Topological Origin of Dynamical Breaking of Chiral Symmetry in QCD and in Gravity.

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Previous work:

Dvali, hep-th/0507215
05100053 PRD 74

Dvali, Jackiw, Pi, PRL 96, 2006
hep-th/0511175

Dvali, Folkerts, Franca, hep-th 1312.7273
PRD 89, 2014

Dvali, Funcke, hep-ph/1602.03191
1608.08969
PRD 93, 2016

Topological susceptibility of vacuum (TSV) in QCD (pure glue):

$$F\tilde{F} \equiv {}^*dC \equiv E$$

$$C \equiv AdA + \frac{2}{3}AAA$$

under QCD gauge transformation:

$$C \to C + d\Omega$$

Ω ← 2-form

$$\langle E\,E\rangle_{p\to 0} \equiv$$

$$\equiv \lim_{p\to 0} \int d^4x\, e^{ipx} \langle T[F\tilde{F}(x),\, F\tilde{F}(0)]\rangle \neq 0$$

In gravity:

$$E_g \equiv R\tilde{R} = dC_g$$

$$C_g \equiv \Gamma d\Gamma + \frac{2}{3}\Gamma\Gamma\Gamma$$

TSV in fermion-free gravity

$$\langle E_g, E_g \rangle_{p\to 0} \equiv \langle R\tilde{R}, R\tilde{R} \rangle_{p\to 0} = ?$$

Role of TSV in QCD

(Witten; Veneziano; Luscher; Aurilia, Takahashi, Townsend; others ...)

* What is the connection with chiral symmetry breaking?

* Can be generalized to gravity?

We would like to show that if in massless-fermion-free version of theory (QCD or Gravity) $TSV \neq 0$, i.e., in QCD

$$\langle F\tilde{F}, F\tilde{F} \rangle_{p \to 0} = const \neq 0,$$

or in Gravity

$$\langle R\tilde{R}, R\tilde{R} \rangle_{p \to 0} = const \neq 0,$$

then when massless fermions are added, the following takes place:

① The chiral flavor symmetry is broken dynamically down to anomaly free subgroup;

② Pseudo-Goldstone of broken $U(1)_A$ is becoming massive;

③ Both, in QCD and in gravity massless fermions (elementary or composite) are eliminated from the spectrum.

Implications:

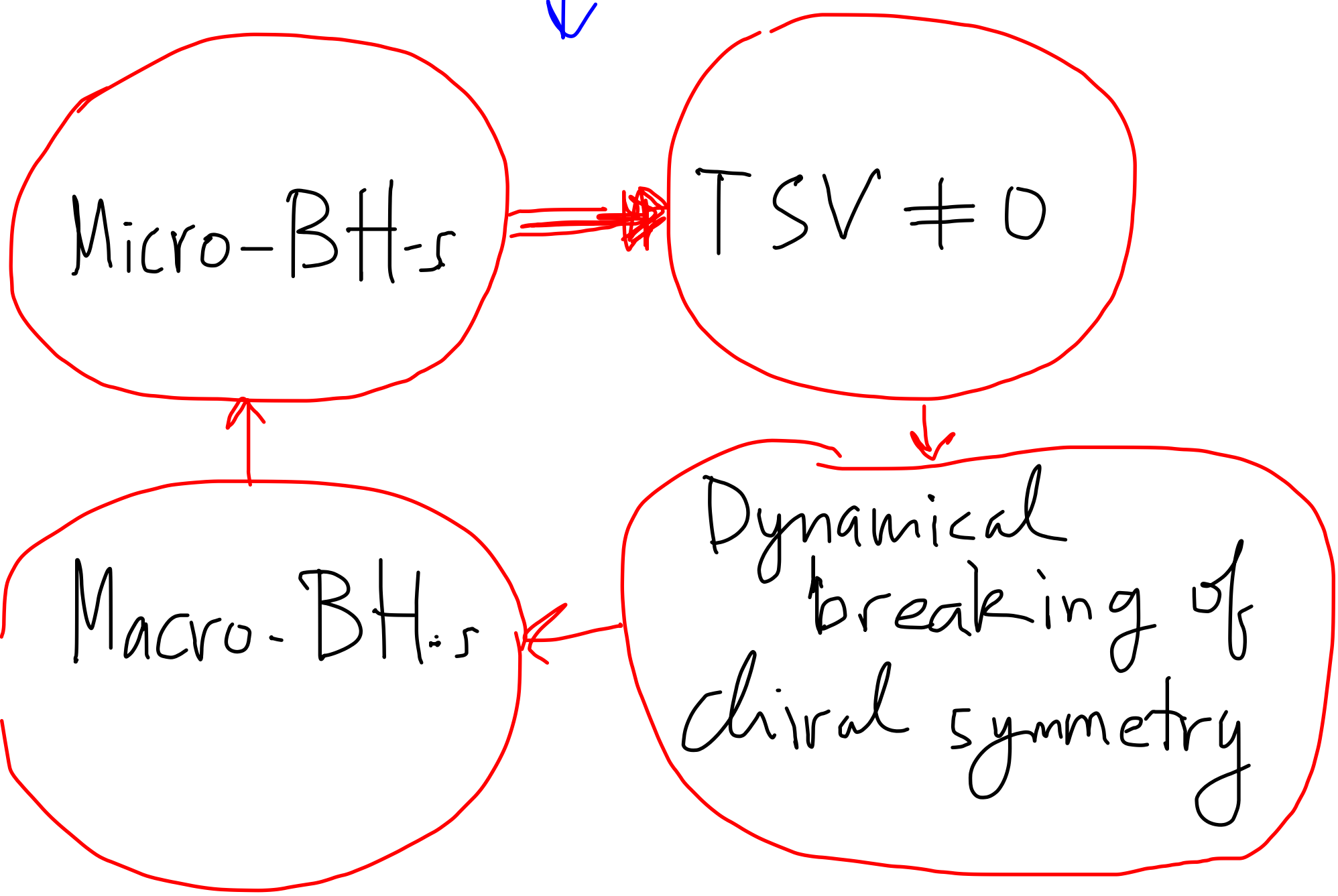
① Role of confinement in chiral symmetry breaking. Is topology central?

② Fundamental resolution of incompatibility of global symmetries with black holes:

Dynamical breaking of chiral symmetry via contribution of micro-BH-s in TSV!

$$\langle R\tilde{R},\ R\tilde{R}\rangle =$$

$$= \sum_{BH} \langle R\tilde{R} \mid BH\rangle\langle BH \mid R\tilde{R}\rangle \neq 0$$



③ Origin of neutrino masses from neutrino condensate.

$$\langle \bar{\nu}\nu \rangle \neq 0 \longrightarrow M_\nu \neq 0.$$

Induced by gravity.
(Lena Funcke & G.D)

④ Protection of axion from gravity by neutrinos
(Sarah Folkerts, Andre Franca, & G.D.)

TSV≠0 ⟷ Physical Θ

Θ F F̃

Θ_g R R̃

TSV≠0 ⟶ ~~CP~~

Alice and Bob must agree on breaking of CP-symmetry

$$\langle E, E \rangle_{p\to 0} \equiv \langle F\tilde{F}, F\tilde{F} \rangle_{p\to 0} = \omega_M \neq 0$$

For Alice this means that there exist θ-vacua

With measurable CP-violation.

Bob's view:

$$\langle E, E \rangle_{p \to 0} = \text{const} \neq 0$$

$$E \equiv dC$$

$$\langle C, C \rangle_{p \to 0} = \frac{1}{p^2}$$

$C =$ massless 3-form $+$ massive glueballs, ...

C

$$\mathcal{L}_{Bob} = \frac{1}{2} E^2 + \cdots$$

$$\partial_\mu E = 0 \longrightarrow E = \theta$$ ← integration constant

Thus,

$$(\theta\text{-vacua})_{\text{Alice}} = (E\text{-vacua})_{\text{Bob}}$$

$$\Delta\theta_{\text{Alice}} = \Delta E_{\text{Bob}}$$

CP-odd "electric" field E.

Now let us introduce

N_f massless quark flavors

(all left-handed) $\psi_j, \psi_{\dot{j}}$

$\dot{j} = 1 \ldots N_f$.

$$\langle F\tilde{F}, F\tilde{F} \rangle_{p \to 0} = 0 .$$

Alice and Bob agree

that CP is conserved,

But interpret differently.

Alice: $U(1)_A$ - transformation

$$\psi \to e^{i\alpha}\psi$$

$$\downarrow$$

$$\delta\mathcal{L}_{Axion} = \alpha F\tilde{F}$$

$$\downarrow$$

$$\theta \to \theta + \alpha$$

$$\downarrow$$

θ - unphysical!

Bob: Higgs effect for 3-form!

$$\langle E, E \rangle_{p\to 0} = 0 \rightarrow \langle C, C \rangle_{p\to 0} = \frac{1}{p^2+m^2}$$

C became massive. Needs a pseudoscalar!

$$\mathcal{L}_{Bob} = \frac{1}{2}E^2 - \frac{\eta'}{f_\eta}E + \frac{1}{2}(\partial\eta')^2$$

$$\partial_\mu\left(E - \frac{\eta'}{f_\eta}\right) = 0 \rightarrow E = \frac{\eta'}{f_\eta} - \theta$$

$$\Box\eta' - \frac{1}{f_\eta}E = 0$$

$$\Box\eta' + \frac{1}{f_\eta}\left(\frac{\eta'}{f_\eta} - \theta\right) = 0$$

Thus,

$$\Box\eta' + \frac{1}{f_\eta}\left(\frac{\eta'}{f_\eta} - \theta\right) = 0$$

$$E = \frac{\eta'}{f_\eta} - \theta$$

$$m_{\eta'} = \frac{1}{f_\eta^2}$$

under $U(1)_A$: $\psi \to e^{i\alpha}\psi$

$\theta \to \theta + \alpha$, $\frac{\eta'}{f_\eta} \to \frac{\eta'}{f_\eta} + \alpha$

$$\frac{\eta'}{f_{\eta'}} \equiv \frac{1}{2N_f}\arg\left(\det(\text{44}')\right)$$

Thus, $\langle \det(\psi\psi) \rangle \neq 0$

and η' is a (pseudo)-

Goldstone of $U(1)_A$.

But, it is also a

Stückelberg field

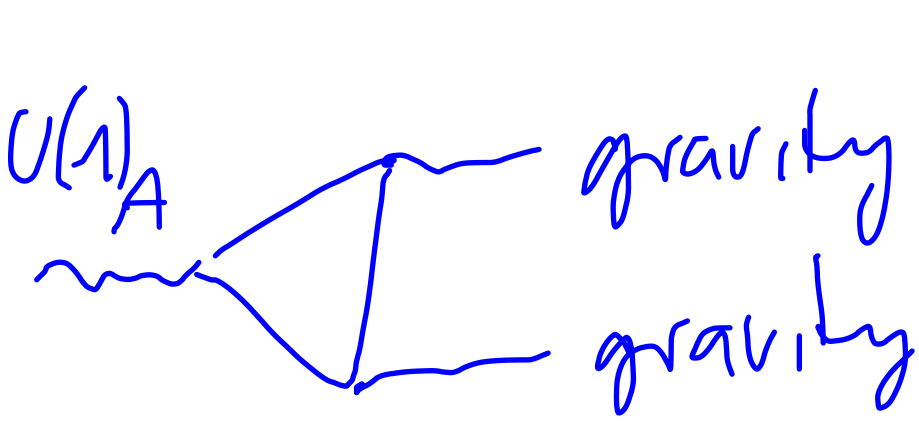
for 3-form!

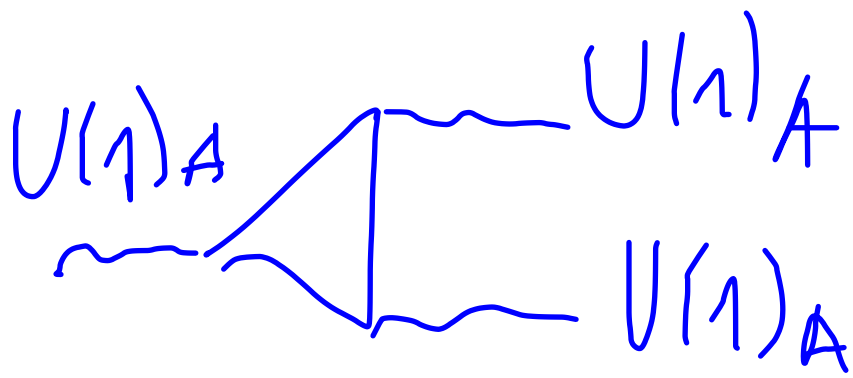
Next step: Gauge $U(1)_A$ and use gravity as spectator.

$$\psi \to e^{i\alpha}\psi, \quad \chi_\mu \to \chi_\mu + \frac{1}{g_\chi}\partial_\mu \alpha(x)$$

$$\frac{\eta'}{f_\eta} \to \frac{\eta'}{f_\eta} + \alpha$$

GS-axion $\quad \frac{a}{f_a} \to \frac{a}{f_a} - \alpha$





$$\frac{a}{f_a}\left(R\tilde{R} + F_\chi \tilde{F}_\chi + F\tilde{F}\right)$$

Anomaly matching:

Alice: $U(1)_A \quad \psi \to e^{i\alpha}\psi$

$$\frac{a}{f_a} \to \frac{a}{f_a} - \alpha$$

$$\mathcal{L}_{Alice} = \frac{a}{f_a}\left(R\tilde{R} + F_x\tilde{F}_x + F\tilde{F}\right)$$

Bob: $U(1)_A \quad \frac{\eta'}{f_\eta} \to \frac{\eta'}{f_\eta} + \alpha$

$$\frac{a}{f_a} \to \frac{a}{f_a} - \alpha$$

$$\mathcal{L}_{Bob} = \left(\frac{\eta'}{f_\eta} + \frac{a}{f_a}\right)\left(R\tilde{R} + F_x\tilde{F}_x + F\tilde{F}\right)$$

No room for massless fermions!

Because χ' is Stückelberg, it takes care of anomaly cancellation.

Thus, in theory of Bob there is no room for mirror fermions due to 't Hooft's anomaly matching!

Absence of massless fermions

$U(N_f)_L \otimes U(N_f)_R$ flavor symmetry is dynamically broken down to an anomaly-free subgroup.

follows from 't Hooft anomaly matching

The same conclusion applies to gravity: Flavor group $U(N)$ must be broken to an anomaly-free subgroup.

E.g.

$$U(N) \longrightarrow SO(N)$$

$\downarrow$

Universal fermion condensate in gravity?