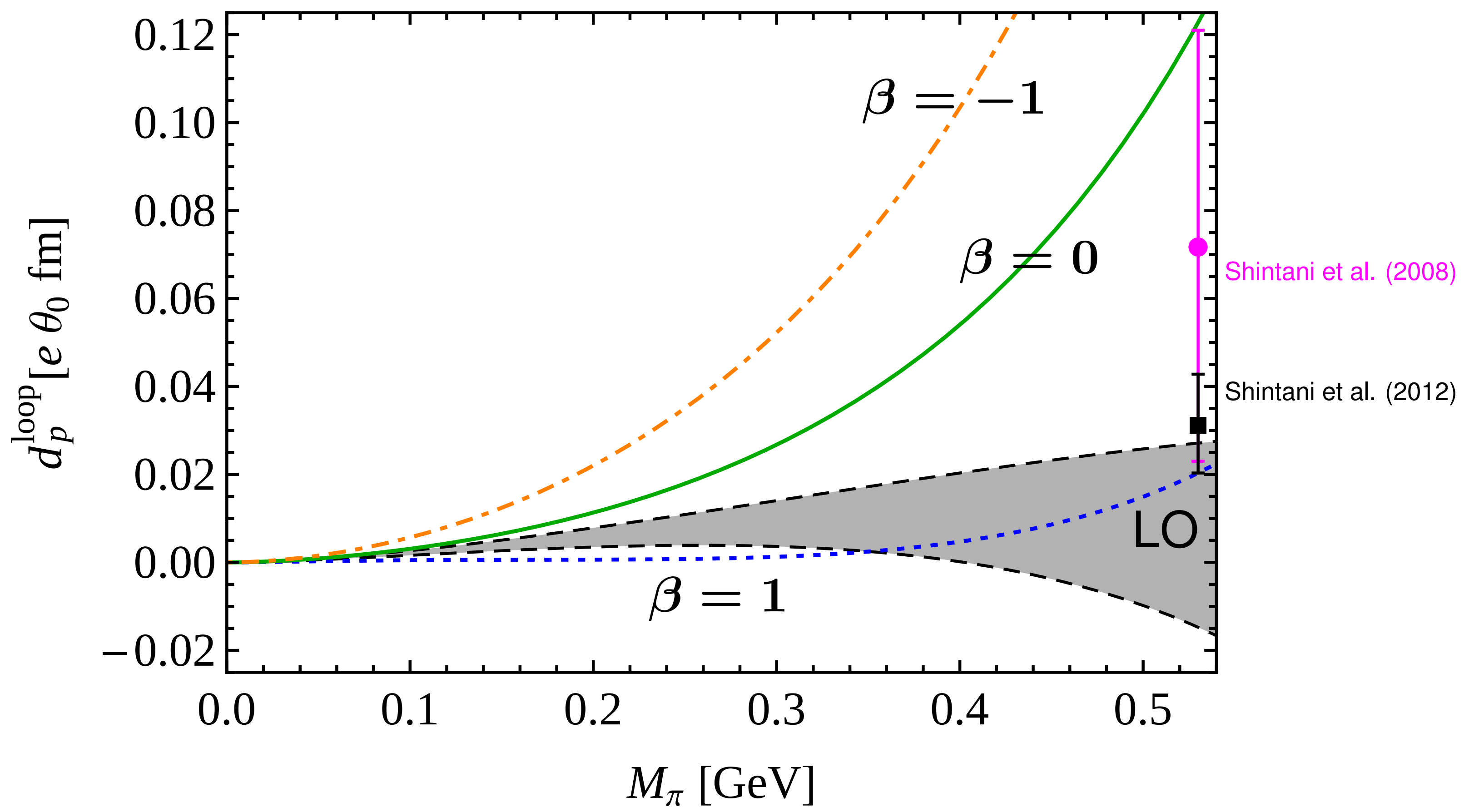
- Pion mass dependence of the neutral baryons loop contributions:



- preliminary data from Shintani et al just for illustration

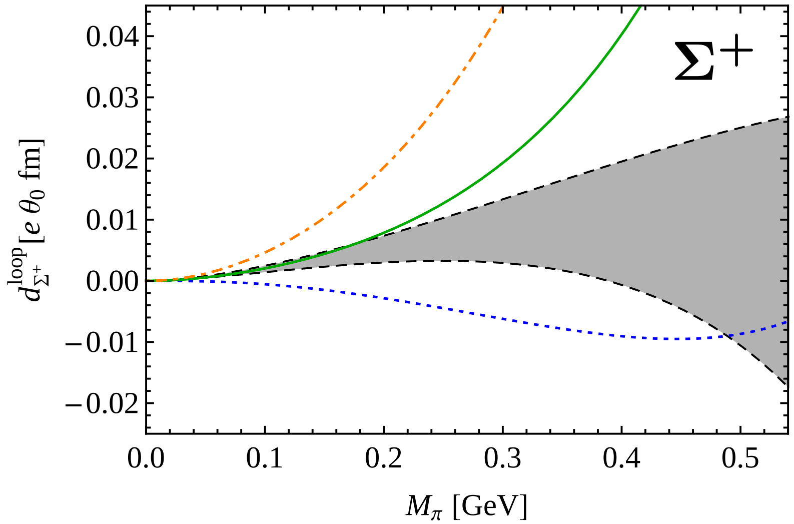
EDMS of CHARGED BARYONS

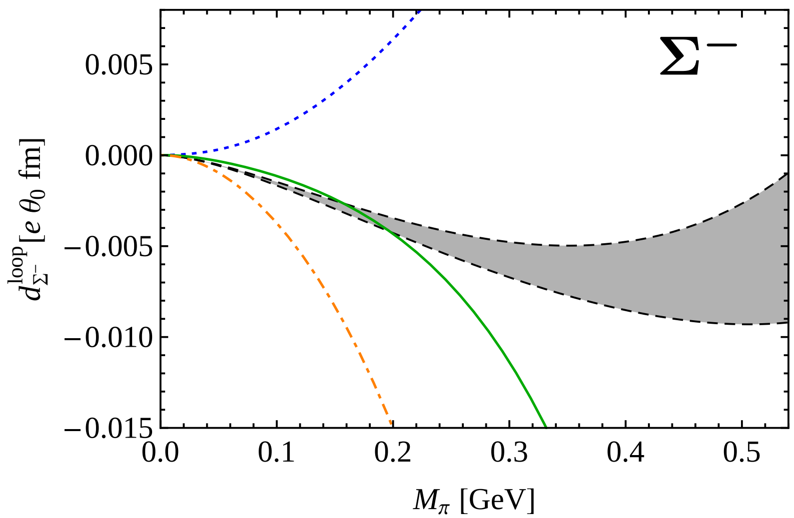
- Pion mass dependence of the proton loop contributions (β in 1/GeV):

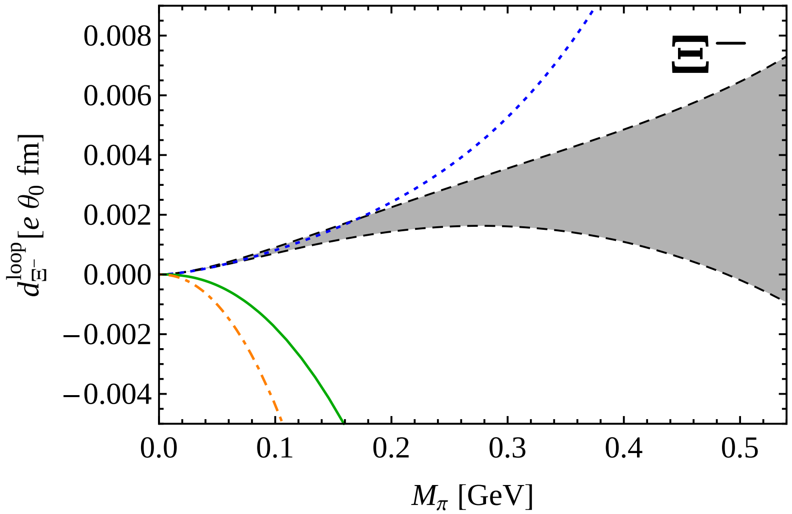


- other charged baryons show similar sensitivity $\rightarrow$ calculate all!

EDMS of CHARGED BARYONS cont'd

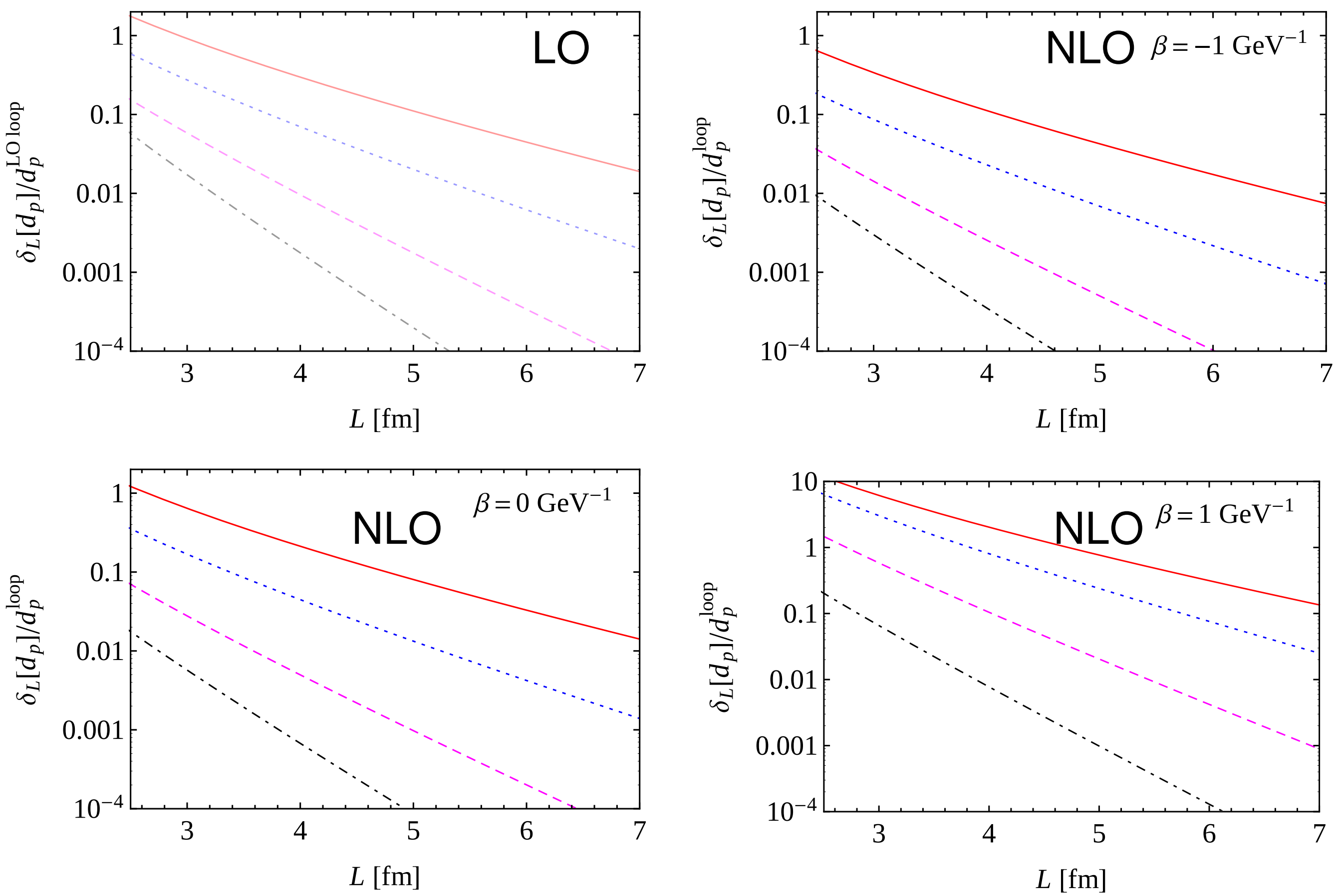






RESULTS for the PROTON EDM

- again large sensitivity to the LEC β



- calcs for the ffs more difficult, in progress   Tiburzi, Greil et al., Akan et al.