$B \to \pi\pi$ Form Factors: Calculations and Applications

Towards a coherent approach to B decays into unstable particles

Javier Virto

Massachussets Institute of Technology

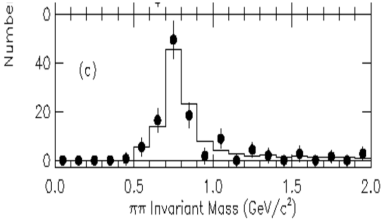
and

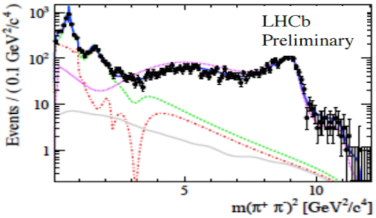
Technische Universität München

Tbilisi, September 25, 2017

What is a ρ meson?

- **Experimentalist version** : Some bump in a $\pi\pi$ distribution





Left: current source for $B^- \to D^0 \rho^-$ (CLEO). Right: $B \to D\pi^+\pi^-$ (LHCb).

- **Theorist version** : A pole in some correlation function

$$\mathrm{Im}\left[\int d^4x\, e^{iq\cdot x}\langle 0|T\{j^\mu(x), j_\mu(0)\}|0\rangle\right] \sim \delta(q^2 - m_\rho^2)\cdot f_\rho^2 + \cdots$$

- Be not surprised that: **Experimentalist version $\neq$ Theorist version**

Why do we care?

- Measurement of V_{ub}

$$\begin{pmatrix} B \to \rho\,\ell\nu \\ B_s \to K^\star\ell\nu \end{pmatrix} \longrightarrow \text{ Really are } \begin{pmatrix} B \to \pi\pi\,\ell\nu \\ B_s \to K\pi\,\ell\nu \end{pmatrix}$$

- Rare penguin decays (NP)

$$\begin{pmatrix} B \to \rho\,\ell\ell \\ B \to K^\star\ell\ell \end{pmatrix} \longrightarrow \text{ Really are } \begin{pmatrix} B \to \pi\pi\,\ell\ell \\ B \to K\pi\,\ell\ell \end{pmatrix}$$

- quasi-two-body decays (α, CP violation, NP ...)

$$\begin{pmatrix} B \to \rho\pi \\ B \to K^\star\pi \\ \dots \end{pmatrix} \longrightarrow \text{ Really are } \begin{pmatrix} B \to \pi\pi\pi \\ B \to K\pi\pi \\ \dots \end{pmatrix}$$

- Huge experimental programs for these modes at LHCb and Belle-2
Huge data sets will require theory precision.

Main theory objects

$B \to \rho$ form factors	$\longleftarrow \cdots \longrightarrow$	$B \to \pi\pi$ form factors
$\langle \rho \vert \bar{q}(x)\Gamma b(0) \vert \bar{B} \rangle$		$\langle \pi\pi \vert \bar{q}(x)\Gamma b(0) \vert \bar{B} \rangle$
ρ-LCDAs...	$\longleftarrow \cdots \longrightarrow$	2π-LCDAs...
$\langle \rho \vert \bar{q}(x)\Gamma q(0) \vert 0 \rangle_{x^2 \to 0}$		$\langle \pi\pi \vert \bar{q}(x)\Gamma q(0) \vert 0 \rangle_{x^2 \to 0}$
... their normalization f_ρ	$\longleftarrow \cdots \longrightarrow$	... their normalization $F_\pi(s)$
... their moments a_n^ρ	$\longleftarrow \cdots \longrightarrow$	... their moments $B_{n\ell}(s)$

OUTLINE of the talk:

- $B \to \rho$ and $B \to \pi\pi$ form factors from B-meson LCSRs
- $B \to \pi\pi$ form factors from 2π LCDAs
- Some extensions

QCD Sum Rules : crash course

- Imagine you have some correlation function

$$\Pi(q^2, \dots) = \int d^4x\, e^{iq\cdot x} \langle \alpha | T\{j_1(x), j_2(0)\} | \beta \rangle$$

with (only) a cut for real $q^2 > s_{th}$, and calculable via some OPE at $q^2 = \bar{q}^2$.

- One can write a dispersion relation:

$$\Pi_{\mathrm{OPE}}(\bar{q}^2, \dots) = \frac{1}{\pi} \int_{s_{th}}^{\infty} ds \frac{\mathrm{Im}\Pi(s, \dots)}{s - \bar{q}^2}$$

- The L.H.S. can be calculated perturbatively (by assumption) in conjunction with a power expansion, and the R.H.S. is given by unitarity:

$$2\mathrm{Im}\Pi(s) = (2\pi)\delta(s - m_\lambda^2) \langle \alpha | j_1 | \lambda \rangle \langle \lambda | j_2 | \beta \rangle + \text{higher states}$$

- Finally, a Borel Transformation $\bar{q}^2 \to M^2$ + duality takes care of possible subtractions, convergence of the OPE and higher states:

$$\boxed{\Pi_{\mathrm{OPE}}(M^2, \dots) = \langle \alpha | j_1 | \lambda \rangle \langle \lambda | j_2 | \beta \rangle e^{-m_\lambda^2 / M^2}}$$

$B \to \rho$ form factors from B-meson LCDAs

Khodjamirian, Mannel, Offen 2006

- Correlation function

$$F_\mu(k,q) = i\int d^4x e^{ik\cdot x}\langle 0|\mathrm{T}\{\bar d(x)\gamma_\mu u(x), \bar u(0) i m_b\gamma_5 b(0)\}|\bar B^0(q+k)\rangle$$

- Unitarity relation

$$2\mathrm{Im}F_\mu(k,q) = \sum_\lambda (2\pi)\delta(k^2-m_\rho^2)\underbrace{\langle 0|\bar d\gamma_\mu u|\rho_\lambda(k)\rangle}_{m_\rho f_\rho\,\varepsilon(\lambda)_\mu}\underbrace{\langle\rho_\lambda(k)|\bar u i m_b\gamma_5 b|\bar B^0(q+k)\rangle}_{(\varepsilon(\lambda)^\star\cdot q)\ A_0^{B\rho}(q^2)} + \cdots$$

$$= q_\mu\ 4\pi\, m_\rho f_\rho A_0^{B\rho}(q^2) + \cdots$$

- Dispersion relation + LCOPE + Borel + duality

$$2m_\rho f_\rho A_0^{B\rho}(q^2)\, e^{-m_\rho^2/M^2} = F_{OPE}(M^2,q^2)$$

$$F_{OPE}(M^2,q^2) = f_B m_B^2 m_b\left\{\int_0^{\sigma_0^{2\pi}} d\sigma\, e^{-s(\sigma,q^2)/M^2}\left[\frac{\sigma}{\bar\sigma}\phi_-^B(\sigma m_B) - \frac{\bar\Phi_\pm^B(\sigma m_B)}{\bar\sigma m_B}\right] + \Delta A_0^{BV}(q^2,\sigma_0^{2\pi},M^2)\right\}$$

- Correlation function

$$F_\mu(k,q) = i \int d^4x\, e^{ik\cdot x} \langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), \bar{u}(0) i m_b \gamma_5 b(0)\}|\bar{B}^0(q+k)\rangle$$

- Unitarity relation

$$2\mathrm{Im}F_\mu(k,q) = m_b \int d\tau_{2\pi} \underbrace{\langle 0|\bar{d}\gamma_\mu u|\pi(k_1)\pi(k_2)\rangle}_{F_\pi^\star(k^2)} \underbrace{\langle \pi(k_1)\pi(k_2)|\bar{u}\gamma_5 b|\bar{B}^0(q+k)\rangle}_{F_t(k^2,q^2,\cos\theta_\pi)} + \cdots$$

$$= q_\mu \frac{s\sqrt{q^2}[\beta_\pi(s)]^2}{4\sqrt{6}\pi\sqrt{\lambda}} F_\pi^\star(k^2)\, F_t^{(\ell=1)}(k^2,q^2) + \cdots$$

Corollary : $F_\pi^\star(s)\, F_t^{(\ell=1)}(s,q^2)$ is real for all $s < 16 m_\pi^2 \Rightarrow$

Phase($F^{B\to\pi\pi}_{P-wave}$) = Phase(vector pion form factor)

Important for CP violation!!!

[See also Kang, Kubis, Hanhart, Meissner '13]

$B \to \pi\pi$ form factors from B-meson LCDAs

Cheng, Khodjamirian, JV, 1701.01633

- Dispersion relation + LCOPE + Borel + duality

$$-\int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2}\, \frac{s\,\sqrt{q^2}\,[\beta_\pi(s)]^2}{4\sqrt{6}\pi^2\sqrt{\lambda}}\, F_\pi^\star(s)\, F_t^{(1)}(s,q^2) = F_{OPE}(M^2,q^2)$$

- ρ-dominance + zero-width limit:

$$F_\pi^\star(s) \simeq \frac{f_\rho g_{\rho\pi\pi} m_\rho/\sqrt{2}}{m_\rho^2 - s + i\sqrt{2}\Gamma_\rho(s)} \quad , \quad F_t^{(1)}(s,q^2) \simeq -\frac{\beta_\pi(s)\sqrt{\lambda}}{\sqrt{3q^2}}\, \frac{m_\rho g_{\rho\pi\pi} A_0^{B\rho}(q^2)}{m_\rho^2 - s - i\sqrt{2}\Gamma_\rho(s)}$$

$$LHS = 2f_\rho m_\rho A_0^{B\rho}(q^2) \int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2} \underbrace{\left[\frac{\sqrt{s}\,\Gamma_\rho(s)/\pi}{(m_\rho^2-s)^2 + s\Gamma_\rho^2(s)}\right]}_{\xrightarrow{\Gamma_\rho\to 0}\ \delta(s-m_\rho^2)} \xrightarrow{\Gamma_\rho\to 0} 2f_\rho m_\rho A_0^{B\rho}(q^2)\, e^{-m_\rho^2/M^2}$$

hep-ph/0611193 ✓

$B \to \pi\pi$ form factors from B-meson LCDAs

Cheng, Khodjamirian, JV, 1701.01633

- The same for other (axial-)vector form factors

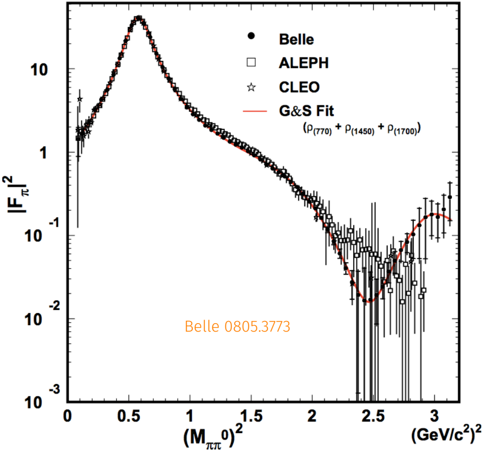
$$
\begin{aligned}
i\langle\pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = {} & F_\perp(k^2, q^2, q\cdot\bar{k})\,\frac{2}{\sqrt{k^2}\sqrt{\lambda}}\, i\epsilon_{\nu\alpha\beta\gamma}\, q^\alpha\, k^\beta\, \bar{k}^\gamma \\
& + F_t(k^2, q^2, q\cdot\bar{k})\,\frac{q_\nu}{\sqrt{q^2}} + F_0(k^2, q^2, q\cdot\bar{k})\,\frac{2\sqrt{q^2}}{\sqrt{\lambda}}\left(k_\nu - \frac{k\cdot q}{q^2} q_\nu\right) \\
& + F_\parallel(k^2, q^2, q\cdot\bar{k})\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda}\, k_\nu + \frac{4k^2(q\cdot\bar{k})}{\lambda}\, q_\nu\right)
\end{aligned}
$$

Similar sum rules and good narrow-width limit

$B \to \pi\pi$ form factors from B-meson LCDAs

Cheng, Khodjamirian, JV, 1701.01633

- Main input to the sum rule: Vector pion form factor $F_\pi(s)$



- Other inputs : f_B, λ_B, M^2, s_0^{th}.

Probing resonance models for $B \to \pi\pi$ form factors

- Sum-rules contains weighted integral of form factors
  $\Rightarrow$ useful to constrain models

E.g. Three-resonance model :

$$F_t^{(\ell=1)}(s,q^2) = -\frac{\beta_\pi(s)\sqrt{\lambda}}{\sqrt{3}\sqrt{q^2}} \sum_{\rho,\rho',\rho''} \frac{m_R\, g_{R\pi\pi}\, A_0^{BR}(q^2)\, e^{i\phi_R(s,q^2)}}{[m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)]}$$

and similar for $F_\perp$, $F_\parallel$, F_0.

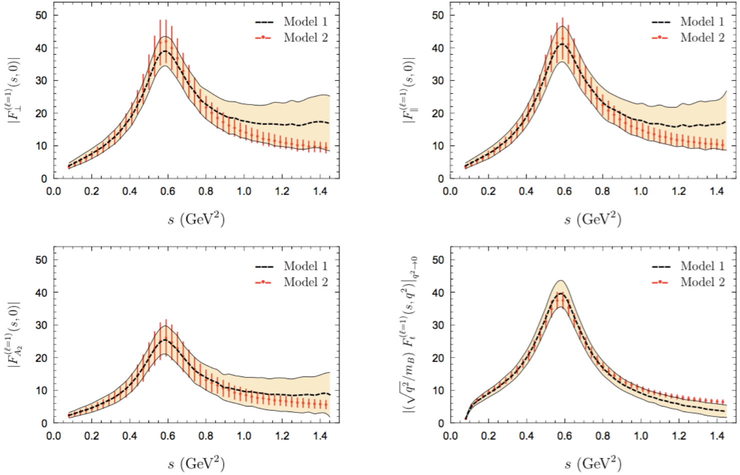
- One-resonance : Finite-width effects in $B \to \rho$ form factors

	$V^{B\rho}(0)$	$A_1^{B\rho}(0)$	$A_2^{B\rho}(0)$	$A_0^{B\rho}(0)$
Inputs of KMO'06	0.31	0.23	0.19	0.26
Updated inputs	0.34	0.26	0.21	0.30
Gaussian scan	0.36 ± 0.17	0.27 ± 0.13	0.22 ± 0.15	0.30 ± 0.06
BSZ'15 (ρ-DAs)	0.33 ± 0.03	0.26 ± 0.03	0.23 ± 0.04	0.36 ± 0.04
Full F_π, $M^2 = 1\,\mathrm{GeV}^2$	0.40 ± 0.19	0.30 ± 0.14	0.24 ± 0.16	0.33 ± 0.07
Final results for ρ-model	0.41 ± 0.11	0.31 ± 0.08	0.25 ± 0.10	0.34 ± 0.04

$\Rightarrow$ Finite-width effects at the level of $\sim 10\%$

 Cheng, Khodjamirian, JV, 1701.01633

- Other models with two or three resonances :



The suppression of F_π outside the ρ hinders sensitivity to ρ', ρ''.

Generalized Distribution Amplitudes

Polyakov'98, and others

- Definition: $[k_{12} = k_1 + k_2 \,;\, s = k_{12}^2 \,;\, k_1 = \zeta k_{12} \,;\, k_2 = (1-\zeta) k_{12}]$

$$\Phi_{\parallel}(u,\zeta,s) = \int \frac{dx^-}{2\pi} e^{iu(k_{12}^+ x^-)} \langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x^- n_-)\not{n}_+ d(0)|0\rangle$$

- Normalization (local correlator):

$$\int du\, \Phi_{\parallel}(u,\zeta,s) = (2\zeta - 1)\, F_\pi(s) \quad \text{(pion vector FF)}$$

- Double Gegenbauer + Partial Wave Expansion:

$$\Phi_{\parallel}^{I=1}(u,\zeta,k^2) = 6u\bar{u} \sum_{n=0,2,\cdots}^{\infty} \sum_{\ell=1,3,\cdots}^{n+1} B_{n\ell}^{\parallel}(k^2)\, C_n^{3/2}(u-\bar{u})\, \beta_\pi(k^2) P_\ell^{(0)}(\cos\theta_\pi)$$

where $B_{01}^{\parallel}(k^2) = F_\pi(k^2)$

$B \to \pi\pi$ form factors from 2π-LCDAs

Cheng, Khodjamirian, JV, 1709.00173

- Correlation function

$$\Pi^5(p^2, k^2, q^2, q\cdot\bar{k}) = i\int d^4x\, e^{iq\cdot x}\langle\pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{\bar{u}(x)im_b\gamma_5 b(x), \bar{b}(0)im_b\gamma_5 d(0)\}|0\rangle$$

- Unitarity relation

$$2\mathrm{Im}\Pi^5 = (2\pi)\delta(p^2 - m_B^2)\underbrace{\langle\pi^+(k_1)\pi^0(k_2)|\bar{u}im_b\gamma_5 b|\bar{B}(p)\rangle}_{\sqrt{q^2}F_t(q^2,k^2,q\cdot k)}\underbrace{\langle\bar{B}(p)|\bar{b}im_b\gamma_5 d|0\rangle}_{m_B^2 f_B} + \cdots$$

$$= (2\pi)\,\delta(p^2 - m_B^2)\, m_B^2 f_B\,\sqrt{q^2}\,F_t(q^2, k^2, q\cdot k) + \cdots$$

- Dispersion relation + LCOPE + Borel + duality

$$m_B^2 f_B\,\sqrt{q^2}\,F_t(q^2, k^2, q\cdot\bar{k})\,e^{-m_B^2/M^2} = \Pi^5_{OPE}(M^2, q^2, k^2, q\cdot\bar{k})$$

$B \to \pi\pi$ form factors from 2π-LCDAs

Cheng, Khodjamirian, JV, 1709.00173

- In this case:

$$\Pi^5_{OPE}(M^2, q^2, k^2, q\cdot\bar{k}) = \frac{m_b^2}{\sqrt{2}} \int_{u_0}^1 \frac{du}{u^2}\, e^{-s(u)/M^2} \left(m_b^2 - q^2 + u^2 k^2\right) \Phi_\parallel^{I=1}(u, q\cdot\bar{k}, k^2)$$

- **SUM RULE :**

$$\sqrt{q^2} F_t(q^2, k^2, \zeta) = \frac{m_b^2}{\sqrt{2} m_B^2 f_B} \int_{u_0}^1 \frac{du}{u^2}\, e^{\frac{m_B^2 - s(u)}{M^2}} \left(m_b^2 - q^2 + u^2 k^2\right) \Phi_\parallel^{I=1}(u, \zeta, k^2)$$

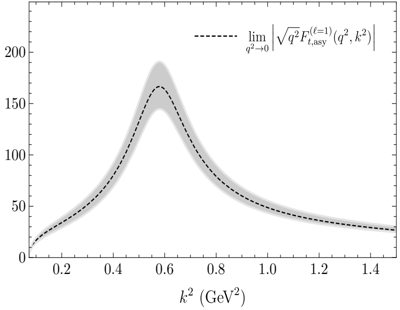
- **Gegenbauer + Partial Wave Expansions :**

$$\sqrt{q^2} F_t^{(\ell)}(q^2, k^2) = -\frac{6 m_b^2}{\sqrt{2} f_B m_B^2} \frac{\beta_\pi(k^2)}{\sqrt{2\ell+1}} \sum_{\substack{n=\ell-1 \\ n\ \text{even}}}^{\infty} B_{n\ell}^{\parallel}(k^2) \int_{u_0}^1 \frac{du}{u}\, \bar{u}\, e^{\frac{m_B^2 - s(u)}{M^2}} \left(m_b^2 - q^2 + u^2 k^2\right) C_n^{3/2}(u - \bar{u})$$

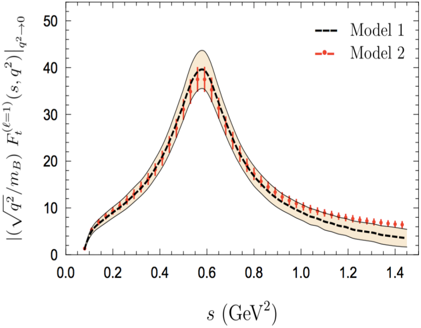
- $B_{01}^{\parallel}(k^2) = F_\pi(k^2)$ -- but for the sum rule we need higher moments.
- $B_{n1}^{\parallel}(k^2)$ for $n > 0$ not known

$B \to \pi\pi$ form factor ($F_t^{\ell=1}$)

Cheng, Khodjamirian, JV, 1709.00173



Cheng, Khodjamirian, JV, 1701.01633



- Both approaches give consistent results
- These results complement Hambrock, Khodjamirian 2015 for $F_\perp, F_\parallel$

$B \to K\pi$ form factors

- Extension to $B \to K\pi$ -- Relevant for $B \to K^* \ell\ell$ Deskotes-Genon, Khodjamirian, JV, w.i.p.

$$\int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2} \frac{\sqrt{q^2}\,\lambda_K}{4\sqrt{6}\pi^2\, s\,\sqrt{\lambda}} F_{K\pi}^\star(s)\, F_t^{(\ell=1)}(s,q^2) = -f_B m_B^2 m_b\, A_{0\,\mathrm{OPE}}^{BV}(q^2,\sigma_0^{2\pi},M^2)$$

Recovers $\pi\pi$ case when $\lambda_K \to s^2[\beta_\pi(s)]^2$.

- More complicated for F_0 :

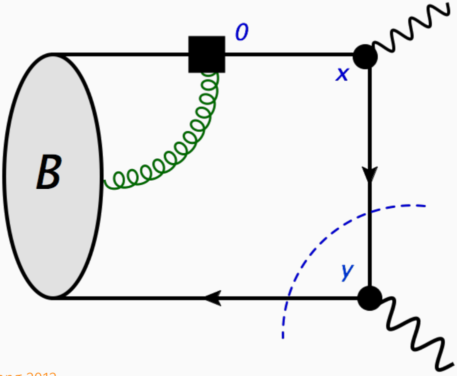
$$\int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2} \frac{\lambda_K^{3/2}}{2\sqrt{6}\pi^2 s^{5/2}\lambda} F_{K\pi}^\star(s) \left[\frac{(m_B^2 - q^2 - s)}{2} F_\parallel^{(\ell=1)}(s,q^2) + \frac{s^{3/2}\sqrt{q^2}}{\lambda_k^{1/2}} F_0^{(\ell=1)}(s,q^2) \right.$$

$$\left. + \frac{\sqrt{3}(m_K^2 - m_\pi^2) q^2 s}{\sqrt{\lambda\lambda_k}} \sum_{\ell=1}^{\infty} \mathcal{I}_{0\ell}\, F_\parallel^{(\ell)}(s,q^2) \right] = f_B m_B\, A_{2,\,\mathrm{OPE}}^{BV}(q^2,\sigma_0^{2\pi},M^2)$$

Mixes partial waves.

Non-local effects in $B \to K^*(\to K\pi)\ell\ell$

$$\mathcal{H}^\mu(q,k) \equiv i \int \mathrm{d}^4x\, e^{iq\cdot x} \langle \bar{K}(k_1)\pi(k_2)|T\{\mathcal{J}^\mu_{\text{em}}(x), \mathcal{C}_i\mathcal{O}_i(0)\}|\bar{B}(k+q)\rangle$$



Khodjamirian, Mannel, Wang 2012

Can do the same generalization in non-local MEs as in form factors!

Summary

- Describe B decays to unstable particles in terms of their underlying multi-body decays
- LCSRs provide a means to perform this generalization
- E.g. we can estimate finite-width effects of $\mathcal{O}(10\%)$ in $B \to \rho$ form factors
- Other applications underway. Good potential.