Zurab Tavartkiladze (Ilia State Univ, Georgia)

The Methods of the Quantum Field Theory: Particle Physics of Standard Model and Beyond; Intersection with Cosmology,

II Semester

1) Introduction to II Semester. (3 hours)
2) Problems and Puzzles with SM. . (2 hours)
3) Supersymmetric (SUSY) Theories and Their Main Properties. (8 hours)
4) Minimal Supersymmetric Standard Model (MSSM) and Its Phenomenology. (8 hours)
5) Need for Going Beyond SM and MSSM. . (2 hours)
6) Neutrino Masses and Mixings. . (3 hours)
7) Grand Unified Theories (GUT). . (10 hours)
8) Flavor Problem. . (5 hours)
9) Various Scenarios and Models of Neutrino Masses and Mixings. Application of Flavor Symmetries. . (4 hours)
10) Constraints on New Physics. (3 hours).
11) Baryon Asymmetry. . (8 hours)

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Flavor beyond the standard model

Three Family SU(5) GUT
& Inverted Neutrino Mass Hierarchy

arXiv:1303.1211
Phys.Lett.B706 (2012) 398;

Volkswagen meeting 14, 15 March 2013, Tbilisi

Neutrino Data

global 3ν oscillation analysis

Parameter	Best fit	1σ range
$\delta m^2/10^{-5}\ \mathrm{eV}^2$ (NH or IH)	7.54	7.32 – 7.80
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25
$\Delta m^2/10^{-3}\ \mathrm{eV}^2$ (NH)	2.43	2.33 – 2.49
$\Delta m^2/10^{-3}\ \mathrm{eV}^2$ (IH)	2.42	2.31 – 2.49
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10
$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	[illegible]
δ/π (NH)	1.08	[illegible]
δ/π (IH)	1.09	[illegible]

Fogli et al.
1205.5254

Daya Bay Discovery (2012)

Schwetz et al.

parameter	best fit $\pm 1\sigma$
$\Delta m^2_{21}\ [10^{-5}\mathrm{eV}^2]$	$7.59^{+0.20}_{-0.18}$
$\Delta m^2_{31}\ [10^{-3}\mathrm{eV}^2]$	$2.50^{+0.09}_{-0.16}$ $-(2.40^{+0.08}_{-0.09})$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$
$\sin^2\theta_{23}$	$0.52^{+0.06}_{-0.07}$ 0.52 ± 0.06
$\sin^2\theta_{13}$	$0.013^{+0.007}_{-0.005}$ $0.016^{+0.008}_{-0.006}$
δ	$\left(-0.61^{+0.75}_{-0.65}\right)\pi$ $\left(-0.41^{+0.65}_{-0.70}\right)\pi$

Evidences for New Physics:

Atmospheric & Solar Neutrino 'scales'

$$\Delta m^2_{\rm atm} = 2.4 \cdot 10^{-3} {\rm eV}^2 \qquad \Delta m^2_{\rm sol} = 7.9 \cdot 10^{-5} {\rm eV}^2$$

- **Origin of these scales and mixings?**

Unexplained in SM / MSSM $\leftarrow \ \mathbf{m}_\nu \lesssim \mathbf{10}^{-4}\ \mathrm{eV}$

Without New Physics $m_\nu \sim \dfrac{M^2_{EW}}{M_{Pl}}$

- Charged fermion masses & mixings

Observed Noticeable Hierarchies:

$$\lambda_t \sim 1 \ , \qquad \lambda_u : \lambda_c : \lambda_t \sim \lambda^8 : \lambda^4 : 1$$

$$\lambda_b \sim \lambda_\tau \sim \frac{m_b}{m_t} \tan\beta \ , \qquad \lambda_d : \lambda_s : \lambda_b \sim \lambda^4 : \lambda^2$$

With λ=0.2

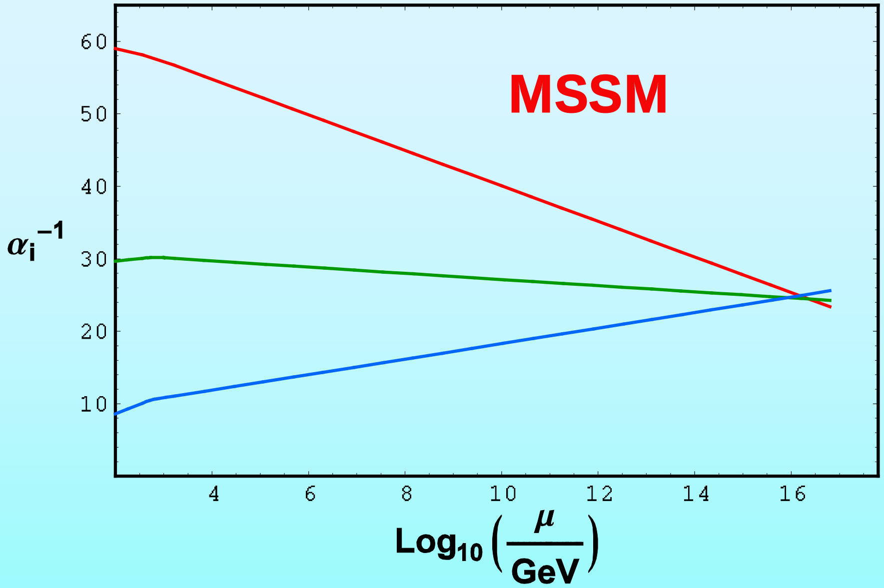
$$\lambda_e : \lambda_\mu : \lambda_\tau \sim \lambda^5 : \lambda^2 : 1$$

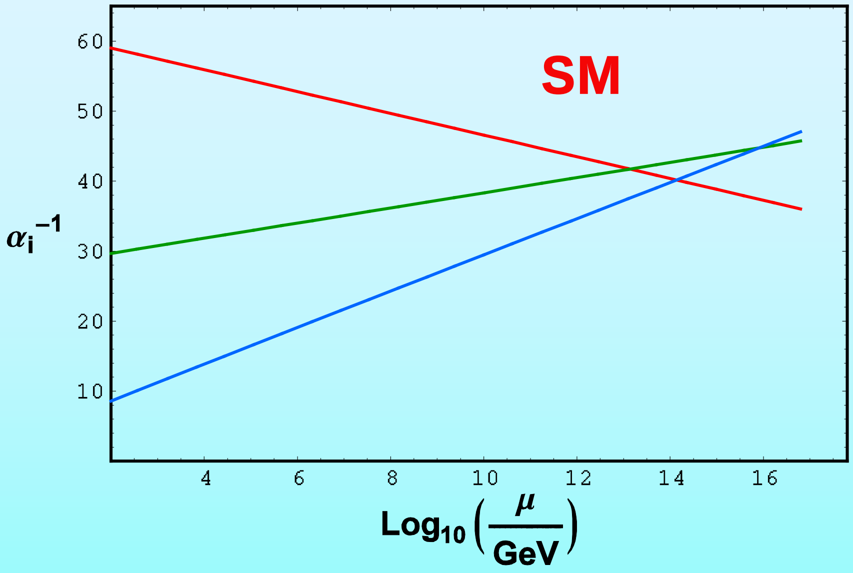
$$V_{us} \approx \lambda \ , \qquad V_{cb} \approx \lambda^2 \ , \qquad V_{ub} = \lambda^4 - \lambda^3$$

What is origin of these hierarchies?
Is there any relation or sum rule?
Why three families?

Within SM no answer to these questions...

SUSY Unification vs. non-SUSY





- In MSSM→ Good coupling unification

  → GUT is revamped!

Within SUSY & GUT, the problem of flavor remains unsolved.

Flavor symmetry G_F distinguishing families can explain hierarchies

Simplest possibility: $G_F = U(1)_F$ (Froggatt, Nielsen'79)

$$U(1)_F \ : \qquad \phi_i \to e^{iQ(\phi_i)}\phi_i$$

$$Q(F_i) = n_i \ , \qquad Q(F_i^c) = \bar{n}_i \ , \qquad Q(H) = 0 \ , \qquad Q(X) = -1$$

'flavon'

With $n_i + \bar{n}_j \neq 0 \ :$ **coupling** $\cancel{F_i F_j^c H}$ **forbidden!**

$$\left(\frac{X}{M_*}\right)^{n_i+\bar{n}_j} F_i F_j^c H \longrightarrow \epsilon^{n_i+\bar{n}_j} F_i F_j^c H$$

→Suppressed couplings emerge

$\frac{\langle X \rangle}{M_*} \equiv \epsilon \ll 1$ $\qquad M_*$ **- cut off scale (simplest possibility $M_* \sim M_{\rm Pl}$)**

Several/multiple flavons also can be considered

New SUSY SU(5) x $U(1)_{Flavor}$ Models

$U(1)_{Flavor}$: Non-Anomalous Flavor Symmetry

SU(5) Matter: 10_i , 5_i^*

$U(1)_{Flavor}$ Charge: $Q[10_i]$, $Q[5_i^*]$

SU(5) Scalars: H(5) , H*(5*) , $\Sigma(24)$

Charge: Q[H] , Q[H*] , 0

Extra matter : Only SU(5) Singlets (# = or < 3)
For anomaly cancellation & RH Neutrinos

Flavons: X[q] , X*[-q] For $U(1)_{Flavor}$ Breaking

Search for Economical Setup..

Anomaly Cancellation

$SU(5)^3$ Vanishes with:

$$10+\bar{5} \qquad + \quad \psi(R)+\bar{\psi}(\bar{R}) \quad + \quad 1$$

$H(5)+\bar{H}(\bar{5})+\Sigma(24)$ $\qquad R\in SU(5)$ $\qquad$ Singlets

Minimal Setup:

No $\psi+\bar{\psi}$ **states beyond min. SUSY SU(5)**

i.e. Three 10's + four 5* 's + one H(5) + one $\Sigma(24)$ $\qquad Q(\Sigma)=0$

$$SU(5)^2\cdot U(1)_F: \quad A_{551}=\frac{3}{2}\sum_{i=1}^{3}Q(10_i)+\frac{1}{2}\left(\sum_{k=1}^{4}Q(\bar{5}_k)+Q(5)\right)=0\ ,$$

$$(U(1)_F)^3: \quad A_{111}=10\sum_{i=1}^{3}Q(10_i)^3+5\left(\sum_{k=1}^{4}Q(\bar{5}_k)^3+Q(5)^3\right)+\sum_s Q_s^3=0\ ,$$

$$(\mathrm{Gravity})^2\cdot U(1)_F: \quad A_{GG1}=\mathrm{Tr}Q=10\sum_{i=1}^{3}Q(10_i)+5\left(\sum_{k=1}^{4}Q(\bar{5}_k)+Q(5)\right)+\sum_s Q_s=0\ ,$$

Finding : **Embedding** $SU(5) \times U(1) \in G$

a. $G = SO(N)$ $-$ Orthogonal group

b. $G -$ Exceptional group, such as $E_6, E_8 \cdots$

c. $G = SU(N)$ $-$ Unitary group

Let's start with SU(5)xU(1) ⊂ SU(N) Embedding

→ Interesting family pattern

c. * $\mathbf{SU(5)\times U(1)_F \subset SU(7)}$ Embedding

$\mathbf{SU(7)}$ – plets : $\mathbf{35} \sim \Psi_{[\mathrm{ijk}]}$, $\bar{\mathbf{7}} \sim \chi^{\mathrm{i}}$ $(i, j, k = 1, \cdots, 7)$

Anomaly free: $35 + 2 \times \bar{7}$

$$SU(7) \to SU(6) \times U(1)_7 \to SU(5) \times U(1)_6 \times U(1)_7$$

$$Y_{U(1)_6} = \frac{1}{\sqrt{60}} \mathrm{Diag}\,(1, 1, 1, 1, 1, -5)$$

$$Y_{U(1)_7} = \frac{1}{\sqrt{84}} \mathrm{Diag}\,(1, 1, 1, 1, 1, 1, -6)$$

$$35 = 20_3 + 15_{-4} = (10_{-3} + \overline{10}_3)_3 + (10_2 + 5_{-4})_{-4}$$

$$\bar{7} = \bar{6}_{-1} + 1_6 = (\bar{5}_{-1} + 1_5)_{-1} + (1_0)_6 \ ,$$

** Started with SU(7) because SU(6)⊂ E6 (already considered)*

c. $\mathbf{SU(5)\times U(1)_F \subset SU(7)}$ Embedding (cntd.)

With 'flips': $\overline{10} \to 10,\ 5 \to \bar{5}$ **all anomalies remain intact**

Three family of $(10+\bar{5})$ **(anomaly free):**

$$(10_{-3}+10_{3})_{3}+(10_{2}+\bar{5}_{-4})_{-4}+2\times[(\bar{5}_{-1}+1_{5})_{-1}+(1_{0})_{6}]$$

-- Possible to build superpositios of U(1)s: $\bar{a}Q_{U(1)_6}+\bar{b}Q_{U(1)_7}$

$$10_{-3\bar{a}+3\bar{b}}+10_{3\bar{a}+3\bar{b}}+10_{2\bar{a}-4\bar{b}}+\bar{5}_{-4\bar{a}-4\bar{b}}+2\times(\bar{5}_{-\bar{a}-\bar{b}}+1_{5\bar{a}-\bar{b}}+1'_{6\bar{b}})$$

RHN's for see-saw

Finding : **Embedding** $SU(5) \times U(1) \in G$

E6->SO(10)xU(1)''→SU(5)xU(1)'':

27= 16[1] + 10[-2]+1'[4]=
(10+ 5*+1)[1]+(5+5'*)[-2]+1'[4]

Consider superposition

$$\bar{Q}_{sup} = \bar{a}Q_{U(1)_6} + \bar{b}Q_{U(1)_7} + \bar{c}Q_{U(1)_{E_6}}$$

Model

Content:

$$10_{-3\bar{a}+3\bar{b}+p\bar{c}} + 10_{3\bar{a}+3\bar{b}-p\bar{c}} + 10_{2\bar{a}-4\bar{b}+\bar{c}} + \bar{5}_{-4\bar{a}-4\bar{b}-2\bar{c}} + \bar{5}_{-\bar{a}-\bar{b}+\bar{c}} + \bar{5}_{-\bar{a}-\bar{b}-2\bar{c}}$$

$$+1_{5\bar{a}-\bar{b}+\bar{c}} + 1_{5\bar{a}-\bar{b}+4\bar{c}} + 1'_{6\bar{b}+k\bar{c}} + 1'_{6\bar{b}-k\bar{c}}$$

$$5_q + \bar{5}_{-q}$$

$$\text{with} \quad 30\bar{a}(3+2p) = \bar{c}(2k^2 + 10p^2 - 27)$$

Selection: $\{\bar{a}, \bar{b}, \bar{c}\} = \{-\frac{1}{2}, \frac{1}{6}, \frac{5}{3}\}, \quad p = q = k = 0$

Identification:

$$Q_{10_i} = \{2. - 1, 0\} \ , \ Q_{\bar{5}_i} = \{0, -2, -3\} \ , \ Q_H = 0 \ , \ Q_{\bar{H}} = 2$$

$$Q_{\mathbf{1}} = \{1, 1, -1, 4\}$$

Yukawa couplings are fixed:

$$\begin{array}{c|ccc} & 10_1 & 10_2 & 10_3 \\ \hline 10_1 & & & \\ 10_2 & & & \\ 10_3 & & & \end{array} \quad \begin{array}{c} \\ 10_1 \\ 10_2 \\ 10_3 \end{array}\begin{array}{c} 10_1 \quad 10_2 \quad 10_3 \\ \begin{pmatrix} \epsilon^4 & \epsilon & \epsilon^2 \\ \epsilon & \bar{\epsilon}^2 & \bar{\epsilon} \\ \epsilon^2 & \bar{\epsilon} & 1 \end{pmatrix} \end{array} H \ , \qquad \begin{array}{c} \\ 10_1 \\ 10_2 \\ 10_3 \end{array}\begin{array}{c} \bar{5}_1 \quad \bar{5}_2 \quad \bar{5}_3 \\ \begin{pmatrix} \epsilon^4 & \epsilon & \epsilon^2 \\ \epsilon & \bar{\epsilon}^2 & \bar{\epsilon} \\ \epsilon^2 & \bar{\epsilon} & 1 \end{pmatrix} \end{array} \bar{H}$$

Hierarhical, good fit with: $\bar{\epsilon} \sim 1/10 \ , \ \epsilon \sim 0.2\bar{\epsilon}^2$

→ $(1,1), (1,3), (3,1)$ elements ≈ 0

→Texture zeros:

$$Y_{U,D,E} \propto \begin{pmatrix} 0 & \epsilon & 0 \\ \epsilon & \bar{\epsilon}^2 & \bar{\epsilon} \\ 0 & \bar{\epsilon} & 1 \end{pmatrix}$$

Quark Sector

Basis: $q^T Y_U u^c h_u$ $\qquad$ $q^T Y_D d^c h_d$

$$Y_U \simeq \begin{pmatrix} 0 & \lambda\bar{\epsilon}^{\,2} & 0 \\ \lambda\bar{\epsilon}^{\,2} & a_u\bar{\epsilon}^{\,2}e^{i\xi_u} & \bar{\epsilon} \\ 0 & \bar{\epsilon} & 1 \end{pmatrix} \lambda_t^0$$

$$Y_D \simeq \begin{pmatrix} e^{i\varphi'} & 0 & 0 \\ 0 & e^{i\varphi} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & c\lambda\bar{\epsilon}^{\,2} & 0 \\ kc\lambda\bar{\epsilon}^{\,2} & a_d\bar{\epsilon}^{\,2}e^{i\xi_d} & b\bar{\epsilon} \\ 0 & b'\bar{\epsilon} & 1 \end{pmatrix} \lambda_b^0$$

φ and φ' do not contribute to masses. Relevant for CP

Fit for: $\tan\beta = 5 - 15$

input:

$$m_t(m_t) = 163.68 \text{ GeV} \qquad m_b(m_b) = 4.24 \text{ GeV}$$

$$\bar\epsilon = 0.0847\ , \quad \lambda = 0.0176\ , \quad a_u = 0.6\ , \quad a_d = 3.7\ ,$$

$$b = -0.798\ , \quad b' = -7.13\ , \quad c = 27.07\ , \quad k = 0.864$$

$$\xi_u = 0\ , \quad \xi_d = -0.065\ , \quad \varphi = -2.696\ , \quad \varphi' = -0.97$$

output:

$$(m_u,\ m_d,\ m_s,\ m_c)\,(2\text{ GeV}) = (2.1,\ 4.67,\ 91.44,\ 1082)\,\text{MeV}$$

$$\mu = M_Z: \quad |V_{us}| = 0.2252\ , \quad |V_{cb}| = 0.042$$

$$|V_{ub}| = 0.00349\ , \quad \overline{\rho} = 0.117\ , \quad \overline{\eta} = 0.34$$

Neutrino Sector: **Dirac & Majorana Couplings**

$$m_D \propto \begin{array}{c c} & \begin{array}{ccc} \mathbf{1}_1 & \mathbf{1}_2 & \mathbf{1}_3 \end{array} \\ \begin{array}{c} \bar{5}_1 \\ \bar{5}_2 \\ \bar{5}_3 \end{array} & \begin{pmatrix} \epsilon & \epsilon & \bar{\epsilon} \\ \bar{\epsilon}^{\,2} & \bar{\epsilon}^{\,2} & \bar{\epsilon}^{\,4} \\ \bar{\epsilon} & \bar{\epsilon} & \bar{\epsilon}^{\,3} \end{pmatrix} \end{array} H$$

$$M_R \propto \begin{array}{c c} & \begin{array}{ccc} \mathbf{1}_1 & \mathbf{1}_2 & \mathbf{1}_3 \end{array} \\ \begin{array}{c} \mathbf{1}_1 \\ \mathbf{1}_2 \\ \mathbf{1}_3 \end{array} & \begin{pmatrix} \epsilon^2 & \epsilon^2 & 0 \\ \epsilon^2 & \epsilon^2 & 1 \\ 0 & 1 & \bar{\epsilon}^2 \end{pmatrix} \end{array} M_*$$

See-saw→

$$\bar{M}_\nu = m_D M_R^{-1} m_D^T \propto \begin{pmatrix} e & c & d \\ c & b^2 & ab \\ d & ab & a^2 \end{pmatrix}$$

$$\bar{M}_\nu^{(2,2)} \bar{M}_\nu^{(3,3)} - (\bar{M}_\nu^{(2,3)})^2 \simeq 0$$

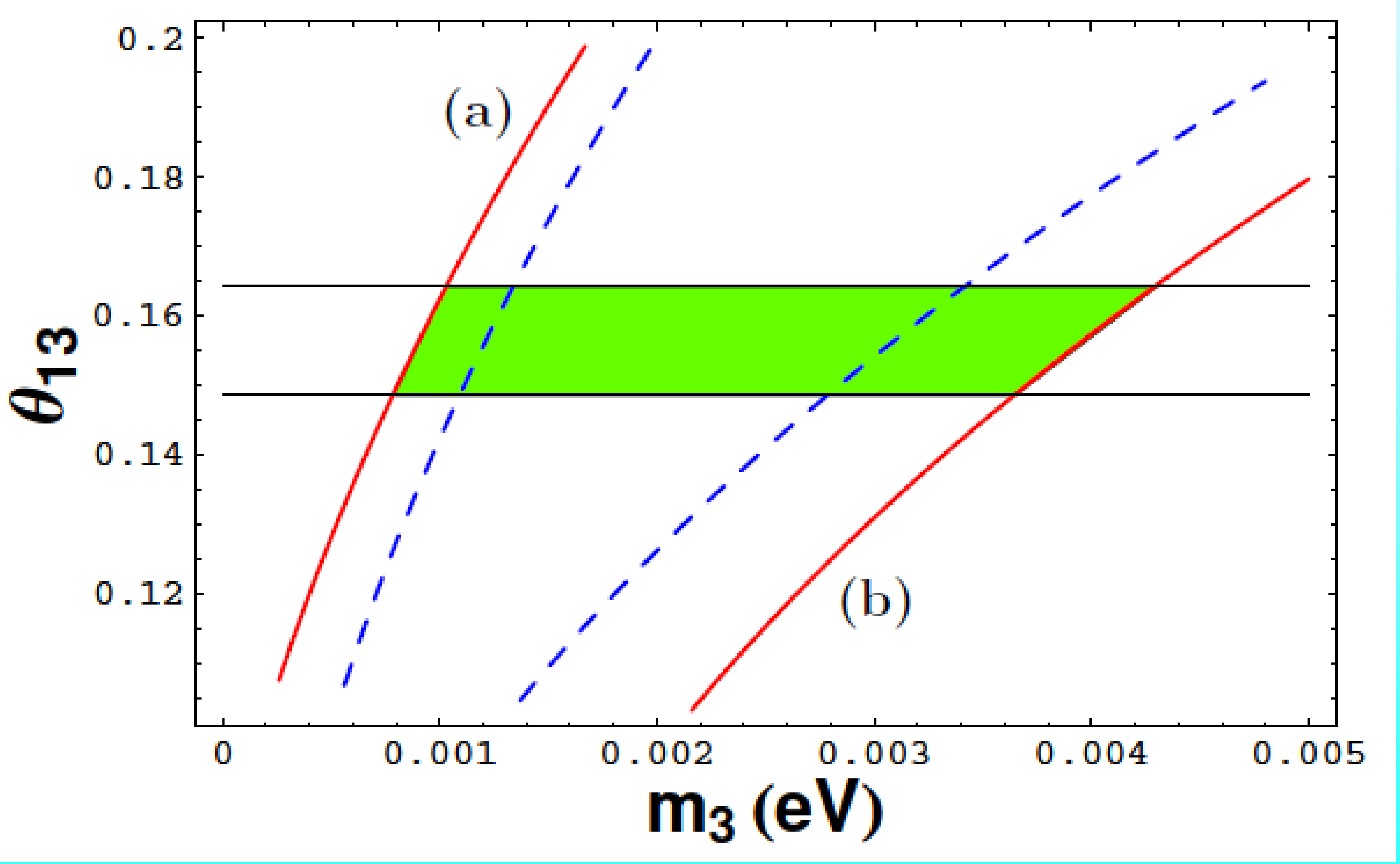
Relations→

$$\tan^2\theta_{13} = \frac{m_3}{m_2}\left| s_{12}^2 e^{i\rho_1} + \frac{m_2}{m_1} c_{12}^2 \right|$$

$$2\delta = \pi - \rho_2 + \mathrm{Arg}\left(s_{12}^2 e^{i\rho_1} + \frac{m_2}{m_1} c_{12}^2 \right)$$

Predict inverted hierarchical neutrinos

$$\tan^2\theta_{13} = \left|\frac{m_3}{m_2}\left|s_{12}^2 e^{i\rho_1} + \frac{m_2}{m_1}c_{12}^2\right| + \frac{K^2}{m_1 m_2}e^{i\kappa}\right|$$



SUMMARY

- SUSY $SU(5) \times U(1)_{Flavor}$ model proposed:
  - Non-anomalous flavor sym.→ texture zeros;
  - successful ch. fermion mass hierarchies;
  - predictive neutrino sector– inverted hierarchical

THANK YOU!

Backup Slides:

$$\tan^2\theta_{13} = \left| \frac{m_3}{m_2} \left| s_{12}^2 e^{i\rho_1} + \frac{m_2}{m_1} c_{12}^2 \right| + \frac{K^2}{m_1 m_2} e^{i\kappa} \right|$$

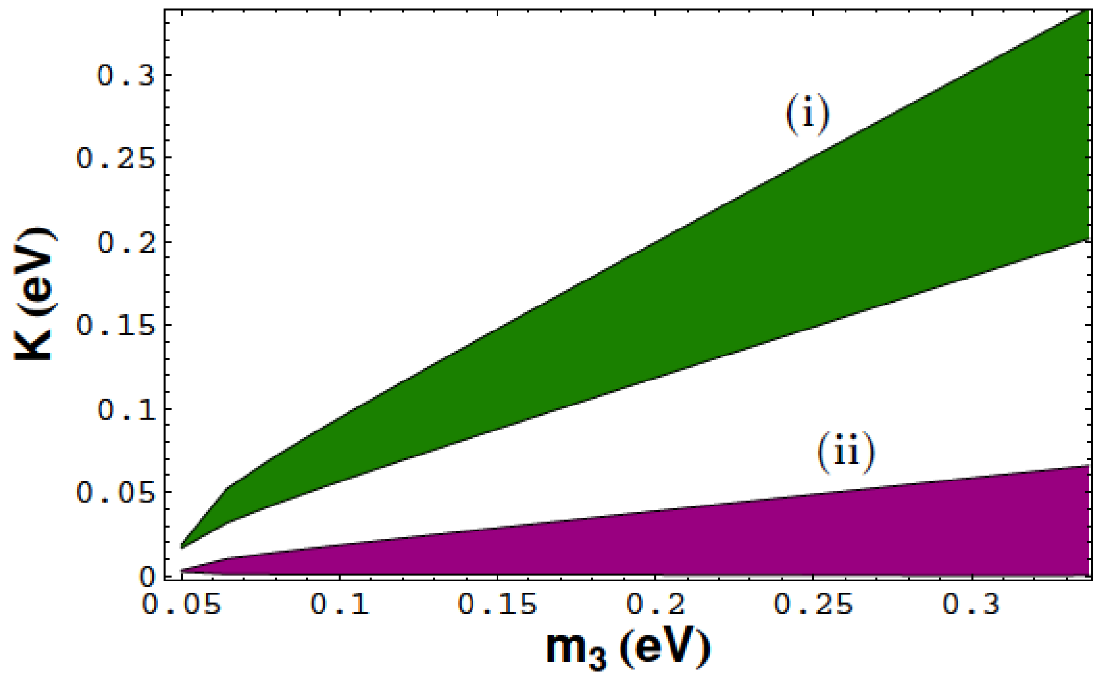


Figure 1: Region (**i**): Needed values of K, realizing normal hierarchical neutrino masses. Region (**ii**): Values of K within considered scenario with normal ordering of neutrino masses of Eq. (56).

13 possible options for the pairs $(Q_H, Q_{\bar{H}})$:

$$
\begin{aligned}
(Q_H, Q_{\bar{H}})^{(i)} = \{ &(q, -q),\ (q, -4\bar{a} - 4\bar{b} - 2\bar{c}),\ (q, -\bar{a} - \bar{b} + \bar{c}),\ (q, -\bar{a} - \bar{b} - 2\bar{c}),\\
&(-4\bar{a} - 4\bar{b} - 2\bar{c}, -\bar{a} - \bar{b} + \bar{c}), (-4\bar{a} - 4\bar{b} - 2\bar{c}, -\bar{a} - \bar{b} - 2\bar{c}), (-4\bar{a} - 4\bar{b} - 2\bar{c}, q),\\
&(-\bar{a} - \bar{b} + \bar{c}, -4\bar{a} - 4\bar{b} - 2\bar{c}), (-\bar{a} - \bar{b} + \bar{c}, -\bar{a} - \bar{b} - 2\bar{c}), (-\bar{a} - \bar{b} + \bar{c}, q),\\
&(-\bar{a} - \bar{b} - 2\bar{c}, -4\bar{a} - 4\bar{b} - 2\bar{c}), (-\bar{a} - \bar{b} - 2\bar{c}, -\bar{a} - \bar{b} + \bar{c}), (-\bar{a} - \bar{b} - 2\bar{c}, q)\}\ ,
\end{aligned}
$$

In comb. of different mass matrices leads to many cases to be investigated

Two flavons offer interesting possibilities (to be explored in details)

$$Q[X] = -\beta\ , \quad Q[\overline{X}] = \beta$$

some more U(1)$_F$ assignments:

$$\mathbf{A}: \quad 10_0 + \bar{5}_0 \ ,$$

$$\mathbf{B}: \quad 10_\alpha + \bar{5}_{-3\alpha} + 1_{5\alpha} \ , \quad (\alpha \neq 0) \ ,$$

$$\mathbf{C}: \quad 10_{a+b} + \bar{5}_{a-3b} + 1_{a+5b} + 5_{-2a-2b} + \bar{5}'_{-2a+2b} + 1'_{4a} \ , \quad (a \neq 0 \ , \ a \neq -5b) \ ,$$

$$\mathbf{D}: \quad 10_{-3\bar{a}+3\bar{b}} + 10_{3\bar{a}+3\bar{b}} + 10_{2\bar{a}-4\bar{b}} + \bar{5}_{-4\bar{a}-4\bar{b}} + 2 \times \left(\bar{5}_{-\bar{a}-\bar{b}} + 1_{5\bar{a}-\bar{b}} + 1'_{6\bar{b}}\right) , \ (\bar{b} \neq 0).$$

Or with $30\bar{a}(3+2p) = \bar{c}(2k^2 + 10p^2 - 27)$

$$1_{5\bar{a}-\bar{b}+\bar{c}} + 1_{5\bar{a}-\bar{b}+4\bar{c}} + 1'_{6\bar{b}+k\bar{c}} + 1'_{6\bar{b}-k\bar{c}}$$

$$10_{-3\bar{a}+3\bar{b}+p\bar{c}} + 10_{3\bar{a}+3\bar{b}-p\bar{c}} + 10_{2\bar{a}-4\bar{b}+\bar{c}} + \bar{5}_{-4\bar{a}-4\bar{b}-2\bar{c}} + \bar{5}_{-\bar{a}-\bar{b}+\bar{c}} + \bar{5}_{-\bar{a}-\bar{b}-2\bar{c}}$$

Many combinations.. , but restrictions – no extra `exotics'

Give 6 combined options:

ABB **BBB** **D**

ABC **BBC**

Three family SU(5) GUT!

U(1)$_F$ Breaking

Flavon(s) needed for U(1)$_F$ Breaking & for generating Yukawa couplings

Tempting to use singlet(s) (1 or/and 1') responsible for anomaly cancellation.
-However, no realistic model has been found.

-- Introduce $X_{-\beta} + \bar{X}_{\beta}$ **flavons –** ***Minimal flavon setup***

In SU(5)x U(1)$_F$, the FI-term $\xi \int d^4\theta V_{U(1)F}$ **is allowed**

D-term: $D_{U(1)F} = \xi - \beta |X|^2 + \beta |\bar{X}|^2$

& superpotential: $W_{Flavon} = \lambda S \left(X\bar{X} - \mu^2 \right) + \frac{1}{2} m_S S^2 + \sigma \frac{1}{3} S^3$

***Without S, higher order superpotential/Kahler terms may do the job**

$U(1)_F$ Breaking (contd.)

All F's & D = 0 ➔ unique solution (no degeneracy)

fixed VEVs: $\langle S\rangle = 0,\ \ X, \overline{X} \neq 0$

a) $\frac{\xi}{\beta} < 0, \qquad |\mu|^2 \ll -\frac{\xi}{\beta}, \qquad |X| \ll |\overline{X}|$

b) $\frac{\xi}{\beta} > 0, \qquad |\mu|^2 \ll \frac{\xi}{\beta}, \qquad |X| \gg |\overline{X}|$

c) $|\mu|^2 \sim \frac{\xi}{\beta}, \qquad |X| \sim |\overline{X}|$

Expansion Parameters:

$$\epsilon = \frac{\langle X\rangle}{M_{Pl}}, \quad \overline{\epsilon} = \frac{\langle \overline{X}\rangle}{M_{Pl}}, \quad \epsilon \neq \overline{\epsilon}$$