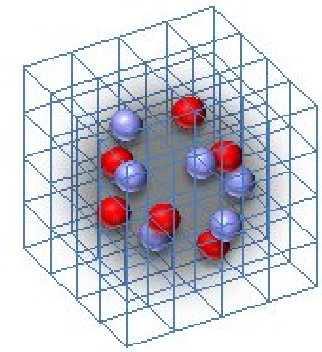
Hadrons and Nuclei: Mass without Higgs

Ulf-G. Meißner, Univ. Bonn & FZ Jülich



NLEFT

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CONTENTS

- Some basic facts
- Anatomy of the nucleon mass
- Ab initio calculation of atomic nuclei
- The fate of carbon-based life as a function of the light quark mass
- Summary & outlook

Some basic facts

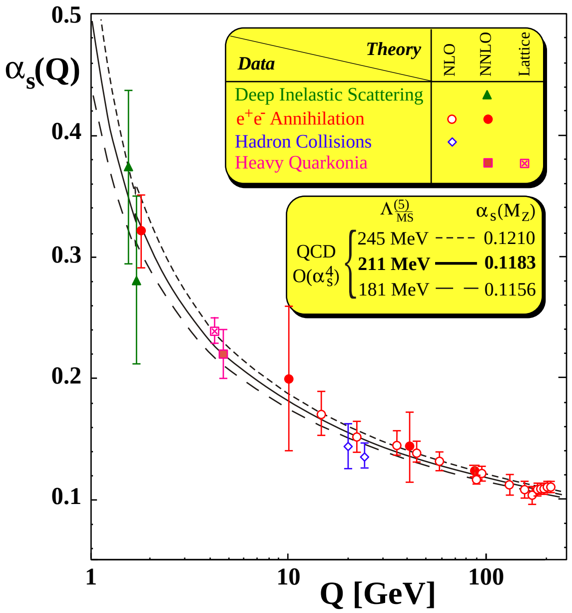
STRUCTURE FORMATION in QCD

- The strong interactions are described by QCD
- Quarks and gluons are confined within **hadrons**
- Protons and neutrons form **atomic nuclei**

⇒ This requires the inclusion of electromagnetism

⇒ Atomic nuclei make up the **visible** matter in the Universe

- **up** and **down** quarks are very light, a few MeV



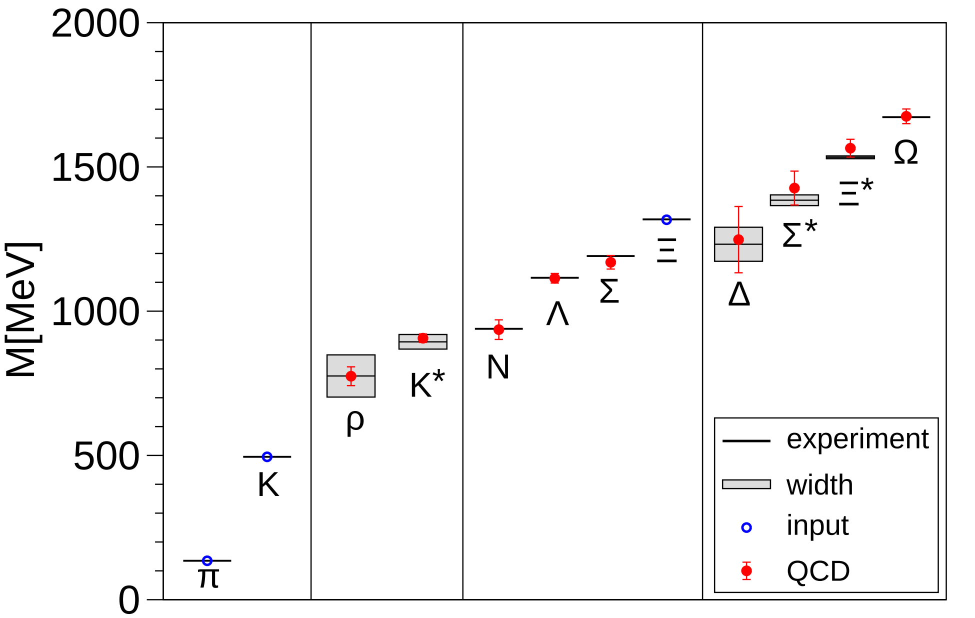
So how is the mass of these strongly interacting composites generated?

Anatomy of the nucleon mass

NUCLEON MASS

- $m_N = (m_p + m_n)/2 = 938.92$ MeV , $m_n - m_p = 1.3$ MeV

- The (strong) nucleon mass can be precisely calculated in **Lattice QCD**
  [BMW 2008 and many others by now]



- The (strong) proton-neutron mass **splitting** can be calculated with some precision
  [NPLQCD 2007 and others by now]

- **EM** effects presently under active investigation
  [BMW, JLQCD, MILC, . . .]

$\Rightarrow$ But can we understand these numbers?

TRACE ANOMALY

- Classical massles QCD is invariant under scale transformations

  $x \to \lambda x \;,\; q(x) \to \lambda^{3/2} q(\lambda x) \;,\; A_\mu(x) \to \lambda A_\mu(\lambda x)$ (*dilatations*)

- Quantization/renormalization generates a scale $\Lambda_{\rm QCD}$ that breaks scale invariance: **dimensional transmutation**

⇒ **trace anomaly**

$$\theta^\mu_\mu = \frac{\beta_{\rm QCD}}{2g} G^a_{\mu\nu} G^{\mu\nu}_a + m_u \bar u u + m_d \bar d d + m_s \bar s s + \ldots$$

∗ trace anomaly = signal for the *generation of hadron masses*

∗ the mass of any hadron made of light quarks mass is essentially field energy ("binding")

"Mass without mass" (Wheeler, 1962)

Anatomy of the nucleon mass

$$m_N \bar{u}(p) u(p) = \langle N(p) | \theta^\mu_\mu | N(p) \rangle$$
$$= \langle N(p) | \frac{\beta_{\rm QCD}}{2g} G^a_{\mu\nu} G^{\mu\nu}_a + m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s | N(p) \rangle$$

- Dissect the various contributions:

  ⋆ $\langle N(p) | m_u \bar{u} u + m_d \bar{d} d | N(p) \rangle = 40 \ldots 70 \text{ MeV}$

  ⋆ $\langle N(p) | m_s \bar{s} s | N(p) \rangle = 0 \ldots 150 \text{ MeV}$

  from the analysis of the pion-nucleon sigma term and lattice QCD

  Gasser, Leutwyler, Sainio; Borasoy & UGM, Büttiker & UGM, Pavan et al., Alarcon et al. . . .

  ⇒ bulk of the nucleon mass is generated by the **gluon fields / field energy**

  ⇒ this is a central result of QCD

  ⇒ requires better Roy-Steiner analysis of πN and lattice data

  Hoferichter, Ditsche, Kubis, UGM, JHEP 1206 (2012) 043, JHEP 1206 (2012) 063 and on-going

Ab initio calculations of atomic nuclei

Ingredients

- Nuclear binding is shallow: $E/A \leq 8$ MeV

$\Rightarrow$ Nuclei can be calculated from the A-body Schrödinger equation: $H\Psi_A = E\Psi_A$

- Forces are of (dominant) two- and (subdominant) three-body nature:

$$V = V_{\rm NN} + V_{\rm NNN}$$

$\Rightarrow$ can be calculated **systematically** and to **high-precision**

Weinberg, van Kolck, Epelbaum, UGM, Entem, Machleidt, . . .

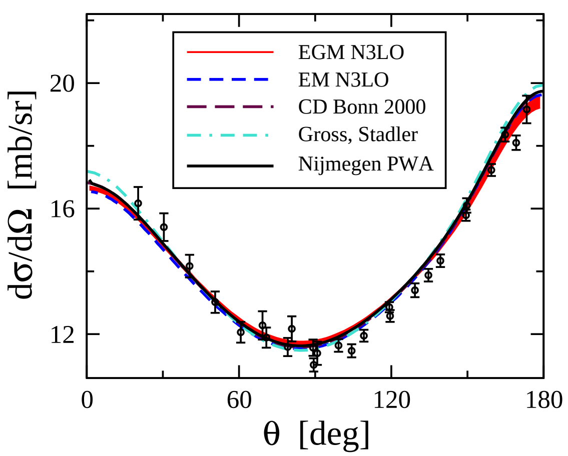
$\Rightarrow$ fit all parameters in $V_{\rm NN} + V_{\rm NNN}$ from 2- and 3-body data

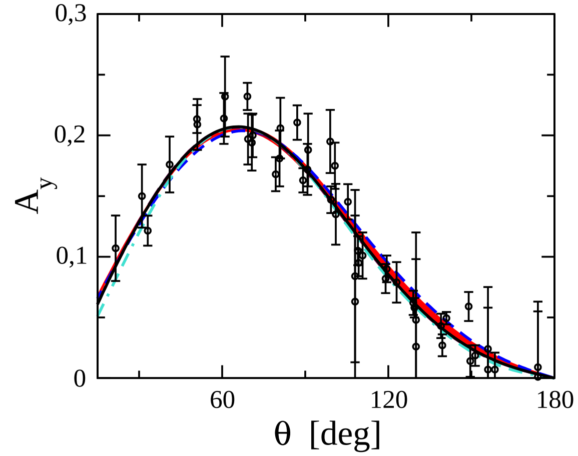
$\Rightarrow$ exact calc's of systems with $A \leq 4$ using Faddeev-Yakubowsky machinery

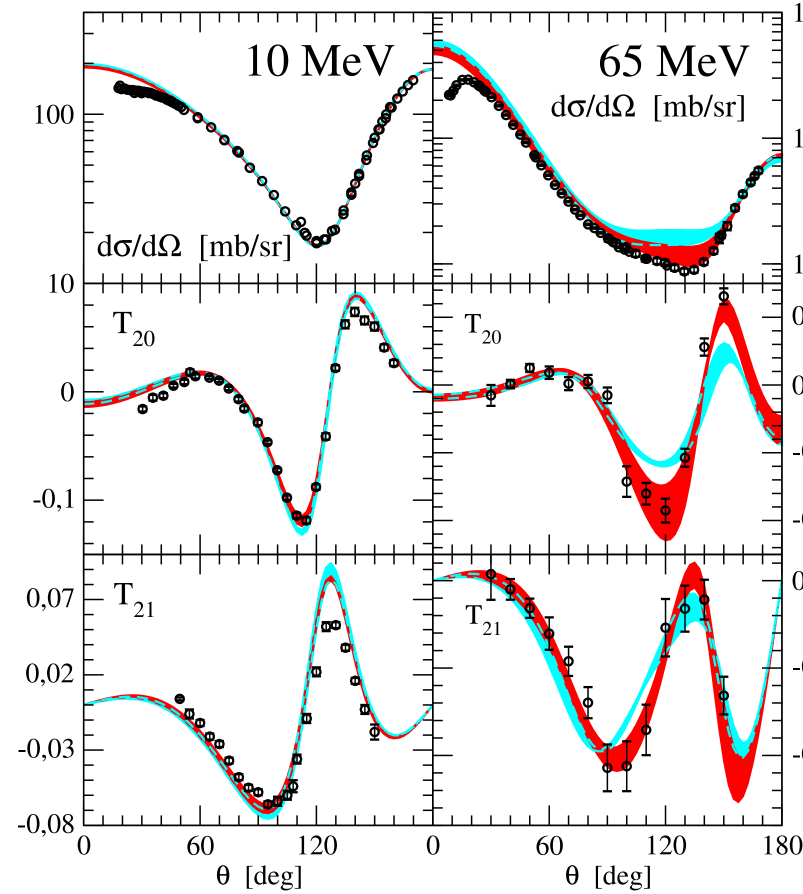
see fig.

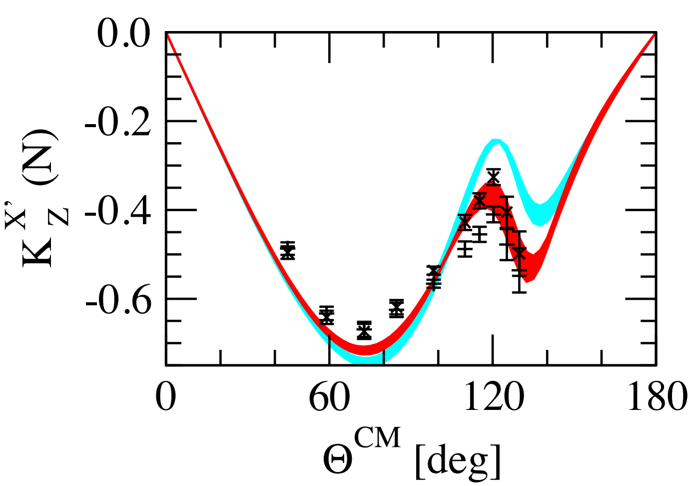
But how about *ab initio* calculations for systems with $A \geq 5$?

Examples


EGM N3LO
EM N3LO
CD Bonn 2000
Gross, Stadler
Nijmegen PWA
dσ/dΩ [mb/sr]
20
16
12
0
60
120
180
θ [deg]


0,3
0,2
0,1
0
A_y
60
120
180
θ [deg]


10 MeV
65 MeV
dσ/dΩ [mb/sr]
dσ/dΩ [mb/sr]
100
10
100
10
1
T_20
T_20
0
-0,1
0,2
0
-0,2
-0,4
T_21
T_21
0,07
0,02
-0,03
-0,08
0
-0,1
-0,2
0
60
120
0
60
120
180
θ [deg]
θ [deg]

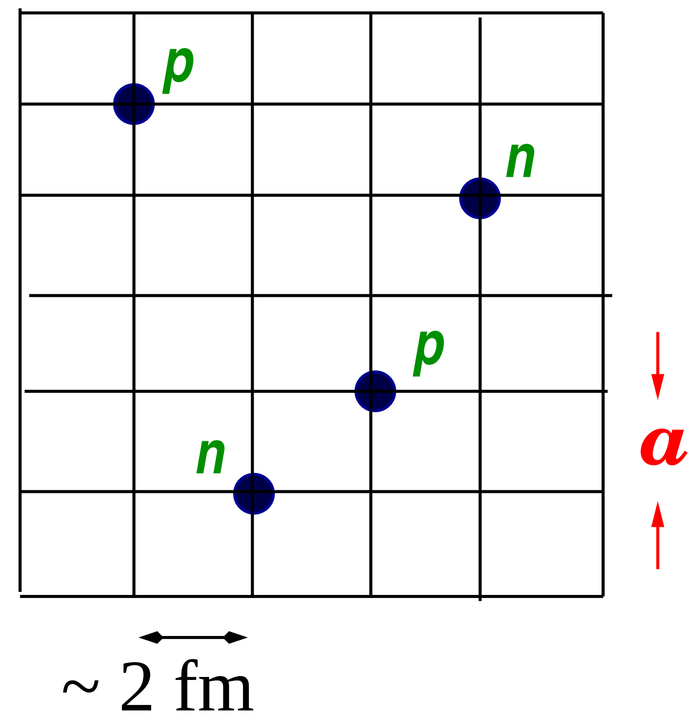

0.0
-0.2
-0.4
-0.6
K_Z^X' (N)
0
60
120
180
Θ^CM [deg]

NUCLEAR LATTICE SIMULATIONS

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

- *new method* to tackle the nuclear many-body problem
- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like fields on the sites
- discretized chiral potential w/ pion exchanges
  and contact interactions
- typical lattice parameters

$$\Lambda = \frac{\pi}{a} \simeq 300\,\text{MeV [UV cutoff]}$$

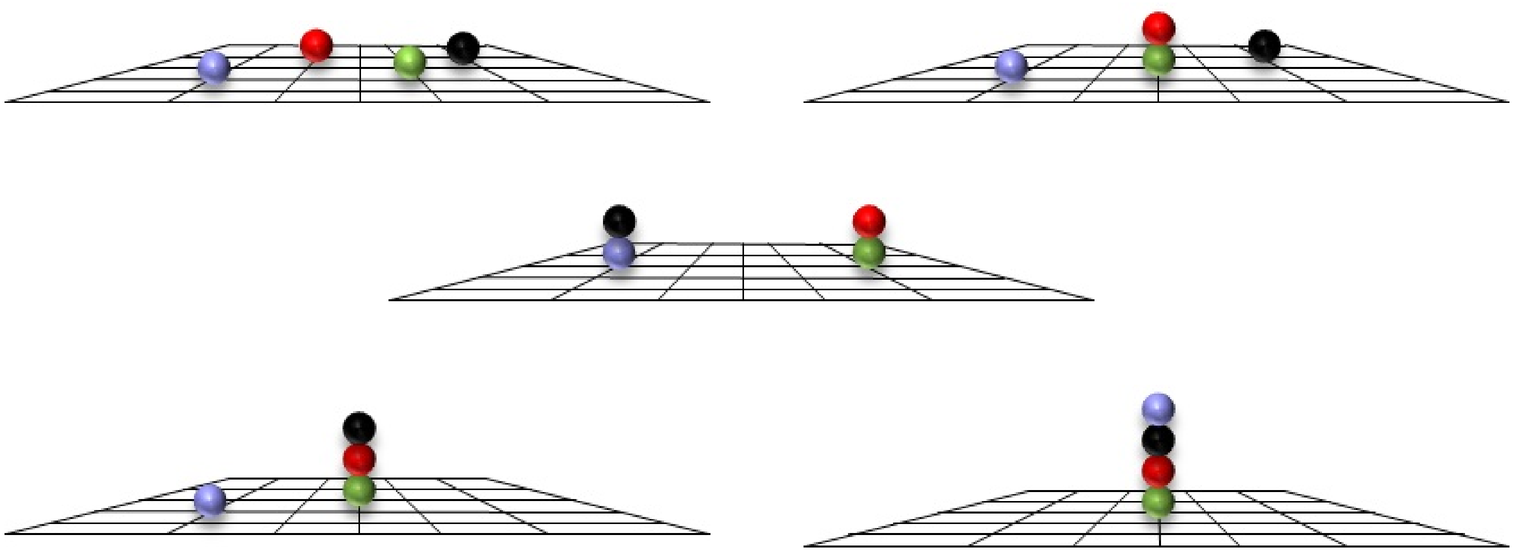


- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

J. W. Chen, D. Lee and T. Schäfer, Phys. Rev. Lett. **93** (2004) 242302

- hybrid Monte Carlo & transfer matrix (similar to LQCD)

CONFIGURATIONS



⇒ all *possible* configurations are sampled

⇒ *clustering* emerges *naturally*

⇒ perform *ab initio* calculations using only V_{NN} and V_{NNN} as input

⇒ grand challenge: the spectrum of ^{12}**C**

A SHORT HISTORY of the HOYLE STATE

- Heavy element generation in massive stars: triple-α process

Bethe 1938, Öpik 1952, Salpeter 1952, Hoyle 1954, ...

$$
\begin{aligned}
&{}^4\text{He} + {}^4\text{He} \rightleftharpoons {}^8\text{Be} \\
&{}^8\text{Be} + {}^4\text{He} \rightleftharpoons {}^{12}\text{C}^* \rightarrow {}^{12}\text{C} + \gamma \\
&{}^{12}\text{C} + {}^4\text{He} \rightleftharpoons {}^{16}\text{O} + \gamma
\end{aligned}
$$

- Hoyle's contribution: calculation of relative abundances of ^{4}He, ^{12}C and ^{16}0
  $\Rightarrow$ need a resonance close to the ^{8}Be + ^{4}He threshold at $E_R = 0.35$ MeV
  $\Rightarrow$ this corresponds to a 0^+ excited state 7.7 MeV above the g.s.

- a corresponding state was experimentally confirmed at Caltech at
  $E - E(\text{g.s.}) = 7.653 \pm 0.008$ MeV Dunbar et al. 1953, Cook et al. 1957

- still on-going experimental activity, e.g. EM transitions at SDALINAC

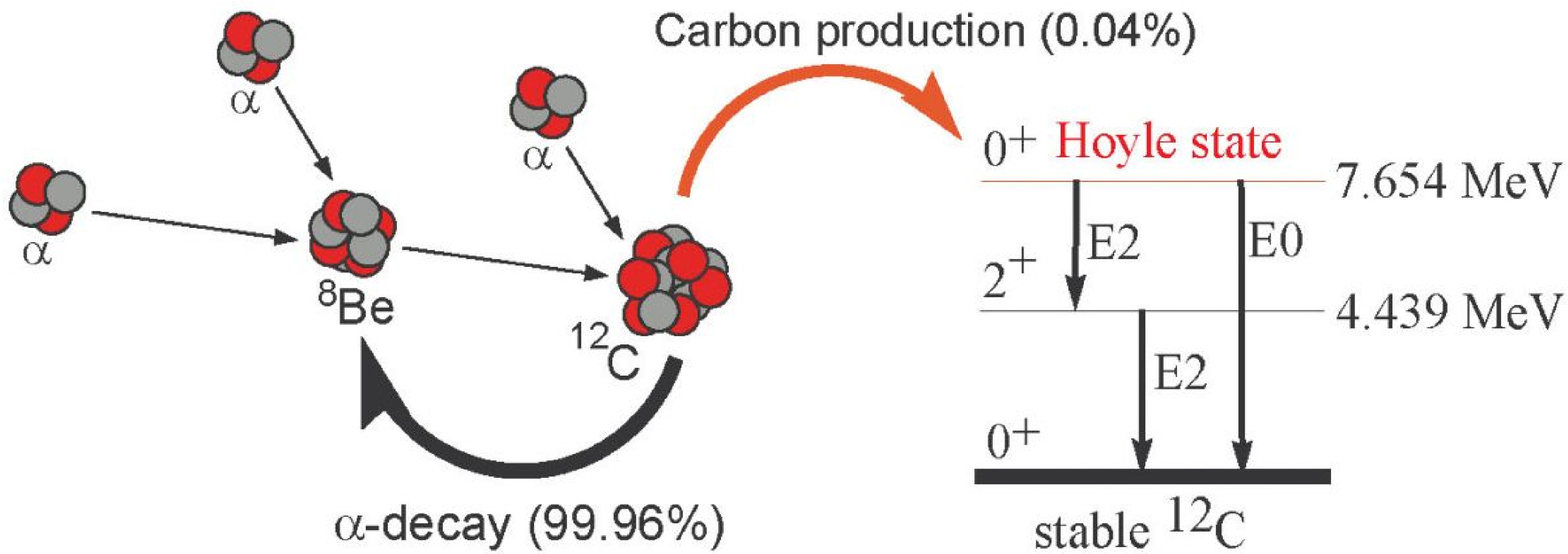
M. Chernykh et al., Phys. Rev. Lett. 98 (2007) 032501

- and how about theory ? $\rightarrow$ this talk

- side remark: NOT driven by anthropic considerations

H. Kragh, Arch. Hist. Exact Sci. 64 (2010) 721

THE TRIPLE-ALPHA PROCESS $\rightarrow$ MOVIE



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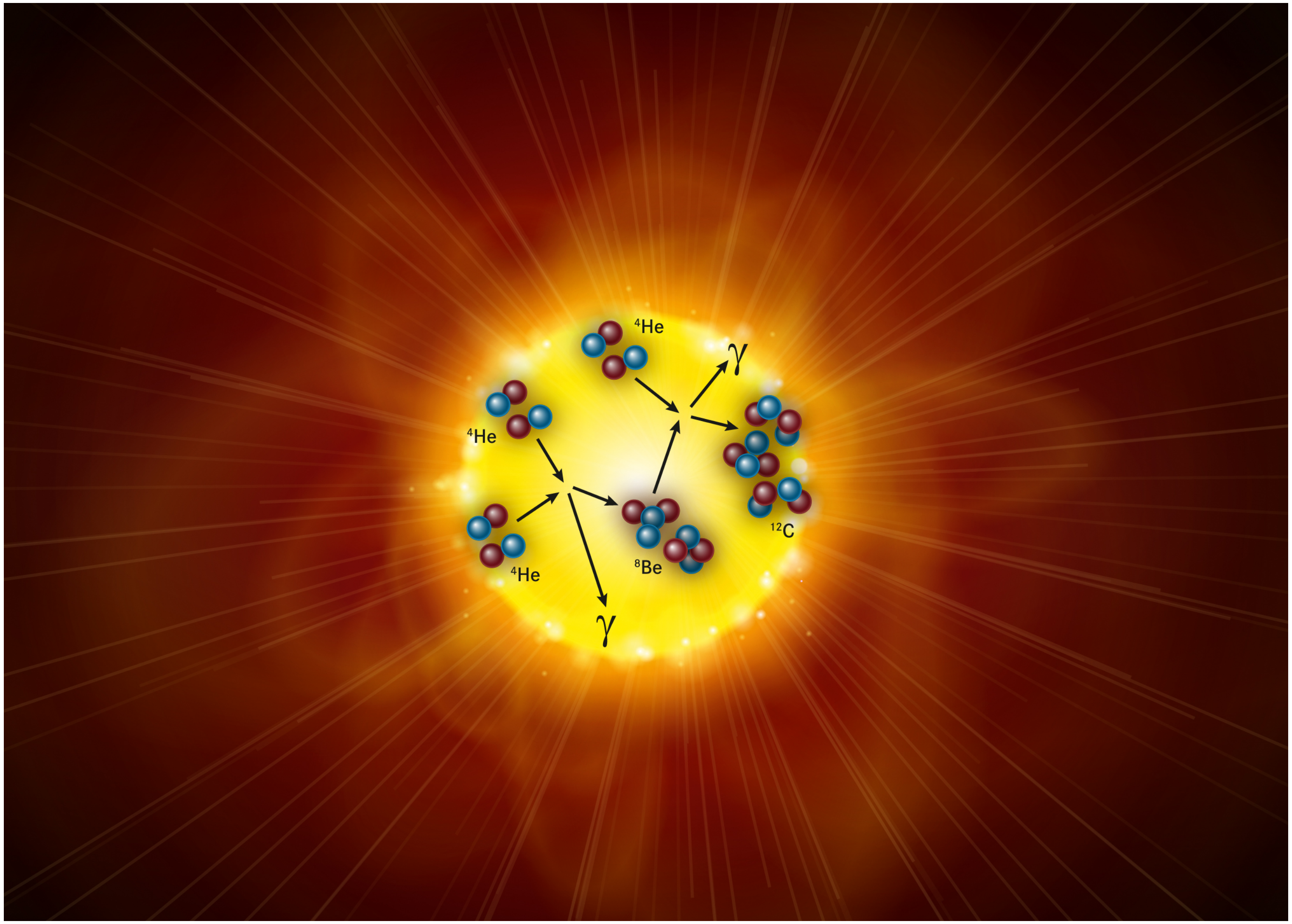
- the ^{8}Be nucleus is instable, long lifetime $\rightarrow$ 3 alphas must meet
- the Hoyle state sits just above the continuum threshold
  $\rightarrow$ most of the excited carbon nuclei decay
  (about 4 out of 10000 decays produce stable carbon)
- carbon is further turned into oxygen but w/o a resonant condition

$\Rightarrow$a triple wonder !

SPECTRUM OF ^{12}C & the HOYLE STATE

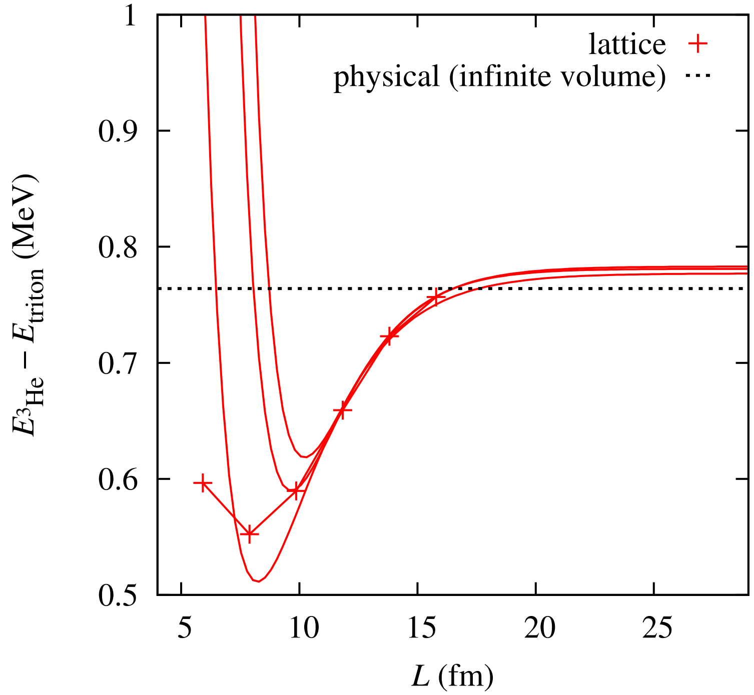
Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **106** (2011) 192501
Epelbaum, Krebs, Lähde, Lee, UGM, Phys. Rev. Lett. **109** (2012) 252501
Viewpoint: Hjorth-Jensen, Physics **4** (2011) 38



RESULTS

- some groundstate energies and differences

E [MeV]	NLEFT	Exp.
^{3}He - ^{3}H	0.78(5)	0.76
^{4}He	−28.3(6)	−28.3
^{8}Be	−55(2)	−56.5
^{12}C	−92(3)	−92.2
^{16}O	−135(6)	−127.6

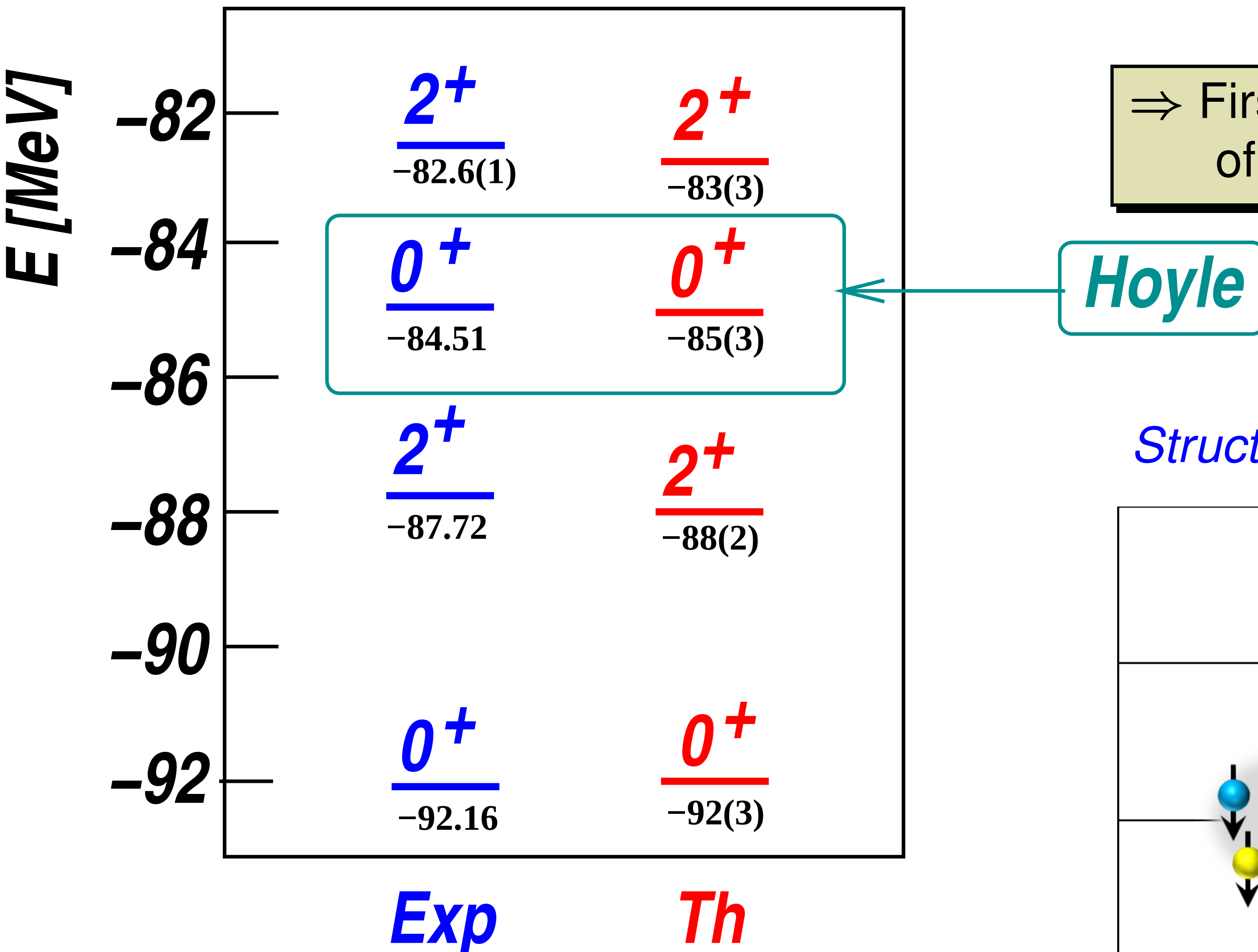


- promising results
- excited states more difficult

⇒ new projection MC method

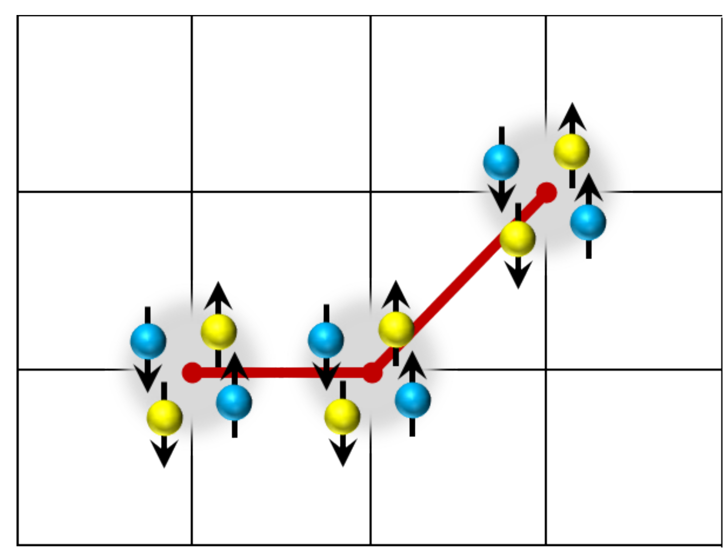
The SPECTRUM of CARBON-12

- After $8 \cdot 10^6$ hrs JUGENE/JUQUEEN (and "some" human work)



⇒ First ab initio calculation of the Hoyle state ✓

Structure of the Hoyle state:



SPECTRUM OF ^{12}C

- Summarizing the results for carbon-12:

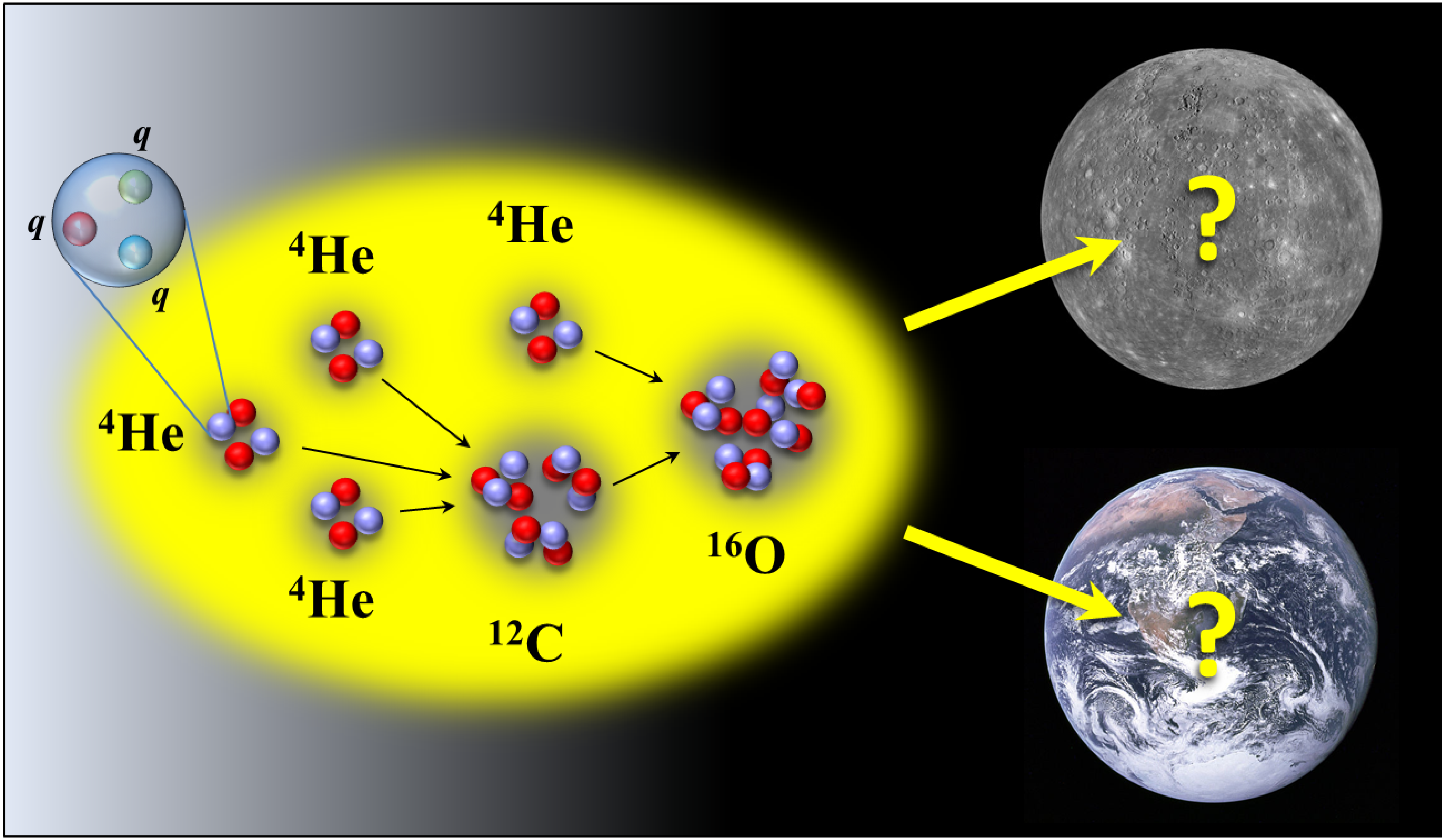
	0_1^+	2_1^+	0_2^+	2_2^+
LO NLO NNLO	$-96(2)$ MeV $-77(3)$ MeV $-92(3)$ MeV	$-94(2)$ MeV $-74(3)$ MeV $-89(3)$ MeV	$-89(2)$ MeV $-72(3)$ MeV $-85(3)$ MeV	$-88(2)$ MeV $-70(3)$ MeV $-83(3)$ MeV
Exp.	-92.16 MeV	-87.72 MeV	-84.51 MeV	$-82.6(1)$ MeV [1,2] $-82.32(6)$ MeV [3] $-81.1(3)$ MeV [4] $-82.13(11)$ MeV [5]

[1] Freer et al., Phys. Rev. C 80 (2009) 041303
[2] Zimmermann et al., Phys. Rev. C 84 (2011) 027304
[3] Hyldegaard et al., Phys. Rev. C 81 (2010) 024303
[4] Itoh et al., Phys. Rev. C 84 (2011) 054308
[5] Weller et al., in preparation

- importance of consistent 2N & 3N forces
- good agreement w/ experiment, can be improved

The fate of carbon-based life as a function of the quark mass

FINE-TUNING of FUNDAMENTAL PARAMETERS



FINE-TUNING: MONTE-CARLO ANALYSIS

Epelbaum, Krebs, Lähde, Lee, UGM, PRL **110** (2013) 112502

- consider first QCD only → calculate $\partial \Delta E / \partial M_\pi$
- relevant quantities (energy *differences*)

$$\boxed{\Delta E_h \equiv E^*_{12} - E_8 - E_4, \quad \Delta E_b \equiv E_8 - 2E_4} \qquad \Delta E_c \equiv E^*_{12} - E_{12}$$

- energy differences depend on parameters of QCD (LO analysis)

$$\boxed{E_i = E_i\Big(M_\pi^{\rm OPE}, m_N(M_\pi), \tilde{g}_{\pi N}(M_\pi), C_0(M_\pi), C_I(M_\pi)\Big)}$$

$$\tilde{g}_{\pi N} \equiv \frac{g_A}{2F_\pi}$$

- remember: $M^2_{\pi^\pm} \sim (m_u + m_d)$

$\Rightarrow$ quark mass dependence $\equiv$ pion mass dependence

PION MASS VARIATIONS

- consider pion mass changes as *small perturbations*

$$\frac{\partial E_i}{\partial M_\pi}\bigg|_{M_\pi^{\rm phys}} = \frac{\partial E_i}{\partial M_\pi^{\rm OPE}}\bigg|_{M_\pi^{\rm phys}} + x_1 \frac{\partial E_i}{\partial m_N}\bigg|_{m_N^{\rm phys}} + x_2 \frac{\partial E_i}{\partial \tilde{g}_{\pi N}}\bigg|_{\tilde{g}_{\pi N}^{\rm phys}} + x_3 \frac{\partial E_i}{\partial C_0}\bigg|_{C_0^{\rm phys}} + x_4 \frac{\partial E_i}{\partial C_I}\bigg|_{C_I^{\rm phys}}$$

with

$$x_1 \equiv \frac{\partial m_N}{\partial M_\pi}\bigg|_{M_\pi^{\rm phys}}, \quad x_2 \equiv \frac{\partial \tilde{g}_{\pi N}}{\partial M_\pi}\bigg|_{M_\pi^{\rm phys}}, \quad x_3 \equiv \frac{\partial C_0}{\partial M_\pi}\bigg|_{M_\pi^{\rm phys}}, \quad x_4 \equiv \frac{\partial C_I}{\partial M_\pi}\bigg|_{M_\pi^{\rm phys}}$$

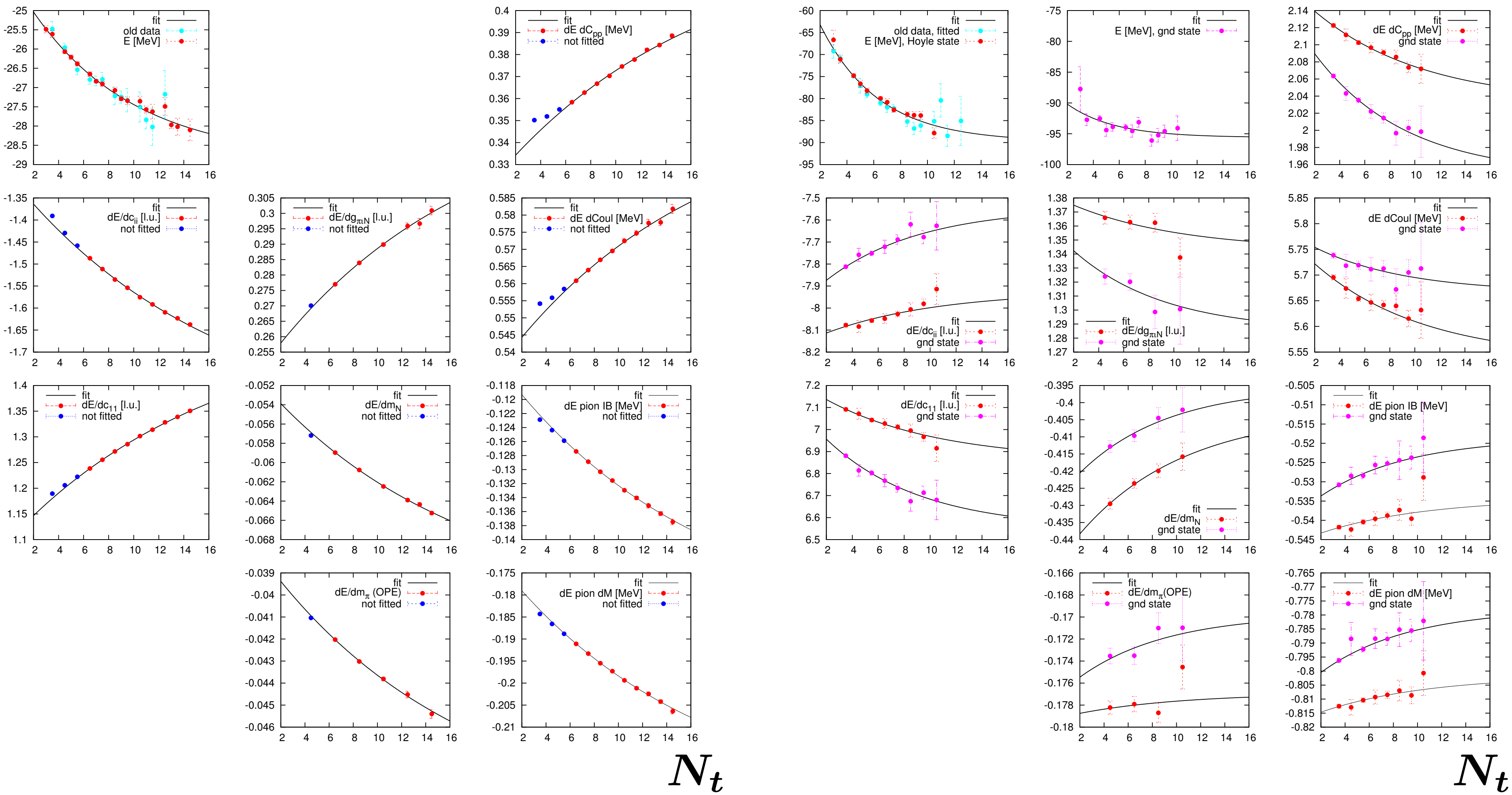
$\Rightarrow$ problem reduces to the calculation of the various derivatives using AFQMC and the determination of the x_i

- x_1 and x_2 can be obtained from LQCD plus CHPT
- x_3 and x_4 can be obtained from two-body scattering and its M_π-dependence

AFQMC RESUTS for the DERIVATIVES

- ^{4}He
- ^{12}C(0_2^+)

$$E(N_t) = E(\infty) + \text{const}\,\exp(-N_t/\tau)$$



DETERMINATION of the x_i

- x_1 from the quark mass expansion of the nucleon mass: $x_1 \simeq 0.8 \pm 0.2$

- x_2 from the quark mass expansion of the pion decay constant and the nucleon axial-vector constant: $x_2 \simeq -0.056 \ldots 0.008$

- x_3 and x_4 can be obtained from a two-nucleon scattering analysis & can be deduced from:

$$-\frac{\partial a^{-1}}{\partial M_\pi} \equiv \frac{A}{a M_\pi} = \frac{1}{\pi L} S'(\eta) \frac{\partial \eta}{\partial M_\pi} \, , \quad \eta \equiv m_N E \left(\frac{L}{2\pi}\right)^2$$

$\Rightarrow$ while this can straightforwardly be computed, we prefer to use a representation that substitutes x_3 and x_4 by:

$$\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} , \qquad \left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

$\Rightarrow$ we are ready to study the pertinent energy differences

RESULTS

- putting pieces together:

$$\left.\frac{\partial \Delta E_h}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.455(35)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.744(24)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.056(10)$$

$$\left.\frac{\partial \Delta E_b}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.117(34)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.189(24)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.012(9)$$

$$\left.\frac{\partial \Delta E_c}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.07(3)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.14(2)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.017(9)$$

- x_1 and x_2 only affect the small constant terms
- also calculated the shifts of the individual energies (not shown here)

INTERPRETATION

- $(\partial\Delta E_h/\partial M_\pi)/(\partial\Delta E_b/\partial M_\pi) \simeq 4$
  $\Rightarrow$ ΔE_h and ΔE_b cannot be independently fine-tuned

- Within error bars, $\partial\Delta E_h/\partial M_\pi$ & $\partial\Delta E_b/\partial M_\pi$ appear unaffected by the choice of x_1 and x_2 $\rightarrow$ indication for α-clustering

- For ΔE_h & ΔE_b, the dependence on M_π is small when

$$\boxed{\partial a_s^{-1}/\partial M_\pi \simeq -1.6 \times \partial a_t^{-1}/\partial M_\pi}$$

- the triple alpha process is controlled by :

$$\Delta E_{h+b} \equiv \Delta E_h + \Delta E_b = E_{12}^\star - 3E_4$$

$$\boxed{\left.\frac{\partial\Delta E_{h+b}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.571(14)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.934(11)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.069(6)}$$

$\Rightarrow$ so what can we say about the quark mass dependence of the scattering lengths?

CONSTRAINTS on the SCATTERING LENGTHS

- Quark mass dependence of hadron properties: $\dfrac{\delta O_H}{\delta m_f} \equiv K_H^f \dfrac{O_H}{m_f}\,, \quad f = u, d, s$

- NN scattering lengths as a function of M_π: $-\dfrac{\partial a_{s,t}^{-1}}{\partial M_\pi} \equiv \dfrac{A_{s,t}}{a_{s,t} M_\pi}, \quad A_{s,t} \equiv \dfrac{K^q_{a_{s,t}}}{K^q_\pi}$

- earlier determinations from chiral EFT at NLO

Beane, Savage (2003), Epelbaum, Glöckle, UGM (2003)

- new determination at NNLO:

Epelbaum et al. (2012)

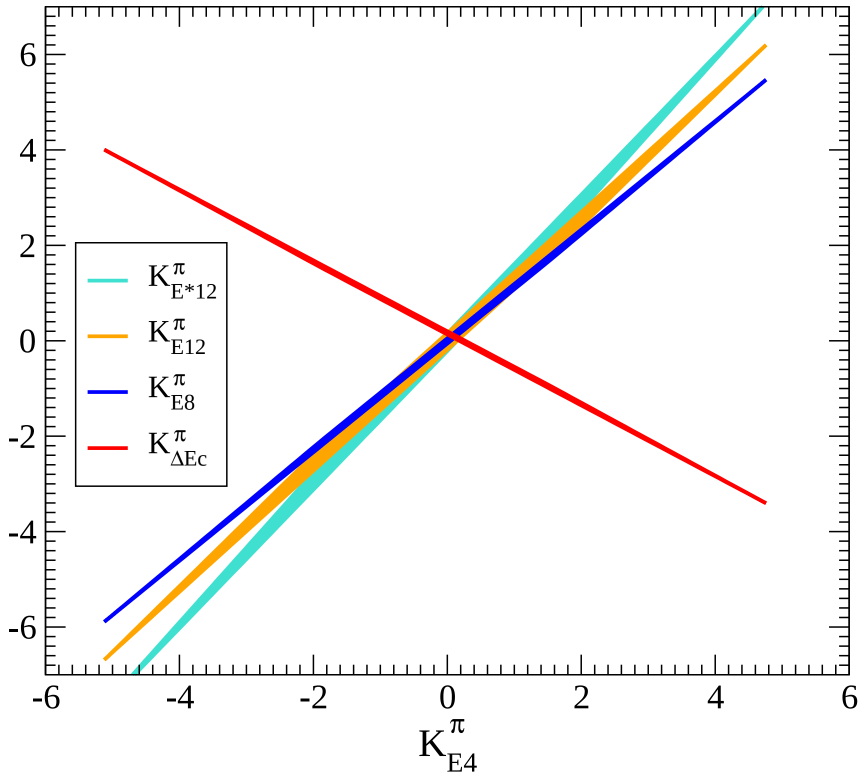
$$K^q_{a_s} = 2.3^{+1.9}_{-1.8}\,,\ K^q_{a_t} = 0.32^{+0.17}_{-0.18} \to \frac{\partial a_t^{-1}}{\partial M_\pi} = -0.18^{+0.10}_{-0.10}\,,\ \frac{\partial a_s^{-1}}{\partial M_\pi} = 0.29^{+0.25}_{-0.23}$$

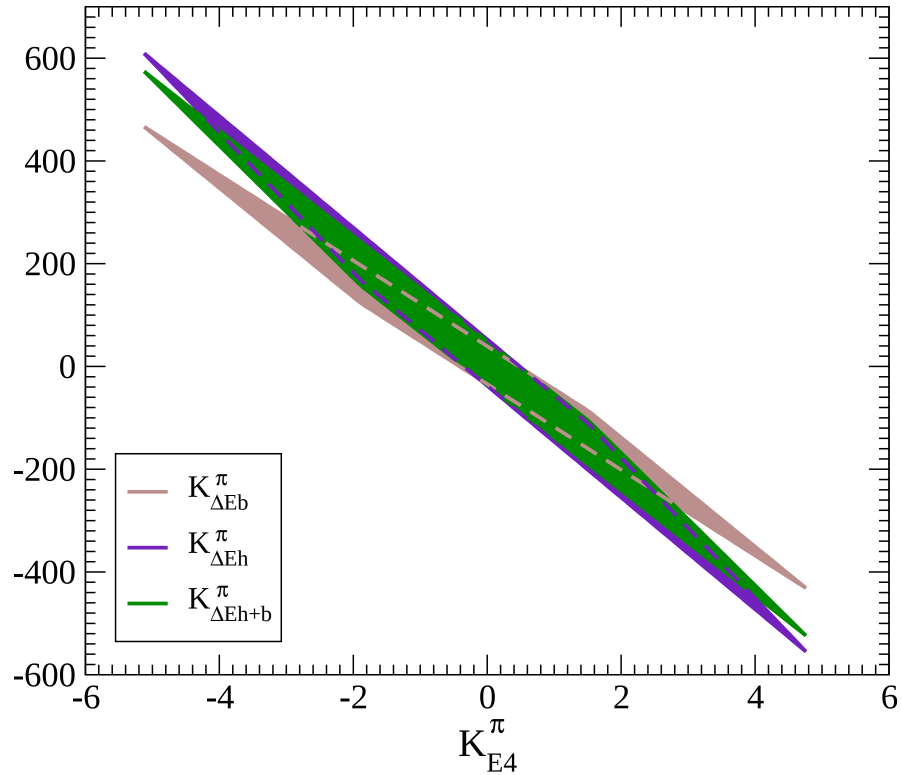
- note the *magical* central value:

$$\boxed{\frac{\partial a_s^{-1}/\partial M_\pi}{\partial a_t^{-1}/\partial M_\pi} \simeq -1.6^{+1.0}_{-1.7}}$$

CORRELATIONS

- vary the quark mass derivatives of $a_{s,t}^{-1}$ within $-1, \ldots, +1$:





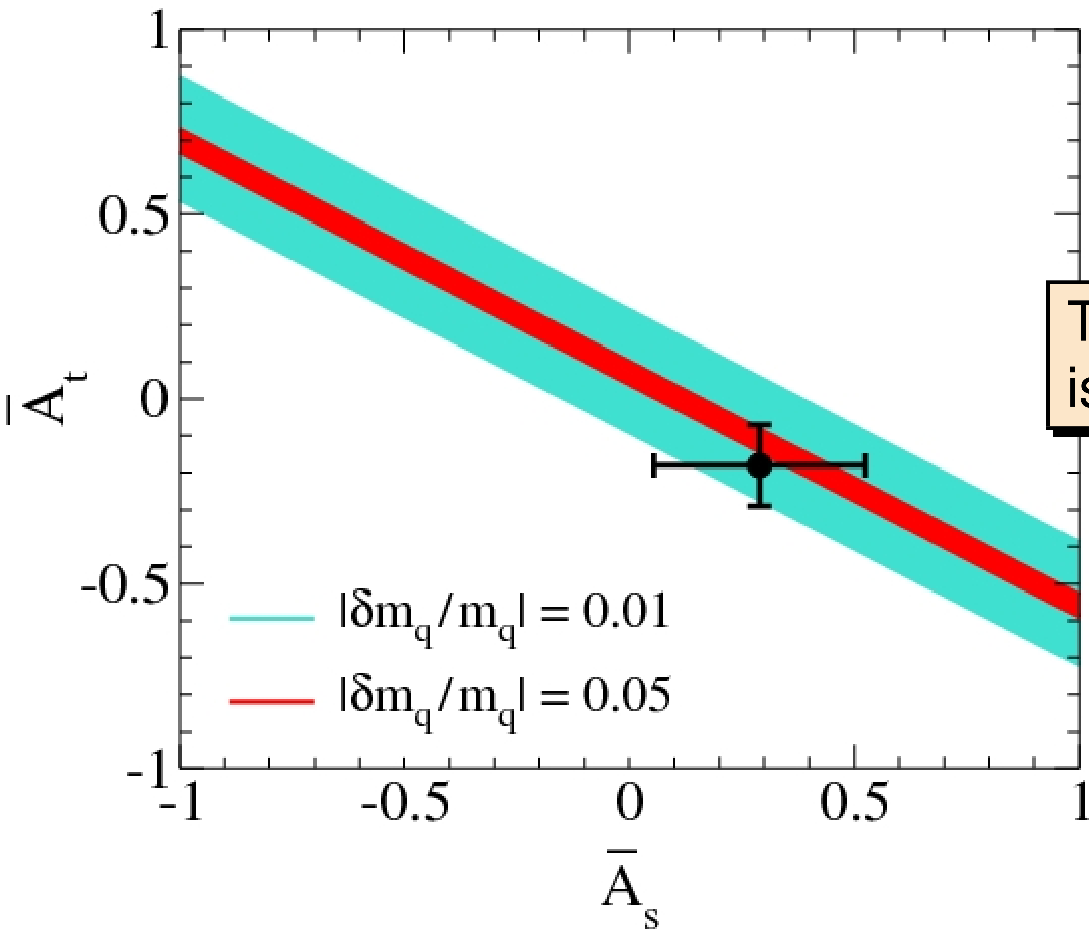
- clear correlations: α-particle BE and the energies/energy differences

$\Rightarrow$ the anthropic view of the Universe depends on whether the ^{4}He BE moves!

THE END-OF-THE-WORLD PLOT

- $|\delta(\Delta E_{h+b})| < 100$ keV   Oberhummer et al., Science (2000)

$$\rightarrow \boxed{\left|\left(0.571(14)\bar{A}_s + 0.934(11)\bar{A}_t - 0.069(6)\right)\frac{\delta m_q}{m_q}\right| < 0.0015}$$



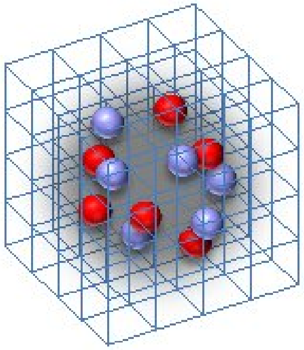
$$\bar{A}_{s,t} \equiv \left.\frac{\partial a_{s,t}^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

The light quark mass is fine-tuned to $\simeq 2\%$

Similarly: $\alpha_{\rm EM}$ is fine-tuned to $\simeq 2.5\%$

SUMMARY & OUTLOOK

- Masses of hadrons and nuclei made from light quarks *feel very little Higgs*
- Nucleon mass be calculated accurately, precise understanding requires better determination of σ-terms
- Nuclear lattice simulations as a new quantum many-body approach
- Fix parameters in few-nucleon systems $\rightarrow$ predictions (*ab initio* calculations)



- ^{12}C spectrum at NNLO $\rightarrow$ Hoyle state and its structure
- Fine-tuning of $m_{\rm quark}$ and $\alpha_{\rm EM}$ $\rightarrow$ viability of life

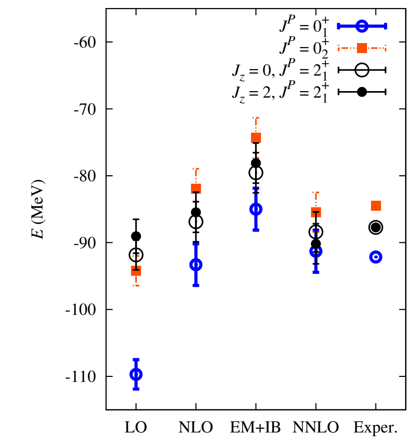
  $\Rightarrow$ changes in $m_{\rm quark}$ of about 2% and in $\alpha_{\rm EM}$ of about 2.5% are allowed
- must improve and extend these calculations, e.g. $\alpha +^{12} C \rightarrow^{16} O + \gamma$

$\Rightarrow$ the strong interactions remain a challenge

SPARES

The RELEVANT QUESTION

Date: Sat, 25 Dec 2010 20:03:42 -0600
From: Steven Weinberg ⟨weinberg@zippy.ph.utexas.edu⟩
To: Ulf-G. Meissner ⟨meissner@hiskp.uni-bonn.de⟩
Subject: Re: Hoyle state in 12C



Dear Professor Meissner,

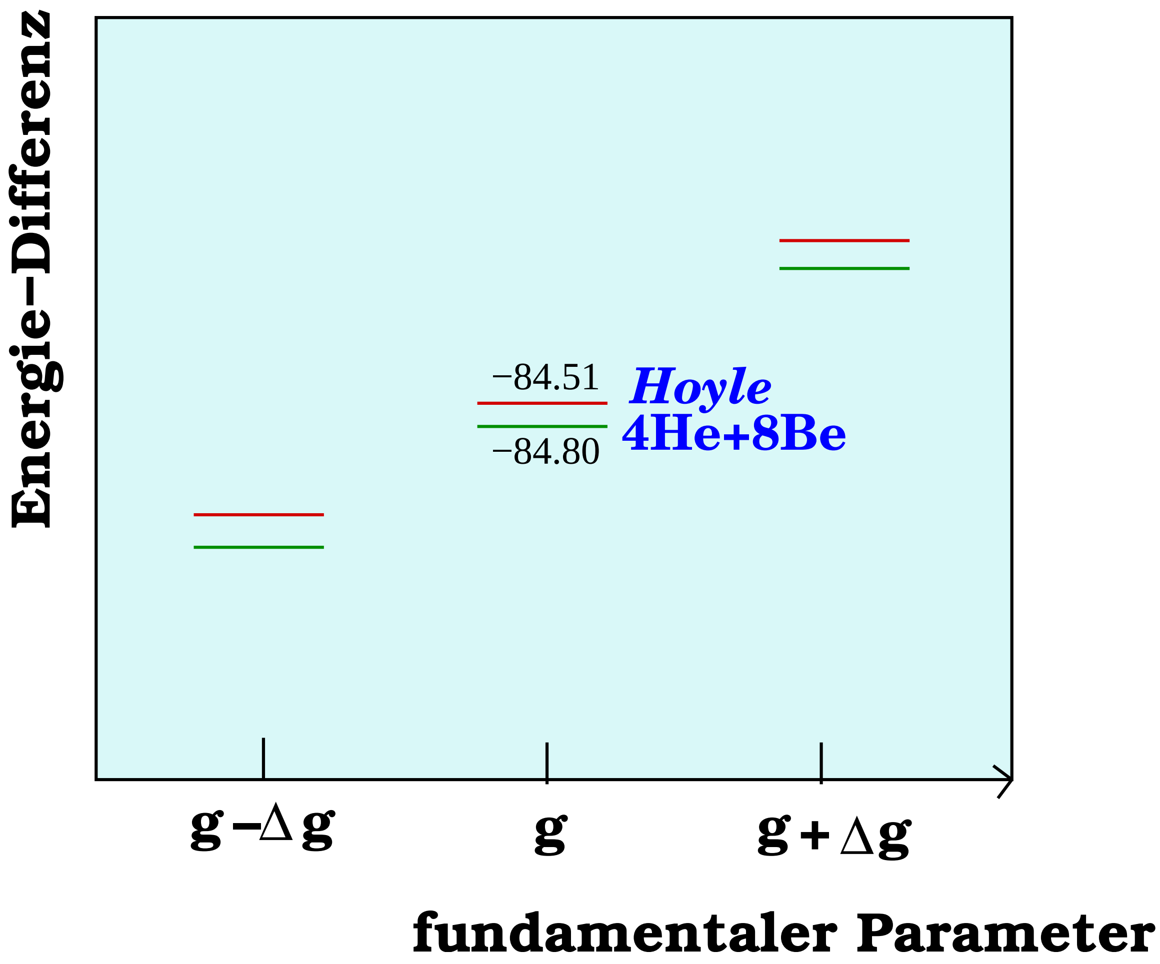
Thanks for the colorful graph. It makes a nice Christmas card. But I have a detailed question. Suppose you calculate not only the energy of the Hoyle state in C12, but also of the ground states of He4 and Be8. How sensitive is the result that the energy of the Hoyle state is near the sum of the rest energies of He4 and Be8 to the parameters of the theory? I ask because I suspect that for a pretty broad range of parameters, the Hoyle state can be well represented as a nearly bound state of Be8 and He4.

All best,
Steve Weinberg

- How does the Hoyle state relative to the 4He+8Be threshold, if we change the fundamental parameters of QCD+QED?
- not possible in nature, *but on a high-performance computer!*

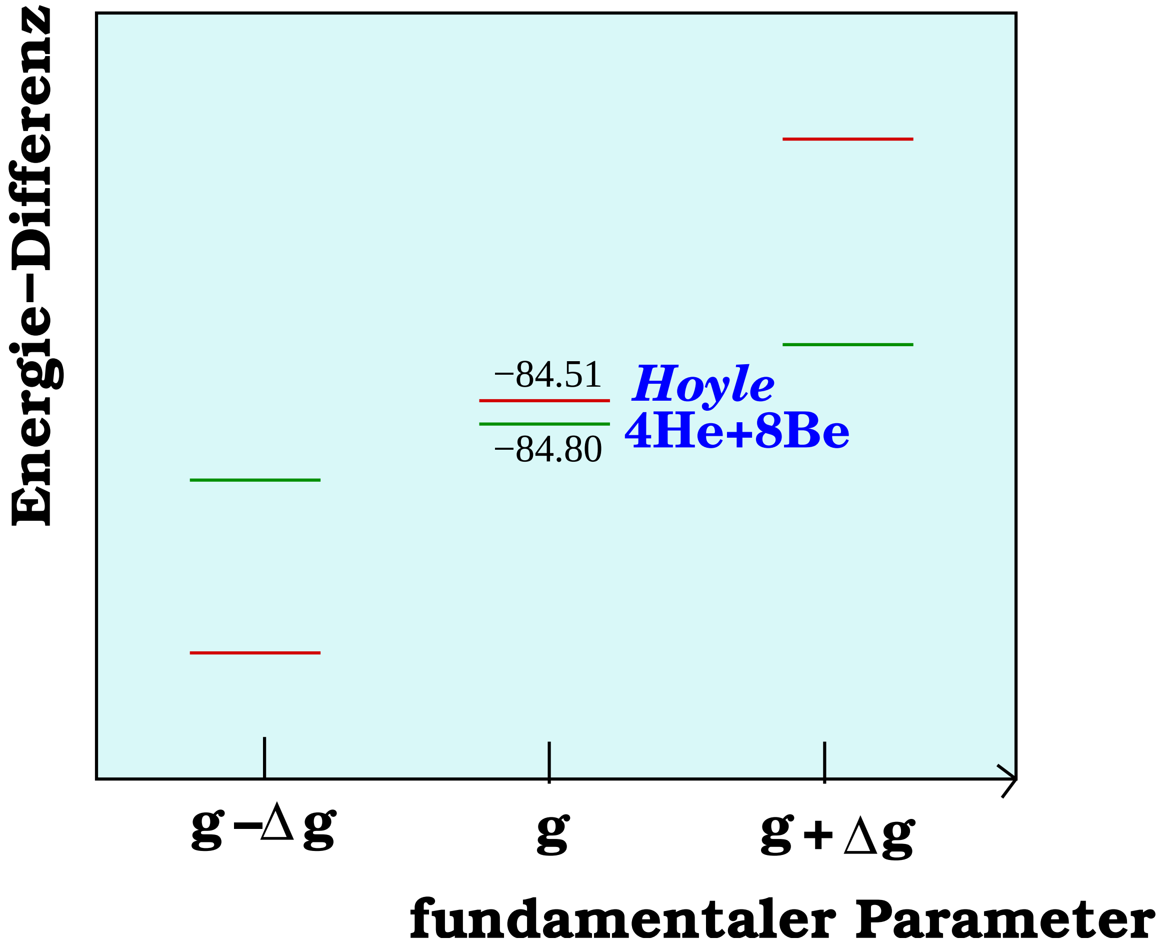
The NON-ANTHROPIC SCENARIO

- Weinberg's assumption: The Hoyle state stays close to the 4He+8Be threshold



The ANTHROPIC SCENARIO

- The AP strikes back: The Hoyle state moves away from the 4He+8Be threshold



EARLIER STUDIES of the AP

- By hand modification of the energy diff. & network calcs in massive stars

  Livio et al., Nature 340 (1989) 281

  $\hookrightarrow$ a 60 keV increase does not significantly alter carbon production

  $\hookrightarrow$ a 60 keV decrease roughly doubles the carbon production rate

  $\hookrightarrow$ a $\pm\mathbf{277}$ keV change leaves essentially no carbon (just oxygen)

  $\hookrightarrow$ weak conclusion: the strong AP might be in trouble

- Changing NN and em interactions in a microscopic model & network calcs

  Oberhummer et al., Science 289 (2000) 88

  $\hookrightarrow$ modified NN strength & fine structure constant in $[\mathbf{0.996}, \mathbf{1.004}]$

  $\hookrightarrow$ no influence on the width but on the relative position of the Hoyle state

  $\hookrightarrow$ use up-to-date stellar evolution model

  $\hookrightarrow$ more than $\mathbf{0.5}[\mathbf{4}]\%$ in the strong coupling $[\alpha_{\rm QED}]$ would destroy all carbon (oxygen) in stars

  $\hookrightarrow$ "should be of interest to AP considerations"

Introduction II: Effective Field Theory for Nuclear Physics

only a brief reminder $\rightarrow$ details in

E. Epelbaum, H.-W. Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773 [arXiv:0811.1338 [nucl-th]]

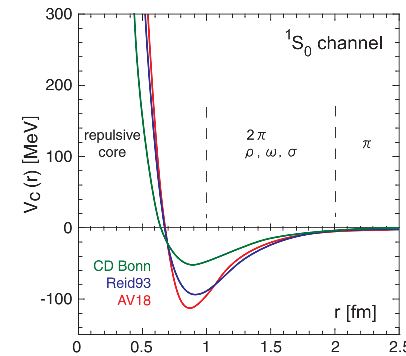
CHIRAL EFT FOR FEW-NUCLEON SYSTEMS

Gasser, Leutwyler, Weinberg, van Kolck, Epelbaum, Bernard, Kaiser, UGM, . . .

- Scales in nuclear physics:

Natural: $\lambda_\pi = 1/M_\pi \simeq 1.5$ fm (Yukawa 1935)

Unnatural: $|a_{np}(^1S_0)| = 23.8\,\text{fm}\,,\; a_{np}(^3S_1) = 5.4\,\text{fm} \gg 1/M_\pi$



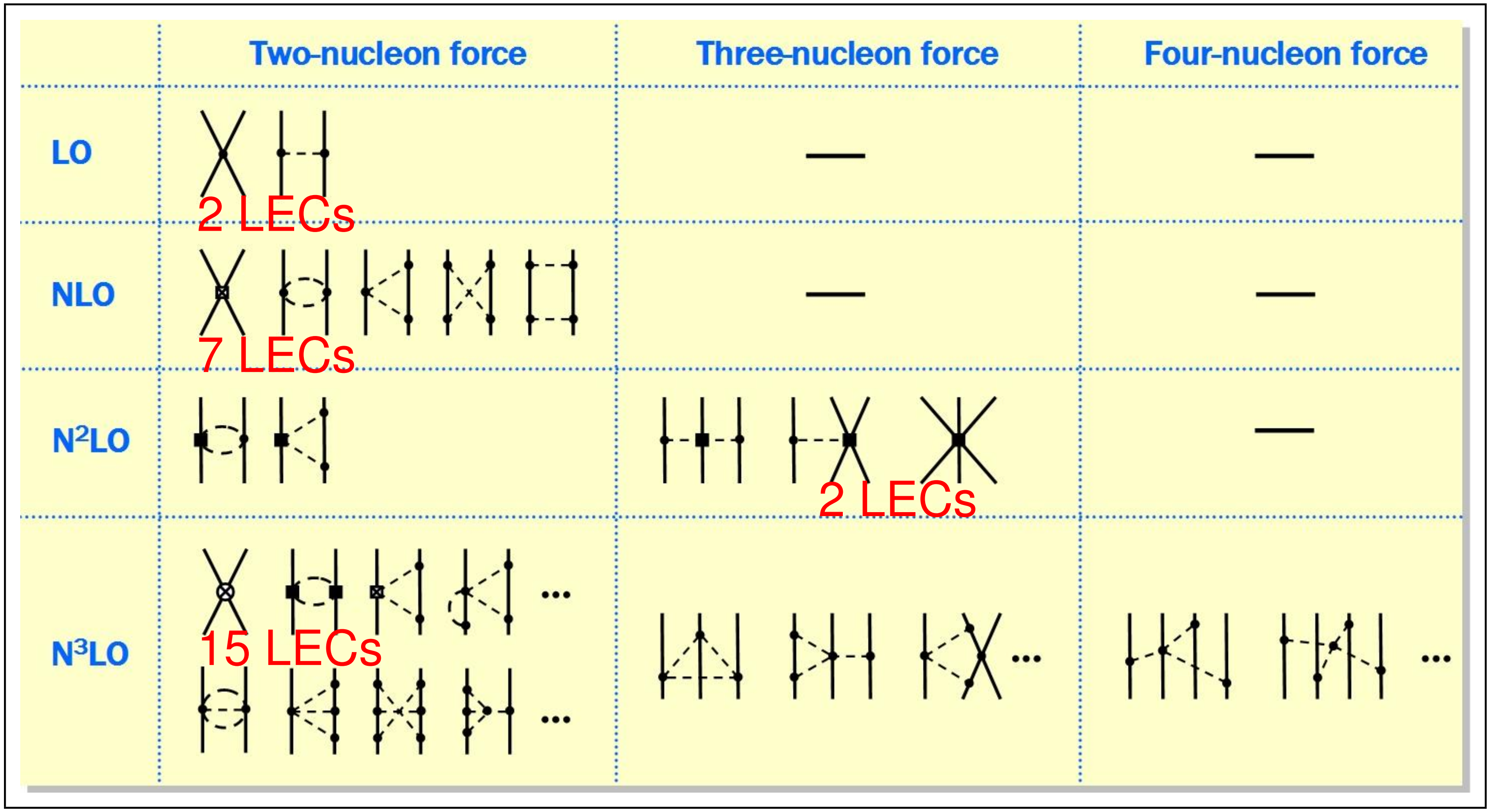
- this can be analyzed in a suitable EFT based on

$$\mathcal{L}_{\rm QCD} \to \mathcal{L}_{\rm EFF} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \ldots$$

- pion and pion-nucleon sectors are perturbative in Q/Λ_χ → chiral perturbation th'y
- $\mathcal{L}_{NN}$ collects short-distance contact terms, to be fitted
- NN interaction requires non-perturbative resummation

→ chirally expand $V_{\rm NN(N)}$, use in regularized LS/FY equation

CHIRAL POTENTIAL and NUCLEAR FORCES



$\mathcal{O}((Q/\Lambda_\chi)^0)$

$\mathcal{O}((Q/\Lambda_\chi)^2)$

$\mathcal{O}((Q/\Lambda_\chi)^3)$

$\mathcal{O}((Q/\Lambda_\chi)^4)$

- explains naturally the observed hierarchy of nuclear forces
- MANY successfull tests in few-nucleon systems (continuum calc's)

Nuclear lattice simulations – Formalism –

NUCLEAR LATTICE SIMULATIONS

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

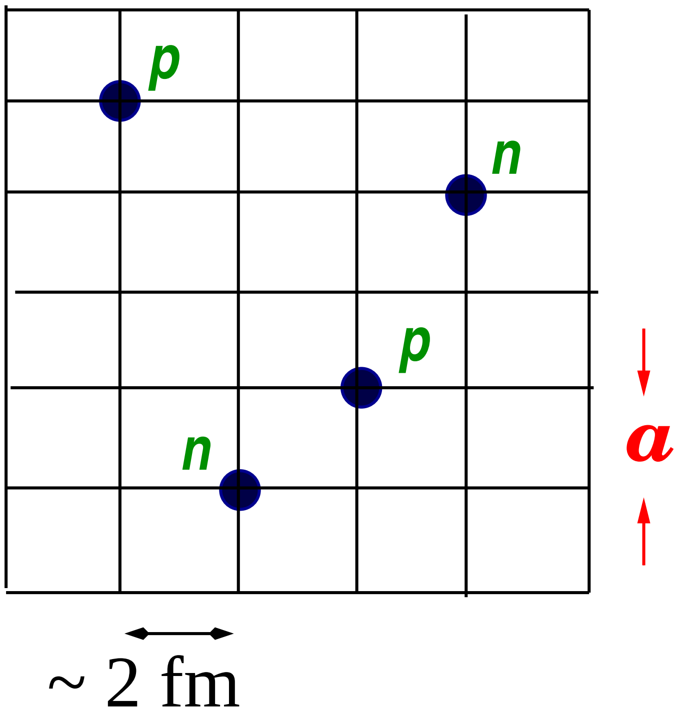
- *new method* to tackle the nuclear many-body problem

- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like fields on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions

- typical lattice parameters

$$\Lambda = \frac{\pi}{a} \simeq 300\,\text{MeV [UV cutoff]}$$

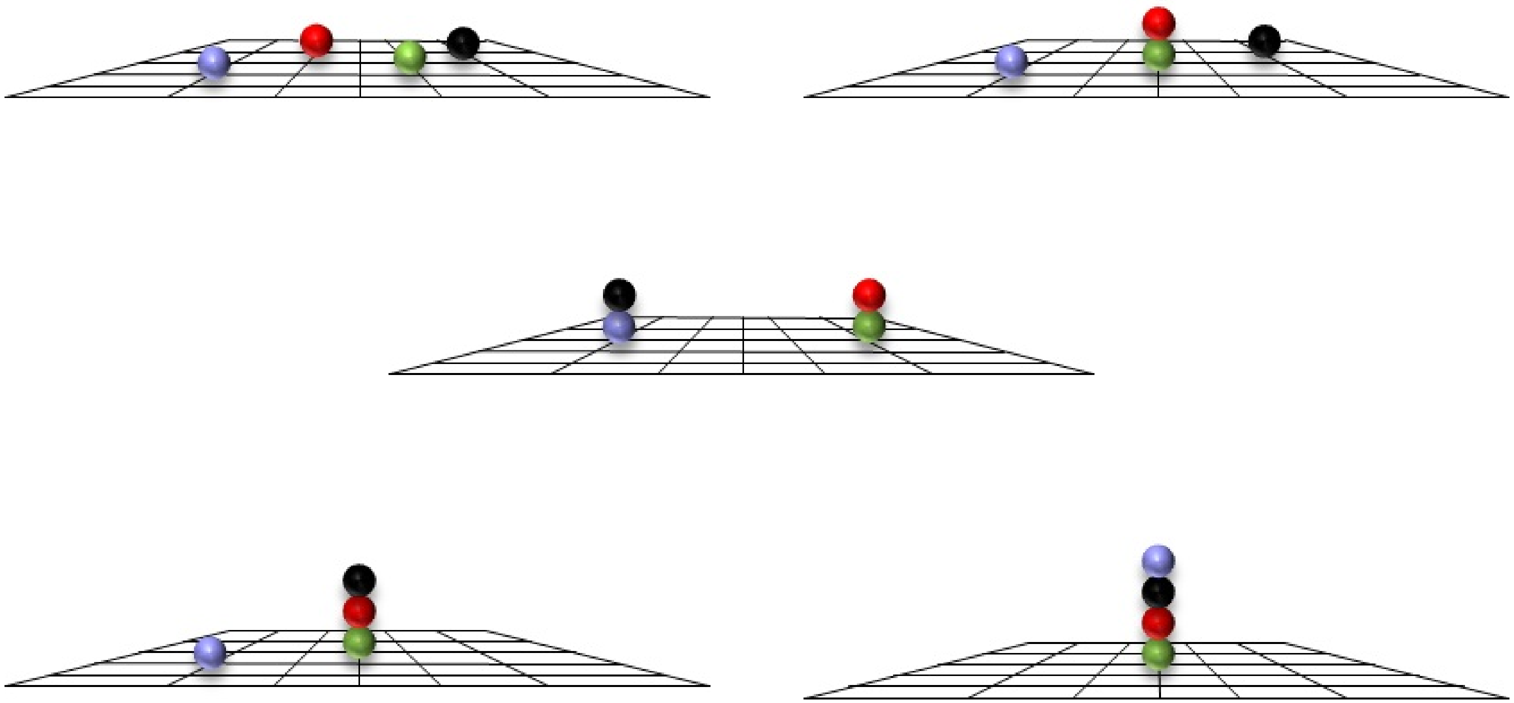


- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

J. W. Chen, D. Lee and T. Schäfer, Phys. Rev. Lett. **93** (2004) 242302

- hybrid Monte Carlo & transfer matrix (similar to LQCD)

CONFIGURATIONS



⇒ all *possible* configurations are sampled

⇒ *clustering* emerges *naturally*

TRANSFER MATRIX METHOD

- Correlation–function for A nucleons: $Z_A(t) = \langle \Psi_A | \exp(-tH) | \Psi_A \rangle$

  with Ψ_A a Slater determinant for A free nucleons

Euclidean time

- Ground state energy from the time derivative of the correlator

$$E_A(t) = -\frac{d}{dt} \ln Z_A(t)$$

$\rightarrow$ ground state filtered out at large times: $E_A^0 = \lim_{t\to\infty} E_A(t)$

- Expectation value of any normal–ordered operator $\mathcal{O}$

$$Z_A^{\mathcal{O}} = \langle \Psi_A | \exp(-tH/2)\, \mathcal{O}\, \exp(-tH/2)\, | \Psi_A \rangle$$

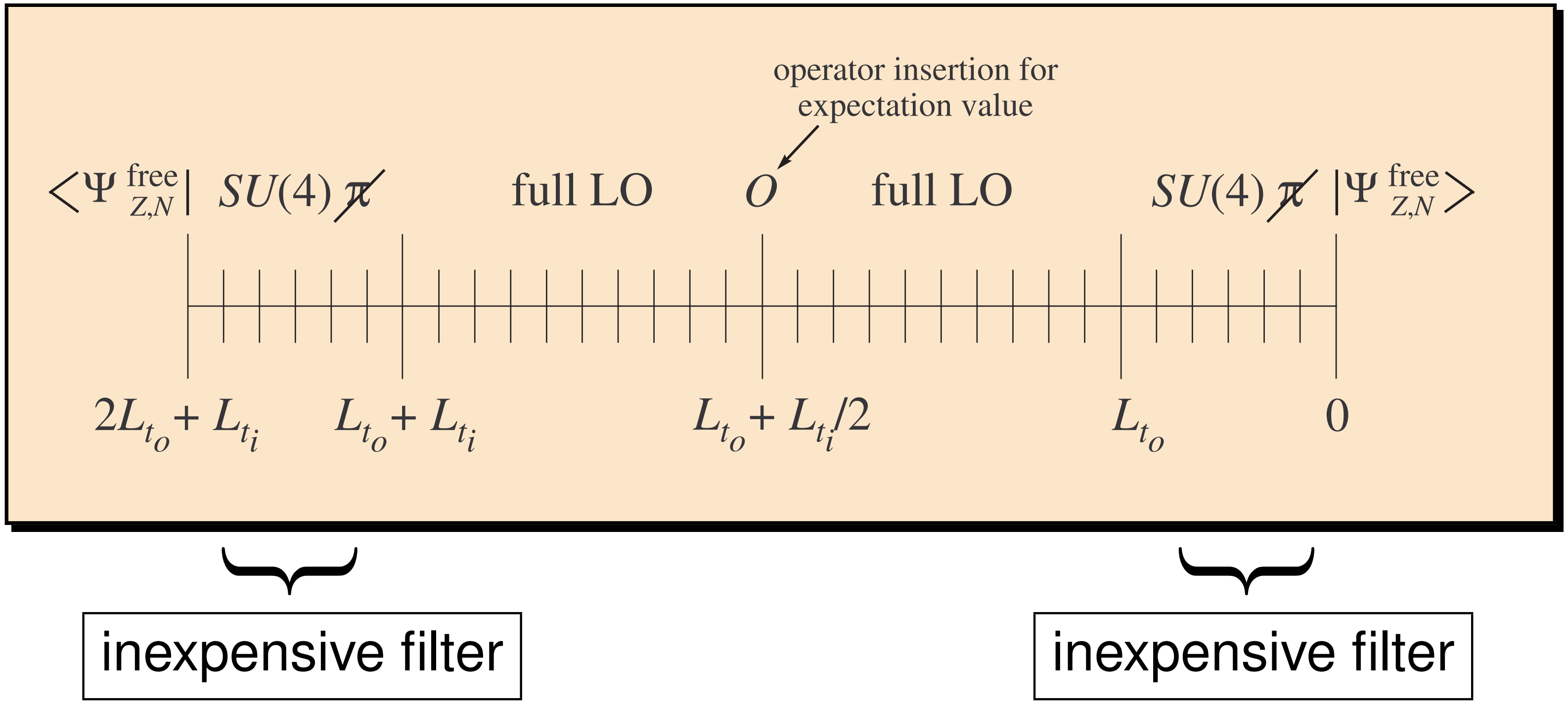
$$\lim_{t\to\infty} \frac{Z_A^{\mathcal{O}}(t)}{Z_A(t)} = \langle \Psi_A | \mathcal{O} \, | \Psi_A \rangle$$

TRANSFER MATRIX CALCULATION

- Expectation value of any normal–ordered operator $\mathcal{O}$

$$\langle \Psi_A | \mathcal{O} | \Psi_A \rangle = \lim_{t \to \infty} \frac{\langle \Psi_A | \exp(-tH/2)\, \mathcal{O}\, \exp(-tH/2) | \Psi_A \rangle}{\langle \Psi_A | \exp(-tH) | \Psi_A \rangle}$$

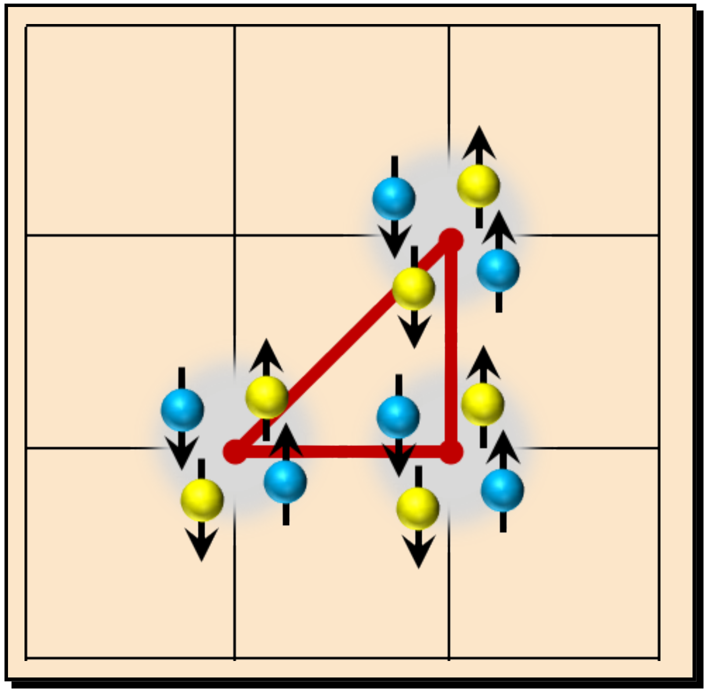
- Anatomy of the transfer matrix

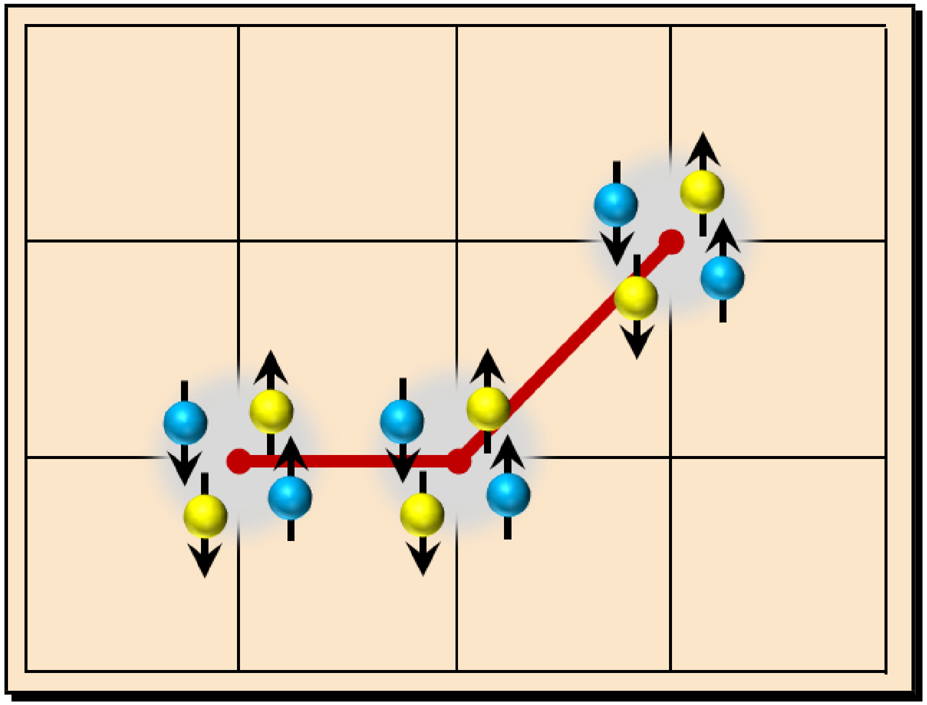


PROJECTION MONTE CARLO TECHNIQUE

- Insert clusters of nucleons at initial/final states (spread over some time interval)
  $\rightarrow$ allows for all type of wave functions (shell model, clusters, ...)
  $\rightarrow$ removes directional bias

- Example: two basic configurations in the spectrum of ^{12}C

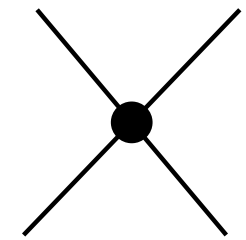




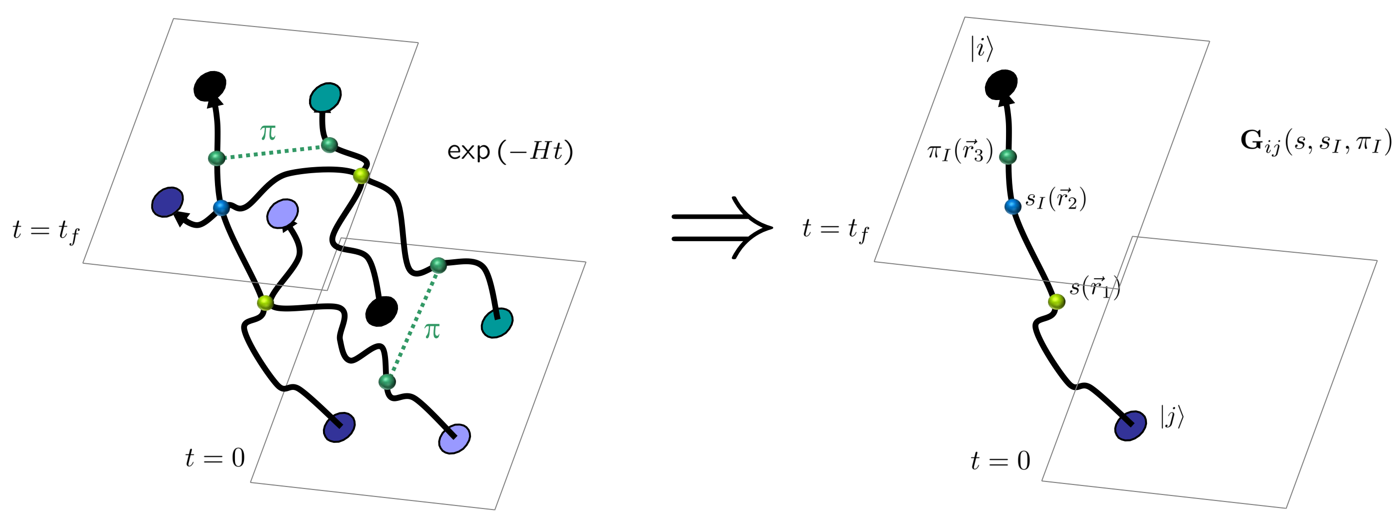
MONTE CARLO with AUXILIARY FILEDS

- Contact interactions represented by auxiliary fields s, s_I

$$\exp(\rho^2/2) \propto \int_{-\infty}^{+\infty} ds\, \exp(-s^2/2 - s\rho)\ , \quad \rho \sim N^\dagger N$$



- Correlation function = path-integral over pions & auxiliary fields



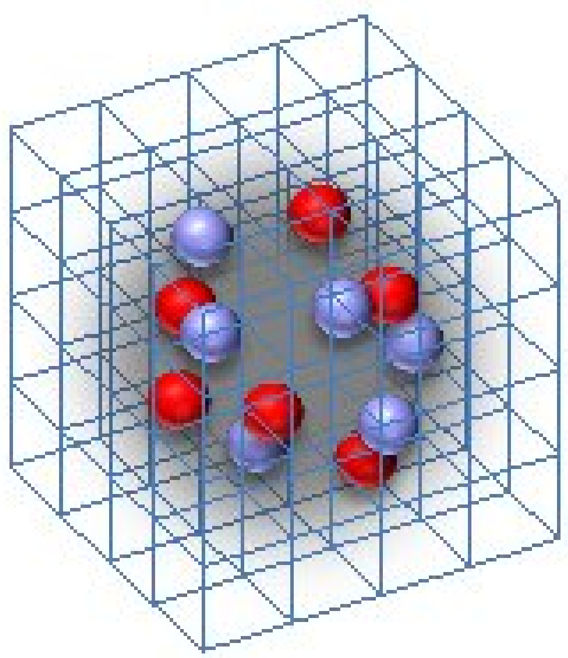
COMPUTATIONAL EQUIPMENT

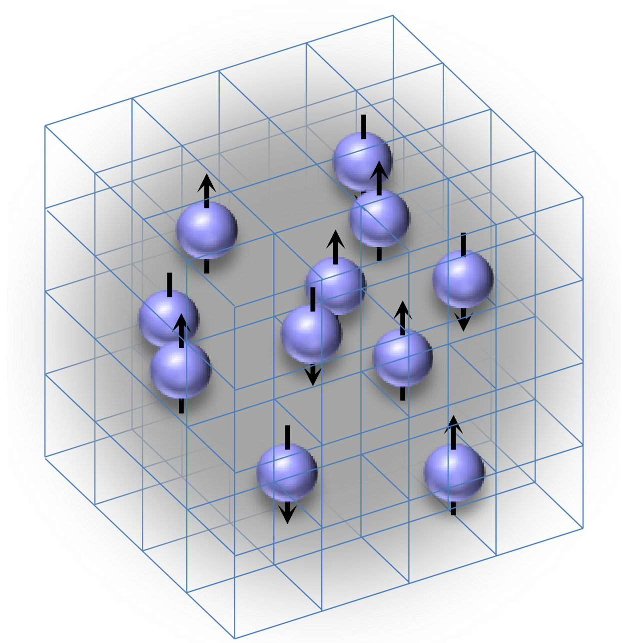
- Past = JUGENE (BlueGene/P)
- Present = JUQUEEN (BlueGene/Q)

Nuclear lattice simulations – Results –

nuclei

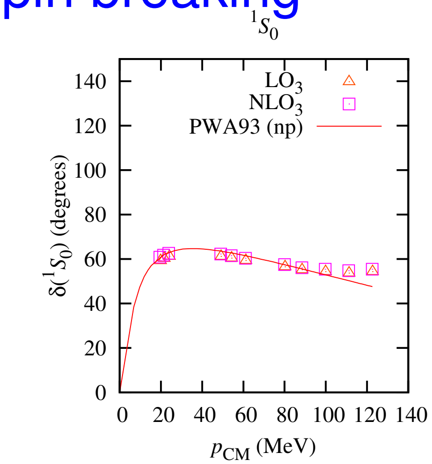
neutron matter

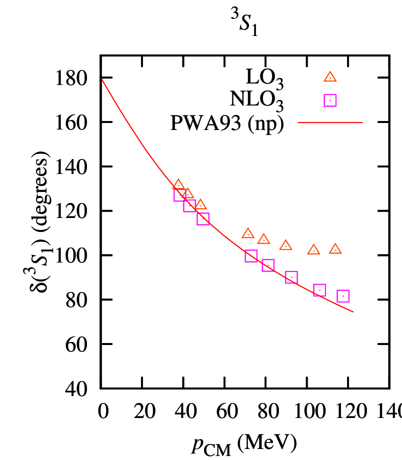




FIXING PARAMETERS & FIRST PREDICTIONS

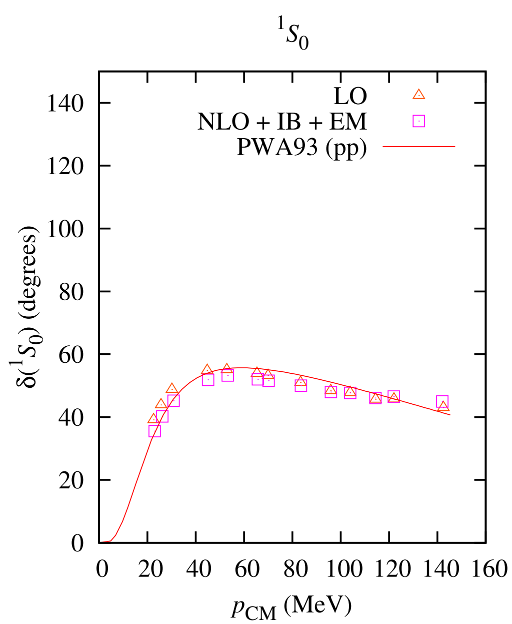
- work at NNLO including strong and em isospin breaking
- 9 NN LECs from np scattering and Q_d
- 2 LECs for isospin-breaking (np, pp, nn)
- 2 LECs D, E related to the leading 3NF

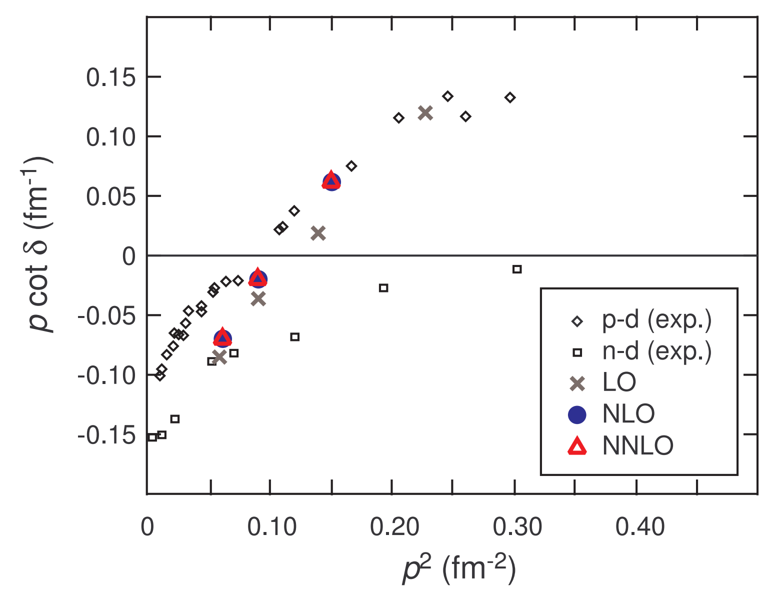




$\Rightarrow$ make predictions

- pp vs np scattering
- nd spin-3/2 quartet channel
- . . .





Ground states

Epelbaum, Krebs, Lähde, Lee, UGM, arxiv:1208.1328

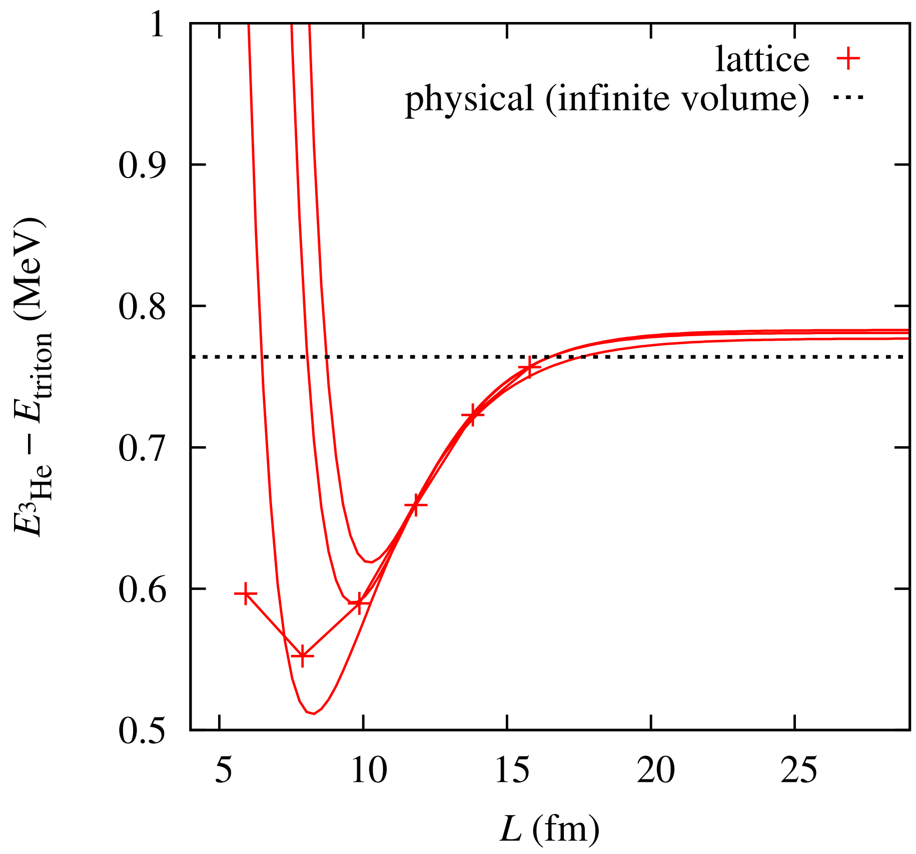
PREDICTIONS: TRITON & HELIUM-3

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **104** (2010) 142501; Eur. Phys. J. **A 45** (2010) 335

- binding energies of 3N systems: $E(L) = \text{B.E.} - \frac{a}{L}\exp(-bL)$

see also Hammer, Kreuzer (2011)

$\Rightarrow$ predict the energy difference $E(^3He) - E(^3H)$

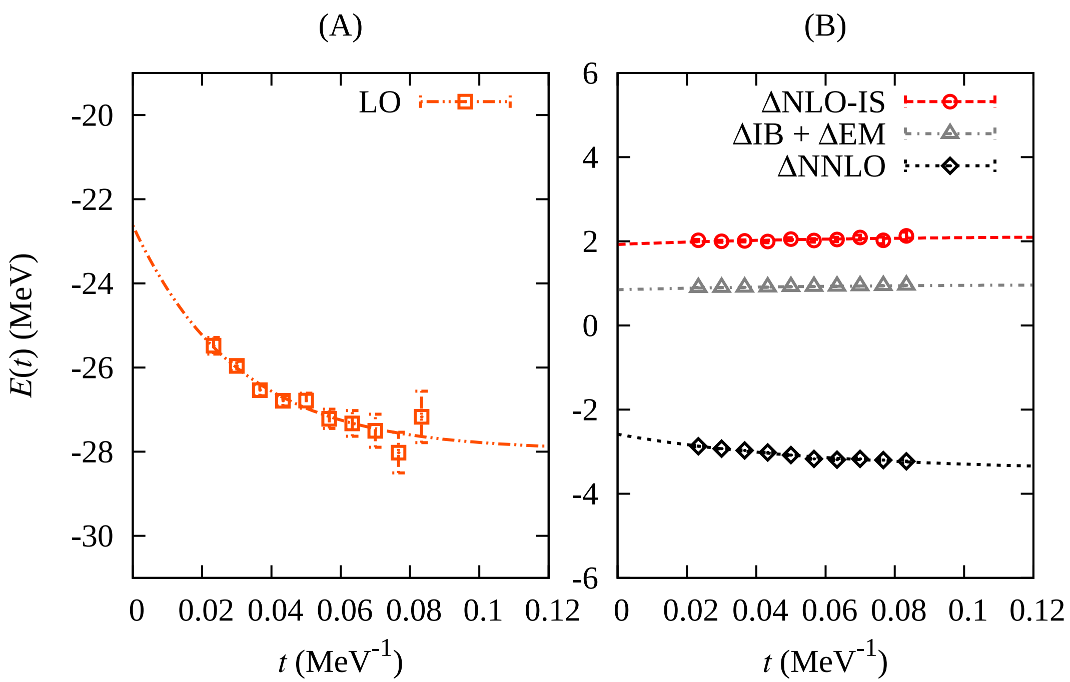


0.78(5) MeV

0.76 MeV [exp.]

Ground state of ^{4}He

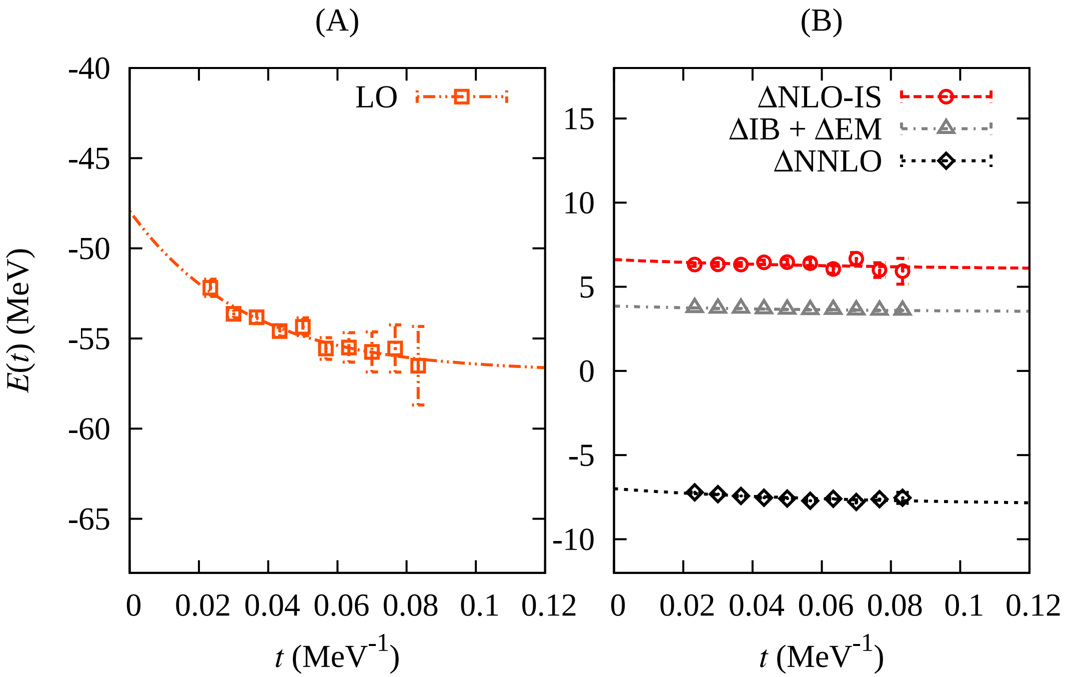
$$L = 11.8 \text{ fm}$$



LO ($\mathcal{O}(Q^0)$)	$-28.0(3)$ MeV
NLO ($\mathcal{O}(Q^2)$)	$-24.9(5)$ MeV
NNLO ($\mathcal{O}(Q^3)$)	$-28.3(6)$ MeV
Exp.	-28.3 MeV

Ground state of ^{8}Be

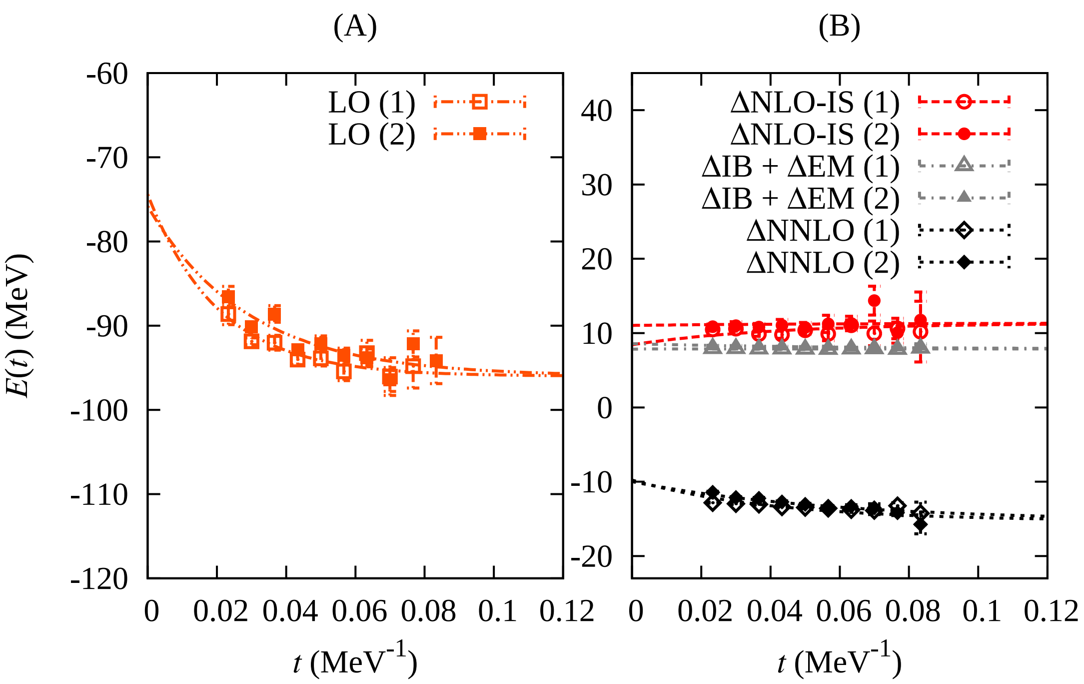
$$L = 11.8 \text{ fm}$$



LO ($\mathcal{O}(Q^0)$)	$-57(2)$ MeV
NLO ($\mathcal{O}(Q^2)$)	$-47(2)$ MeV
NNLO ($\mathcal{O}(Q^3)$)	$-55(2)$ MeV
Exp.	-56.5 MeV

Ground state of ^{12}C

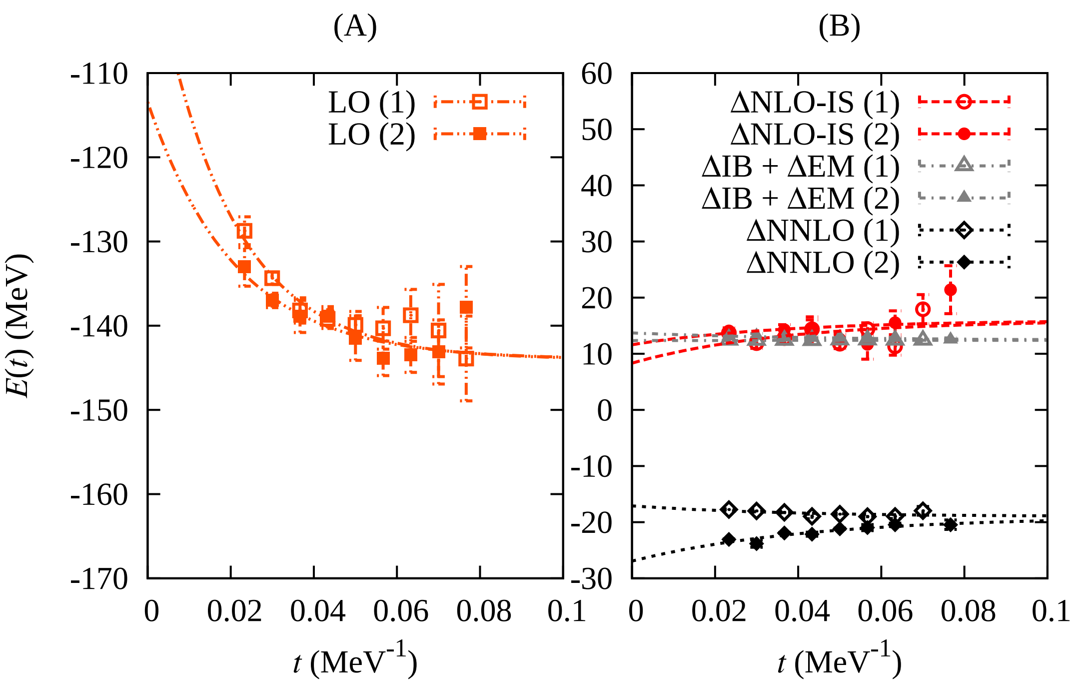
$$L = 11.8 \text{ fm}$$



LO ($\mathcal{O}(Q^0)$)	$-96(2)$ MeV
NLO ($\mathcal{O}(Q^2)$)	$-77(3)$ MeV
NNLO ($\mathcal{O}(Q^3)$)	$-92(3)$ MeV
Exp.	-92.2 MeV

Ground state of ^{16}O

$$L = 11.8 \text{ fm}$$



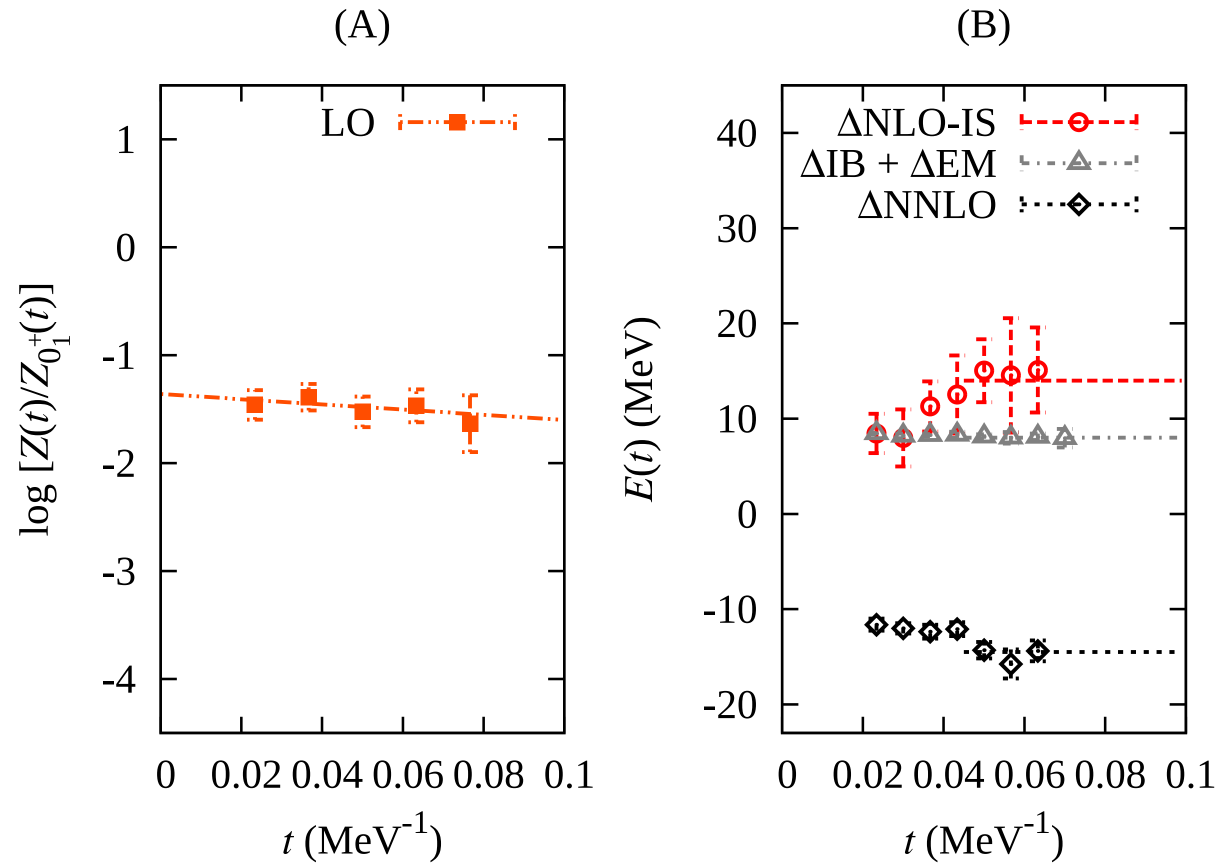
to be published

LO ($\mathcal{O}(Q^0)$)	$-144(4)$ MeV
NLO ($\mathcal{O}(Q^2)$)	$-116(6)$ MeV
NNLO ($\mathcal{O}(Q^3)$)	$-135(6)$ MeV
Exp.	-127.6 MeV

EXCITED STATES of ^{12}C

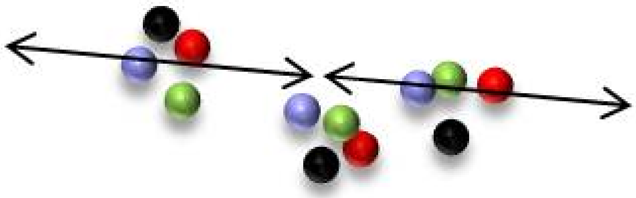
- Lowest excited state is 2_1^+ (as in nature)

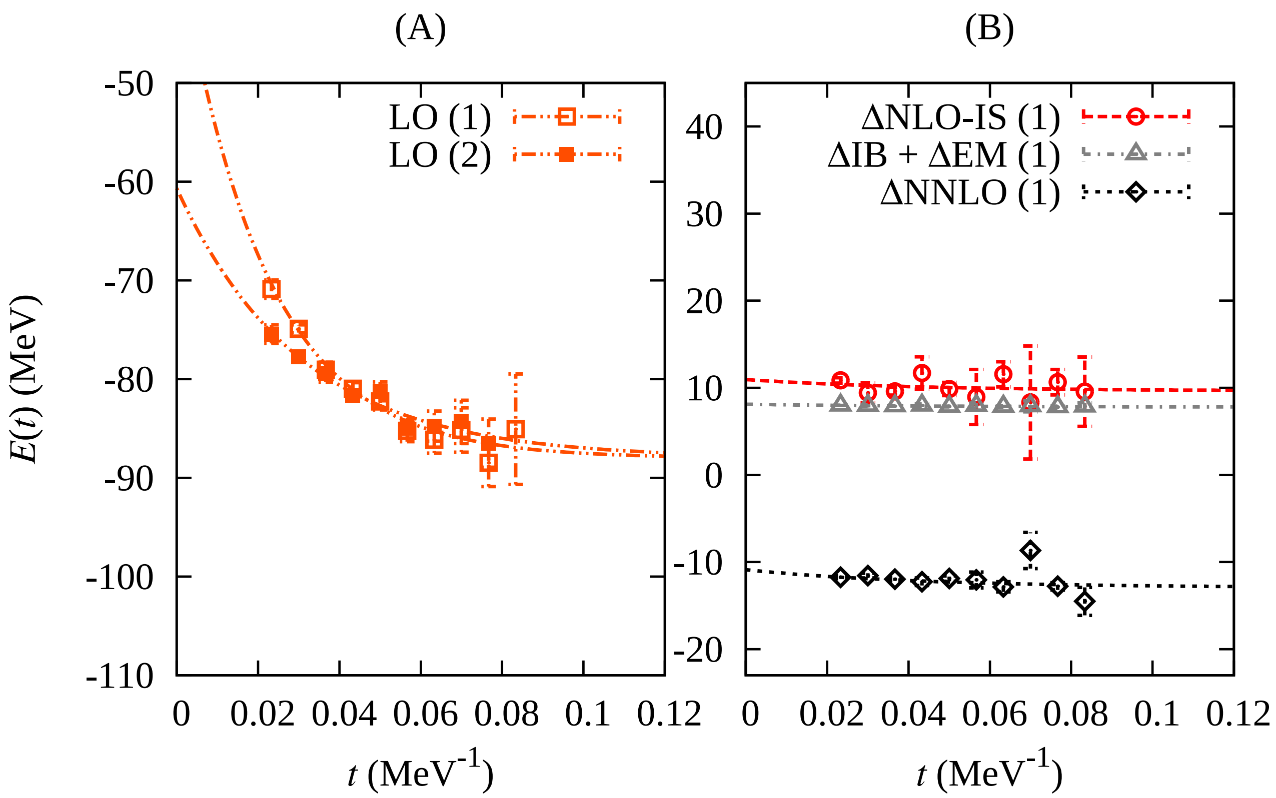
$E(2_1^+) = -89(3)$ MeV
[-87.7 MeV]



THE HOYLE STATE (0_2^+)

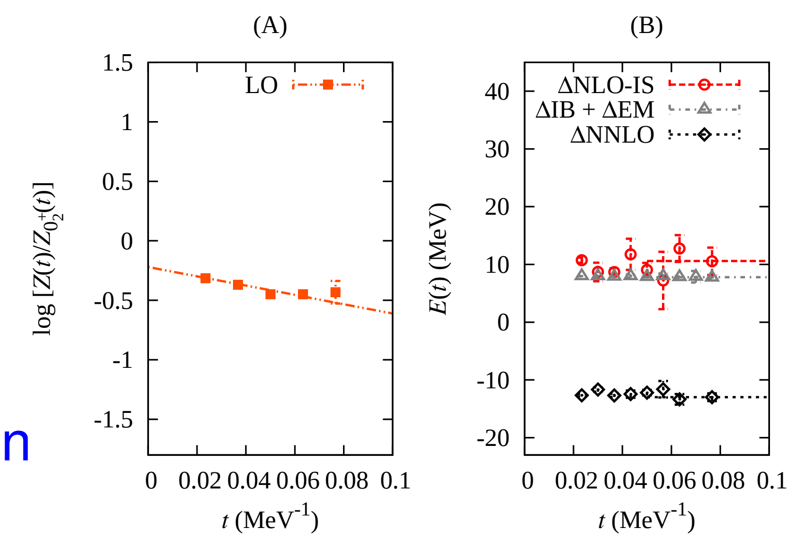
- energy: $E(0_2^+) = -85(3)$ MeV
- close to $E(^4\text{He}) + E(^8\text{Be}) = -83.3(2.0)$ MeV
- structure: “bent” alpha-chain like (not “BEC”)





A HOYLE STATE EXCITATION (2_2^+)

- a 2^+ state 2 MeV above the Hoyle state



- interpretation:
  a rotational band of the Hoyle state
  generated from excitations of the alpha-chain

- what's in the data ?
  a 2^+ state 3.51 MeV above the Hoyle state seen in $^{11}B(d,n)^{12}C$
  not included in the level scheme! Ajzenberg-Selove, Nucl. Phys. A506 (1990) 1

  a 2^+ state 3.8(4) MeV above the Hoyle state seen in $^{12}C(\alpha,\alpha)^{12}C$
  Bency John et al., Phys. Rev. C 68 (2003) 014305

- and much more, see next slide and: $\rightarrow$ talk by Henry Weller

$\Rightarrow$ ab initio prediction requires experimental confirmation

SPECTRUM OF ^{12}C

- Summarizing the results for carbon-12:

	0_1^+	2_1^+	0_2^+	2_2^+
LO	$-96(2)$ MeV	$-94(2)$ MeV	$-89(2)$ MeV	$-88(2)$ MeV
NLO	$-77(3)$ MeV	$-74(3)$ MeV	$-72(3)$ MeV	$-70(3)$ MeV
NNLO	$-92(3)$ MeV	$-89(3)$ MeV	$-85(3)$ MeV	$-83(3)$ MeV
Exp.	-92.16 MeV	-87.72 MeV	-84.51 MeV	$-82.6(1)$ MeV [1,2] $-82.32(6)$ MeV [3] $-81.1(3)$ MeV [4] $-82.13(11)$ MeV [5]

[1] Freer et al., Phys. Rev. C 80 (2009) 041303
[2] Zimmermann et al., Phys. Rev. C 84 (2011) 027304
[3] Hyldegaard et al., Phys. Rev. C 81 (2010) 024303
[4] Itoh et al., Phys. Rev. C 84 (2011) 054308
[5] Weller et al., in preparation

- importance of consistent 2N & 3N forces

- good agreement w/ experiment, can be improved

Testing the Anthropic Principle

MC ANALYSIS of the AP

- consider QCD only → calculate $\partial\Delta E/\partial M_\pi$

- relevant quantities (energy *differences*)

$$\boxed{\Delta E_h \equiv E^*_{12} - E_8 - E_4, \quad \Delta E_b \equiv E_8 - 2E_4} \qquad \Delta E_c \equiv E^*_{12} - E_{12}$$

- energy differences depend on parameters of QCD (LO analysis)

$$\boxed{E_i = E_i\left(M_\pi^{\rm OPE}, m_N(M_\pi), \tilde{g}_{\pi N}(M_\pi), C_0(M_\pi), C_I(M_\pi)\right)}$$

$$\tilde{g}_{\pi N} \equiv \frac{g_A}{2F_\pi}$$

- remember: $M^2_{\pi^\pm} \sim (m_u + m_d)$

  ⇒ quark mass dependence ≡ pion mass dependence

PION MASS VARIATIONS

- consider pion mass changes as *small perturbations*

$$\frac{\partial E_i}{\partial M_\pi}\bigg|_{M_\pi^{\rm phys}} = \frac{\partial E_i}{\partial M_\pi^{\rm OPE}}\bigg|_{M_\pi^{\rm phys}} + x_1 \frac{\partial E_i}{\partial m_N}\bigg|_{m_N^{\rm phys}} + x_2 \frac{\partial E_i}{\partial \tilde{g}_{\pi N}}\bigg|_{\tilde{g}_{\pi N}^{\rm phys}} + x_3 \frac{\partial E_i}{\partial C_0}\bigg|_{C_0^{\rm phys}} + x_4 \frac{\partial E_i}{\partial C_I}\bigg|_{C_I^{\rm phys}}$$

with

$$x_1 \equiv \frac{\partial m_N}{\partial M_\pi}\bigg|_{M_\pi^{\rm phys}}, \; x_2 \equiv \frac{\partial \tilde{g}_{\pi N}}{\partial M_\pi}\bigg|_{M_\pi^{\rm phys}}, \; x_3 \equiv \frac{\partial C_0}{\partial M_\pi}\bigg|_{M_\pi^{\rm phys}}, \; x_4 \equiv \frac{\partial C_I}{\partial M_\pi}\bigg|_{M_\pi^{\rm phys}}$$

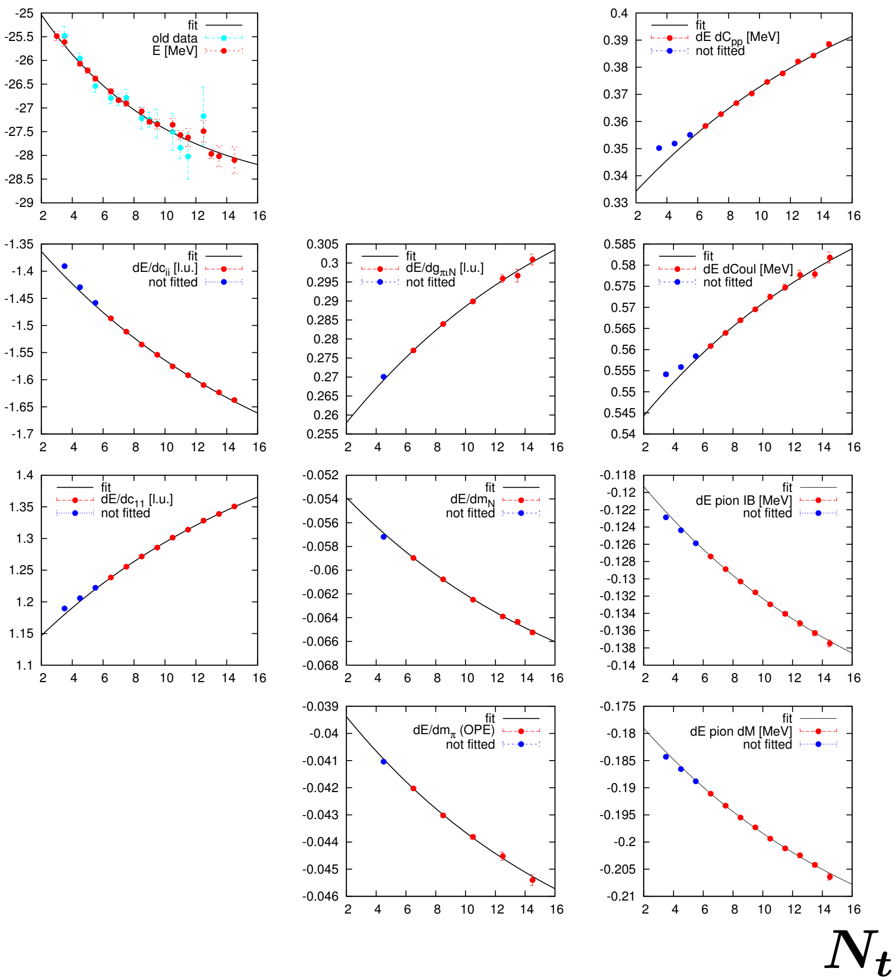
⇒ problem reduces to the calculation of the various derivatives
using AFQMC and the determination of the x_i

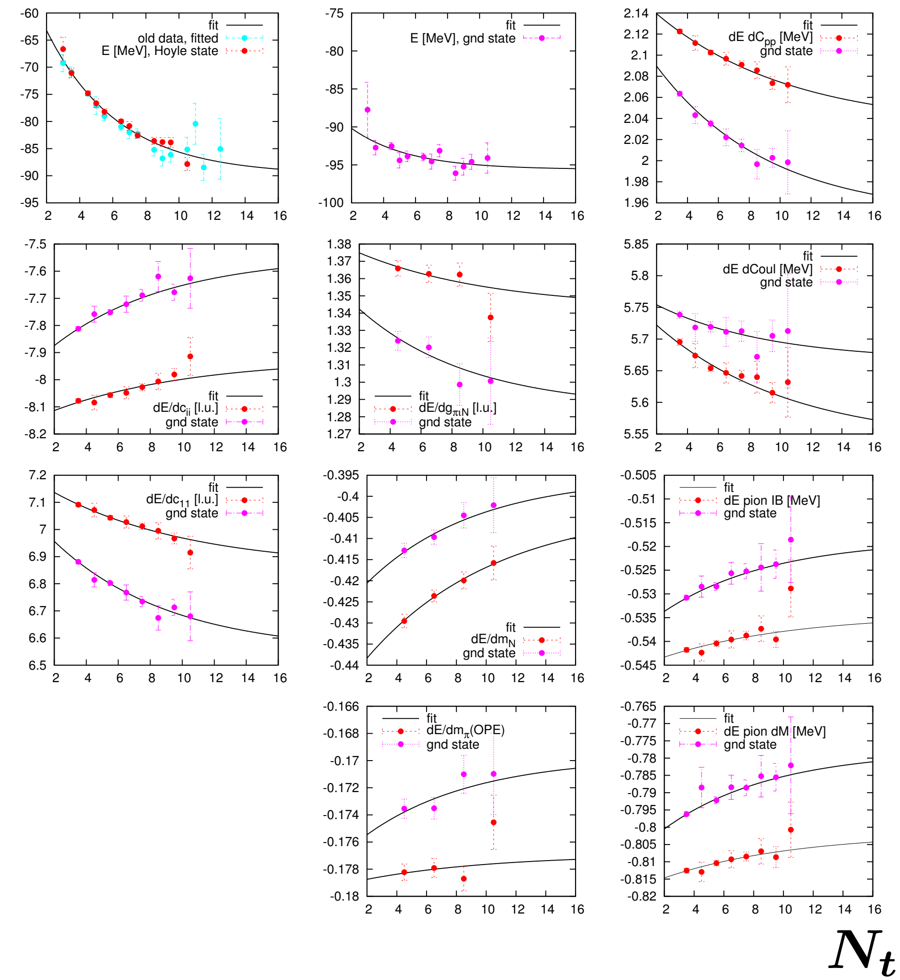
- x_1 and x_2 can be obtained from LQCD plus CHPT
- x_3 and x_4 can be obtained from two-body scattering and its M_π-dependence

AFQMC RESUTS for the DERIVATIVES

- ^{4}He
- ^{12}C(0_2^+)

$$E(N_t) = E(\infty) + \text{const}\ \exp(-N_t/\tau)$$





DETERMINATION of the x_i

- x_1 from the quark mass expansion of the nucleon mass: $x_1 \simeq 0.8 \pm 0.2$

- x_2 from the quark mass expansion of the pion decay constant and the nucleon axial-vector constant: $x_2 \simeq -0.056 \ldots 0.008$

- x_3 and x_4 can be obtained from a two-nucleon scattering analysis & can be deduced from:

$$-\frac{\partial a^{-1}}{\partial M_\pi} \equiv \frac{A}{aM_\pi} = \frac{1}{\pi L} S'(\eta) \frac{\partial \eta}{\partial M_\pi} \, , \quad \eta \equiv m_N E \left(\frac{L}{2\pi}\right)^2$$

$\Rightarrow$ while this can straightforwardly be computed, we prefer to use a representation that substitutes x_3 and x_4 by:

$$\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} , \quad \left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

$\Rightarrow$ we are ready to study the pertinent energy differences

RESULTS

- putting pieces together:

$$\left.\frac{\partial \Delta E_h}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.455(35)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.744(24)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.056(10)$$

$$\left.\frac{\partial \Delta E_b}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.117(34)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.189(24)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.012(9)$$

$$\left.\frac{\partial \Delta E_c}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.07(3)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.14(2)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.017(9)$$

- x_1 and x_2 only affect the small constant terms
- also calculated the shifts of the individual energies (not shown here)

INTERPRETATION

- $(\partial\Delta E_h/\partial M_\pi)/(\partial\Delta E_b/\partial M_\pi) \simeq 4$
  $\Rightarrow$ ΔE_h and ΔE_b cannot be independently fine-tuned

- Within error bars, $\partial\Delta E_h/\partial M_\pi$ & $\partial\Delta E_b/\partial M_\pi$ appear unaffected by the choice of x_1 and x_2 $\rightarrow$ indication for α-clustering

- For ΔE_h & ΔE_b, the dependence on M_π is small when

$$\boxed{\partial a_s^{-1}/\partial M_\pi \simeq -1.6 \times \partial a_t^{-1}/\partial M_\pi}$$

- the triple alpha process is controlled by :

$$\Delta E_{h+b} \equiv \Delta E_h + \Delta E_b = E_{12}^\star - 3E_4$$

$$\boxed{\left.\frac{\partial\Delta E_{h+b}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.571(14)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.934(11)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.069(6)}$$

$\Rightarrow$ so what can we say about the quark mass dependence of the scattering lengths?

CONSTRAINTS on the SCATTERING LENGTHS

- Quark mass dependence of hadron properties: $\dfrac{\delta O_H}{\delta m_f} \equiv K_H^f \dfrac{O_H}{m_f}\,, \quad f = u, d, s$

- NN scattering lengths as a function of M_π: $-\dfrac{\partial a_{s,t}^{-1}}{\partial M_\pi} \equiv \dfrac{A_{s,t}}{a_{s,t} M_\pi}, \quad A_{s,t} \equiv \dfrac{K^q_{a_{s,t}}}{K^q_\pi}$

- earlier determinations from chiral EFT at NLO

Beane, Savage (2003), Epelbaum, Glöckle, UGM (2003)

- new determination at NNLO:

Epelbaum et al. (2012)

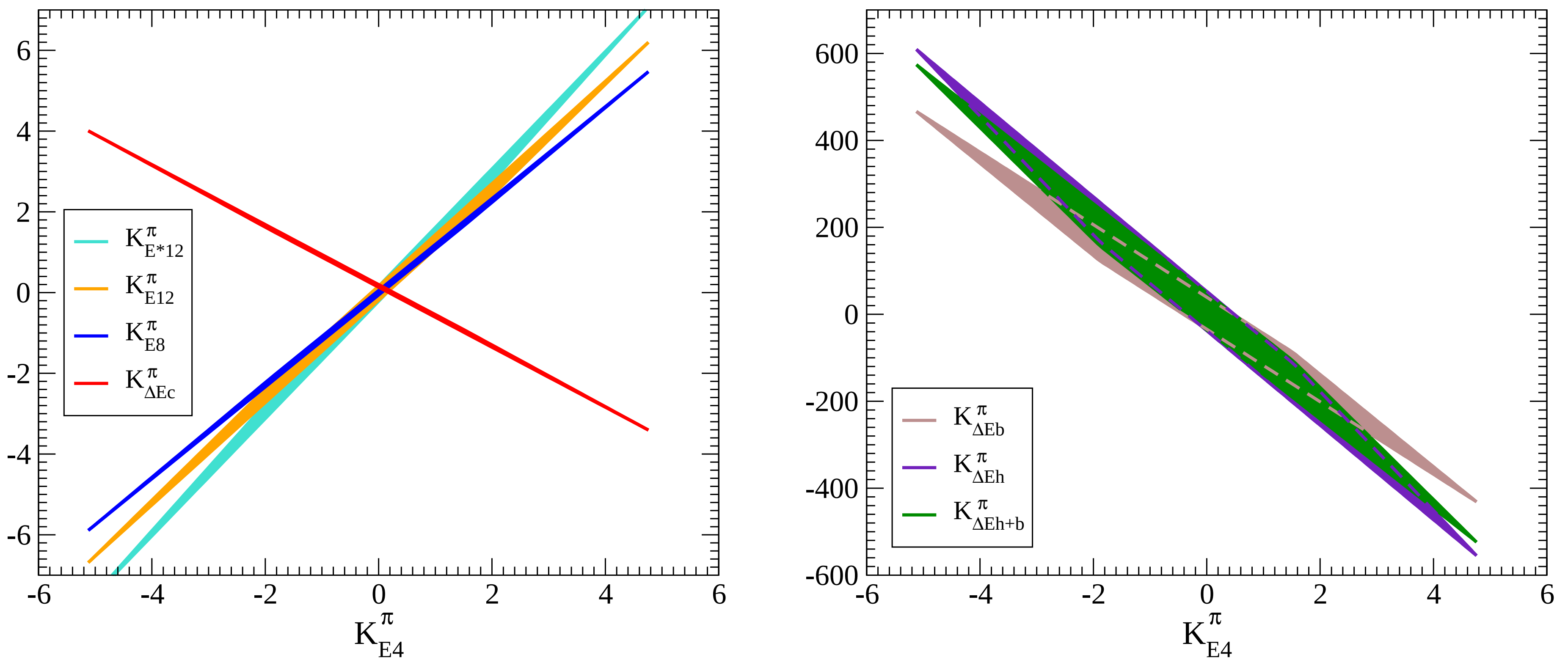
$$K^q_{a_s} = 2.3^{+1.9}_{-1.8}\,,\ K^q_{a_t} = 0.32^{+0.17}_{-0.18} \to \frac{\partial a_t^{-1}}{\partial M_\pi} = -0.18^{+0.10}_{-0.10}\,,\ \frac{\partial a_s^{-1}}{\partial M_\pi} = 0.29^{+0.25}_{-0.23}$$

- note the *magical* central value:

$$\boxed{\frac{\partial a_s^{-1}/\partial M_\pi}{\partial a_t^{-1}/\partial M_\pi} \simeq -1.6^{+1.0}_{-1.7}}$$

CORRELATIONS

- vary the quark mass derivatives of $a_{s,t}^{-1}$ within $-1, \ldots, +1$:



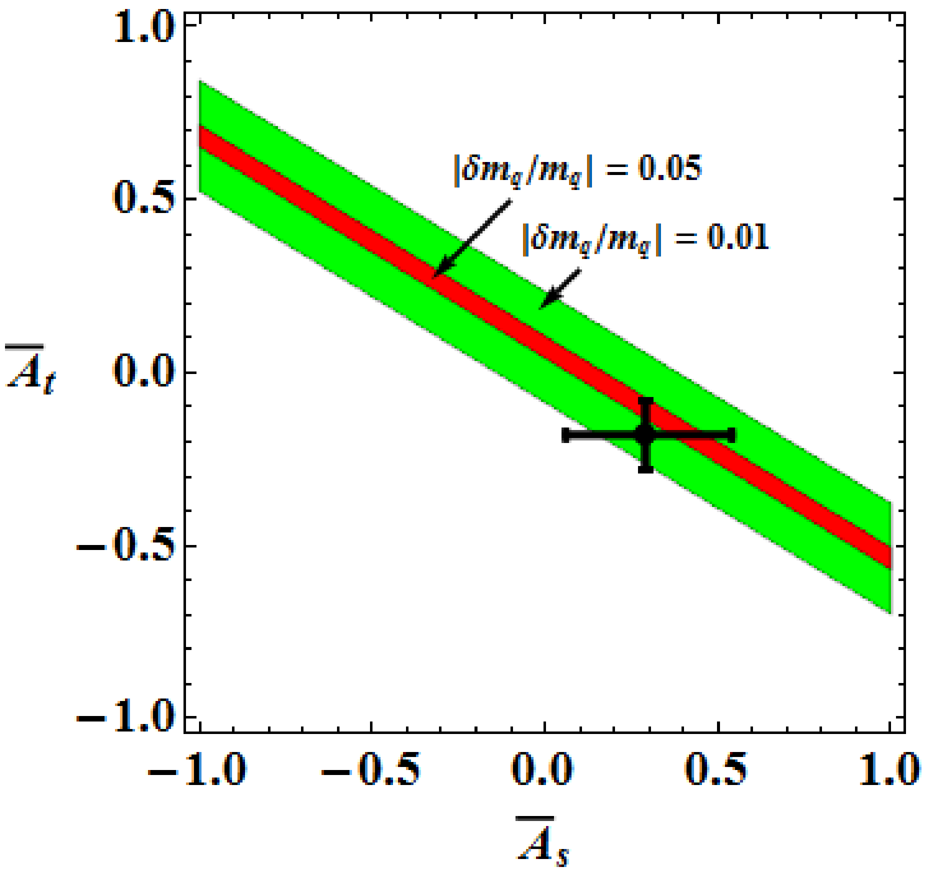
- clear correlations: α-particle BE and the energies/energy differences

$\Rightarrow$ anthropic or non-anthropic scenario depends on whether the ^{4}He BE moves!

THE END-OF-THE-WORLD PLOT

- $|\delta(\Delta E_{h+b})| < 100$ keV

$$\rightarrow \boxed{\left|\left(0.571(14)\,\bar{A}_s + 0.934(11)\,\bar{A}_t - 0.069(6)\right)\frac{\delta m_q}{m_q}\right| < 0.0015}$$



$$\bar{A}_{s,t} \equiv \left.\frac{\partial a_{s,t}^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$