Effective field theory and triviality of quantum electrodynamics

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Outline

- Introduction to the problem of triviality;
- A simple demonstration of the problem;
- Running coupling;
- Cutoff-dependent bare coupling;
- Summary;

Introduction

The concept of triviality in QFT originates from papers by Landau and collaborators studying the asymptotic behavior of the photon propagator in QED

L. D. Landau, A. A. Abrikosov and J. M. Khalatnikov, Dokl. Akad. Nauk SSSR 95(1954) 773; 95(1954)1177; 96 (1954) 261.
L. D. Landau, in: Niels Bohr and the Development of Physics, ed. W. Pauli (Pergamon, London, 1955); and works cited in there.

Resumming the leading logarithms they found that the photon propagator has a pole at large momentum transfer. If this pole persists in non-perturbative calculations then to avoid the apparent inconsistency QED has to be a non-interacting, i.e. *trivial*, theory.

In calculations applying a finite cutoff this problem manifests itself as a singularity in the bare coupling for a finite value of the cutoff. It is therefore impossible to remove the cutoff unless the renormalized coupling vanishes. For a review see e.g.

D. J. E. Callaway, Phys. Rept. **167**, 241 (1988).

Triviality of QED has been investigated on the lattice in

M. Gockeler, R. Horsley, V. Linke, P. E. L. Rakow, G. Schierholz and H. Stuben, Phys. Rev. Lett. **80**, 4119 (1998).

It was found that while the Landau pole lies beyond the accessible region of the parameter space due to spontaneous chiral symmetry breaking, spinor QED (with four flavors) does not exist as an interacting theory.

The standard model, in a modern point of view, is a LO approximation to an EFT

S. Weinberg, “The Quantum Theory Of Fields. Vol. 1,2.*Cambridge, UK: Univ. Pr. (1995)*.

While the effective Lagrangian contains an infinite number of local interactions, at low energies the contributions in physical quantities of the non-renormalizable interactions (in the traditional sense) are suppressed by powers of the energy divided by a large scale.

In EFT the solution of the LO Wilson renormalization group equations might be obstructed at very large cutoffs, however, this should not be a severe problem because of irrelevant interactions or omitted fields being important at short distances. Therefore inconsistencies in the renormalization group analysis of renormalizable quantum field theories, like QED or ϕ^4 theory of self-interacting scalars, might be absent in the corresponding EFT.

We address the consequences of treating QED as a leading order approximation of an EFT for the problem of *triviality*.

To that end we analyze the contributions of the next-to-leading order interaction, i.e. dimension five operator, the well-known Pauli term.

A simple demonstration of the problem

Consider the LS equation for the S-wave scattering amplitude

$$T\left(p,p',q\right) = V\left(p,p'\right) + m\int_0^\infty \frac{dk\,k^2}{(2\,\pi)^3}\,\frac{V\left(p,k\right)\,T\left(k,p',q\right)}{q^2-k^2+i\,0^+}$$

with the potential

$$V_{NLO} = c + c_2\left(p^2+p'^2\right).$$

The corresponding on-shell amplitude reads:

$$T_{NLO}(q) = \frac{c_2\left[c_2\left(I_3 q^2 - I_5\right) - 2q^2\right] - c}{I\left(q^2\right)\left[c_2\left(c_2\left(I_5 - I_3 q^2\right) + 2q^2\right) + c\right] - \left(I_3 c_2 - 1\right)^2},$$

where using the cutoff regularization loop integrals are given by

$$I_n = -m\int \frac{d^3\vec{k}}{(2\,\pi)^3}\,k^{n-3}\,\theta(\Lambda - k) = -\frac{m\,\Lambda^n}{2\,n\,\pi^2},$$

$$I(p^2) = \int \frac{d^3\vec{k}}{(2\,\pi)^3}\,\frac{m\theta(\Lambda-k)}{p^2-k^2+i\,0^+} = -\frac{i\,p\,m}{4\pi} - \frac{m\,\Lambda}{2\,\pi^2} + O\left(\frac{1}{\Lambda}\right).$$

Couplings c and c_2 can be fixed by demanding that the scattering length and the effective range are reproduced.

This leads to the amplitude

$$T_\Lambda(q) = -\frac{4i\pi a\left[4a\Lambda + \pi\left(aq^2 r_e + 2\right)\right]}{m\left[\pi\left(a^2q^3 r_e + 2aq - 2i\right) + 2a\Lambda(aq(2 + iq\, r_e) - 2i)\right]}.$$

This expression is finite in $\Lambda \to \infty$ limit:

$$T(q) = -\frac{4\pi/m}{-\frac{1}{a} + \frac{q^2 r_e}{2} - iq}.$$

However ...

$T_\Lambda(q)$ is restricted by Wigner bound - cutoff cannot be taken very large unless $r_e \leq 0$ (otherwise c and c_2 become complex).

S. R. Beane, T. D. Cohen and D. R. Phillips, Nucl. Phys. A **632**, 445 (1998)

$$\begin{aligned}
c(\Lambda) &= \frac{6\pi^2}{5\Lambda^2 m^2 \left(-16a^2\Lambda^2 + \pi a\Lambda \left(a\Lambda^2 r_e + 12\right) - 3\pi^2\right)} \\
&\times \Bigg[\Lambda m \left(-64a^2\Lambda^2 + \pi a\Lambda \left(3a\Lambda^2 r_e + 62\right) - 18\pi^2\right) \\
&+ 6\sqrt{3}\sqrt{-4\pi a^4\Lambda^7 m^2 r_e + 4a^3\Lambda^6 m^2 \left(16a + \pi^2 r_e\right) + \cdots}\Bigg], \\
c_2(\Lambda) &= \cdots .
\end{aligned}$$

Running coupling

We start with the most general U(1) locally gauge invariant effective Lagrangian of the electron field ψ interacting with the e.m. field A_μ

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}\,(i\not{D} - m)\,\psi + \frac{i\kappa}{2}\,\bar{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\psi\, F_{\mu\nu} + \mathcal{L}_{\rm ho}\,, \quad (1)$$

where m is the electron mass, e is the e.m. charge, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $D_\mu = \partial_\mu - ieA_\mu$ and $\mathcal{L}_{\rm ho}$ contains an infinite number of terms with operators of dimension six and higher.
We assume that the contributions of these terms in the photon self-energy are suppressed compared to those of the Pauli term.

The standard QED describes the experimental data very well because the contributions of higher order operators are beyond the current accuracy of the data.
E.g. the calculated value of the anomalous magnetic moment of the electron in QED agrees with the experiment very well suggesting that the Pauli term is suppressed by a scale larger than 4×10^7 GeV.

Running coupling $e_R(q^2)$ can be defined by the following relation

$$D^{\mu\nu}(q)\, e^2 = -\frac{1}{q^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) e_R^2(q^2), \tag{2}$$

where $D^{\mu\nu}(q)$ is the dressed propagator of the bare photon field.

At one-loop order we obtain for $-q^2 \gg m^2$:

$$e_R^2(q^2) = e_r^2 \left[1 - \frac{e_r^2 + 2\kappa^2 q^2}{12\pi^2} \ln \frac{-q^2}{m^2} + c_R\, q^2\right]^{-1} . \tag{3}$$

e_r is the renormalized coupling at $q^2 = -m^2$ for $c_R = 0$, where c_R is a coupling of $\mathcal{L}_{\rm ho}$, suppressed by two orders of some large scale.

For $\kappa = c_R = 0$ the running coupling has the well-known pole singularity at (the Landau pole)

$$q_L^2 = -m^2 \exp\left[12\pi^2/e_r^2\right] . \tag{4}$$

While this pole appears at extremely high energies, it is still a problem if present in the full theory.

For reasonable values of $\kappa \gg 1/\sqrt{-q_L^2}$ the Landau pole is absent remedying the inconsistency at the level of an EFT.

In renormalizable theories only logarithmic divergences contribute to the renormalization of coupling constants and therefore there is a direct correspondence between the Gell-Mann-Low and the Wilsonian RG approaches.

As a result, the presence of the Landau pole in the expression of the running coupling authomatically leads to the pole in the bare coupling as a function of the cutoff parameter.

However, in an EFT with non-renormalizable interactions the direct link between the two RG equations is lost and therefore the Wilsonian RG approach requires additional study of the cutoff dependence.

Cutoff-dependent bare coupling

We investigate the cutoff dependence of the bare electromagnetic coupling by applying the higher derivative regularization which preserves the local U(1) gauge invariance.

L. D. Faddeev and A. A. Slavnov, "Gauge Fields. Introduction To Quantum Theory," Front. Phys. **50**, 1 (1980) [Front. Phys. , 1 (1991)].

Dimensional regularization is not suitable here as it discards the power-law divergences.

In addition to the fields in conventional QED, we introduce scalar ghost fields $\bar{\xi}$ and ξ which regulate the one-loop counter term diagrams contributing to the photon self-energy at two-loop order.

The effective Lagrangian generating one and two-loop diagrams, contributing to our calculation of the photon self-energy up to two-loop $e^2\kappa^2$ order, which are all finite for finite Λ, is given by:

$$\begin{aligned}\mathcal{L}_{\mathrm{HDR}} = & -\frac{1}{4} F^{\mu\nu} \left(1 + \frac{\partial^2}{\Lambda^2}\right)^2 F_{\mu\nu} \\ & + \frac{1}{2} \bar{\psi} (i\not{D} - m) \left(1 + \frac{D^2}{\Lambda^2}\right)^3 \psi + \mathrm{h.c.} \\ & + \frac{1}{2} \bar{\xi} (i\not{D} - m) \left(1 + \frac{D^2}{\Lambda^2}\right)^3 \xi + \mathrm{h.c.} \\ & + \frac{i\kappa}{2} \bar{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\psi F_{\mu\nu} + \mathcal{L}_{\mathrm{ho}}. \end{aligned} \tag{5}$$

The bare electromagnetic coupling as a function of the cutoff satisfies a renormalization group equation which up to the level of accuracy of our calculation has the form:

$$\frac{d\alpha(\Lambda)}{d\ln\Lambda} = A_1\,\alpha^2(\Lambda) + \kappa^2\alpha^2(\Lambda)\,\Lambda^2(2A_2 + A_3 + 2A_3\ln\Lambda/m), \qquad (6)$$

where $\alpha(\Lambda) = e^2/(4\pi)$, the coefficient A_1 is given by one-loop diagrams and A_2 and A_3 are extracted from two-loop calculations.

Notice that there are no power-law divergences at one-loop order and all terms suppressed by powers of m/Λ have been dropped in our calculations as they are negligible for large values of Λ.

The solution to Eq. (6) is given by

$$\alpha(\Lambda) = \alpha_0\left[1 - \alpha_0 \ln\frac{\Lambda}{m}\left(A_1 + A_3\kappa^2\Lambda^2\right) + \alpha_0 \ln\frac{\Lambda_0}{m}\left(A_1 + A_3\kappa^2\Lambda_0^2\right) - \alpha_0 A_2\kappa^2(\Lambda^2 - \Lambda_0^2)\right]^{-1}, \tag{7}$$

where $\alpha_0 = \alpha(\Lambda_0)$ is the bare coupling at some fixed cutoff $m < \Lambda_0 < \Lambda$.

For $\Lambda \gg \Lambda_0$ we have

$$\alpha(\Lambda) = \alpha_0\left[1 - \alpha_0 \ln\frac{\Lambda}{m}\left(A_1 + A_3\kappa^2\Lambda^2\right) - \alpha_0 A_2\kappa^2\Lambda^2\right]^{-1}. \tag{8}$$

For $\kappa = 0$ and A_1 positive $\alpha(\Lambda)$ has a pole at

$$\Lambda_{\mathrm{P}} = m \exp\left[\frac{1}{A_1\alpha_0}\right]. \tag{9}$$

This pole, if remaining in the full expressions of the bare coupling, prevents the $\Lambda \to \infty$ limit unless $\alpha_0 \equiv 0$, thus leaving us with a non-interacting theory.

Using FeynCalc

R. Mertig, M. Bohm and A. Denner, Comput. Phys. Commun. **64**, 345 (1991).
V. Shtabovenko, R. Mertig and F. Orellana, Comput. Phys. Commun. **207**, 432 (2016).

and Form

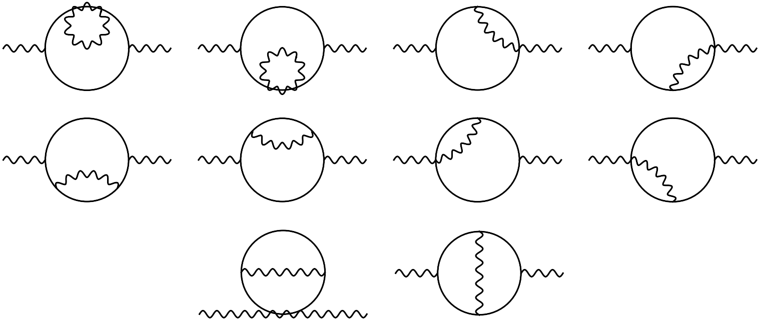
J. A. M. Vermaseren, math-ph/0010025.

and applying the method of dimensional counting of

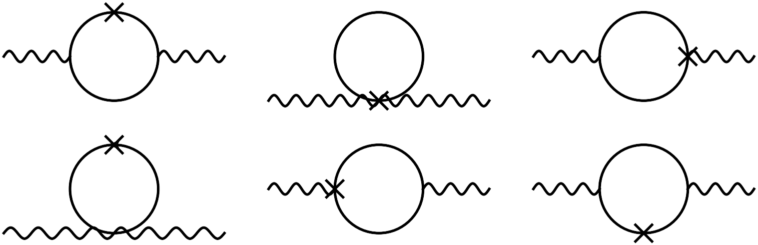
J. Gegelia, G. S. Japaridze and K. S. Turashvili, Theor. Math. Phys. **101**, 1313 (1994) [Teor. Mat. Fiz. **101**, 225 (1994)].

we have calculated the logarithmically divergent contributions to the photon self-energy generated by one-loop diagrams, and the quadratic divergences generated by the two-loop diagrams and by the corresponding counter term diagrams, shown below

Two-loop diagrams contributing in the photon self-energy



One loop counter term diagrams contributing in the photon self-energy at two-loop order



Our results read:

$$A_1 = \frac{2}{3\pi} \simeq 0.212,$$
$$A_2 = -\frac{5491889}{201600\pi^3} + \frac{1181}{1296\pi} + \frac{319\,\psi^{(1)}\left(\frac{1}{6}\right)}{648\pi^3} \simeq 0.0040\,,$$
$$A_3 = -\frac{7}{40\,\pi^3} \simeq -0.0056, \tag{10}$$

where $\psi^{(1)}$ is the trigamma function.

For the above values of A_1, A_2 and A_3, and for *natural* values of $\kappa \gg 1/\Lambda_{\rm P}$, the A_3 term is larger than the A_1 and A_2 terms and the negative sign of A_3 guarantees that $\alpha(\Lambda)$ has no pole.

Summary

- The problem of *triviality* in QED can be attributed to QED being a leading order approximation of an effective field theory.
- Already at NLO, i.e. adding the Pauli term to QED, the Landau pole and the pole in the bare coupling, as a function of the cutoff, disappear thus obviating the need for QED to be *trivial*.
- While the triviality of QED is not a settled issue, from the modern point of view which considers the Standard Model as an EFT, the issue of triviality can be of academic interest only as the higher order operators qualitatively change the UV behavior of QED.